DR. HUTTON'S

PHILOSOPHICAL RECREATIONS,

By EDWARD RIDDLE.

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PRINTED BY NUTTALL AND HODGSON,
GOUGH SQUARE, LONDON.

RECREATIONS

IN

MATHEMATICS

AND

NATURAL PHILOSOPHY:

TRANSLATED

FROM MONTUCLA'S EDITION OF OZANAM,

BY

CHARLES HUTTON, LL.D. F.R.S. &c.

A NEW AND REVISED EDITION, WITH NUMEROUS ADDITIONS,

AND ILLUSTRATED WITH UPWARDS OF FOUR HUNDRED WOODCUTS,

BY EDWARD RIDDLE,

MASTER OF THE MATHEMATICAL SCHOOL, ROYAL HOSPITAL, GREENWICH.

LONDON:

PRINTED FOR THOMAS TEGG, 73, CHEAPSIDE.

M.DCCC.XL.

PREFATORY NOTICE.

Jacques Ozanam, the original composer of the "Recreations in Mathematics and Natural Philosophy," was born in 1640, at Bouligneu, in what is now called the department of Ain, in France. His family was of Jewish extraction, but had long been members of the Church of Rome; and having landed property to which some ecclesiastical patronage was attached, Jacques, who was the younger of two sons, was intended for the church; and he accordingly entered on a course of study suitable for his destination.

He is said to have been of gay, lively, and expensive habits, and to have shewn no inclination for theological pursuits. Chemistry, mechanics, &c. which have a more obvious connection with the business of life, attracted his attention; and after his father's death, which took place about four years after he began to read for the church, he abandoned theology, and attached himself to science.

His opportunities of receiving assistance in his scientific studies, were so scanty, that he may be considered as having been self-taught; and though he cannot be regarded as having attained great eminence as a mathematician, even among his contemporaries, he was the author of a good many useful works, whose popularity carried them through several editions.

It would appear, however, that in attaching himself to science, he did not at first look to it as a means of living; for soon after his father's death he removed to Lyons, where he taught mathematics *gratuitously*, considering it a degradation to receive pay for his instructions.

It is probable, however, that he soon changed his opinion on the subject. He was addicted to gaming; his private pecuniary resources were limited; and the stern realities of distress would speedily dissipate all illusions about the dignity of teaching science for its own sake.

An act of striking and disinterested liberality, which he performed towards two strangers, having been mentioned to the chancellor of France, that distinguished personage invited the Lyonnese mathematician to Paris; where, after some time spent in dissipation, he married a young woman without fortune, but who proved to him a most excellent wife. After bearing to him twelve children, all of whom died young, she died in 1701, deeply lamented by her husband.

Ozanam subsisted in Paris by teaching mathematics, and met with considerable success, especially among foreigners. But, upon the breaking out of the war of the Spanish succession, most of his pupils quitting France, his professional income became both small and precarious.

He lived for some years in comparative indigence, but, towards the close of his life, his difficulties were somewhat alleviated by his being

admitted an élève of the Academy of Sciences. He died of apoplexy, at Paris, April 3rd, 1717, aged 77 years.

He was of a mild and cheerful temper, generous to the full extent of his means, and of an inventive genius; and his conduct after marriage was irreproachable. He was devout, but averse to disputations about points of faith. On this subject he used to say, "It is the business of the Sorbonne to discuss, of the Pope to decide, and of *a mathematician to go straight to heaven in a perpendicular line.*"

Jean Etienne Montucla (who so greatly enlarged and improved the "Recreations" of Ozanam, that he may be said to have made the work his own,) was the son of a merchant at Lyons, where he was born on Sept. 5th, 1725. He was left an orphan at the age of sixteen, and was educated at the Jesuit's College in his native town. His attention was chiefly directed to the ancient classics; but having a natural taste for philological studies, and a powerful memory, he was enabled to acquire an accurate knowledge of several modern languages; among which Italian, German, Dutch, and English are mentioned. Under Le Père Bèraud, who was subsequently the tutor of Lalande, he made considerable proficiency in the study of mathematics and physics.

Having completed his course of general education, he studied for the legal profession, first at Toulouse, and afterwards in Paris; where at the scientific soirées of M. Jambert, he became acquainted with Diderot, D'Alembert, Lalande, and other scientific men of the highest character.

Having published several scientific works, by which he acquired much reputation, he began to be employed by the government. He was sent as Intendant Secretary to Grenoble, where he married the daughter of M. Roland in 1763; and in the following year he was sent as secretary and astronomer royal to the expedition for colonising Cayenne.

On his return to France, after a few years' absence, he obtained the situation of "Premier Commis des Bâtiments," and in addition the office of "Censor Royal of mathematical works," an appointment which was merely honorary.

It would appear, that though the income which he derived from his official appointment was not large, yet, from his prudent and economical habits, it was sufficient for the immediate wants of himself and his family. He employed his leisure in educating his children, and in scientific pursuits; following the latter, it is said, in secrecy, lest he should be suspected of neglecting his official duties.

It was at this time that he edited the new edition of the "Recreations;" and so carefully had he concealed his connection with the work, that, on its completion, a copy of it was sent to him, in his capacity of censor, for examination and approval.

Besides expunging from the work of Ozanam much that was absurd, puerile, and obsolete, he enriched his edition with dissertations upon almost every branch of practical science; and much of what he added is valuable even at the present day.

But the name of Montucla is best known from his "History of the Mathematics," which contains, besides what is strictly historical, treatises upon all the leading departments of the pure and applied sciences; and abounds with interesting details respecting the discoveries and improvements which have contributed to their progress.

The French Revolution put an end at once to his office and the little savings which his regularity and economy had enabled him to make from his income,—throwing him on the world in his old age, stripped of every thing but his integrity, and the love and respect of his friends. He died on the 18th of December, 1799.

In 1803, a translation into English of Montucla's Edition of Ozanam's Recreations, by Dr. Charles Hutton, of Woolwich, was published in London. In this Edition were incorporated many valuable additions and observations by the learned and judicious translator, who lived to superintend a second edition, which, with still further improvements, was published in 1814.

Dr. Hutton was born in Percy Street, Newcastle-upon-Tyne, on August 14th, 1737. His father, who was employed in the coal works in the neighbourhood, was understood to be descended from a respectable family in Westmoreland. He died when Charles was only five years old; and his widow married a person named Fraim, whose employment was that of a colliery over-man.

From an accident which happened to Charles at play, he was not sent when a boy to work in the pits, as his brothers were; but kept at school for some years, in the hope that he might be enabled to earn his bread by his scholarship. He was taught to read by an old woman who conducted a little school in the neighbourhood, and to write by a schoolmaster named Robson, near Benwell, a village near Newcastle; and he attended afterwards a school at Jesmond, kept by the Rev. Mr. Ivison, a clergyman of the English Church; and on Mr. Ivison's removal to a curacy in the county of Durham, Mr. Hutton succeeded him in his school at Jesmond.

It would appear that between his being the pupil and the successor of Mr. Ivison, Hutton had worked for some time (probably not long) as a miner at Old Long Benton colliery.

Mr. Hutton's school at Jesmond soon increased so much that he was obliged to remove to a larger room in the neighbourhood.

While conducting with such success his village seminary, he attended in the evenings the school of a Mr. James, at Newcastle, to prosecute his studies in mathematics; and Mr. James some time after declining his school, Mr. Hutton embraced the opportunity of settling in Newcastle as a teacher. In that town, the metropolis of the northern counties, his success was very great: and though his previous associates had been chiefly among the humbler classes of society, his manners, as well as his talents, rendered him acceptable as a private teacher in the families of the higher classes. Among others, he had for his pupils the late Lord Chancellor Eldon, and his lady, who was the daughter of a wealthy banker in Newcastle.

While in that town he published his Arithmetic, his Mensuration, and his Tract on the principles of Bridges; and he made for the corporation a survey and plan of the town. He became also a leading writer in the Ladies' and Gentleman's Diaries, and other scientific periodicals of the day.

On the death of Mr. Cowley, professor of Mathematics in the Royal Military Academy at Woolwich, Mr. Hutton offered himself as a candidate for the situation; and after an examination, which lasted several days, the

examiners (Bishop Horsley, Dr. Makeslyne, and Col. Watson) unanimously recommended him as preferable to all the other candidates, and peculiarly well qualified to fill the situation, and he received his appointment accordingly on May 24th, 1773.

Soon after his settlement at Woolwich he was elected a Fellow of the Royal Society; and at a later period he received the degree of LL.D. from the University of Edinburgh.

The Stationers' Company appointed him general Editor of all their Almanacs, except the Ladies' Diary and Poor Robin, and he held the appointment for forty-six years.

The editorship of the Ladies' Diary afforded him an opportunity of becoming acquainted with the talents and acquirements of many ingenious individuals, who were improving themselves in science by endeavouring to solve the mathematical questions proposed in the Diary; and as opportunity occurred, many of them were drawn by his kind discrimination from obscurity, and placed in situations in which they have been eminently useful to society. Indeed it has been justly said, that "of this class of men he was eminently the patron."

After filling with distinguished ability the situation at Woolwich for thirty-four years, he was permitted, at his own request, to retire; and the Board of Ordnance assigned him a pension of £500 per annum, in testimony of regard for his long and faithful services.

He settled in London, and enjoyed for the last sixteen years of his life the society of all the leading men distinguished for science and worth in the metropolis.

He died on January 27th, 1823, and was buried in the family vault, in the Churchyard of Charlton, near Woolwich.

For a full account of the various scientific labours of Dr. Hutton, and of the peculiarities by which he was distinguished as a teacher of science, we must refer our readers to a memoir of him by his friend, and eventual successor in the chair at Woolwich, Dr. Olinthus Gregory, published in the Imperial Magazine for March 1823.

Both Editions having been long out of print, the present Editor was induced to undertake the superintendance of a new one; in which, by omitting what appeared trifling or of doubtful utility, and introducing in its stead a popular account of the more interesting discoveries in modern science, the work might continue to be to the present generation a useful manual of Scientific Recreation, as its predecessors have been to the generation which has passed.

Greenwich Hospital,
14th September, 1840.

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Erratum.—P. 550, line 7, for 'name,' read 'value.'

MATHEMATICAL AND PHILOSOPHICAL

RECREATIONS.

PART FIRST.

CONTAINING THE MOST CURIOUS PROBLEMS AND MOST INTERESTING TRUTHS IN REGARD TO ARITHMETIC.

ARITHMETIC and geometry, according to Plato, are the two wings of the mathematician. The object indeed of all mathematical questions, is to determine the ratios of numbers, or of magnitudes; and it may even be said, to continue the comparison of the ancient philosopher, that arithmetic is the mathematician's right wing; for it is an incontestible truth, that geometrical determinations would, for the most part, present nothing satisfactory to the mind, if the ratios thus determined could not be reduced to numerical ratios. This justifies the common practice, which we shall here follow, of beginning with arithmetic.

This science affords a wide field for speculation and curious research: but in the collection which we here present to the mathematical reader, we have confined ourselves to what is best calculated to excite the curiosity of those who have a taste for mathematical pursuits.

CHAPTER I.

OF OUR NUMERICAL SYSTEM, AND THE DIFFERENT KINDS OF ARITHMETIC.

IT has been generally remarked, that all or most of the nations with which we are acquainted, reckon by periods of ten; that is to say, after having counted the units from 1 to 10, they begin and add units to the ten; having attained to two tens, or 20, they continue to add units as far as 30, or three tens; and so on, in succession, till they come to ten tens, or a hundred; of ten times a hundred they form a thousand, and so on. Did this arise from necessity; was it occasioned by any physical cause; or was it merely the effect of chance?

No person, after the least reflection on this unanimous agreement, will entertain any idea of its being the effect of chance. There can be no doubt that this system derives its origin from our physical conformation. All men have ten fingers, a very few excepted, who, by some lusus naturæ, have twelve. The first men begin

to reckon on their fingers. When they had exhausted them by reckoning the units, it was necessary that they should form a first total, and again begin to reckon the same figures till they had exhausted them a second time; and so on in succession. Hence the origin of tens, which being confined, like the units, to the number of the fingers, could not be carried beyond it, without forming a new total, called a hundred; then another called a thousand, and so on.

From these observations, a curious consequence may be drawn. If nature, instead of ten fingers, had given us twelve, our system of numeration would have been different. After 10, instead of saying ten plus one or eleven, ten plus two or twelve, we should have ascended by simple denominations to twelve, and should then have counted twelve plus one, twelve plus two, &c., as far as two dozens; our hundred would have been twelve dozens, &c. A six-fingered people would certainly have had an arithmetic of this kind, which indeed would have sufficiently answered every arithmetical purpose, and indeed would have been attended with some advantages, which our numerical system does not possess.

In consequence of an idea of this kind, philosophers have been induced to examine the properties of other numerical systems. The celebrated Leibnitz proposed one, in which only two characters, 1 and 0, were to be employed. In this system of arithmetic, the addition of an 0 multiplied every thing by two, as it does by ten in common arithmetic, and the numbers were expressed as follow:

One	1	Eleven	1011
Two	10	Twelve	1100
Three	11	Thirteen	1101
Four	100	Fourteen	1110
Five	101	Fifteen	1111
Six	110	Sixteen	10,000
Seven	111	Thirty-two	100,000
Eight	1000	Sixty-four	1,000,000
Nine	1001	Two thousand three huudred	
Ten	1010	and seventy-nine	100,101,001,011

As Leibnitz found in the above mode of expressing numbers some peculiar advantages, he published, in the Memoirs of the Academy of Sciences at Berlin, rules for performing, in this kind of arithmetic, the usual operations of common arithmetic. But it may be readily perceived, that this new system, if introduced into practice, would be attended with the inconvenience of requiring too many figures: twenty would be necessary to express a number equal to about a million.

One curious circumstance in regard to this binary arithmetic must not be here omitted. It serves to explain, as some pretend, a Chinese symbol, which has occasioned great embarrassment to the learned who have applied to the study of the Chinese antiquities. This symbol, which is highly revered by the Chinese, who ascribe it to their ancient emperor Fohi, consists of certain characters, formed by the different combinations of a small whole line and a broken one. Father Bouset, a celebrated Jesuit, who resided some time in China as a missionary, having heard of Leibnitz' ideas, observed, that if the whole line were supposed to represent our 1, and the broken line our 0, these characters would be nothing else than a series of numbers expressed by binary arithmetic. It is very singular, that a Chinese enigma should find its Œdipus only in Europe; but perhaps in this explanation there is more of ingenuity than truth.

If the binary arithmetic of Leibnitz is entitled to no farther notice, than to be classed among the curious arithmetical speculations, the case however is not the same with duodenary arithmetic, or that kind which, as already observed, would have been brought into use had men been born with twelve fingers. This arithmetic would indeed have been as expeditious as the arithmetic now employed, and even somewhat

more so; the number of the characters, which would have received an increase only of two, to express ten and eleven, would have been as little burthensome to the memory as the present characters, and would have been attended with advantages which ought to make us regret that this system was not originally adopted.

It is not improbable, however, that the duodenary system would have been preferred, had philosophy presided at the invention; for it would have been readily seen that *twelve*, of all the numbers from 1 to 20, is that which possesses the advantage of being small, and of having the greatest number of divisors; for there are no less than four divisors by which it can be divided without a fraction, viz., 2, 3, 4, and 6. The number 18 indeed has four divisors also; but being larger than 12, the latter deserves to be preferred for measuring the periods of numeration. The first of these periods, from 1 to 12, would have had the advantage of being divisible by 2, 3, 4, 6; and the second, from 1 to 144, by 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72; whereas, in our system, the first period, from one to ten, has only two divisors, 2 and 5; and the second, from one to a hundred, has only 2, 4, 5, 10, 20, 25, 50. It is evident, therefore, that fractions would less frequently have occurred in the designation of numbers in that way, namely by twelves.

But what would have been most convenient in this mode of numeration, is that, in the divisions and sub-divisions of measures, it might have introduced a duodecimal progression. Thus, as the foot has by chance been divided into 12 inches, the inch into 12 lines, and the line into 12 points; the pound might have been divided into 12 ounces, the ounce into 12 drams, and the dram into 12 grains, or parts of any other denominations; the day might have been divided into 12 equal portions called hours, the hour into 12 other parts, each equal to 10 minutes, each of these parts 12 others, and so on successively. The case might have been the same in regard to measures of capacity.

Should it be asked, what would be the advantages of such a division? we might reply as follows. It is well known, by daily experience, that when it is necessary to divide any measure into 3, 4, or 6 parts, an integer number in the measures of a lower denomination cannot be found, or at least only by chance. Thus, the third or the sixth of a pound averdupois does not give an exact number of ounces; and the third of a pound sterling does not give an integer number of shillings. The case is the same in regard to the bushel, and the greater part of the other measures of capacity. These inconveniences, which render calculations exceedingly complex, would not take place if the duodecimal progression were every where followed.

There is still another advantage which would result from a combination of duodenary arithmetic, with this duodecimal progression. Any number of pounds, shillings, and pence; of feet, inches, and lines; or of pounds, ounces, &c., being given, they would be expressed as whole numbers of the same kind usually are in common arithmetic. Thus, for example, supposing the fathom to consist of 12 feet, as must necessarily be the case in this system of numeration, if we had to express 9 fathoms 5 feet 3 inches and 8 lines, we should have no occasion to write $9^f\ 5^f\ 3^i\ 8^l$, but merely 9538; and whenever we had a similar number expressing any dimension in fathoms, feet, inches, &c., the first figure on the right would express lines, the second inches, the third feet, the fourth fathoms, and the fifth dozens of fathoms, which might be expressed by a simple denomination, for example a perch, &c. In the last place, when it might be necessary to add, or subtract, or multiply, or divide, similar quantities, we might operate as with whole numbers, and the result would in like manner express, according to the order of the figures, lines, inches, feet, &c.

It may easily be conceived how convenient this would be in practice. On this account Stevin, a Dutch mathematician, proposed to adapt the subdivisions of weights and measures to our present system of numeration, by making them decrease in decimal progression. According to this plan, the fathom would have contained

10 feet, the foot 10 inches, the inch 10 lines, &c. But he did not reflect on the inconvenience of depriving himself of the advantage of being able to divide his measures, &c. by 3, 4, and 6, without a fraction, which is indeed of some importance.

On the suggestion of Borda, the centesimal division of the quadrant has been adopted in France. Having divided the quadrant into 100 equal parts, called *grades*, each grade is divided into 100 parts for minutes, and each minute into 100 parts for seconds, &c.

Let S = any arc in the sexagesimal division, and D the same arc in the decimal division; then

$$\frac{S}{D} = \frac{90}{100} = \frac{9}{10}; \text{ or } S = \frac{9\,D}{10}, \text{ and } D = \frac{10\,S}{9} = S + \frac{S}{9}$$

Therefore, to reduce an arc expressed decimally to sexagesimals, multiply by 9, and divide the product by 10. To reduce sexagesimals into decimals, to the arc expressed sexagesimally, add its ninth part.

Example.—What is the sexagesimal value of 34ᵍ· 2896?

$$\begin{array}{r} 34{\cdot}2896 \\ 9 \\ \hline 30^{\circ}{\cdot}86064 \\ 60 \\ \hline 51{\cdot}63840 \\ 60 \\ \hline 38{\cdot}30400 \\ \hline\hline \end{array}$$

Answer.—30° 51′ 38″ 304.

Conversely, it is required to express 30° 51′ 38″ 304 decimally?

$$\begin{array}{r|r} 60 & 38{\cdot}304 \\ 60 & 51{\cdot}6384 \\ & 30{\cdot}86064 \\ \text{Add 1-9th} & 3{\cdot}42896 \\ \hline \text{Answer} & 34{\cdot}28960 \\ \hline\hline \end{array}$$

On the basis of the decimal division the French have also constructed their system of national measures. The distance from the pole to the equator being determined by computations founded on an extensive series of trigonometrical operations; its ten millionth part, or the tenth part of a decimal angular second, equal to 39·371 English inches, constitutes the *metre*, which is the French unit of length. From this metre the several measures of surface and capacity are derived, and a given measure of water at its greatest concentration furnishes the standard of weights.

Whether, with its many apparent advantages, this method of division is ever likely to be generally adopted seems very doubtful In the mean time its *partial* introduction is productive of much inconvenience, as it not only deranges our habits, but lessens the utility of our instruments and tables, all of which have been adapted to the old system.

It is evident that in the duodenary arithmetic, the nine first numbers might be expressed as usual, by the nine known characters, 1, 2, 3, &c.; but as the period ought to terminate only at twelve, it would be necessary to express ten and eleven by simple characters. In this case we might choose φ to denote ten, and 3 to denote eleven, and then it is evident that,

10 would express twelve.
11 .... thirteen.
13 .... fifteen.
15 .... seventeen.
18 .... twenty.
1φ .... twenty-two.

1Ɛ .... twenty-three.
20 .... twenty-four.
40 .... forty-eight.
60 .... seventy-two.
100 .... a hundred and forty-four.
300 .... four hundred and thirty-two.
1000 .... one thousand seven hundred and twenty-eight.
2000 .... three thousand four hundred and fifty-six.
10,000 .... twenty thousand seven hundred and thirty-six.
100,000, &c. .... two hundred and forty-eight thousand eight hundred and thirty-two.

Thus the number denoted by the figures φ943 would be eighteen thousand six hundred and twenty-seven; for φ000 is eighteen thousand two hundred and eighty, 900 is one thousand two hundred and ninety-six, 40 is forty-eight, and 3 is three numbers, which if added will form the above sum.

It would be easy to form a set of rules for this new arithmetic, similar to those of common arithmetic; but as it does not seem likely that this mode of calculation will ever be brought into general use, we shall confine ourselves to what has been already said on the subject, and only add, that a book was printed in Germany, in which the common rules of arithmetic were explained in all the systems, the binary, ternary, quaternary, and so on, to the duodenary inclusively.

ON THE ARITHMETIC OF THE GREEKS.

The Greeks divided all their numbers, as we do ours, into periods of tens; but for the want of the happy idea of giving a local value to their numerical symbols, they were obliged to employ thirty-six characters, most of which were derived from their alphabet.

Thus our digits 1, 2, 3, 4, 5, 6, 7, 8, 9,
Were represented by..α, β, γ, δ, ε, ς, ζ, η, θ,
And 10, 20, 30, 40, 50, 60, 70, 80, 90
by ι, κ, λ, μ, ν, ξ, ο, π, ϙ
For hundreds they had ρ, σ, τ, υ, φ, χ, ψ, ω, ϡ

And for the thousands they had recourse again to the characters of the simple units, with the addition of a little dash below. Thus ͵α = 1000, ͵β = 2000, &c.

With these characters they could express any number under 10,000, or a myriad. Thus 991 was ϡϙα; 7382, ͵ζτπβ; and 4001, ͵δα.

In order to express myriads, they placed the letter M below the character representing the number of myriads they intended to indicate, as $\overset{\alpha}{\mathrm{M}}$ for 10000, $\overset{\delta\tau o\beta}{\mathrm{M}}$ for 43720000. This is the notation employed by Eutocius, in his commentaries on Archimedes.

Diophantus and Pappus represented their myriads by the letters Μυ, or more simply still, by a point, placed after the number, thus 43728097 is expressed by δτοβ. Μυ ͵ηϙζ, or δτοβ. ͵ηϙζ.

The number 100,000,000 was the greatest extent of the Greek arithmetic; but Archimedes indefinitely increased it, when he invented his system of Octates, or periods of eight. He assumed 100,000,000 as a new unit, and called the numbers which he formed with it numbers of the second order; then assuming the square, cube, &c. of 100,000,000 successively as a new unit, he obtained numbers of the third, fourth, and higher orders.

This idea of Archimedes, we are informed by Pappus, Apollonius greatly improved;

by reducing the octates to periods of fours, and dividing all numbers into orders of myriads; thus the number .... 7932. 3846. 2643; written according to the notation of Appollonius, is .... ζϡλβ. γωμϛ. βχμγ; the first period of four to the right being units, the next myriads, the next double myriads, or numbers of the second order. The next would be numbers of the third order, and so on indefinitely.

Having given a local value to his periods of fours, it was only necessary to have done the same for single digits, to have arrived at the system in present use; and it is astonishing that he did not perceive the advantages of doing so; and the more singular, as the use of the cipher was not unknown to the Greeks, being always employed in their sexagesimal operations, when it was necessary, as in the division of the circle, of which ours is still a representative, as is evident from the following example:

$$0\ \nu\theta'\ \eta''\ \iota\zeta'''\ \iota\gamma'''' = 0^\circ\ 59'\ 8''\ 17'''\ 13''''$$

Having given an idea of the Grecian notation for integer numbers, we next proceed to their method of representing fractions; which was by placing the denominator above, and to the right of the numerator; thus $\iota\epsilon^{\xi\delta}$ represented $\frac{15}{64}$. But when the numerator was unity, a small dash was placed to the right of the denominator, as γ' for $\frac{1}{3}$, ζ' for $\frac{1}{7}$. And the fraction $\frac{1}{2}$ had a particular character, as c or ∠, c′ or κ′.

It now remains for us to explain the method employed by the ancients in performing the common rules of arithmetic with this complicated system of notation.

The examples below, of addition and subtraction, require no explanation, being performed exactly as we do ours, proceeding from right to left; but to this method, though so clearly the most simple, the Greeks did not constantly adhere, as there are many instances which make it evident that they did both addition and subtraction from left to right.

Example in Addition.		Example in Subtraction.	
ωμζ. ͵γϡκα	847.3921	θ. ͵γχλϛ	93636
ξ. ͵ηυ	60.8400	β. γυ θ	23409
ϡη. ͵βτκα	908.2321	ζ. σκζ	70227

In multiplication they usually proceeded from left to right, as we do in multiplication of algebra, and placed their successive products without much apparent order; but as each of their characters retained its own proper value wherever it was placed, this want of order only rendered the addition a little more troublesome.

In the examples which follow we will mark the myriads by an m, the thousands by ‴, the hundreds by ″, &c., and so make the partial products in the Greek, and the translation identical. With this arrangement the reader will find it extremely easy to follow the work.

ρνγ	1″ 5′ 3
ρνγ	1″ 5′ 3
α. ͵ετ	1m 5‴ 3″
͵ε ͵β φ ρν	5‴ 2‴ 5″ 1″ 5′
τρνθ	3″ 1 5′ 9
β. ͵γυθ	2m 3‴ 4″ 9 = 23409

The division of the Greeks was still more intricate than their multiplication; for which reason, it seems, they generally preferred the sexagesimal division; and no example is left at length by any of their writers, except in the latter form; but these are sufficient to throw some light on the process they followed in the division of common numbers; and Delambre, in an essay subjoined to the French translation of Archimedes, has accordingly supposed the following example:

τλβ. ,γτκθ (,αωκγ	$332^{m}\ 3'''\ 3''\ 2'\ 9$ ($1'''\ 8''\ 2'\ 3$	
ρπβ. ,γ ‍ ‍ ,αωκγ	182 3	$1'''\ 8''\ 2'\ 3$
ρν. τκθ	150 0 3 2 9	
ρμε. ,ηυ	145 8 4	
δ. ,αϡκθ	4 1 9 2 9	
γ. ,ϛυξ	3 6 4 6	
,ευξθ	5 4 6 9	
,ευξθ	5 4 6 9	

The example will be found, on a slight inspection, to resemble that sort of division in which we divide feet, inches, and parts by similar denominations. which, together with the number of characters they employed, must have rendered this rule extremely laborious; and that for the extraction of the square root must have been equally difficult; the principle of which was the same as ours, except in the difference of the notation; though it appears that they frequently, instead of making use of the rule, found the root by successive trials, and then squared it to prove the truth of their assumption.

CHAPTER II.

OF SOME SHORT METHODS OF PERFORMING ARITHMETICAL OPERATIONS.

SECTION I.

Method of Subtracting several Numbers from several other given Numbers, without making partial Additions.

$$\left.\begin{array}{r}56243\\84564\\3252\\26848\end{array}\right\}A$$

$$\left.\begin{array}{r}2942\\3654\\2308\end{array}\right\}B$$

$$162003$$

To give the reader an idea of this operation, one example will be sufficient. Let it be proposed to subtract all the sums below the line at B, from all those above it at A. Add, in the usual manner, all the lower figures of the first column on the right, which will make 14, and subtract their sum from the next highest number of tens, or 20. Add the remainder 6 to the corresponding column above at A, and the sum total will be 23. Write down 3 at the bottom, and because there were here two tens, as before, there is nothing to be reserved or carried. Add, in like manner, the figures of the second lower column, which will amount to 9, and this sum taken from 10 will leave 1; add 1 therefore to the second column of the upper numbers, the sum of which will be 20; write down 0 at the bottom, and because there were here two tens, while in the lower column there was only one, reserve the difference, and substract it from the next column of the numbers marked B before you begin to add. In the contrary case, that is to say when there are more tens in any one of the columns marked B than in the corresponding column above it, the difference must be added. In the last place, when it happens that this difference cannot be taken from the next column below, for want of more significant figures, as is the case here in the fifth column, we must add it to the upper one, and write down the whole sum below the line. By proceeding in this manner, we shall have, in the present instance, 162003 for the remainder of the subtraction required.

SECTION II.

Some Short Methods of performing Multiplication and Division.

I. Every one, in the least acquainted with arithmetic, knows, that to multiply and

number by 10, nothing is necessary but to add to it a cipher; that to multiply by 100, two must be added, and so on.

Hence it follows, that to multiply by 5, we have only to suppose a cipher added to the number, and then to divide it by 2. Thus, if it were required to multiply 127 by 5; suppose a cipher added to the former, which will give 1270, and then divide by 2: the quotient 635 will be the product required.

In like manner, to multiply any number by 25, we must suppose it multiplied by 100, or increased by two ciphers, and then divide by 4. Thus 127 multiplied by 25, will give 3175. For 127 when increased by two ciphers makes 12700, which being divided by 4, produces 3175.

According to the same principle, to multiply by 125, it will be sufficient to add three ciphers to the multiplicand, or to suppose them added, and then to divide by 8. The reason of these operations may be so readily conceived, that it is not necessary to explain it.

II. The multiplication of any number by 11 may be reduced to simple addition. For it is evident that to multiply a number by 11, is nothing else than to add the number to its decuple, that is to say, to itself followed by a cipher.

Let the proposed number, for ex., be 67583
To multiply this number by eleven, say 3 and 0 make 3; write down 3 in the units place; then add 8 and 3, which makes 11; write down 1 in the place of tens, and carry 1; then 5 and 8 and 1 carried make 14; write down 4 in the third place, or that of hundreds, and carry 1. Continue in this manner, adding every figure to its next following one, till the operation is finished, and the product will be 743413, as above.

$$\begin{array}{r} 67583 \\ \hline 743413 \\ \hline \end{array}$$

The same number may be multiplied in like manner, by 111, if we first write down the 3, then the sum of 8 and 3, then that of 5, 8, and 3, then that of 7, 5, and 8, and so on, adding always three contiguous figures together

III. We shall only farther observe. that to multiply any number by 9, simple subtraction may be employed. Let us take, for example, the same number as before, ... 67583

$$\begin{array}{r} 67583 \\ \hline 608247 \\ \hline \end{array}$$

To multiply this number by 9, nothing is necessary but to suppose a cipher added to the end of it, and then to subtract each figure from that which precedes it, beginning at the right. Thus 3 from 0 or 10, leaves 7; 8 from 2 or 12, leaves 4; and if we continue in this manner, taking care to borrow 10 when the right-hand figure is too small to admit of the preceding one being subtracted from it, we shall find the product to be 608247.

The reason of these operations may be readily perceived. For it is evident, that in the first we only add the number itself to its decuple; and in the latter, we subtract it from its decuple. But in order to form a clearer idea of the matter, it may perhaps be worth while to perform the operation at full length.

Concise operations of a similar kind may be employed in certain cases of division; as in dividing, for example, a given number by any power whatever of 5. Thus, if it were required to divide 128 by 5; we must double it, which will give 256; if we then cut off the last figure, which will be a decimal, the quotient will be 25·6 or $25\frac{6}{10}$. To divide the same number by 25, we must quadruple it, which will give 512; and if we then cut off the two last figures as decimals, we shall have for the quotient 5 and $\frac{12}{100}$. To divide by 125, we must multiply the dividend by 8, and cut off three figures. In like manner we may divide a given number by any other power of 5; but it must be confessed that such short methods of calculation are attended with no great advantage.

SECTION III.

Short Method of performing Multiplication and Division by Napier's Rods or Bones.

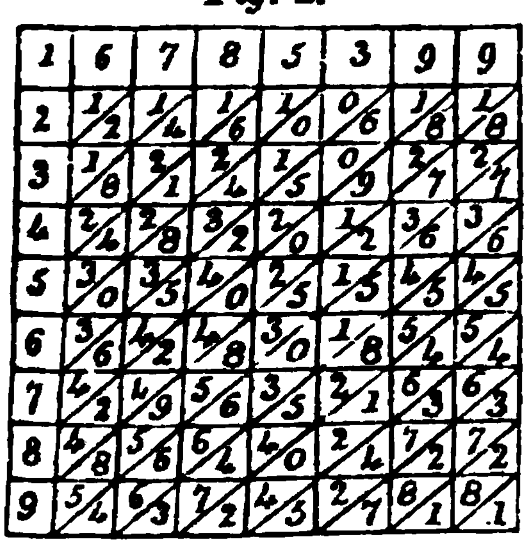
When large numbers are to be multiplied, it is evident that the operation might be performed much more readily, by having a table previously formed of each number of the multiplicand, when doubled, tripled, quadrupled, and so on. Such a table indeed might be procured by simple addition, since nothing would be necessary but to add any number to itself, and we should have the double; then to add it to the double, and we should have the triple, &c. But unless the same figure should frequently recur in the multiplicand, this method would be more tedious than that which we wished to avoid.

The celebrated Napier, the sole object of whose researches seems to have been to shorten the operations of arithmetic and trigonometry, and to whom we are indebted for the ingenious and ever-memorable invention of logarithms, devised a method of forming a table of this kind in a moment, by means of certain rods, which he has described in his work entitled *Rabdologia;* printed at Edinburgh in 1617. The construction of them is as follows:

Fig. 1.

Provide several slips of card or ivory or metal rods, about nine times as long as they are broad, and divide each of them into 9 equal squares. (Fig. 1.) Inscribe at the top, that is to say in the first square of each slip or rod, one of the numbers of the natural series 1, 2, 3, 4, &c., as far as 9 inclusively. Then divide each of the lower squares into two parts by a diagonal, drawn from the upper angle on the right hand to the lower one on the left, and inscribe in each of these triangular divisions, proceeding downwards, the double, triple, quadruple, &c. of the number inscribed at the top; taking care, when the multiple consists of only one figure, to place it in the lower triangle, and when it consists of two to place the units in the lower triangle, and the tens in the upper one, as seen in the figure. It will be necessary to have one of these slips or rods, the squares of which are not divided by a diagonal, but inscribed with the natural numbers from 1 to 9. This one is called the index rod. It will be proper also to have several of these slips or rods for each figure.

Fig. 2.



The rods being prepared as above, let us suppose that it is required to multiply the number 6785399. Arrange the seven rods inscribed at the top with the figures 6785, &c., close to each other, and apply to them on the left hand the index rod, or that inscribed with the single figures (Fig. 2); by which means we shall have a table of all the multiples of each figure in the multiplicand; and scarcely any thing more will be necessary but to transcribe them. Thus, for example, to multiply the above number by 6; looking for 6 on the index rod, and opposite to it in the first square, on the right hand, we find 54; writing down the 4 found in the lower triangle, and adding the 5 in the upper one to the 4 in the lower triangle of the next square on the left, which makes 9; write down the 9, and then add the 5 in the upper triangle of the same square to the 8 in the lower triangle of the next one; and proceed in this manner, taking care to carry as in common addition, and we shall find the result to be 40712394, or the product of 6785399 multiplied by 6.

Compound multiplication, or by several figures, may be performed in the same manner, and with equal facility. Let us suppose, for example, that the same number is to be multiplied by 839938. Write down the multiplicand, and the multiplier below it in the usual manner; and as the first figure of the multiplier is 8, look for it in the index rod, and by adding the different figures in the triangles of the horizontal column opposite to it, the sum will be found to be 54283192, or the product of the above number by 8, which must be written down. Then find the sum of the figures in the horizontal column opposite to 3, and write the sum down as before, but carrying it one place farther to the left. Continue in this manner till you have gone through all the figures of the multiplier, and if the several partial products be then added as usual, you will have the total product, as above expressed.

```
      6785399
       839938
     --------
     54283192
    20356197
   61068591
  61068591
 20356197
54283192
-------------
5699314465262
```

A similar artifice may be employed to shorten division, especially when large sums are to be often divided by the same divisor. Thus, for example, if the number 1492992 is to be divided by 432, and if the same divisor must frequently occur, construct, in the manner above described, a table of the multiples of 432, which will scarcely require any farther trouble than that of transcribing the numbers, as may be seen here on the left.

1 ...	432
2 ...	864
3 ...	1296
4 ...	1728
5 ...	2160
6 ...	2592
7 ...	3024
8 ...	3456
9 ...	3888

```
1492992 ( 3456
1296
 ----
 1969
 1728
  ----
  2419
  2160
   ----
   2592
   2592
   ----
   0000
```

When this is done, it may be readily perceived, that since 432 is not contained in the first three figures of the dividend, some multiple of it must be contained in the first four figures, viz., 1492. To find this multiple, you need only cast your eye on the table, to observe that the next less multiple of 432 is 1296, which stands opposite to 3; write down 3 therefore in the quotient, and 1296 under 1492, then subtract the former from the latter, and there will remain 196, to which if you bring down the next figure of the dividend, the result will be 1969. By casting your eye again on the table, you will find that 1728, which stands opposite to 4, is the greatest multiple of 432 contained in 1969; write down 4 therefore in the quotient, and subtract as before. By continuing the operation in this manner, it will be found that the following figures of the quotient are 5 and 6; and as the last multiple leaves no remainder, the division is perfect and complete.

Remark.—Mathematicians have not confined themselves to endeavouring to simplify the operations of arithmetic by such means: they have attempted something more, and have tried to reduce them to mere mechanical operations. The celebrated Pascal was the first who invented a machine for this purpose, a description of which may be seen in the fourth volume of the *Recueil des Machines presèntées à l'Académie*. Sir Samuel Morland, without knowing perhaps what Pascal had done in this respect, published, in 1673, an account of two arithmetical machines which he invented, one of them for addition and subtraction, and the other for multiplication, but without explaining their internal construction. The same object engaged the attention of the celebrated Leibnitz about the same time; and afterwards that of

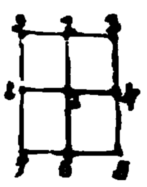
the marquis Poleni. A description of their machines may be seen in the *Theatrum Arithmeticum* of Leupold, printed in 1727, together with that of a machine invented by Leupold himself, and also in the *Miscell. Berol.* for 1709. We have likewise the *Abaque rabdologique* of Perrault, in the collection of his machines published in 1700. It serves for addition, subtraction, and multiplication. The *Recueil des Machines présentées à l'Académie Royale des Sciences* contains also an arithmetical machine, by Lespine, and three by Boistissandeau. Finally, Mr. Gersten, professor of mathematics at Giessen, transmitted, in the year 1735, to the Royal Society of London, a minute description of a machine of the same kind, invented by himself. We shall not enlarge further on this subject, but proceed to give an account, which we hope will be acceptable to the curious reader, of an ingenious method of performing the operations of arithmetic, invented by Dr. Saunderson, a celebrated mathematician, who was blind from his infancy.

SECTION IV.

Palpable Arithmetic, or a method of performing arithmetical operations, which may be practised by the blind, or in the dark.

What is here announced may, on the first view, appear to be a paradox; but it is certain that this method of performing arithmetical operations was practised by the celebrated Dr. Saunderson, who, though he had lost his sight when a child of a year old, made so great progress in the mathematics, that he was elected to fill the professor's chair of that science, in the university of Cambridge. The apparatus he employed, to supply the deficiency of sight, was as follows:

Fig. 3.

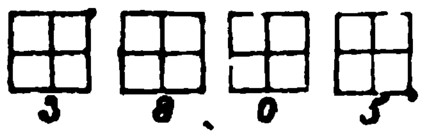


Let the square (Fig. 3.) be divided into four other squares, by two lines parallel to the sides, and intersecting each other in the centre. These two lines form with the sides of the square four points of intersection, and these added to the four angles of the primitive square, give altogether nine points. If a hole be made in each of these points, into which a pin or peg can be fixed, it is evident that there will be nine distinct places for the nine simple and significant figures of our arithmetical system, and nothing further will be necessary but to establish some order in which these points or places, destined to receive a moveable peg, ought to be counted. To mark 1, it may be placed in the centre; to express 2, it may be placed immediately above the centre; to express 3, at the upper angle on the right; and so on in succession, round the sides of the square, as marked by the numbers opposite to each point.

But there is still another character to be expressed, viz., the 0, which in our arithmetic is of very great importance. This character might be expressed in a manner exceedingly simple, by leaving the holes empty; but Saunderson preferred placing in the middle one a large-headed pin, unless when having unity to express, he was obliged to substitute in its stead a small-headed pin. By these means he obtained the advantage of being better able to direct his hands, and to distinguish with more ease, by the relative position of the small-headed pins, in regard to the large one in the centre, what the former expressed. This method therefore ought to be adopted; for Saunderson no doubt made choice of those means which were most significant to his fingers.

As the reader has here seen with what ease a simple number may be expressed in this manner, we shall now shew that a compound number may be expressed with equal facility. If we suppose several squares to be constructed like the preceding, ranged in a line, and separated from each other by small intervals, that they may be better distinguished by the touch, any person acquainted with common arithmetic may

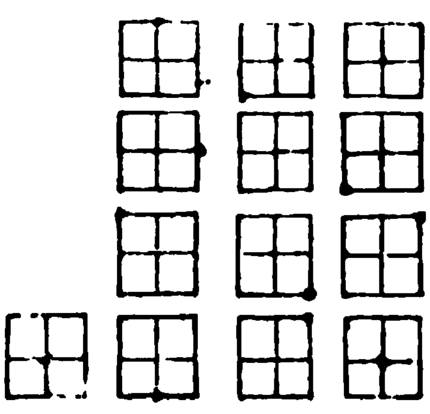
Fig. 4.



perceive, that the first square on the right will serve to express units; the next towards the left to express tens; the third to express hundreds, &c. Thus the four squares, with the pegs arranged as represented (Fig. 4.) will express the number 3805.

If you therefore provide a board, or table, divided into several horizontal bands, on each of which are placed seven or eight similar squares, according to circumstances; if these bands be separated by proper intervals, that they may be better distinguished; and if all the squares of the same order, in each of the bands, be so arranged, as to correspond with each other in a perpendicular direction; you may perform, by means of this machine, all the different operations of arithmetic. The reader will find (Fig. 5.) a representation of the method of adding three numbers, and expressing their sum by a machine of this kind.

Fig. 5.



271
407
953
——
1631
——

Saunderson employed this ingenious machine, not only for arithmetical operations, but also for representing geometrical figures, by arranging his pins in a certain order, and extending threads from the one to the other. But what has been said is sufficient on this subject; those persons who are desirous of farther information respecting it, may consult Saunderson's Algebra, or the French translation of Wolff's Elements Abridged, where this palpable arithmetic is explained at full length.

PROBLEM.

To multiply £11. 11*s*. 11*d*. *by* £11. 11*s*. 11*d*.

This problem was once proposed by a sworn accountant to a young man who had been recommended to him as perfectly well acquainted with arithmetic. And indeed, besides the difficulty which results from the multiplication of quantities of different kinds, and from their reduction, it is well calculated to try the ingenuity of an arithmetician. But it is not improbable that the proposer would have been embarrassed by the following simple question: What is the nature of the product of pounds shillings and pence multiplied by pounds shillings and pence? We know that the product of a yard by a yard represents a square yard, because geometricians have agreed to give that appellation to a square surface one yard in length and one in breadth; and 6 yards multiplied by 4 yards make 24 square yards; for a rectangular superficies 6 yards in length and 4 in breadth, contains 24 square yards, in the same manner as the product of 4 by 6 contains 24 units. But who can tell what the product of a penny by a penny is, or of a penny by a pound?

The question considered in this point of view, is therefore absurd, though ordinary arithmeticians sometimes are not sensible of it.

CHAPTER III.

OF CERTAIN PROPERTIES OF NUMBERS.

We do not here mean to examine those properties of numbers which engaged so much the attention of the ancients, and in which they pretended to find so many mysterious virtues. Every one, whose mind is not tinctured with the spirit of credulity, must laugh to think of the good canon of Cezene, Peter Bungus, collecting in a large quarto volume, entitled *De Mysteriis Numerorum*, all the ridiculous ideas which Nichomachus, Ptolemy, Porphyry, and several more of the ancients, childishly propagated respecting numbers. How could it enter the minds of reasonable beings, to ascribe physical energy to things entirely metaphysical? For numbers are mere conceptions of the mind, and consequently can have no influence in nature.

None therefore but people of weak minds can believe in the virtues of numbers. Some imagine, that if thirteen persons sit down at the same table, one of them will die in the course of the year; but there is a much greater probability that one will die if the number be twenty-four.

I. The number 9 possesses this property, that the figures which compose its multiples, if added together, are always a multiple of 9; so that by adding them, and rejecting 9 as often as the sum exceeds that number, the remainder will always be 0. This may be easily proved by trying different multiples of 9, such as 18, 27, 36, &c.

This observation may be of utility, to enable us to discover whether a given number be divisible by 9, for in all cases, when the figures which express any number, on being added together, form 9, or one of its multiples, we may be assured that the number is divisible by 9, and consequently by 3 also.

But this property does not exclusively belong to the number 9; for the number 3 has a similar property. If the figures which express any multiple of 3 be added, we shall find that their sum is always a multiple of 3; and when any proposed number is not such a multiple, whatever the sum of the figures by which it is expressed exceed a multiple of 3, will be the quantity to be deducted from the number, in order that it may be divisible by 3 without a remainder.

We must not omit to take notice here, of a very ingenious observation of the author of the History of the Academy of Sciences, for the year 1726, which is, that if a system of numeration, different from that now in use, had been adopted, that for example of duodecimal progression, the number eleven, or, in general, that preceding the first period, would have possessed the same property as the number nine does in our present system of numeration. By way of example, let us take a multiple of eleven, as nine hundred and fifty-seven, and let us express it according to that system by the characters 7 φ 5: it will here be seen that 7 and φ make seventeen, and 5 added makes twenty-two, which is a multiple of eleven.

This property of 9 and 3, in the decimal notation, admits of a very simple proof. For let a be the digit in the units place, b, c, d, &c. those in the place of tens, hundreds, &c.; then the number will be represented analytically by $1000\,d + 100\,c + 10\,b + a$; or by $\overline{999+1}.\,d + \overline{99+1}.\,c + \overline{9+1}.\,b + a$; or by $999\,d + 99\,c + 9\,b + d + c + b + a$. But $999\,d + 99\,c + 9\,b$ is divisible both by 9 and 3; therefore, if the whole number represented by $1000\,d + 100\,c + 10\,b + a$ be divisible by 9 or 3, the remaining part, $d + c + b + a$, must also be divisible by 9 or 3. And a like proof would apply to the digit and its factors preceding the last digit of the first period, in any system of numeration.

In addition to the foregoing observations of the French author, may be added the following remarks on the same subject, lately made by an ingenious English gentleman. He first expresses all the products of 9 by the other figures, in the following manner, and then enumerates the curious properties.

$$
\begin{array}{l}
9 \\
\underline{1} \\
9 \,..\, 9 \\
\quad \underline{2} \\
\quad 18 \,..\, 1+8=9 \\
\qquad \underline{3} \\
\qquad 27 \,..\, 2+7=9 \\
\qquad\quad \underline{4} \\
\qquad\quad 36 \,..\, 3+6=9 \\
\qquad\qquad \underline{5} \\
\qquad\qquad 45 \,..\, 4+5=9 \\
\qquad\qquad\quad \underline{6} \\
\qquad\qquad\quad 54 \,..\, 5+4=9 \\
\qquad\qquad\qquad \underline{7} \\
\qquad\qquad\qquad 63 \,..\, 6+3=9 \\
\qquad\qquad\qquad\quad \underline{8} \\
\qquad\qquad\qquad\quad 72 \,..\, 7+2=9 \\
\qquad\qquad\qquad\qquad \underline{9} \\
\qquad\qquad\qquad\qquad 81 \,..\, 8+1=9
\end{array}
$$

The component figures of the product, made by the multiplication of every digit into the number 9, when added together, make NINE.

The order of those component figures is reversed, after the said number has been multiplied by 5.

The component figures of the amount of the multipliers (viz. 45), when added together make NINE.

The amount of the several products, or multiples of 9 (viz. 405), when divided by 9, gives for a quotient 45 ; that is $4+5=$ NINE.

The amount of the first product (viz. 9), when added to the other products, whose respective component figures make 9, is 81 ; which is the *square of* NINE.

The said number 81, when added to the above-mentioned amount of the several products, or multiples of 9 (viz. 405), makes 486 ; which, if divided by 9, gives for a quotient 54 ; that is, $5+4=$ NINE.

It is also observable that the number of changes that may be rung on *nine* bells, is 362880 ; which figures, added together, make 27 ; that is, $2+7=$ NINE.

And the quotient of 362880, divided by 9, is 40320 ; that is, $4+0+3+2+0=$ NINE.

II. Every square number necessarily ends with one of these figures, 1, 4, 5, 6, 9 ; or with an even number of ciphers preceded by one of these figures. This may be easily proved, and is of great utility in enabling us to discover when a number is not a square ; for though a number may end as above mentioned, it is not always however a perfect square ; but, at any rate, when it does not end in that manner, we are certain that it is not a square, which may prevent useless labour. In regard to cubic numbers, they may end with any figure whatever ; but if they terminate with ciphers, they must be in number either three, or six, or nine, &c.

If a square number end with 4, the last figure but one will be even, as in 64, 144, and 97344.

If a square number end with 5, it will end with 25 ; as 625, 1225.

If a square number end with an odd figure, the last figure but one will be even, as 81529. But if it end with any even digit, except 4, the last figure but one will be odd, as 36, 576, 13456.

No square number can end with two even digits except two ciphers, or two fours, as 100, 144, 40000, 44944.

A square number cannot end in three equal digits, except they be three fours; nor in more than three equal digits unless they be ciphers.

III. Every square number is divisible by 3, or becomes so when diminished by unity. This may be easily tried on any square number at pleasure. Thus 4 less 1, 16 less 1, 25 less 1, 121 less 1, &c. are all divisible by 3; and the case is the same with other square numbers.

Every square number is divisible also by 4, or becomes so when diminished by unity. This may be proved with the same case as the former.

Every square number is divisible likewise by 5, or becomes so when increased, or else diminished by unity. Thus, for example, $36 - 1$, $49 + 1$, $64 + 1$, $81 - 1$, &c., are all divisible by 5.

Every odd square number is a multiple of 8 increased by unity. We have examples of this property in the numbers 9, 25, 49, 81, &c.; from which if 1 be deducted the remainders will be divisible by 8.

If a square number be either multiplied or divided by a square, the product or the quotient will be a square.

If a number be not a complete square, its square root cannot be represented either by an integer, or by a rational fraction, either proper or improper.

IV. Every number is either a square, or divisible into two, or three, or four squares. Thus 30 is equal to $25 + 4 + 1$; $31 = 25 + 4 + 1 + 1$; $33 = 16 + 16 + 1$; $63 = 49 + 9 + 4 + 1$, or $36 = 25 + 1 + 1$.

We shall here add, by anticipation, though we have not yet informed the reader what triangular, or pentagonal, &c., numbers are, that

Every number is either triangular, or composed of two or of three triangular numbers. And that

Every number is either pentagonal, or composed of two, or three, or four, or five pentagonals, and so of the rest.

We shall add also, that every even square, after the first square 1, may be resolved at least into four equal squares; and that every odd square may be resolved into three, if not into two. Thus $81 = 36 + 36 + 9$; $121 = 81 + 36 + 4$; $169 = 144 + 25$; $625 = 400 + 144 + 81$.

V. Every power of 5, or of 6, necessarily ends with 5 or with 6.

VI. If we take any two numbers whatever; then either one of them, or their sum, or their difference, is necessarily divisible by 3. Let the numbers assumed be 20 and 17; though neither of these numbers, nor their sum 37, is divisible by 3, yet their difference is, for it is three.

It might easily be demonstrated, that this must necessarily be the case, whatever be the numbers assumed.

VII. If two numbers are of such a nature, that their squares when added together form a square, the product of these two numbers is divisible by 6.

Of this kind, for example, are the numbers 3 and 4, the squares of which, 9 and 16, when added, make the square number 25: their product 12 is divisible by 6.

From this property a method may be deduced, for finding two numbers, the squares of which, when added together, shall form a square number. For this purpose, multiply any two numbers together; the double of their product will be one of the numbers sought, and the difference of their squares will be the other.

Thus if we multiply together 2 and 3, the squares of which are 4 and 9, their product will be 6; if we then take 12 the double of this product, and 5 the difference of their squares, we shall have two numbers, the sum of whose squares is equal to another square number; for these squares are 144 and 25, which when added make 169, the square of 13.

VIII. When two numbers are such, that the difference of their squares is a square

number; the sum and difference of these numbers are themselves square numbers, or the double of square numbers.

Thus, for example, the numbers 13 and 12, when squared, give 169 and 144, the difference of which 25, is also a square number; then 25, the sum of these numbers, is a square number, and also their difference 1.

In like manner, 6 and 10, when squared produce 36 and 100, the difference of which 64 is also a square number; then it will be found, that their sum 16 is a square number, as well as their difference 4.

The numbers 8 and 10 give for the difference of their squares 36; and it may be readily seen, that 18, the sum of these numbers, is the double of 9, which is a square number, and that their difference 2 is the double of 1, which is also a square number.

IX. If two numbers, the difference of which is 2, be multiplied together, their product increased by unity will be the square of the intermediate number.

Thus, the product of 12 and 14 is 168, which being increased by 1, gives 169, the square of 13, the mean number between 12 and 14.

Nothing is easier than to demonstrate, that this must always be the case; and it will be found in general, that the product of two numbers increased by the square of half their difference, will give the square of the mean number.

X. A *prime* number is that which has no other divisor but unity. Numbers of this kind, the number 2 excepted, can never be even, nor can any of them terminate in 5, except 5 itself; hence it follows, that except those contained in the first period of ten, they must necessarily terminate in 1 or 3, or 7 or 9.

One curious property of prime numbers is, that every prime number, 2 and 3 excepted, if increased or diminished by unity, is divisible by 6. This may be readily seen in any numbers taken at pleasure, as 5, 7, 11, 13, 17, 19, 23, 29, 31, &c.; but I do not know, that any one has ever yet demonstrated this property *à priori*. But the inverse of this is not true, that is, every number when increased or diminished by unity is divisible by 6, is not, on that account, necessarily a prime number.

As it is often of utility to be able to know, without having recourse to calculation, whether a number be prime or not, we have here subjoined a table of all the prime numbers from 1 to 10,000.

Table of the Prime Numbers from 1 *to* 10,000.

2	71	163	263	373	479	601	719	853	977	1093	1223
3	73	167	269	379	487	607	727	857	983	1097	1229
5	79	173	271	383	491	613	733	859	991		1231
7	83	179	277	389	499	617	739	863	997	1103	1237
11	89	181	281	397		619	743	877		1109	1249
13	97	191	283		503	631	751	881	1009	1117	1259
17		193	293	401	509	641	757	883	1013	1123	1277
19	101	197		409	521	643	761	887	1019	1129	1279
23	103	199	307	419	533	647	769		1021	1151	1283
29	107		311	421	541	653	773	907	1031	1153	1289
31	109	211	313	431	547	659	787	911	1033	1163	1291
37	113	223	317	433	557	661	797	919	1039	1171	1297
41	127	227	331	439	563	673		929	1049	1181	
43	131	229	337	443	569	677	811	937	1051	1187	1301
47	137	233	347	449	571	683	821	941	1061	1193	1303
53	139	239	349	457	577	691	823	947	1063		1307
59	149	241	353	461	587		827	953	1069	1201	1319
61	151	251	359	463	593	701	829	967	1087	1213	1321
67	157	257	367	467	599	709	839	971	1091	1217	1327

1361	1787	2251		3191	3643	4129	4639	5113	5651	6143	6661
1367	1789	2267	2707		3659	4133	4643	5119	5653	6151	6673
1373		2269	2711	3203	3671	4139	4649	5147	5657	6163	6679
1381	1801	2273	2713	3209	3673	4153	4651	5153	5659	6173	6689
1399	1811	2281	2719	3217	3677	4157	4657	5167	5669	6197	6691
	1823	2287	2729	3221	3691	4159	4663	5171	5683	6199	
1409	1831	2293	2731	3229	3697	4177	4673	5179	5689		6701
1423	1847	2297	2741	3251			4679	5189	5693	6203	6703
1427	1861		2749	3253	3701	4201	4691	5197		6211	6709
1429	1867	2309	2753	3257	3709	4211			5701	6217	6719
1433	1871	2311	2767	3259	3719	4217	4703	5209	5711	6221	6733
1439	1873	2333	2777	3271	3727	4219	4721	5227	5717	6229	6737
1447	1877	2339	2789	3299	3733	4229	4723	5231	5737	6247	6761
1451	1879	2341	2791		3739	4231	4729	5233	5741	6257	6763
1453	1889	2347	2797	3301	3761	4241	4733	5237	5743	6263	6779
1459		2351		3307	3767	4243	4751	5261	5749	6269	6781
1471	1901	2357	2801	3313	3769	4253	4759	5273	5779	6271	6791
1481	1907	2371	2803	3319	3779	4259	4773	5279	5783	6277	6793
1483	1913	2377	2819	3323	3793	4261	4787	5281	5791	6287	
1487	1931	2381	2833	3329	3797	4271	4789	5297		6299	6803
1489	1933	2383	2837	3331		4273	4793		5801		6823
1493	1949	2389	2843	3343	3803	4283	4799	5303	5807	6301	6827
1499	1951	2393	2851	3347	3821	4289		5309	5813	6311	6829
	1973	2399	2857	3359	3823	4297	4801	5323	5821	6317	6833
1511	1979		2861	3361	3833		4813	5333	5827	6323	6841
1523	1987	2411	2879	3371	3847	4327	4817	5347	5839	6329	6857
1531	1993	2417	2887	3373	3851	4337	4831	5351	5843	6337	6863
1543	1997	2423	2897	3389	3853	4339	4861	5381	5849	6343	6869
1549	1999	2437		3391	3863	4349	4871	5387	5851	6353	6871
1553		2441	2903		3877	4357	4877	5393	5857	6359	6883
1559	2003	2447	2909	3407	3881	4363	4889	5399	5861	6361	6899
1567	2011	2459	2917	3413	3889	4373			5867	6367	
1571	2017	2467	2927	3433		4391	4903	5407	5869	6373	6907
1579	2027	2473	2939	3449	3907	4397	4909	5413	5879	6379	6911
1583	2029	2477	2953	3457	3911		4919	5417	5881	6389	6917
1597	2039		2957	3461	3917	4409	4931	5419	5897	6397	6947
	2053	2503	2963	3463	3919	4421	4933	5431			6949
1601	2063	2521	2969	3467	3923	4423	4937	5437	5903	6421	6959
1607	2069	2531	2971	3469	3929	4441	4943	5441	5923	6427	6961
1609	2081	2539	2999	3491	3931	4447	4951	5443	5927	6449	6967
1613	2083	2543		3499	3943	4451	4957	5449	5939	6451	6971
1619	2087	2549	3001		3947	4457	4967	5471	5953	6469	6977
1621	2089	2551	3011	3511	3967	4463	4969	5477	5981	6473	6983
1627	2099	2557	3019	3517	3989	4481	4973	5479	5987	6481	6991
1637		2579	3023	3527		4483	4987	5483		6491	6997
1657	2111	2591	3037	3529	4001	4493	4993		6007		
1663	2113	2593	3041	3533	4003		4999	5501	6011	6521	7001
1667	2129		3049	3539	4007	4507		5503	6029	6529	7013
1669	2131	2609	3061	3541	4013	4513	5003	5507	6037	6547	7019
1693	2137	2617	3067	3547	4019	4517	5009	5519	6043	6551	7027
1697	2141	2621	3079	3557	4021	4519	5011	5521	6047	6553	7039
1699	2143	2633	3083	3559	4027	4523	5021	5527	6053	6563	7043
	2153	2647	3089	3571	4049	4547	5023	5531	6067	6569	7057
1709	2161	2657		3581	4051	4549	5039	5557	6073	6571	7069
1721	2179	2659	3109	3583	4057	4561	5051	5563	6079	6577	7079
1723		2663	3119	3593	4073	4567	5059	5569	6089	6581	
1733	2203	2671	3121		4079	4583	5077	5573	6091	6599	7103
1741	2207	2677	3137	3607	4091	4591	5081	5581			7109
1747	2213	2683	3163	3613	4093	4597	5087	5591	6101	6607	7121
1753	2221	2687	3167	3617	4099		5099		6113	6619	7127
1759	2237	2689	3169	3623		4603		5623	6121	6637	7129
1777	2239	2693	3181	3631	4111	4621	5101	5639	6131	6653	7151
1783	2243	2699	3187	3637	4127	4637	5107	5641	6133	6659	7159
								5647			

7177	7451	7643	7883	8147	8377	8627	8831	9059			9749
7187	7457	7649		8161	8387	8629	8837	9067	9311	9511	9767
7193	7459	7669	7901	8167	8389	8641	8839	9091	9319	9521	9769
7207	7477	7673	7907	8171		8647	8849		9323	9533	9781
7211	7481	7681	7917	8179	8419	8663	8861	9103	9337	9539	9787
7213	7487	7687	7927	8191	8423	8669	8863	9109	9341	9547	9791
7219	7489	7691	7933		8429	8677	8867	9127	9343	9551	9803
7229	7499	7699	7937	8209	8431	8681	8887	9133	9349	9587	9811
7237	7507	7703	7949	8219	8443	8689	8893	9137	9371		9817
7243	7517	7717	7951	8221	8447	8693		9151	9377	9601	9829
7247	7523	7723	7963	8231	8461	8699	8923	9157	9391	9613	9833
7253	7529	7727	7993	8233	8467		8929	9161	9399	9619	9839
7283	7537	7741	8009	8237		8707	8933	9173		9623	9851
7297	7541	7753	8011	8243	8501	8713	8941	9181	9403	9629	9857
	7547	7757	8017	8263	8513	8719	8951	9187	9413	9631	9859
7307	7549	7759	8039	8269	8521	8731	8963	9199	9419	9643	9871
7309	7559	7789	8053	8273	8527	8737	8969		9421	9649	9883
7321	7561	7793	8059	8287	8537	8741	8971	9203	9431	9661	9887
7331	7573		8069	8291	8539	8747	8999	9209	9433	9677	
7333	7577	7817	8081	8293	8543	8753		9221	9437	9679	9901
7349	7583	7823	8087	8297	8563	8761	9001	9227	9439	9689	9907
7351	7589	7829	8089		8573	8779	9007	9239	9461	9697	9923
7369	7591	7841	8093	8311	8581	8783	9011	9241	9463		9929
7393		7853		8317	8597		9013	9257	9467	9719	9931
	7603	7867	8101	8329	8599	8803	9027	9277	9473	9721	9941
7411	7607	7873	8111	8353		8807	9041	9281	9479	9733	9949
7417	7621	7877	8117	8363	8609	8819	9043	9283	9491	9739	9967
7433	7639	7879	8123	8369	8623	8821	9049	9293	9497	9743	9973

Eratosthenes invented what he called a *seive* for excluding from a series of odd numbers those which are not prime. The principle of the *seive* is this:

Having written down in consecutive order all odd numbers, from one to any required extent, as

$$1, 3, 5, 7, \dot{9}, 11, 13, \ddot{15}, 17, 19, \ddot{21}, 23, \dot{25}, \dot{27}, \text{\&c.}$$

We begin with three, the first prime number, and over every subsequent third in the series put a point, and from 5 a point is placed over every fifth number; from 7 over every seventh number, and so on.

Then the numbers which remain without points are prime numbers,—and adding 2, the only even prime, we obtain all the prime numbers included in the series.

Every prime number greater than 3 is of one of the forms $6n+1$, or $6n-1$.

For every number is either divisible by 6, or leaves, when divided by it, a remainder of 1, 2, 3, 4, or 5; that is, every number is of one of the forms $6n$, $6n+1$, $6n+2$, $6n+3$, $6n+4$, or $6n+5$. But the first and fifth of these are divisible by 2, and the fourth is divisible by 3, and are therefore not prime. Hence all prime numbers greater than 3 are of the form $6n+1$ or $6n+5$. But $6n+5$ is of the same form, or would produce the same number, as $6n-1$. For taking $n=2$, $6n+5=17$, and taking $n=3$, $6n-1=17$. Therefore all prime numbers above 3 are of the form $6n+1$, or $6n-1$.

But though all prime numbers are included in these two forms, they include also many numbers which are not primes. For example, if $n=4$, $6n+1=25$, which is not prime, and if $n=6$, $6n-1=35$, which is not prime. Indeed, it may be demonstrated that no algebraic formula can contain prime numbers only.

With reference to the two forms under consideration, it has been proved that whenever n is of the form $6n'n'' \pm n' + n''$, $6n+1$ is not prime; and whenever n is not of that form $6n+1$ is prime.

Also that when n is of the form $6n'n'' \pm n' \backsim n''$, $6n-1$ is not prime, while it is always prime when n is not of that form.

XI. Another kind of numbers, which possess a singular and curious property, are those called *perfect numbers*. This name is given to every number, the aliquot parts of which, when added together, form exactly that number itself. Of this we have an example in the number 6; for its aliquot parts are 1, 2, 3, which together make 6. The number 28 possesses the same property; for its aliquot parts are 1, 2, 4, 7, 14, the sum of which is 28.

To find all the perfect numbers of the numerical progression, take the double progression 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, &c.; and examine those terms of it, which when diminished by unity, are prime numbers. Those to which this property belongs, will be found to be 4, 8, 32, 128, 8192; for these terms when diminished by unity, are 3, 7, 31, 127, 8191. Multiply therefore each of these numbers by that number in the geometrical progression which preceded the one from which it is deduced, for example 3 by 2, 7 by 4, 31 by 16, 127 by 64, 8191 by 4096, &c.; and the result will be 6, 28, 496, 8128, 33550336, which are perfect numbers.

These numbers however are far from being so numerous as some authors have believed.* The following is a series of numbers either perfect, or, for want of proper attention, supposed to be so, taken from a memoir of Mr. Krafft, published in the 7th volume of the Transactions of the Academy of Petersburgh. Those to which this property really belongs are marked with an asterisk.

*	6
*	28
*	496
*	8128
	130816
	2096128
*	33550336
	536854528
*	8589869056
*	137438691328
	2199022206976
	35184367894528
	562949936644096
	9007199187632128
	144115187807420416
*	2305843008139952128
	36893488143124135936

Thus we find that between 1 and 10 there is only one perfect number; that there is one between 10 and 100, one between 100 and 1000, and one between 1000 and 10000; but those would be mistaken who should believe that there is one also between ten thousand and a hundred thousand, one between a hundred thousand and a million, &c.; for there is only one between ten thousand and eight hundred millions. The rarity of perfect numbers, says a certain author, is a symbol of that of perfection.

All the perfect numbers terminate with 6 or 28.

XII. There are some numbers called *amicable numbers*, on account of a certain property which gives them a kind of affinity or reciprocity, and which consists in their being mutually equal to the sum of each other's aliquot parts. Of this kind are the numbers 220 and 284; for the first 220 is equal to the aliquot parts of 284, viz. 1, 2, 4, 71, 142; and, reciprocally, 284 is equal to the aliquot parts, 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, of the other number 220.

* The rule given by Ozanam is incorrect, and produces a multitude of numbers, such as 130816, 2096128, &c., which are not perfect numbers. When Ozanam wrote his rule, he did not recollect that one of the multipliers must be a prime number. But 511 and 2047 are not prime numbers.

Amicable numbers may be found by the following method. Write down, as in the subjoined example, the terms of a double geometrical progression, or having the ratio 2, and beginning with 2; then triple each of these terms, and place these triple numbers each under that from which it has been formed; these numbers diminished by unity, 5, 11, 23, &c. if placed each over its corresponding number in the geometrical progression, will form a third series above the latter. In the last place, to obtain the numbers of the lowest series, 71, 287, &c. multiply each of the terms of the series, 6, 12, 24, &c., by the one preceding it, and subtract unity from the product.

5	11	23	47	95	191	383
2	4	8	16	32	64	128
6	12	24	48	96	192	384
	71	287	1151	4607	18431	73727

Take any number of the lowest series, for example 71, of which its corresponding number in the first series, viz. 11, and the one preceding the latter, viz. 5, as well as 71, are prime numbers: multiply 5 by 11, and the product 55 by 4, the corresponding term of the geometrical series, and the last product 220, will be one of the numbers required. The second will be found by multiplying the number 71 by the same number 4, which will give 284.

In like manner, with 1151, 47, and 23, which are prime numbers, we may find two other amicable numbers, 17296 and 18416; but 4607 will not produce any amicable numbers, because, of the two other corresponding numbers, 47 and 95, the latter is not a prime number. The case is the same with the number 18431, because 95 is among its corresponding numbers; but the following number 73727, with 383 and 191, will give two more amicable numbers, 9363584 and 9437056.

By these examples it may be seen, that if perfect numbers are rare, amicable numbers are much more so, the reason of which may be easily conceived.

XIII. If we write down a series of the squares of the natural numbers, viz. 1, 4, 9, 16, 25, 36, 49, &c.; and take the difference between each term and that which follows it, and then the differences of these differences; the latter will each be equal to 2, as may be seen in the following example.

	1	4	9	16	25	36	49
1st. Diff.	3	5	7	9	11	13	
2d. Diff.	2	2	2	2	2		

It hence appears, that the square numbers are formed by the continual addition of the odd numbers 1, 3, 5, &c., which exceed each other by 2.

In the series of the cubes of the natural numbers, viz. 1, 8, 27, &c., the third, instead of the second differences, are equal, and are always 6, as may be seen in the following example.

Cubes	1	8	27	64	125	216
1st. Diff.	7	19	37	61	91	
2d. Diff.	12	18	24	30		
3d. Diff.	6	6	6			

In regard to the series of the fourth powers, or biquadrates, of the natural numbers, the fourth differences only are equal, and are always 24. In the fifth powers, the fifth differences only are equal, and are invariably 120.

All this may be readily shewn, by taking the successive differences of the expanded terms of the series x^n, $\overline{x+1}^n$, $\overline{x+2}^n$, &c., giving to n the values 2, 3, 4, &c. in succession.

These differences, 2, 6, 24, 120, &c. may be found by multiplying the series of the numbers 1, 2, 3, 4, 5, 6, &c. For the second power, multiply the two first; for the third power, the three first, and so on.

XIV. The progression of the cubes 1, 8, 27, 64, 125, &c. of the natural numbers, 1, 2, 3, 4, 5, 6, &c. possesses this remarkable property, that if any number of its

terms whatever, from the beginning, be added together, their sum will always be a square. Thus, 1 and 8 make 9; if we add to this sum 27, we shall have 36, which is still a square number; and if we add 64, we shall have 100, and so on.

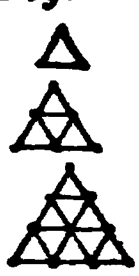
The root of each square so formed is the sum of the roots of all the component cubes. Thus $1^3+2^3+3^3=36=\overline{1+2+3}^2$.

XV. The number 120 has the property of being equal to half the sum of its aliquot parts, or divisors, viz, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, which together make 240. The number 672 is also equal to half the sum of its aliquot parts, 1344. Several other numbers of the like kind may be found, and some even which would form only a third, or fourth, of the sum of their aliquot parts, or which would be the double, triple, or quadruple of that sum; but what has been here said, will be sufficient to exercise those who are fond of such researches.

CHAPTER IV.

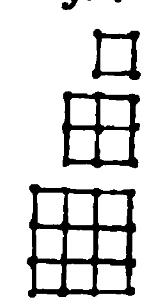
OF FIGURATE NUMBERS.

Fig. 6.



If there be taken any arithmetical progression, as for instance, the most simple of all, or that of the natural numbers 1, 2, 3, 4, 5, 6, 7, &c.; and if we take the first term, the sum of the two first, that of the three first, and so on; the result will be a new series of numbers, 1, 3, 6, 10, 15, 21, 28, &c. called *triangular numbers*, because they can always be ranged in such a manner as to form an equilateral triangle, as may be seen Fig 6.

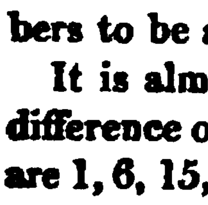
Fig. 7.



The square numbers, as 1, 4, 9, 16, 25, 36, &c. arise from a like addition of the first terms of the arithmetical progression, 1, 3, 5, 7, 9, 11, &c., the common difference of which is 2. These numbers, as is well known, may be arranged so as to form square figures. Fig. 7.

A similar addition of the terms of the arithmetical progression 1, 4, 7, 10, 13, &c., the common difference of which is 3, will produce the numbers 1, 5, 12, 22, &c., which are called pentagonal numbers, because they represent the number of points which may be arranged on the sides and in the interior part of a regular pentagon; as may be seen Fig. 8; where there are three pentagons, having one common angle, representing the number of points which increase arithmetically; the first having two points on each side, the second three, and the third four; and which progression, it is evident, might be continued ever so far.

Fig. 8.



It is in this sense, and in this manner, that we must conceive the figurate numbers to be arranged.

It is almost needless to say, that the progression 1, 5, 9, 13, 17, &c., the common difference of which is 4, produces, by a similar addition, the hexagonal numbers, which are 1, 6, 15, 28, 45, &c.; and that in like manner may be found the heptagonals, the octagonals, &c.

There is another kind of polygonal numbers, which result from the number of points that can be ranged in the middle, and on the sides, of one or more similar polygons, having a common centre. These are different from the preceding; for the series of the triangulars of this kind is 1, 4, 10, 19, 31, &c., which are formed by the successive addition of the numbers 1, 3, 6, 9, 12.

The central square numbers are 1, 5, 13, 25, 41, 61, &c.; formed, in like manner, by the successive addition of the numbers 1, 4, 8, 12, 16, 20, &c.

The central pentagonal numbers are 1, 6, 16, 31, 51, 76, &c.; formed by the addition of the numbers 1, 5, 10, 15, 20, &c.

But we shall not enlarge farther on this kind of polygonal numbers, because they are not those to which mathematicians usually give that name. Let us return therefore to the ordinary polygonal numbers.

The radix of a polygonal number is the number of the terms of the progression necessary to be added in order to obtain that number. Thus the radix of the triangular number 21, is 6, because that number results from the successive addition of the six numbers 1, 2, 3, 4, 5, 6. In like manner, 4 is the radix of the square number 16, considered as a figurate number, because that number is produced by adding the *four* terms 1, 3, 5, 7, of the progression of the odd numbers.

Having given this explanation of the nature of polygonal numbers, we shall now present the reader with a few problems respecting them.

PROBLEM I.

To find whether any proposed Number is Triangular, or Square, or Pentangular, &c.

The method of finding whether a number be square, is well known, and serves as a foundation for discovering the other figurate numbers. This being supposed; then to determine whether any given number is a polygonal number, the following general rule may be employed.

Multiply by 8 the number of the angles of the polygon less 2; multiply this first product by the proposed number, and to the new product add the square of a number equal to that of the angles of the polygon less 4: if the sum be a perfect square, the given number is a polygon of the kind proposed.

It may easily be seen, that as the number of the angles in the triangle are 3, in the square 4, in the pentagon 5, &c., we shall have, as the multiplier of the proposed number, in the case of the triangular number, 8; in that of the quadrangular number, 16; in that of the pentagonal, 24; and in that of the hexagonal, 32.

In like manner, as the number of the angles less 4, gives for the triangle — 1; for the square 0; for the pentagon, 1; for the hexagon, 2, &c.; the numbers to be added to the product, as before mentioned, will be, for the triangle, 1 (because the square of — 1 is 1); for the square, 0; for the pentagon, 1; for the hexagon, 4; for the heptagon, 9; &c. From these principles we may deduce the following rules, which we shall illustrate by examples.

Suppose it were required to know whether 21 be a triangular number.

Multiply 21 by 8, to the product add 1, and the sum will be 169, which is a perfect square: consequently 21 is a triangular number.

If we are desirous of knowing whether 35 be a pentagonal number, we must multiply 35 by 24, and the product will be 840; to this product if 1 be added we shall have 841, which is a square number: we may therefore rest assured that 35 is a pentagonal number.

PROBLEM II.

A Triangular, or any Figurate Number whatever, being given; to find its Radix, or the Number of the Terms of the Arithmetical Progression of which it is the Sum.

First perform the operation described in the preceding problem; and having found the square root, the possibility of which will indicate whether the number be figurate or not, add to this root a number equal to that of the angles of the proposed polygon less 4, and divide the sum by the double of the same number of angles less 2: the quotient will be the radix of the polygon.

The number to be added is, for the triangle — 1, that is to say 1 to be deducted; for the square it is 0; for the pentagon 1; for the hexagon 2; &c.

As to the divisor, it may be easily seen that for the triangle it is 2 (because the double of 3 less 2 is 2), for the square 4, for the pentagon 6, for the hexagon 8, &c.

Let it be required therefore to find the radix of the triangular number 36.

Having performed the operation explained in the preceding problem, and found the product 289, the square root of which is 17, subtract unity from this number, and divide the remainder by 2; the quotient 8 will be the radix or side of the triangular number 36.

Let the radix of the pentagonal number 35 be required.

Having found, as before, the radix 29, add to it 1, which will give 30, and divide by 6; the quotient 5 will be the radix of this pentagonal number, that is to say, of the number formed by the addition of the 5 terms of the series 1, 4, 7, 10, 13.

PROBLEM III.

The Radix of a Polygonal Number being given; to find that Number.

The rule for this purpose is exceedingly simple. From the square of the given radix, subtract the product of the same radix by a number equal to that of the angles less 4; the half of the remainder will be the polygonal number required.

For example, what is the triangular number the radix of which is 12?

The square of 12 is 144; the number equal to that of the angles less 4 is − 1, which being multiplied by 12 gives − 12: but according to the rule, − 12 ought to be subtracted, which is the same thing as adding 12; in that case you will have 156, which being divided by 2 gives 78.

What is the heptagonal number the radix of which is 20?

To find the number required, take the square of 20, which is 400; then multiply 20 by 3, which is the number of the angles less 4, and subtract 60, the product, from 400; if you then divide the remainder 340 by 2, the quotient 170 will be the number sought, or the heptagon the radix of which is 20.

It may not be improper here to remark, that the same number may be a polygon or figurate number in different ways. Every number greater than 3 is a polygon, of a number of sides or angles equal to that of its units.

Thus 36 is a polygon of 36 sides, the radix of which is 2; for the two first terms of the progression are 1, 35. The same number 36 is a square; and lastly it is triangular, having 8 for its radix.

In the like manner, 21 is a polygon of 21 sides; it is also triangular; and lastly it is octagonal.

PROBLEM IV.

To find the Sum of as many Triangular, or of as many Square, or of as many Pentagonal Numbers, as we choose.

As by the successive addition of the terms of different arithmetical progressions, we obtain new progressions of numbers, called triangular numbers, square numbers, pentagonals, &c.; we can add also these last progressions, which will give rise to new figurate numbers, of a higher order, called *pyramidal numbers*. Those which arise from the progression of triangular numbers, are called pyramidals of the first order; those produced by the addition of the square numbers, pyramidals of the second order; and those by the progression of the pentagonal numbers, pyramidals of the third order. The same operation may be performed with the pyramidals; which gives rise to the pyramido-pyramidals. But as these numbers are of little utility, and can answer no other purpose than that of exercising the genius of such as are fond of analytical investigation, we shall not enlarge farther on the subject. We shall therefore confine ourselves to giving a general rule for adding as many figurate numbers as the reader may choose.

Multiply the cube of the number of terms to be added, by the number of the angles of the polygon less 2; to the sum add three times the square of the said number of terms, and subtract from it the product of the same number multiplied by that of the angles less 5: if you divide the remainder by 6, you will have the sum of the terms of the progression.

For example, suppose it were required to find the sum of the first eight triangular numbers.

The cube of 8 is 512; which being multiplied by the number of the angles of the polygon less 2, or by 1, gives still 512; add to this number the triple of the square of 8, or 192, which will make 704; then, as the number of the angles less 5, is -2, multiply 8 by -2, and you will have -16; if you then add 16 to 704 you will have 720, which being divided by 6, gives 120, for the sum of the eight first triangular numbers.

The same result may be obtained, with more ease, by multiplying the number of the terms 8, by 9, and the product by 10, which gives also 720; which divided by 6, the quotient is 120, as before.

In the case of a series of squares, the number of which we shall here suppose to by 10, we have only to multiply the number of terms, viz. 10, by the same number plus unity, or by 11, and then by the double of the same number plus unity, that is to say by 21: the product of these three numbers, 2310, if divided by 6, gives 385, for the sum of the first ten square numbers 1, 4, 9, 16, &c.

CHAPTER V.

OF RIGHTANGLED TRIANGLES IN NUMBERS; OR RIGHTANGLED TRIANGULAR NUMBERS.

RIGHTANGLED triangular numbers, are rational numbers so related to each other, that the sum of the squares of two of them is equal to the square of the third.

The numbers 3, 4, and 5 have this property, $3^2 + 4^2$ being equal to 5^2.

Right-angled triangular numbers must be severally unequal; for, if the two less ones could each be represented by a, and the third or greatest by b, then $2a^2 = b^2$, $b = a\sqrt{2}$, an irrational number, whatever is the value of a.

The area of a rightangled triangle, whose sides are rational, cannot be equal to a rational square.

If a, b, and c represent the sides of a triangle, and C be the angle opposite c; then if $C = 90^\circ$, $a^2 + b^2 = c^2$: if $C = 120^\circ$, $a^2 + ab + b^2 = c^2$, and if $C = 60^\circ$, $a^2 - ab + b^2 = c^2$.

If n represent any number, and m any other number less than n, then $n^2 + m^2$ will represent the hypothenuse of a rightangled plane triangle, of which the other two sides are respectively $n^2 - m^2$, and $2nm$.

For example, if $n = 2$, and $m = 1$, then $n^2 + m^2 = 5$, $n^2 - m^2 = 3$, and $2nm = 4$, which are rightangled triangular numbers.

If $n = 7$ and $m = 2$, the formulæ give 53, 45, and 28 for the numbers, and $53^2 = 2809 = 45^2 + 28^2$.

We shall now propose and solve a few of the most easy and curious problems respecting right-angled triangular numbers.

PROBLEM I.

To find as many Rightangled Triangles in Numbers as we please.

This may be effected by the concluding formulæ which we have just given, but we think it right to add the following methods.

Take any two numbers at pleasure, for example 1 and 2, which we shall call generating numbers; multiply them together; then having doubled the product, we obtain one of the sides of the triangle, which in this case will be 4. If we then square each of the generating numbers, which in the present example will give 4 and 1, their difference 3 will be the second side of the triangle, and their sum 5 will be the hypothenuse. The sides of the triangle, therefore, having 1 and 2 for their generating numbers, are 3, 4, 5.

If 2 and 3 had been assumed as generating numbers, we should have found the sides to be 5, 12, and 13; and the numbers 1 and 3 would have given 6, 8, and 10.

Another Method.—Take a progression of whole and fractional numbers, as $1\frac{1}{3}$, $2\frac{2}{5}$, $3\frac{3}{7}$, $4\frac{4}{9}$, &c., the properties of which are; 1st, The whole numbers are those of the common series, and have unity for their common difference. 2nd, The numerators of the fractions, annexed to the whole numbers, are also the natural numbers. 3rd, The denominators of these fractions are the odd numbers 3, 5, 7, &c.

Take any term of this progression, for example $3\frac{3}{7}$, and reduce it to an improper fraction, by multiplying the whole number 3 by 7, and adding to 21, the product, the numerator 3, which will give $\frac{24}{7}$. The numbers 7 and 24 will be the sides of a rightangled triangle, the hypothenuse of which may be found by adding together the squares of these two numbers, viz. 49 and 576, and extracting the square root of the sum. The sum in this case being 625, the square root of which is 25, this number will be the hypothenuse required. The sides therefore of the triangle produced by the above term of the generating progression, are 7, 24, 25.

In like manner, the first term $1\frac{1}{3}$ will give the rightangled triangle 3, 4, 5.

The second term $2\frac{2}{5}$ will give 5, 12, 13.

The fourth $4\frac{4}{9}$ will give 9, 40, 41. All these triangles have the ratio of their sides different; and they all possess this property, that the greatest side and the hypothenuse differ only by unity.

The progression $1\frac{7}{8}$, $2\frac{11}{12}$, $3\frac{15}{16}$, $4\frac{19}{20}$, &c., is of the same kind as the preceding. The first term of it gives the rightangled triangle 8, 15, 17; the second term gives the triangle 12, 35, 37; the third the triangle 16, 63, 65, &c. All these triangles, it is evident, are different in regard to the proportion of their sides; and they all have this peculiar property, that the difference between the greater side and the hypothenuse, is always the number 2.

PROBLEM II.

To find any Number of Rightangled Triangles in Numbers, the sides of which shall differ only by Unity.

To resolve this problem, we must find out such numbers that the double of their squares plus or minus unity shall also be square numbers. Of this kind are the numbers 1, 2, 5, 12, 29, 70, &c.; for twice the square of 1 is 2, which diminished by unity leaves 1, a square number. In like manner, twice the square of 2 is 8, to which if we add 1, the sum 9 will be a square number. And so on.

Having found these numbers, take any two of them which immediately follow each other, as 1 and 2, or 2 and 5, or 12 and 29, for generating numbers. The rightangled triangles arising from them will be of such a nature, that their sides will differ from each other only by unity. The following is a table of these triangles, with their generating numbers.

Gener. Numb.		Sides.		Hypoth.
1	2	3	4	5
2	5	20	21	29
5	12	119	120	169
12	29	696	697	985
29	70	4059	4060	5741
70	169	23660	23661	33461

But if the problem were, to find a series of triangles of such a nature, that the hypothenuse of each should exceed one of the sides only by unity, the solution would be much easier. Nothing in this case would be necessary but to assume, as the generating numbers of the required triangle, any two numbers having unity for their difference. The following is a table similar to the preceding, of the six first rightangled triangles produced by the first numbers of the natural series.

Gener.	Numb.	Sides.		Hypoth.
1	2	3	4	5
2	3	5	12	13
3	4	7	24	25
4	5	9	40	41
5	6	11	60	61
6	7	13	84	85

If we assume, as generating numbers, the respective sides of the preceding series of triangles, we shall have a new series of rightangled triangles, the hypothenuses of which will always be square numbers; as may be seen in the following table.

Gener.	Numb.	Sides.		Hypoth.	Roots.
3	4	7	24	25	5
5	12	119	120	169	13
7	24	336	527	625	25
9	40	720	1519	1681	41
11	60	1320	3479	3721	61
13	84	2184	6887	7225	85

It may here be observed, that the roots of the hypothenuses are always equal to the greater of the generating numbers increased by unity.

But if the second side and the hypothenuse of each triangle in the above table, which differ only by unity, were assumed as the generating numbers, we should have a series of rightangled triangles, the least sides of which would always be squares. A few of these are as follow:

Gener.	Numb.	Sides.		Hypoth.
4	5	9	40	41
12	13	25	312	313
24	25	49	1200	1201
40	41	81	3280	3281

In the last place, if it were required to find a series of rightangled triangles, one of the sides of which shall be always a cube, we have nothing to do but to take, as generating numbers, two following terms in the progression of triangular numbers, as 1, 3, 6, 10, 15, 21, &c. By way of example we shall here give the first four of these triangles:

Gener.	Numb.	Sides.		Hypoth.
1	3	6	8	10
3	6	36	27	45
6	10	120	64	136
10	15	300	125	325

PROBLEM III.

...e different Rightangled Triangles, the Areas of which shall be all Equal.

...are three rightangled triangles which possess this property. The ...are, 40, 42, 58; those of the second 24, 70, 74; and those of the ...3.

The method by which these triangles are found, is as follows:

Add the product of any two numbers to the sum of their squares, and that will give the first number; the difference of their squares will give a second; and double the sum of their product and of the square of the least number, will give the third.

If you then form a rightangled triangle from the two first of the numbers thus found, as generating numbers; a second from the two extremes; and a third from the first and the sum of the other two; these three rightangled triangles will be equal to each other.

No more than three rightangled triangles, equal to each other, can be found in whole numbers; but we may find as many as we choose in fractions or mixt numbers, by means of the following formula:

With the hypothenuse of one of the above triangles, and the quadruple of its area, form another rightangled triangle, and divide it by double the product which arises from multiplying the hypothenuse of the triangle you made choice of by the difference of the squares of the two other sides: the triangle thence produced will be the one required.

PROBLEM IV.

To find a Rightangled Triangle, the Sides of which shall be in Arithmetical Progression.

Take two generating numbers which have to each other the ratio of 1 to 2; the sides of the rectangled triangle thence produced will be in arithmetical progression.

The simplest of these triangles, is that which has for its sides 3, 4, and 5, arising from the numbers 1 and 2 assumed as generating numbers. But it is to be observed, that all the other triangles, which possess the same property, are similar to this one, and are only multiples of it. That there can be no other kind, might easily be demonstrated in a great many different ways.

For let x, $x + a$, and $x + 2a$, be the sides, then $x^2 + \overline{x + a}^2 = \overline{x + 2a}^2$, and this quadratic equation gives $x = 3a$. Therefore the sides are represented by $3a$, $4a$, and $5a$.

Remark—If it were required to find a rightangled triangle, the three sides of which should be in geometrical proportion, we must observe, that none such can be found in whole numbers; for the two generating numbers ought to be in the ratio of 1 to $\sqrt{\sqrt{5} - 2}$, which is an irrational number.

PROBLEM V.

To find a Rightangled Triangle, the Area of which, expressed in Numbers, shall be equal to the Perimeter, or in a given ratio to it.

Of any square number, and the same square increased by 2, form a rightangled triangle, and divide each of its sides by that square number: the quotients will give the sides of a new rightangled triangle, the area of which, expressed numerically, will be equal to the perimeter.

Thus, if we take, as generating numbers, 1 and 3, we shall have the triangle 6, 8, 10, the sides of which, if divided by unity, give the same 6, 8, 10, forming a triangle having the property required; for the area and the perimeter are each equal to 24. In like manner, if we take 4 and 6 as generating numbers, we obtain for the required triangle 5, 12, 13, which on trial will be found to possess the same property.

These triangles are the only two of the kind which can be found in whole num-

bers; but we may find abundance of them in fractional numbers, by means of the squares 9, 16, &c.; such as the following: $\frac{34}{8}$, $\frac{288}{8}$, $\frac{290}{8}$; or $\frac{68}{16}$, $\frac{576}{16}$, $\frac{580}{16}$, or in their least terms, $\frac{17}{4}$, $\frac{144}{4}$, $\frac{145}{4}$.

If it were required that the area of the proposed triangle should be only in a given ratio to the perimeter, for example that of $\frac{1}{2}$; take as generating numbers a square, and the same square increased by 3, and form from them, as before directed, a right-angled triangle: this triangle will possess the required property. Of this kind, in whole numbers, are the two triangles 8, 15, 17, and 7, 24, 25: and numberless others may be found in fractional numbers.

CHAPTER VI.

SOME CURIOUS PROBLEMS RESPECTING SQUARE AND CUBE NUMBERS.

PROBLEM I.

Any Square Number being given, to divide it into Two other Squares.

INNUMERABLE solutions may be found to this problem, in the following manner. Let 16, for example, whose root is 4, be the square to be divided into two other squares, which, as may be easily seen, can be only fractions.

Take any two numbers, as 3 and 2; multiply them together; and by their product multiply the double of 4, the root of the proposed square; the last product, which in this case is 48, will be the numerator of a fraction, the denominator of which will be 13, the sum of the squares of the above numbers 3 and 2: the fraction $\frac{48}{13}$ therefore will give the side of the first square required, which square consequently will be $\frac{2304}{169}$.

To obtain the second, multiply the given square by the above denominator 169, and from the product 2704 subtract the numerator 2304: if we then take 20, the root of 400 the remainder, (which will be always a square,) for a numerator, and 13 for a denominator, we shall have the fraction $\frac{20}{13}$ for the side of the second square.

The two sides of the required squares therefore, are $\frac{48}{13}$ and $\frac{20}{13}$, the squares of which, $\frac{2304}{169}$ and $\frac{400}{169}$, will be found equal to the square number 16.

If we had taken for the primitive numbers 2 and 1, we should have had the roots $\frac{16}{5}$ and $\frac{12}{5}$, the squares of which are $\frac{256}{25}$ and $\frac{144}{25}$; the sum of which is $\frac{400}{25}$ or 16.

The numbers 4 and 3 would have given the roots $\frac{96}{25}$ and $\frac{28}{25}$, the squares of which $\frac{9216}{625}$ and $\frac{784}{625}$ still make up $\frac{10000}{625}$ or 16.

It may here be seen, that by varying the two first supposed numbers at pleasure, the solutions also may be varied without end.

Remark.—Should it be here asked whether a given cube can, in like manner, be divided into two other cubes? we shall reply, on the authority of an eminent analyst, M. de Fermat, that it is not possible. It is equally impossible to divide any power above the square into two parts, which shall be powers of the same kind; for example, a biquadrate into two biquadrates.

PROBLEM II.

To divide a Number, which is the Sum of Two Squares, into Two other Squares.

Let the proposed number be 13, which is composed of the two squares 9 and 4: it is required to divide it into two other squares.

Take any two numbers, for example, 4 and 3; and multiply the former, 4, by 6, the double of 3 the root of one of the above squares; and the second 3 multiply by the

double of 2 the root of the other square; which will give as products 24 and 12. Subtract the latter of these numbers from the former, and their difference 12 will be the numerator of a fraction, the denominator of which will be 25, the sum of the squares of the numbers first assumed. Multiply this fraction $\frac{12}{25}$, by each of the assumed numbers, viz. 4 and 3, and you will have $\frac{48}{25}$ and $\frac{36}{25}$. If you then take the greater of these numbers from the root of the greater square contained in 13, viz. 3, the remainder will be $\frac{27}{25}$: and if you add the other to the side of the smaller square contained in 13, viz. 2, you will have $\frac{86}{25}$. These two fractions then, $\frac{27}{25}$ and $\frac{86}{25}$, will be the sides of the two squares sought, viz. $\frac{729}{625}$ and $\frac{7396}{625}$, which together are equal to 13.

By supposing other numbers, other squares may be obtained; but these are sufficient to shew the method of finding them.

Remark.—For a number to be divisible, in a variety of ways, into two squares, it must be either a square, or composed of two squares. Of this kind, taking them in order, are the numbers 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 25, 26, 29, 32, 34, 36, 37, &c. We do not know, nor do we think it possible to find, any method of dividing into two squares any number which is not a square, or the sum of two squares; and we are of opinion that it may be established as a rule, that every whole number, which is not a square, or composed of two squares, in whole numbers, cannot be divided, in any manner, into two squares. A demonstration of this would be curious.

But every number is divisible, in a great variety of ways, into four squares; for there is no number which is not either a square, or the sum of two, or of three, or of four squares. Bachet de Meziriac advanced this proposition,* the truth of which he ascertained as far as possible by trying all the numbers from 1 to 325. It is added, by M. de Fermat,† that he was able to demonstrate the following general and curious properties of numbers, viz.:

That every number is either triangular, or composed of two or three triangular numbers.

That every number is either square, or composed of two, or three, or four square numbers.

And that every number is either pentagonal, or composed of two, or three, four, or five pentagonal numbers. And so of the rest.

A demonstration of these properties of numbers, if they be real, would be truly curious.

PROBLEM III.

To find Four Cubes, two of which taken together shall be equal to the Sum of the other two.

This problem may be solved by the following simple method. Take any two numbers of such a nature, that double the cube of the less shall exceed that of the greater; then from double the greater cube subtract the less; and multiply the remainder, as well as the sum of the cubes, by the less of the assumed numbers: the two products will be the sides of the two first cubes required.

In like manner, take the cube of the greater of the assumed numbers from double the cube of the less; and multiply the remainder, as well as the sum of these two cubes, by the greater of the assumed numbers: the two new products will be the sides of the other two cubes.

For example, if we assume the numbers 4 and 5, which possess the above property, we obtain, by following the rule, for the sides of the two first cubes, 744, 756; and

* Diophanti Alexandrini Arithmeticorum, lib. vi. cum Comm. C. G. Bacheti. Tolosæ. 1670. fol. p. 179.
† Ibid. p. 180.

for those of the other two, 945 and 15, which being divided by 3, give for the two first 248, 252; and for the two latter, 315, 5.

If the assumed numbers be 5 and 6, we shall have 1535 and 1705 for the sides of the two first cubes; and 2046 and 204 for those of the other two.

Remark.—A number composed of two cubes being given, it is possible to find two other cubes, the sum of which shall be equal to the former two. Vieta was of a contrary opinion; but M. de Fermat, in his Observations on the Arithmetical Questions of Diophantus, with a Commentary by Bachet de Meziriac, has pointed out a method by which such cubes can be found. The calculation indeed extends to numbers which are exceedingly complex, and sufficient to frighten the boldest arithmetician; as may be seen by the following example, where it is required to divide the sum of the two cubes 8 and 1 into two other cubes. By following the method of M. de Fermat, Father de Billy found that the sides of the two new cubes were the following numbers:

$$\frac{1243617773990097836481}{609623835661372974 49},$$

and

$$\frac{487267171714352336560}{60962383566137297449}.$$

We must take these numbers on Father de Billy's word; for we do not know that any one will ever venture to examine whether he has been deceived.

But it is possible to resolve, without much trouble, another question of a similar kind, which is: To find three cubes which, taken together, shall be equal to a fourth. By following the method pointed out in the above-mentioned work, it will be found that the least whole numbers, which resolve the question, are 3, 4, and 5; for their cubes added together make 216, which is the cube of 6.

We have confined ourselves to a few questions of this kind, but they might be varied almost without end. They are attended with a peculiar kind of difficulty which renders them interesting, and on that account they have been an object of attention to various analysts; such as Diophantus of Alexandria, among the ancients, who wrote thirteen books on arithmetical questions, of which the first six only remain, with another on polygonal numbers. Vieta too exercised his ingenuity on questions of this kind; as did also Bachet de Meziriac, who wrote a commentary on the above work of the Greek arithmetician. But this species of analysis was carried farther than ever it had been before by the celebrated M. de Fermat. Father de Billy, about the same time, gave proofs of the acuteness of his talents in this way, by his work entitled *Diophantus Redivivus*, in which he far excelled the ancient analyst. M. Ozanam likewise shewed great ability in this species of analysis, by the resolution of several problems which had been considered as insoluble. He wrote a work on this subject, but it was never published; and the manuscript, after his death, came into the hands of the late M. Daguesseau, as we are informed by the historian of the Academy of Sciences.

The Hindoos also were great adepts in such problems, as we learn from some algebraical works which have lately been found among them, an account of which may be seen in the second volume of Tracts by the late Dr. Charles Hutton.

CHAPTER VII.

OF ARITHMETICAL AND GEOMETRICAL PROGRESSIONS, AND OF CERTAIN PROBLEMS WHICH DEPEND ON THEM.

SECTION I.

Explanation of the most remarkable properties of an Arithmetical Progression.

If there be a series of numbers, either increasing or decreasing, in such a manner, that the difference between the first and the second shall be equal to that between the second and third, and between the third and fourth, and so on successively; these numbers will be in arithmetical progression.

The series of numbers 1, 2, 3, 4, 5, 6, &c.; or 1, 5, 9, 13, &c.; or 20, 18, 16, 14, 12, &c.; or 15, 12, 9, 6, 3, are therefore arithmetical progressions; for in the first, the difference between each term and the following one, which exceeds it, is always 1; in the second it is 4: in like manner this difference is always 2 in the third series, which goes on decreasing; and in the fourth it is 3.

It may be readily seen, that an increasing arithmetical progression may be continued ad infinitum; but this cannot be the case, in one sense, with a decreasing series; for we must always arrive at some term, from which if the common difference be taken, the remainder will be 0, or else a negative quantity. Thus, the progression 19, 15, 11, 7, 3, cannot be carried farther, at least in positive numbers; for it is impossible to take 4 from 3, or if it be taken we shall have, according to analytical expression, -1;* and by continuing the subtraction we should have -5, -9, &c.

The chief properties of arithmetical progressions may be easily deduced from the definitions which we have here given. For a little attention will shew,

1st. That each term is nothing else than the first, plus or minus the common difference multiplied by the number of intervals between that term and the first. Thus, in the progression 2, 5, 8, 11, 14, 17, &c., the difference of which is 3, there are five intervals between the sixth term and the first; and for this reason the sixth term was equal to the first plus 15, the product of the common difference 3 by 5. But as the number of intervals is always less by unity than the number of terms, it thence follows, that we may find any term, the place of which in the series is known, if we multiply the common difference by the number expressing that place less unity. According to this rule, the hundredth term of an increasing progression will be equal to the first plus 99 times the common difference. If it be decreasing, it will be equal to the first term minus that product.

In every arithmetical progression therefore, the common difference being given, to find any term the place of which is known; multiply the common difference by the number which indicates that place less unity, and add the product to the first term, if the progression be increasing, but subtract it if it be decreasing: the sum or remainder will be the term required.

2nd. In every arithmetical progression, the sum of the first and last terms, is equal to that of the second and the last but one; and to that of the third and the last but two, &c.; in a word, to the sum of the middle terms if the number of the terms be even, or to the double of the middle term if the number of the terms be odd.

This may easily be demonstrated from what has been said: for let us call the first term A, and let us suppose that there are twenty terms in the progression; if it be increasing, the twentieth term will be equal to A plus nineteen times the common

* As the quantities called negative are real quantities, taken in a sense contrary to that of the quantities called positive, it is evident that, according to mathematical and analytical strictness, an arithmetical progression may be continued ad infinitum, decreasing as well as increasing; but we here speak agreeably to the vulgar mode of expression.

difference; and their sum will be double the first term plus nineteen times that difference. But the second term is equal to the first plus the common difference, and the nineteenth term, or last but one, according to our supposition, is equal to the first plus eighteen times that difference. The sum therefore of the second and last but one, is twice the first term plus nineteen times the common difference, the same as before. And so of the third and last but two.

3rd. By this last property we are enabled to shew in what manner the sum of all the terms of an arithmetical progression may be readily found; for, as the first and last terms make the same sum as the second and last but one, and as the third and the last but two, &c.; in short as the two middle terms, if the number of terms be even; it thence follows, that the whole progression contains as many times the sum of the first and the last terms, as there are pairs of such terms. But the number of pairs is equal to half the number of terms; consequently the sum of the whole progression is equal to the product of the sum of the first and last terms multiplied by half the number of terms, or, what amounts to the same, to half the product of the sum of the first and the last terms by the number of the terms of the progression.

If the number of the terms be odd, as 9 for example; it may be readily seen that the middle term will be equal to half the sum of the two next to it, and consequently of the sum of the first and the last. But the sum of all the terms, the middle term excepted, is equal to the product of the sum of the first and last terms by the number of terms less unity, for example 8 in the case here proposed, where there are 9 terms; consequently, by adding the middle term, which will complete the sum of the progression, and which is equal to half the sum of the first and the last terms, we shall have, for the sum total of the progression, as many times the half sum above-mentioned, as there are terms in the progression; which is the same thing as the product of half the sum of the first and last terms by the number of the terms, or the product of the whole sum by half the number of terms.

When these rules are well understood, it will be easy to resolve the following questions.

PROBLEM I.

If a hundred stones are placed in a straight line, at the distance of a yard from each other; how many yards must the person walk, who undertakes to pick them up one by one, and to put them into a basket a yard distant from the first stone?

It is evident, that to pick up the first stone, and put it into the basket, the person must walk 2 yards, one in going and another in returning; that for the second he must walk 4 yards; and so on, increasing by two as far as the hundredth, which will oblige him to walk two hundred yards, one hundred in going, and one hundred in returning. It may easily be perceived also, that these numbers form an arithmetical progression, in which the number of terms is 100, the first term 2, and the last 200. The sum total therefore will be the product of 202 by 50, or 10100 yards, which amount to more than five miles and a half.

PROBLEM II.

A gentleman employed a bricklayer to sink a well, and agreed to give him at the rate of three shillings for the first yard in depth, five for the second, seven for the third, and so on increasing till the twentieth, where he expected to find water: how much was due to the bricklayer when he had completed the work.

This question may be easily answered by the rules already given; for the difference of the terms is 2, and the number of terms 20; consequently, to find the twentieth term, we must multiply 2 by 19, and add 38, the product, to the first term 3, which will give 41 for the twentieth term.

If we then add the first and last terms, that is 3 and 41, which will make 44, and multiply this sum by 10, or half the number of terms, the product 440 will be the sum of all the terms of the progression, or the number of shillings due to the bricklayer when he had completed the work. He would therefore have to receive £22.

PROBLEM III.

A gentleman employed a bricklayer to sink a well to the depth of 20 yards, and agreed to give him £20 for the whole; but the bricklayer falling sick, when he had finished the eighth yard, was unable to go on with the work: how much was then due to him?

Those who might imagine that two fifths of the whole sum were due to the workman, because 8 yards are two fifths of the depth agreed on, would certainly be mistaken; for it may be easily seen that, in cases of this kind, the labour increases in proportion to the depth. We shall here suppose, for it would be difficult to determine it with any accuracy, that the labour increases arithmetically as the depth; consequently the price ought to increase in the same manner.

To determine this problem, therefore, £20 or 400 shillings must be divided into 20 terms in arithmetical progression, and the sum of the first eight of these will be what was due to the bricklayer for his labour.

But 400 shillings may be divided into twenty terms, in arithmetical proportion, a great many different ways, according to the value of the first term, which is here undetermined: if we suppose it, for example, to be 1 shilling, the progression will be 1, 3, 5, 7, &c., the last term of which will be 39; and consequently the sum of the first eight terms will be 64 shillings. On the other hand, if we suppose the first term to be $10\frac{1}{2}$, the series of terms will be $10\frac{1}{2}$, $11\frac{1}{2}$, $12\frac{1}{2}$, $13\frac{1}{2}$, $14\frac{1}{2}$, which will give 112 shillings for the sum of the first eight terms.

But to resolve the problem in a proper manner, so as to give to the bricklayer his just due for the commencement of the work, we must determine what is the fair value of a yard of work, similar to the first, and then assume that value as the first term of the progression. We shall here suppose that this value is 5 shillings; and in that case the required progression will be 5, $6\frac{11}{19}$, $8\frac{3}{19}$, $9\frac{14}{19}$, $11\frac{6}{19}$, $12\frac{17}{19}$, &c., the common difference of which is $\frac{30}{19}$, and the last term 35. Now to find the eighth term, which is necessary before we can find the sum of the first eight terms, multiply the common difference $\frac{30}{19}$ by 7, which will give $11\frac{1}{19}$, and add this product to 5 the first term, which will give the eighth term $16\frac{1}{19}$; if we then add $16\frac{1}{19}$ to the first term, and multiply the sum, $21\frac{1}{19}$, by 4, the product, $84\frac{4}{19}$, will be the sum of the first eight terms, or what was due to the bricklayer, for the part of the work he had completed. The bricklayer therefore had to receive $84\frac{4}{19}$ shillings, or £4. 4*s*. 2*d*.

PROBLEM IV.

A merchant being considerably in debt, one of his creditors, to whom he owed £1860, offered to give him an acquittance if he would agree to pay the whole sum in 12 monthly instalments; that is to say, £100 the first month, and to increase the payment by a certain sum each succeeding month, to the twelfth inclusive, when the whole debt would be discharged: by what sum was the payment of each month increased?

In this problem the payments to be made each month ought to increase in arithmetical progression. We have given the sum of the terms, which is equal to the sum total of the debt, and also the number of these terms, which is 12; but their common difference is unknown, because it is that by which the payments ought to increase each month.

To find this difference, we must take the first payment multiplied by the number of terms, that is to say 1200 pounds, from the sum total, and the remainder will

be 660; we must then multiply the number of terms less unity, or 11, by half the number of terms, or 6, and we shall have 66; by which, if the remainder 660 be divided, the quotient 10 will be the difference required. The first payment, therefore, being 100, the second payment must have been 110, the third 120, and the last 210.

SECTION II.

Of Geometrical Progressions, with an explanation of their Principal Properties.

If there be a series of terms, each of which is the product of the preceding by a common multiplier; or what amounts to the same thing, each of which is in the same ratio to the preceding; such a series forms what is called a geometrical progression. Thus 1, 2, 4, 8, 16, &c., form a geometrical progression: for the second is the double of the first, the third the double of the second, and so on in succession. The terms 1, 3, 9, 27, 81, &c. form also a geometrical progression, each term being the triple of that which precedes it.

I. The principal property of geometrical progression is, that if we take any three following terms, as 3, 9, 27, the product, 81, of the extremes will be equal to the square of the middle term 9; in like manner, if we take four following terms, as 3, 9, 27, 81, the product of the extremes, 243, will be equal to the product of the two means or middle terms, 9 and 27.

In the last place, if we take any successive number of terms, as 2, 4, 8, 16, 32, 64, the product of the extremes, 2 and 64, will be equal to the product of any two which are equally distant from them, viz. 4 and 32, or 8 and 16. If the number of the terms were odd, it is evident that there would be only one term equally distant from the two extremes; and in that case, the square of this term would be equal to the product of the extremes, or to that of any two equally distant from them, or from the mean term.

II. Between geometrical and arithmetical progression there is a certain analogy, which deserves here to be mentioned, and which is, that the same results are obtained in the former by employing multiplication and division, as are obtained in the latter by addition and subtraction. When in the latter we take the half or the third, we employ in the former extraction of the square, cube, &c. roots.

Thus, to find an arithmetical mean between any two numbers, for example 3 and 12, we add the two given extremes, and $7\frac{1}{2}$, the half of their sum 15, will be the number required; but to find a geometrical mean between two numbers, we must multiply the two extremes, and extract the square root of their product. Thus, if the given numbers were 3 and 12, by extracting the square root of their product 36, we shall have 6 for the number required.

If we take any geometrical progression whatever, as 1, 2, 4, 8, 16, 32, 64, &c., and write it down as in the subjoined example, with the terms of an arithmetical progression above it, in regular order,

0	1	2	3	4	5	6	7	8	9	10
1	2	4	8	16	32	64	128	256	512	1024

the following properties will be remarked in this combination:

1st. If any two terms whatever of the geometrical progression, for example 4 and 64, be multiplied together, their product will be 256; if we then take the two terms of the arithmetical progression corresponding to 4 and 64, which are 2 and 6, and add them together, their sum 8 will be found over the above sum 256.

2d. If we take four terms of the lower series in geometrical proportion, for example, 2, 16, 64, 512, the numbers of the upper series corresponding to them will be 1, 4, 6, 9, which are in arithmetical proportion; for the difference between 4 and 1 is the same as that between 9 and 6.

3d. In the lower series, if we take any square number, for example 64, and in the

upper series the term corresponding to it, viz. 6, the half of the latter will be found to correspond to the square root of 64, the former, viz. 8.

By taking, in the lower series, a cube, for example 512, and in the upper series the corresponding number 9, it will be found that the third of the latter, which is 3, will correspond to the cube root of the former 512, which is 8.

Thus, it is evident, that what is multiplication in geometrical progression, is addition in arithmetical; that what is division in the former, is subtraction in the latter; and, in the last place, that which is extraction of the square, cube, &c. roots, in geometrical progression, is simple division by 2, 3, &c. in arithmetical.

This remarkable analogy is the foundation of the common theory of logarithms; and on that account seemed worthy of some illustration.

III. It is evident that all the powers of the same number, taken in regular order, form a geometrical progression; as may be seen in the following example, which is a series of the powers of the number 2,

2 4 8 16 32 64 128, &c.

The case is the same with the powers of the number 3, which form the series,

3 9 27 81 243 729, &c.

The first of these series has this peculiar property, that if we take the first, second, fourth, eighth, sixteenth, and thirty-second terms, and to them add unity, the result will be prime numbers.

IV. The common ratio of a geometrical progression, is the number that results from the division of any term by that which precedes it. Thus, in the geometrical progression 2, 8, 32, 128, 512, the ratio is 4; for if we divide 128 by 32, 32 by 8, or 8 by 2, the quotient will be always 4. The ratio therefore acts an important part in geometrical progression; the same that the common difference does in arithmetical, that is to say, it is always constant.

To find any term then, for example the 8th, of a geometrical progression, the ratio and first term of which are known, multiply the ratio by itself 7 times, or as many times as there are units in the place of the required term less one; or, what is the same thing, raise the ratio to the 7th power; then multiply the first term by the product, and the new product will be the eighth term required. For example, let the first term of the progression be 3, and the ratio 2; to find the 8th term, raise 2 to the 7th power, which will be 128, and multiply 128 by the first term 3, the product 384 will give the 8th term of the progression required.

We shall here observe, that had the 8th term of an arithmetical progression been required, the first term and the common difference being given, we should have multiplied that difference by 7, and added the product to the first term; which is a proof of the analogy already mentioned in the second paragraph.

V. The sum of the terms of any given geometrical progression may be found in the following manner:

Multiply the first term by itself, and the last by the second, and take the difference of the two products. Then divide this difference by that of the first two terms, and the quotient will be the sum of all the terms.

Let us take, for example, the progression 3, 6, 12, 24, &c., the eighth term of which is 384, and let it be required to find the sum of these eight terms: the product of the first term by itself is 9, and that of the last by the second is 2304; the difference of these products is 2295; if this difference then be divided by 3, the difference of the first and second terms, we shall have for quotient the number 765, which will be the sum of these eight terms.

VI. A geometrical progression may decrease *in infinitum*, without ever reaching 0; for it is evident that any part of the quantity greater than 0 can never become 0. A decreasing geometrical progression therefore may be extended without end; for by

dividing the last term by the ratio of the progression, we shall have the following term.

We shall here subjoin two of these decreasing progressions, by way of examples :—

$$1,\ \tfrac{1}{2},\ \tfrac{1}{4},\ \tfrac{1}{8},\ \tfrac{1}{16},\ \tfrac{1}{32},\ \tfrac{1}{64},\ \&c.$$

$$1,\ \tfrac{1}{3},\ \tfrac{1}{9},\ \tfrac{1}{27},\ \tfrac{1}{81},\ \tfrac{1}{243},\ \tfrac{1}{729},\ \&c.$$

VII. The sum of an increasing geometrical progression is evidently infinite; but that of a decreasing geometrical progression, whatever be the number of terms assumed, is always finite. Thus the sum of all the terms, *in infinitum*, of the progression 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c., is only 2; that of the progression 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c. *in infinitum*, is only $1\frac{1}{2}$; &c. This necessarily follows from the method already given, for finding the sum of any number of terms whatever of a geometrical progression; for if we suppose it prolongued *in infinitum*, and decreasing, the last term will be infinitely small, or 0; the product of the second term by the last will therefore be 0; and consequently, to find the sum, nothing will be necessary but to divide the square of the first term by the difference of the first and the second. In this manner it will be found that the sum of 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. continued *in infinitum*, is 2; and that of 1, $\frac{1}{3}$, $\frac{1}{9}$, &c. will be $\frac{3}{2}$ or $1\frac{1}{2}$; for the square of 1 is 1, the difference of $1\frac{1}{2}$ and is $\frac{1}{2}$, and unity divided by $\frac{1}{2}$ gives 2; in like manner, 1 being divided by $\frac{2}{3}$, which is the difference of 1 and $\frac{1}{3}$, gives $\frac{3}{2}$.

Remark.—When we say that a progression continued *in infinitum* may be equal to a finite quantity, we do not, like Fontenelle, pretend to assert that infinity can have a real existence. What is here meant, and what ought to be understood by all such expressions, is that, whatever be the number of terms of a progression assumed, their sum never can equal the determined finite quantity, though it may approach to it in such a manner, that their difference will become smaller than any assignable quantity.

PROBLEM I.

If Achilles can walk ten times as fast as a tortoise, which is a furlong before him, can crawl; will the former overtake the latter, and how far must he walk before he does so?

This problem has been thought worthy of notice merely because Zeno, the founder of the sect of the Stoics, pretended to prove by a sophism that Achilles could never overtake the tortoise; for while Achilles, said he, is walking a furlong, the tortoise will have advanced the tenth of a furlong; and while the former is walking that tenth, the tortoise will have advanced the hundredth part of a furlong, and so on *in infinitum;* consequently an infinite number of instants must elapse before the hero can come up with the reptile, and therefore he will never come up with it.

Any person however, possessed of common sense, may readily perceive that Achilles will soon come up with the tortoise, since he will get before it. In what then consists the sophism? It may be explained as follows:

Achilles indeed would never overtake the tortoise, if the intervals of time during which he is supposed to be walking the first furlong, and then the tenth, hundredth, and thousandth parts of a furlong, which the tortoise has successively advanced before him, were equal; but if we suppose that he has walked the first furlong in 10 minutes, he will require only one minute to walk the tenth of a furlong, and $\frac{1}{10}$ of a minute to walk the hundredth, &c. The intervals of time therefore, which Achilles will require to pass over the space gained by the tortoise, during the preceding time, will go on decreasing in the following manner: 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c.; and this series forms a sub-decuple geometrical progression, the sum of which is equal to $11\frac{1}{9}$, or the interval of time at the end of which Achilles will have reached the tortoise.

PROBLEM II.

If the hour and minute hands of a clock both begin to move exactly at noon, at what points of the dial-plate will they be successively in conjunction, during a whole revolution of the twelve hours?

This problem, considered in a certain manner, is in nothing different from the preceding. The minute hand acts here the part which Achilles did in the former, and the hour hand, which moves twelve times slower, that of the tortoise. In the last place, if we suppose the hour hand to be beginning a second revolution, and the minute hand to be beginning a first, the distance which the one has gained over the other will be a whole revolution of the dial-plate. When the minute hand has made one revolution, the hour hand will have made but one twelfth of a revolution, and so on progressively. To resolve the problem therefore, we need only apply to these data, the method employed in the former case, and we shall find that the interval from noon to the point where the two hands come again into conjuction, will be $\frac{1}{11}$ of a whole revolution, or, what amounts to the same thing, one hour and $\frac{1}{11}$ of an hour. They will afterwards be in conjunction at 2 hours and $\frac{2}{11}$, 3 hours and $\frac{3}{11}$, 4 hours and $\frac{4}{11}$, &c.; and, lastly, at 11 hours $\frac{11}{11}$, that is to say at 12 hours.

PROBLEM III.

A courtier having performed some very important service to his sovereign, the latter, wishing to confer on him a suitable reward, desired him to ask whatever he thought proper, promising that it should be granted. The courtier, who was well acquainted with the science of numbers, only requested that the monarch would give him a quantity of wheat equal to that which would arise from one grain doubled sixty-three times successively. What was the value of the reward?

The origin of this problem is related in so curious a manner by Al-Sephadi, an Arabian author, that it deserves to be mentioned. A mathematician named Sessa, says he, the son of Daher, the subject of an Indian prince, having invented the game of chess, his sovereign was highly pleased with the invention, and wishing to confer on him some reward worthy of his magnificence, desired him to ask whatever he thought proper, assuring him that it should be granted. The mathematician however only asked a grain of wheat for the first square of the chess-board, two for the second, four for the third, and so on to the last or sixty-fourth. The prince at first was almost incensed at this demand, conceiving that it was ill-suited to his liberality, and ordered his vizier to comply with Sessa's request; but the minister was much astonished when, having caused the quantity of corn necessary to fulfil the prince's order to be calculated, he found that all the grain in the royal granaries, and that even of all his subjects, and in all Asia, would not be sufficient. He therefore informed the prince, who sent for the mathematician and candidly acknowledged that he was not rich enough to be able to comply with his demand, the ingenuity of which astonished him still more than the game he had invented.

Such is then the origin of the game of chess, at least according to the Arabian historian Al-Sephadi. But it is not our business here to discuss the truth of this story; our business being to calculate the number of grains demanded by the mathematician Sessa.

It will be found by calculation, that the 64th term of the double progression, beginning with unity, is 9223372036854775808. But the sum of all the terms of a double progression, beginning with unity, may be obtained by doubling the last term and subtracting from it unity. The number therefore of the grains of wheat equal to Sessa's demand, will be 18446744073709551615. Now if a standard pint contains

9216 grains of wheat, a gallon will contain 73728, and, as eight gallons make one bushel, if we divide the above result by eight times 73728, we shall have 31274997412295 for the number of the bushels of wheat necessary to discharge the promise of the Indian king; and if we suppose that one acre of land is capable of producing, in one year, thirty bushels of wheat, to produce this quantity would require 1042499913743 acres, which make more than eight times the surface of the globe; for the diameter of the earth being supposed equal to 7930 miles, its whole surface, comprehending land and water, will amount to very little more than 126437889177 square acres.

Dr. Wallis considers the matter in a manner somewhat different, and says, in his Arithmetic, that the quantity of wheat necessary to discharge the promise made to Sessa, would form a pyramid nine miles English in length, breadth, and height; which is equal to a parallelopiped mass having nine square leagues for its base, and of the uniform height of one league. But as one league contains 15840 feet, this solid would be equivalent to another one foot in height, and having a base equal to 142560 square leagues. Hence it follows, that the above quantity of wheat would cover, to the height of one foot, 142560 square leagues; an extent of surface equal to eleven times that of Britain, which, when every reduction is made, will be found to contain little more than 12674 square leagues.

If the price of a bushel of wheat be estimated at ten shillings, the value of the above quantity will amount to £15637498706147. 10*s*., a sum which, in all probability, far surpasses all the riches on the earth.

Another problem of the same kind is proposed in the following manner:

A gentleman taking a fancy to a horse, which a horse-dealer wished to dispose of at as high a price as he could, the latter, to induce the gentleman to become a purchaser, offered to let him have the horse for the value of the twenty-fourth nail in his shoes, reckoning one farthing for the first nail, two for the second, four for the third, and so on to the twenty-fourth. The gentleman thinking he should have a good bargain accepted the offer. What was the price of the horse?

By calculating as before, the twenty-fourth term of the progression 1, 2, 4, 8, &c., will be found to be 8388608, equal to the number of farthings the purchaser ought to give for the horse. The price therefore amounted to £8738. 2*s*. 8*d*., which is more than any Arabian horse, even of the noblest breed, was ever sold for.

Had the price of the horse been the value of all the nails, at a farthing for the first, two for the second, four for the third, and so on, the sum would have been double the above number, minus the first term, or 16777215 farthings, that is £17476. 5*s*. 3¾*d*.

We shall conclude this chapter with some physico-mathematical observations on the prodigious fecundity, and the progressive multiplication, of animals and vegetables, which would take place if the powers of nature were not continually meeting with obstacles.

I. It is not astonishing that the race of Abraham, after sojourning 260 years in Egypt, should have formed a nation capable of giving uneasiness to the sovereigns of that country. We are told in the sacred writings, that Jacob settled in Egypt with 70 persons; now if we suppose that among these 70 persons there were 20 too far advanced in life, or too young, to have children; that of the remaining 50, 25 were males and as many females, forming 25 married couples, and that each couple, in the space of 25 years, produced, one with another, 8 children, which will not appear incredible in a country celebrated for the fecundity of its inhabitants, we shall find that, at the end of 25 years, the above 70 persons may have increased to 270; from which if we deduct those who died, there will perhaps be no exaggeration in making them amount to 210. The race of Jacob therefore, after sojourning 25 years in Egypt, may have been tripled. In like manner, these 210 persons, after 25 years

more, may have increased to 630; and so on in triple geometrical progression: hence it follows that, at the end of 225 years, the population may have amounted to 1377810 persons, among whom there might easily be five or six hundred thousand adults fit to bear arms.

II. If we suppose that the race of the first man, making a proper deduction for those who died, may have been doubled every twenty years, which certainly is not inconsistent with the powers of nature, the number of men, at the end of five centuries, may have amounted to 1048576. Now, as Adam lived about 900 years, he may have seen therefore, when in the prime of life, that is to say about the five hundredth year of his age, a posterity of 1048576 persons.

III. How great would be the multiplication of many animals, did not the difficulty of finding food, the continual war which they carry on against each other, or the numbers of them consumed by man, set bounds to their propagation? It might easily be proved, that the breed of a sow, which brings forth six young, two males and four females, if we suppose that each female produces every year afterwards six young, four of them females and two males, would in twelve years amount to 33554230.

Several other animals, such as rabbits and cats, which go with young only for a few weeks, would multiply with still greater rapidity: in half a century the whole earth would not be sufficient to supply them with food, nor even to contain them!

If all the ova of a herring were fecundated, a very few years would be sufficient to make its posterity fill the whole ocean; for every oviparous fish contains thousands of ova which it deposits in spawning time. Let us suppose that the number of ova amounts only to 2000, and that these produce as many fish, half males and half females; in the second year there would be more than 200000; in the third, more than 200000000; and in the eighth year the number would exceed that expressed by 2 followed by twenty-four ciphers. As the earth contains scarcely so many cubic inches, the ocean, if it occupied the whole globe, would not be sufficient to contain all these fish, the produce of one herring in eight years!

IV. Many vegetable productions, if all their seeds were put into the earth, would in a few years cover the whole surface of the globe. The hyosciamus, which of all the known plants produces perhaps the greatest number of seeds, would for this purpose require no more than four years. According to some experiments, it has been found that one stem of the hyosciamus produces sometimes more than 50000 seeds: now if we admit the number to be only 10000, at the fourth crop it would amount to a 1 followed by sixteen ciphers. But as the whole surface of the earth contains no more than 5507634452576256 square feet; if we allow to each plant only one square foot, it will be seen that the whole surface of the earth would not be sufficient for the plants produced from one hysociamus at the end of the fourth year!

SECTION III.

Of some other Progressions, and particularly Harmonical Progression.

Three numbers are in harmonical proportion, when the first is to the last, as the difference between the first and the second is to that between the second and the third. Thus, the numbers 6, 3, 2, are in harmonical proportion; for 6 is to 2, as 3, the difference between the two first numbers, is to 1, the difference between the two last. This kind of relation is called harmonical, for a reason which will be seen hereafter.

I. Two numbers being given, a third which shall form with them harmonical proportion may be found, by multiplying these two numbers, and dividing their product by the excess of the double of the first over the second. Thus, if 6 and 3 be given, we must multiply 6 by 3, and divide the product 18 by 9, which is the excess of 12 the double of 6 over 3, the second of the numbers given. In this case the quotient will be 2.

It may hence be readily seen that, in one sense, it is not always possible to find a third number in harmonical proportion with two others; for when the first is less, if its double be equal to or less than the second, the result will be an infinite or a negative number. Thus, the third harmonic proportional to 2 and 4 is infinite; for it will be found that the number sought is equal to 8 divided by $4-4$, or 0. But every person, in the least acquainted with arithmetic, knows that the more the denominator of a fraction is inferior to unity, the greater the fraction; consequently, a fraction which has 0 for its denominator is infinite.

If the double of the first number be less than the second, as would be the case were it proposed to find a third harmonical to 2 and 6, the required divisor will be a negative number. Thus, in the proposed example of 2 and 6, it will be -2; and therefore the third harmonical required will be 12 divided by -2, that is -6.*

But this inconvenience, if it be one, is not to be apprehended when the greater number is the first term of the proportion; for if the first exceeds the second, much more will its double exceed it. In this case therefore, the third harmonical will always be a finite and positive number.

II. When three numbers, in decreasing harmonical proportion, are given, for example 6, 3, 2, it is easy to find a fourth: nothing is necessary but to find a third harmonical to the two last, and this will be the fourth. The third and fourth may, in like manner, be employed to find a fifth, and so on; and this will form what is called an harmonical progression, which may be always continued decreasing. In the present example, this series will be found to be

$$6,\ 3,\ 2,\ \tfrac{6}{4},\ \tfrac{6}{5},\ \tfrac{6}{6},\ \tfrac{6}{7},\ \tfrac{6}{8},\ \&c.$$

$$\text{or } 6,\ 3,\ 2,\ \tfrac{3}{2},\ \tfrac{6}{5},\ 1,\ \tfrac{6}{7},\ \tfrac{3}{4},\ \&c.$$

Had the two first numbers been 2 and 1, we should have had the harmonical progression 2, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, &c.

It is a remarkable property, therefore, of the series of fractions, having for their numerators unity, and for their denominators the numbers of the natural progression, that they are in harmonical progression.

Besides the numerical relation already mentioned, we find indeed, in the series of these numbers, all the musical concords possible; for the ratio of 1 to $\frac{1}{2}$ gives the octave; that of $\frac{1}{2}$ to $\frac{1}{3}$, or of 3 to 2, the fifth; that of $\frac{1}{3}$ to $\frac{1}{4}$, or of 4 to 3, the fourth; that of $\frac{1}{4}$ to $\frac{1}{5}$, or of 5 to 4, the third major; that of $\frac{1}{5}$ to $\frac{1}{6}$, or of 6 to 5, the third minor; that of $\frac{1}{8}$ to $\frac{1}{9}$, or of 9 to 8, the tone major, and that of $\frac{1}{9}$ to $\frac{1}{10}$, or of 10 to 9, the tone minor. But this will be explained at more length in that part of this work which treats of music.

PROBLEM.

What is the Sum of the Infinite Series of Numbers in Harmonical Progression, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, *&c.?*

It has been already seen, that a series of numbers in geometrical progression, if continued *in infinitum*, will always be equal to a finite number, which may easily be determined. But is the case the same in the present problem?

We will venture to reply in the negative, though an author, in the *Journal de Trevoux*, has bestowed great labour in endeavouring to prove that the sum of these fractions is finite. But his reasoning consists of mere paralogisms, which he would not have employed had he been more of a geometrician; for it can be demonstrated that the series 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., may be always continued in such a manner as to exceed any finite number whatever.

* See what has been already said in regard to negative quantities, in the article on arithmetical progression.

SECTION IV.

Of various Progressions decreasing in infinitum, the Sums of which are known.

I. A variety of decreasing progressions, which have served to exercise the ingenuity of mathematicians, may be formed according to different laws. Thus, for example, the numerator being constantly unity, the denominators may increase in the ratio of the triangular numbers 1, 3, 6, 10, 15, 21, &c. Of this kind is the following progression:

$$\tfrac{1}{1},\ \tfrac{1}{3},\ \tfrac{1}{6},\ \tfrac{1}{10},\ \tfrac{1}{15},\ \tfrac{1}{21},\ \text{\&c.}$$

Its sum is finite, and exactly equal to 2, or $1\tfrac{1}{1}$.

In like manner, the sum of a progression having unity constantly for its numerators, and the pyramidal numbers for its denominators, as,

$$1,\ \tfrac{1}{4},\ \tfrac{1}{10},\ \tfrac{1}{20},\ \tfrac{1}{35},\ \tfrac{1}{56},\ \text{\&c.}$$

is equal to $1\tfrac{1}{2}$.

That where the denominators are the pyramidals of the second order, as

$$1,\ \tfrac{1}{5},\ \tfrac{1}{15},\ \tfrac{1}{35},\ \tfrac{1}{70},\ \tfrac{1}{126},\ \text{\&c.},$$

is equal to $1\tfrac{1}{3}$.

That where they are the pyramidals of the third order, as

$$1,\ \tfrac{1}{6},\ \tfrac{1}{21},\ \tfrac{1}{56},\ \tfrac{1}{126},\ \tfrac{1}{252},\ \text{\&c.},$$

is equal to $1\tfrac{1}{4}$.

The law therefore which these sums follow, is evident; and if the sum of a similar progression, that, for example, where the denominators are the pyramidals of the tenth order, were required, we might easily reply that it is equal to $1\tfrac{1}{11}$.

II. Let us now assume the following progression,

$$1,\ \tfrac{1}{4},\ \tfrac{1}{9},\ \tfrac{1}{16},\ \tfrac{1}{25},\ \tfrac{1}{36},\ \text{\&c.},$$

in which the denominators are the squares of the numbers of the natural progression.

If the reader is desirous to know its sum, we shall observe, with Mr. John Bernoulli, by whom it was first found, that it is finite, and equal to the square of the circumference of the circle divided by 6, or $\tfrac{1}{6}$ of $3{\cdot}14159^2$.

As to that in which the denominators are the cubes of the natural numbers, Mr. Bernoulli acknowledges that he had not been able to discover it.

Those who are fond of researches of this kind, may consult a work of James Bernoulli, entitled *Tractatus de Seriebus Infinitis*, which is at the end of another published at Bâle in 1713, under the title of *Ars Conjectandi*, where they will find ample satisfaction. They may also consult various other memoirs both of John Bernoulli, to be found in the collection of his works, and of Euler, published in the Transactions of the Imperial Academy of Sciences at Petersburgh.

CHAPTER VIII.

OF COMBINATIONS AND PERMUTATIONS.

Before we enter on the present subject, it will be necessary to explain the method of constructing a sort of table, invented by Pascal,* called the Arithmetical triangle, which is of great utility for shortening calculations of this kind.

* This is a mistake in Montucla, as the triangle was invented some ages before Pascal: see Dr. Hutton's Tracts, 4to, p 69.

First, form a band A B of ten equal squares, and below it another C D of the like kind, but shorter by one square on the left, so that it shall contain only nine squares; and continue in this manner,

A	1	1	1	1	1	1	1	1	1	1	B
	C	1	2	3	4	5	6	7	8	9	D
			1	3	6	10	15	21	28	36	
				1	4	10	20	35	56	84	
					1	5	15	35	70	126	
						1	6	21	56	126	
							1	7	28	84	
								1	8	36	
									1	9	
										1	
										E	

always making each successive band a square shorter. We shall thus have a series of squares disposed in vertical and horizontal bands, and terminating at each end in a single square so as to form a triangle, on which account this table has been called the arithmetical triangle. The numbers with which it is to be filled up, must be disposed in the following manner.

In each of the squares of the first band A B, inscribe unity, as well as in each of those on the diagonal A E.

Then add the number in the first square of the band C D, which is unity, to that in the square immediately above it, and write down the sum 2, in the following square. Add this number, in the like manner, to that in the square above it, which will give 3, and write it down in the next square. By these means we shall have the series of the natural numbers, 1, 2, 3, 4, 5, &c. The same method must be followed to fill up other horizontal bands; that is to say, each square ought always to contain the sum of the number in the preceding square of the same row, and that which is immediately above it. Thus, the number 15, which occupies the fifth square of the third band, is equal to the sum of ten which stands in the preceding square, and of 5 which is in the square above it. The case is the same with 21, which is the sum of 15 and 6; with 35 in the fourth band, which is the sum of 15 and 20; &c.

The first property of this table is, that it contains, in its horizontal bands, the natural, triangular, pyramidal, &c., numbers; for in the second, we have the natural numbers 1, 2, 3, 4, &c.; in the third, the triangular numbers 1, 3, 6, 10, 15, &c.; in the fourth, the pyramidals of the first order 1, 4, 10, 20, 35, &c.; in the fifth, the pyramidals of the second order, 1, 5, 15, 35, 70, &c. This is a necessary consequence of the manner in which the table is formed; for it may be readily perceived that the number in each square is always the sum of those which fill the preceding squares on the left, in the band immediately above.

The same numbers will be found in the bands parallel to the diagonal, or the hypothenuse of the triangle.

But a property still more remarkable, which can be comprehended only by such of our readers as are acquainted with algebra, is, that the perpendicular bands exhibit the co-efficients belonging to the different members of any power to which a binomial, as $a+b$, can be raised. The third band contains those of the three members of the square; the fourth those of the four members of the cube; the fifth, those of the five members of the biquadrate. But, without enlarging farther on this subject, we shall proceed to explain what is meant by combinations.

By combinations are understood the various ways that different things, the number

of which is known, can be chosen or selected, taking them one by one, two by two, three by three, &c., without regard to their order. Thus, for example, if it were required to know in how many different ways the four letters *a*, *b*, *c*, *d*, could be arranged, taking them two and two, it may be readily seen that we can form with them the following combinations *ab*, *ac*, *ad*, *bc*, *bd*, *cd*: four things, therefore, may be combined, two and two, six different ways. Three of these letters may be combined four ways, *abc*, *abd*, *acd*, *bcd*; hence the combinations of four things, taken three and three, are only four.

In combinations, properly so called, no attention is paid to the order of the things; and for this reason we have made no mention of the following combinations, *ba*, *ca*, *da*, *cb*, *db*, *dc*. If, for example, four tickets, marked *a*, *b*, *c*, *d*, were put into a hat, and any one should bet to draw out the tickets *a* and *d*, either by taking two at one time, or taking one after another, it would be of no importance whether *a* should be drawn first or last: the combinations *ad* or *da* ought therefore to be here considered only as one.

But if any one should bet to draw out *a* the first time, and *d* the second, the case would be very different; and it would be necessary to attend to the order in which these four letters may be taken and arranged together, two and two: it may be easily seen that the different ways are *ab*, *ba*, *ac*, *ca*, *ad*, *da*, *bc*, *cb*, *bd*, *db*, *cd*, *dc*. In like manner, these four letters might be combined and arranged, three and three, 24 ways, as *abc*, *acb*, *bac*, *bca*, *cab*, *cba*, *adb*, *abd*, *dba*, *dab*, *bad*, *bda*, *acd*, *adc*, *dac*, *dca*, *cad*, *cda*, *bcd*, *dbc*, *cdb*, *bdc*, *cbd*, *dcb*. This is what is called permutation and change of order.

PROBLEM I.

Any number of things whatever being given; to determine in how many ways they may be combined two and two, three and three, &c., without regard to order.

This problem may be easily solved by making use of the arithmetical triangle. Thus, for example, if there are eight things to be combined, three and three, we must take the ninth vertical band, or in all cases that band, the order of which is expressed by a number exceeding by unity the number of things to be combined; then the fourth horizontal band, or that the order of which is greater by unity than the number of the things to be taken together, and in the common square of both will be found the number of the combinations required, which in the present example will be 56.

But as an arithmetical triangle may not always be at hand, or as the number of things to be combined may be too great to be found in such a table, the following simple method may be employed.

The number of the things to be combined, and the manner in which they are to be taken, viz. two and two, or three and three, &c., being given:

1st. Form two arithmetical progressions, one in which the terms go on decreasing by unity, beginning with the given number of things to be combined; and the other consisting of the series of the natural numbers 1, 2, 3, 4, &c.

2d. Then take from each as many terms as there are things to be arranged together in the proposed combination.

3d. Multiply together the terms of the first progression, and do the same with those of the second.

4th. In the last place, divide the first product by the second, and the quotient will be the number of the combinations required.

I. *In how many ways can 90 things be combined, taking them two and two?*

According to the above rule we must multiply 90 by 89, and divide the

product 8010 by the product of 1 and 2, that is 2: the quotient 4005 will be the number of the combinations resulting from 90 things, taken two and two.

Should it be required, in how many ways the same things can be combined three and three, the problem may be answered with equal ease; for we have only to multiply together 90, 89, 88, and to divide the product 704880 by that of the three numbers 1, 2, 3; the quotient 117480 will be the number required.

In like manner, it will be found that 90 things may be combined by four and four, 2555190 ways; for if the product of 90, 89, 88, and 87, be divided by 24, the product of 1, 2, 3, 4, we shall have the above result.

In the last place, if it be required, what number of combinations the same 90 things, taken five and five, are susceptible of, it will be found, by following the rule, that the answer is 43949268.

II.—Were it asked, how many conjunctions the seven planets could form with each other, two and two, we might reply 21; for, according to the general rule, if we multiply 7 by 6, which will give 42, and divide that number by the product of 1 and 2, that is 2, the quotient will be 21.

If we wished to know the number of all the conjunctions possible of these seven planets, two and two, three and three, &c.; by finding separately the number of the conjunctions two and two, then those of three and three, &c., and adding them together, it will be seen that they amount to 120.

The same result might be obtained by adding the seven terms of the double geometrical progression 1, 2, 4, 8, 16, 32, 64, which will give 127. But from this number we must deduct 7, because when we speak of the conjunction of a planet, it is evident that two of them, at least, must be united; and the number 127 comprehends all the ways in which seven things can be taken one and one, two and two, three and three, &c. In the present example, therefore, we must deduct the number of the things taken one and one; for a single planet cannot form a conjunction.

PROBLEM II.

Any number of things being given; to find in how many ways they can be arranged.

This problem may be easily solved by following the method of induction; for

1st. One thing *a* can be arranged only in one way: in this case therefore the number of arrangements is $= 1$.

2nd. Two things may be arranged together two ways; for with the letters *a* and *b* we can form the arrangements *ab* and *ba*: the number of arrangements therefore is equal to 2, or the product of 1 and 2.

3rd. The arrangements of three things *a*, *b*, *c*, are in number six; for *ab* can form with *c*, the third, three different ones, *bac*, *bca*, *cba*, and there can be no more. Hence it is evident that the required number is equal to the preceding multiplied by 3, or to the product of 1, 2, 3.

4th. If we add a fourth thing, for instance *d*, it is evident that, as each of the preceding arrangements may be combined with this fourth thing four ways, the above number 6 must be multiplied by 4 to obtain that of the arrangements resulting from four things; that is to say, the number will be 24, or the product of 1, 2, 3, 4.

s to enlarge farther on this subject; for it may be easily seen that, e number of the things given, the number of the arrangements they of may be found by multiplying together as many terms of the natural gression as there are things proposed.

Remark.—1st. It may sometimes happen that, of the things proposed, one of them is repeated, as *a*, *a*, *b*, *c*. In this case, where two of the four things proposed are the same, it will be found that they are susceptible only of 12 arrangements instead of 24; and that five, where two are the same, can form only 60, instead of 120.

But if three of four things were the same, there would be only 4 combinations, instead of 24; and five things, if three of them were the same, would give only 20, instead of 120, or a sixth part. But as the arrangements of which two things are susceptible amount to 2, and as those which can be formed with three things are 6, we may thence deduce the following rule:

In any number of things, of which the different arrangements are required, if one of them be several times repeated, divide the number of arrangements, found according to the general rule, by that of the arrangements which would be given by the things repeated, if they were different, and the quotient will be the number required.

2nd. In the number of things, the different arrangements of which are required, if there are several of them which occur several times, one twice for example, and another three times; nothing will be necessary but to find the number of the arrangements according to the general rule, and then to divide it by the product of the numbers expressing the arrangements which each of the things repeated would be susceptible of, if instead of being the same, they were different. Thus, in the present case, as the things which occur twice would be susceptible of two arrangements if they were different; and as those which occur thrice would, under the like circumstances, give six; we must multiply 6 by 2, and the product 12 will be the number by which that found according to the general rule ought to be divided. Thus, for example, the five letters *a*, *a*, *b*, *b*, *b*, can be arranged only 10 different ways: for, if they were different, they would give 120 arrangements; but as one of them occurs twice, and another thrice, 120 must be divided by the product of 2 and 6, or 12, which will give 10.

By observing the precepts given for the solution of this problem, the following questions may be resolved.

I.—*A club of seven persons agreed to dine together every day successively, as long as they could sit down to table differently arranged. How many dinners would be necessary for that purpose?*

It may be easily found that the required number is 5040, which would require 13 years and more than 9 months.

II.—The different anagrams which can be formed with any word, may be found in like manner. Thus, for example, if it be required, how many different words can be formed with the four letters of the word AMOR, which will give all the possible anagrams of it, we shall find that they amount to 24, or the continued product of 1, 2, 3, 4. We shall here give them in their regular order.

AMOR	MORA	ORAM	RAMO
AMRO	MOAR	ORMA	RAOM
AOMR	MROA	OARM	RMAO
AORM	MRAO	OAMR	RMOA
ARMO	MAOR	OMRA	ROAM
AROM	MARO	OMAR	ROMA

Hence it appears that the Latin anagrams of the word *amor*, are in number seven, viz., *Roma*, *mora*, *maro*, *oram*, *ramo*, *armo*, *orma*. But in the proposed word, if one or more letters were repeated, it would be necessary to follow the precepts already

given. Thus, the word *Leopoldus*, where the letter *l* and the letter *o* both occur twice, is susceptible of only 90720 different arrangements, or anagrams, instead of 362880, which it would form, if none of the letters were repeated; for, according to the before-mentioned rule, we must divide this number by the product of 2 by 2, or 4, which will give 90720.

The word *studiosus*, where the *u* occurs twice, and the *s* thrice, is susceptible of only 30240 arrangements; for the arrangements of the 9 letters it contains, which are in number 362880, must be divided by the product of 2 and 6, or twelve, and the quotient will be 30240.

In this manner may be found the number of all the possible anagrams of any word whatever; but it must be observed that however few be the letters of which a word is composed, the number of the arrangements thence resulting will be so great as to require considerable labour to find them.

III.—*How many ways can the following verse be varied, without destroying the measure:*

"Tot tibi sunt dotes, Virgo, quot sidera cælo?"

This verse, the production of a devout Jesuit of Louvain, named Father Bauhuys, is celebrated on account of the great number of arrangements of which it is susceptible, without the laws of quantity being violated; and various mathematicians have exercised or amused themselves with finding out the number. Erycius Puteanus took the trouble to give an enumeration of them in forty-eight pages, making them amount to 1022, or the number of the stars comprehended in the catalogues of the ancient astronomers; and he very devoutly observes, that the arrangements of these words as much exceed the above number as the perfections of the Virgin exceed that of the stars.*

Father Prestet, in the first edition of his Elements of the Mathematics, says that this verse is susceptible of 2196 variations; but in the second edition he extends the number to 3276.

Dr. Wallis, in the edition of his Algebra, printed at Oxford, in 1693, makes them amount to 3096.

But none of them has exactly hit the truth, as has been remarked by James Bernoulli, in his *Ars Conjectandi.* This author says, that the different combinations of the above verse, leaving out the spondees, and admitting those which have no cæsura, amount exactly to 3312. The method by which the enumeration was made may be seen in the above work.

The same question has been proposed respecting the following verse of Thomas Lansius:

"Mars, mors, sors, lis, vis, styx, pus, nox, fex, mala crux, fraus."

It may be easily found, retaining the word *mala* in the antepenultima place, in order to preserve the measure, that this verse is susceptible of 399168000 different arrangements.

PROBLEM III.

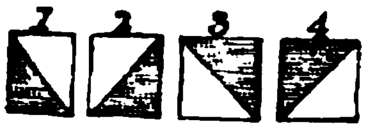
Of the combinations which may be formed with squares divided by a diagonal into two differently coloured triangles.

We are told by Father Sebastian Truchet, of the Royal Academy of Sciences, in a memoir printed among those of the year 1704, that having seen, during the course of a tour which he made to the canal of Orleans, some square porcelain tiles, divided

* See also Vossius de Scient. Math. cap. vii.

by a diagonal into two triangles of different colours, destined for paving a chapel and some apartments, he was induced to try in how many different ways they could be joined side by side, in order to form different figures. In the first place, it may be readily seen that a single square, (Fig. 9.) according to its position can form four different figures, which however may be reduced to two, as there is no difference between the first and the third, or between the second and fourth, than what arises from the transposition of the shaded triangle into the place of the white one.

Fig. 9.

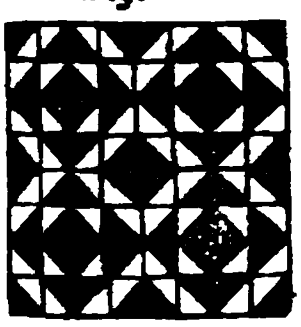


Now, if two of these squares be combined together, the result will be 64 different ways of arrangement; for, in that of two squares, one of them may be made to assume four different situations, in each of which the other may be changed 16 times. The result therefore will be 64 combinations.

We must however observe, with Father Sebastian, that one half of these combinations are only a repetition of the other, in a contrary direction, which reduces them to 32; and if attention were not paid to situation, they might be reduced to 10.

In like manner, we might combine three, four, five, &c., squares together, and in that case it would be found, that three squares are capable of forming 128 figures; that four could form 256, &c.

Fig. 10.



The immense variety of compartments which arise, in this manner, from so small a number of elements, is really astonishing. Father Sebastian gives thirty different kinds, selected from a hundred; and these even are only a very small part of those which might be formed. The annexed figure (10.) exhibits one of the most remarkable.

In consequence of Father Sebastian's memoir, Father Douat, one of his associates, was induced to pursue this subject still farther, and to publish, in the year 1722, a large work, in which it is considered in a different manner. It is entitled "Méthode pour faire une infinité de dessins différents, avec des carreaux mi-partis de deux couleurs par une ligne diagonale; ou, Observations du P. D. Douat, religieux Carme de la P. de T. sur un Mémoire inseré dans l'Hist. de l'Acad. royale des Sciences de Paris, année 1704, par le P. S. Truchet, religieux du même ordre." Paris 1722, in 4to. In this work it may be seen that four squares, each divided into two triangles of different colours, repeated and changed in every manner possible, are capable of forming 256 different figures; and that these figures themselves, taken two and two, three and three, and so on, will form a prodigious multitude of compartments, engravings of which occupy the greater part of the book.

It is rather surprising that this idea should have been so little employed in architecture; as it might furnish an inexhaustible source of variety in pavements, and other works of the like kind. However this may be, it forms the object of a pastime, called by the French *Jeu du Parquet*. The instrument employed for this pastime, consists of a small table, having a border round it, and capable of receiving 64 or a hundred small squares, each divided into two triangles of different colours, with which people amuse themselves in endeavouring to form agreeable combinations.

CHAPTER IX.

APPLICATION OF THE DOCTRINE OF COMBINATIONS TO GAMES OF CHANCE AND TO PROBABILITIES.

Though nothing, on the first view, seems more foreign to the province of the mathematics than chance, the powers of analysis have, as we may say, enchained this Proteus, and subjected it to calculation. It has found means to measure the different degrees of probability; and this has given birth to a curious branch of the mathematics, the principles of which we shall here explain.

When an event can take place different ways, it is evident that the probability of its happening in a certain determinate manner, will be greater when, of the whole of the ways in which it can happen, the greater number determine it to happen in that manner. In a lottery, for example, every one knows that the probability or hope of obtaining a prize, is greater according as the number of prizes is greater, and as the total number of the tickets is less. The probability therefore of an event is in the compound ratio of the number of the cases which can produce it, taken directly, and of the total number of those according to which it may be varied, taken inversely; consequently it may be expressed by a fraction, having for its numerator the number of the favourable cases, and for its denominator the whole of the cases.

Thus, in a lottery consisting of a thousand tickets, 25 of which only are prizes, the chance of obtaining one of the latter will be represented by $\frac{25}{1000}$ or $\frac{1}{40}$; if the number of the prizes were 50, this probability would be double, for in that case it would be equal to $\frac{1}{20}$; but, on the other hand, if the whole number of tickets, instead of a thousand, were two thousand, the probability would be only one half of the former, that is $\frac{1}{80}$. If the whole number of tickets were infinitely great, the number of prizes still remaining the same, the probability would be infinitely small; and if the whole number of tickets were prizes, it would become certainty, and in that case would be expressed by unity.

Another principle of this theory, necessary to be here explained, the enunciation of which will be sufficient to shew the truth of it, is as follows:

We play an equal game, when the money deposited is in direct proportion to the probability of gaining the stake; for, to play an equal game, is nothing else than to deposit a sum so proportioned to the probability of winning, that, after a great number of throws or games, the player may find himself nearly at par; but for this purpose, the sums deposited must be proportioned to the degree of probability, which each of the players has in his favour. Let us suppose, for example, that A bets against B on a throw of the dice, and that the chances are two to one in favour of A; the game will be equal if, after a great number of throws, the parties separate nearly without any loss; but as there are two chances in favour of A, and only one in favour of B, after three hundred throws A will have gained nearly two hundred, and B one hundred; A therefore ought to deposit two and B only one; for by these means, as A in winning two hundred throws will gain 200, B in winning a hundred throws will gain 200 also. In such cases therefore, it is said that two to one may be betted in favour of A.

PROBLEM I.

In tossing up, what probability is there of throwing a head several times successively, or a tail; or, in playing with several pieces, what probability is there that they will be all heads, or all tails?

In this game it is evident, 1st, That as there is no reason why a head should come up rather than a tail, or a tail rather than a head, the probability that one of the two will be the case is equal to $\frac{1}{2}$, or an equal bet may be taken for or against.

But if the game were for two throws, and any one should bet that a head will come up twice, it must be observed, that all the combinations of head or tail, which can take place in two successive throws with the same piece, are *head, head; head, tail; tail, head; tail, tail;* one of which only gives head, head. There is therefore only one case in four which can make the person win who bets to throw a head twice in succession; consequently the probability of this event is only $\frac{1}{4}$; and he who bets in favour of two heads, ought to deposit a crown, and the person who bets against him ought to deposit three; for the latter has three chances of winning, while the former has only one. To play an equal game then, the sums deposited by each ought to be in this proportion.

It will be found also, that he who bets to throw a *head* three times in succession, will have in his favour only one of the eight combinations of head and tail, which may result from three throws of the same piece. The probability of this event therefore is $\frac{1}{8}$, while that in favour of his adversary will be $\frac{7}{8}$. Consequently, to play an equal game, he ought to stake 1 against 7.

It is needless to go over all the other cases; for it may be easily seen, that the probability of throwing a *head* four times successively is $\frac{1}{16}$; five times successively, $\frac{1}{32}$, &c.

It is unnecessary also to enumerate all the different combinations which may result from *head* or *tail;* but in regard to probabilities, the following simple rule may be employed.

The probabilities of two or more single events being known, the probability of their taking place altogether may be found, by multiplying together the probabilities of these events, considered singly.

Thus the probability of throwing a head, considered singly, being expressed at each throw by $\frac{1}{2}$, that of throwing it twice in succession, will be $\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{4}$; that of throwing it three times, and three successive throws, will be $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{8}$, &c.

2d. The problem, to determine the probability of throwing, with two, three, or four pieces, all *heads* or all *tails*, may be resolved by the same means. When two pieces are tossed up, there are four combinations of *head* and *tail*, one of which only is all heads. When three pieces are tossed up together, there are 8, one of which only gives all *heads*, &c. The probabilities of these cases therefore are the same as those of the cases similar to them, which we have already examined.

It may be easily seen indeed, without the help of analysis, that these two questions are absolutely the same; and the following mode of reasoning may be employed to prove it. To toss up the two pieces A and B together, or to toss them up in succession, giving time to A, the first, to settle before the other is tossed up, is certainly the same thing. Let us suppose then, that when A, the first, has settled, instead of tossing up B, the second, A the first is taken from the ground, in order to be tossed up a second time; this will be the same thing as if the piece B had been employed for a second toss; for by the supposition they are both equal and similar, at least in regard to the chance whether head or tail will come up. Consequently, to toss up the two pieces A and B, or to toss up twice in succession the piece A, is the same thing. Therefore, &c.

3d. We shall now propose the following question: What may a person bet, that in two throws a *head* will come up at least once? By the above method it will be found, that the chances are 3 to 1. In two throws, indeed, there are four combinations, three of which give at least a head once in the two throws, and one only which gives all tails; hence it follows, that there are three combinations in favour of the person who bets to bring a head once in two throws, and only one against him.

PROBLEM II.

Any number of dice being given; to determine what probability there is of throwing an assigned number of points.

We shall first suppose that the dice are of the ordinary kind, that is to say, having six faces, marked with the numbers 1, 2, 3, 4, 5, 6; and we shall analyse some of the first cases of the problem, in order that we may proceed gradually to those that are more complex.

1st. *It is proposed to throw a determinate point,* 6 *for example, with one die.*

Here it is evident, that as the die has six faces, one of which only is marked 6, and as any one of them may as readily come up as another, there are 5 chances against the person who proposes to throw a six at one throw, and only one in his favour.

2d. *Let it be proposed to throw the same point* 6 *with two dice.*

To analyse this case, we must first observe that two dice give 36 different combinations; for each of the faces of the die A, for example, may be combined with each of those of the die B, which will produce 36 combinations. But six may be thrown, 1st, by 3 and 3; 2d, by 2 with the die A, and 4 with the die B, which, as may be readily seen, forms two distinct cases: 3d, by 1 with the die A, and 5 with the die B, or 1 with B and 5 with A, which also gives two cases; and these are all that are possible. Hence there are 5 favourable chances in 36; consequently the probability of throwing 6 with two dies is $\frac{5}{36}$, and that of not throwing it is $\frac{31}{36}$. This therefore ought to be the ratio of the stakes or money deposited by the players.

By analysing the other cases, it will be found that, of throwing two with two dice, there is one chance in 36; of throwing three, there are 2; of throwing four, 3; of throwing five, 4; of throwing six, 5; of throwing seven, 6; of throwing eight, 5; of throwing nine, 4; of throwing ten, 3: of throwing eleven, 2; and of throwing sixes, 1.

If three dice were proposed, with which it is evident the lowest point would be three, and the highest eighteen, it will be found, by means of a similar analysis, that in 216, the whole number of the throws possible with three dice, there is 1 chance of throwing three; 3 of throwing four; 6 of throwing five, &c.: as may be seen in the annexed table, the use of which is as follows.

If it be required, for example, to find in how many ways 13 can be thrown with three dice, we must look in the first vertical column, on the left, for the number 13, and at the top of the table for 3, the number of the dice; and in the square below, opposite to 13, will be found 21, the number of ways in which 13 may be thrown with three dice. In like manner, it will be found, that with 4 dice, it may be thrown 140 ways; with five dice, 420; &c.

Table of the different ways in which any point can be thrown with one, two, three, or more dice.

When it is once known how many ways a point can be thrown with a certain number of dice, the probability of throwing it may be easily found: nothing is necessary but to form a fraction, having for its numerator the number of ways in which the point can be thrown, and for denominator the number 6, raised to that power indicated by the number of dice; as the cube of 6, or 216, for three dice; the biquadrate, or 1296, for four dice; &c.

Thus, the probability of throwing 13 with three dice, is $\frac{21}{216}$; of throwing it with four, $\frac{140}{1296}$; &c.

Various other questions may be proposed concerning the throwing of dice, a few of which we shall here examine.

1st. *When two players are engaged; to determine the advantage or disadvantage of the person who undertakes to throw a certain face, that for example marked* 6, *in a certain number of throws.*

Let us suppose that he undertakes it at one throw: to find the probability of his succeeding, it must be considered, that he who holds the die has only one chance of winning, and five of losing; consequently to undertake it at one throw, he ought to stake no more than one to five. There is therefore a great disadvantage in undertaking, on an even bet, to throw six at a single throw of one die.

To determine the probability of throwing the face marked 6 in two throws with a single die, we must observe, as has been already said, in regard to tossing up, that

this is the same thing as to undertake, in throwing two dice together, that one of them shall have the side marked 6 uppermost. He then who holds the dice has only 11 chances, or combinations, by which he can win; for he may throw 6 with the first die, and 1, 2, 3, 4, or 5 with the second; or 6 with the second die, and 1, 2, 3, 4, or 5 with the first, or 6 with each die. But there are 25 combinations or chances unfavourable to his winning, as may be seen in the following table:

1, 1	2, 1	3, 1	4, 1	5, 1
1, 2	2, 2	3, 2	4, 2	5, 2
1, 3	2, 3	3, 3	4, 3	5, 3
1, 4	2, 4	3, 4	4, 4	5, 4
1, 5	2, 5	3, 5	4, 5	5, 5

Hence it may be concluded, that he who undertakes to throw a 6 with two dice, ought to stake no more than 11 to 25; and consequently, that it would be disadvantageous to do it on equal terms.

It must here be observed that 36, the number of all the chances or combinations possible in two throws of the dice, is the square of 6, which is the number of the faces of one die; and that 25, the number of the chances unfavourable to the person who undertakes to throw a determinate face, is the square of 5, or of 1 less than the same number 6. The number of the favourable chances therefore, in this case, is equal to the difference of the squares 36 and 25, or of the square of the number of the faces of one die, and of that of the faces of the same die less one.

In the case of undertaking to bring a 6 in three throws with one die, we must consider, in like manner, that this is the same thing as to undertake that, in throwing three dice at once, one of them shall bring a 6; but of the 216 combinations, which result from three dice, there are 125 without a 6, and 91 among which there is at least one 6; consequently, he who engages to throw a 6, either in three throws with one die, or one throw with three dice, ought to bet no more than 91 to 125; and it would be disadvantageous to undertake it on equal terms.

It is here to be observed, that the number 91 is the difference of the cube of the number of the faces of one die, viz. 216, and of 125, the cube of the same number less unity, or of 5. Hence it appears that, in general, to find the probability of throwing a determinate face, in a certain number of throws, or at one throw with a certain number of dice, we must raise 6, the number of the faces of one die, to that power which is indicated by the number of throws agreed on, or by the number of dice to be thrown at one time; we must then raise 6 less unity, or 5, to the same power, and subtract it from the former; the remainder with this power of 5 will be the respective number of chances for winning or losing.

Thus, if a person should bet to throw at least one 3 with four dice, we must raise 6 to the 4th power, which is 1296, and subtract from it the fourth power of 5, or 625; the remainder 671 will be the number of chances for winning, and 625 that of the chances of losing; consequently there will be an advantage in laying an even bet.

It is advantageous also to undertake, on an even bet, to throw any determinate point, for example 3, in five throws, or with five dice; for if from the 5th power of 6, which is 7776, we deduct the 5th power of 5, or 3125, the remainder 4651 will be the number of favourable chances, and 3125 that of the unfavourable. Consequently, to play an equal game, he who bets on throwing the above point, ought to deposit 4651 to 3125, or nearly 3 to 2.

2d. *In how many throws may one bet, on equal terms, to throw a determinate doublet, for example sixes, with two dice?*

It has been already shewn, that the probability of not throwing sixes with two two dice, is $\frac{35}{36}$; consequently the probability of their not coming up in two throws,

will be the square of that fraction; in three throws, the cube, &c. But as the powers of every number greater than unity, however small the excess, go on always increasing, those of a number less than unity, however small the defect, go on always decreasing: the consecutive powers therefore of $\frac{35}{36}$ will go on always decreasing. Now let us conceive $\frac{35}{36}$ to be raised to such a power as to be equal to $\frac{1}{2}$; it will be found that the 24th power of $\frac{35}{36}$ is somewhat greater than $\frac{1}{2}$; and that the 25th power is somewhat less;* hence it follows that one may lay an even bet with some advantage, that another will not bring sixes in 24 throws with two dice, but that there is some disadvantage in taking an even bet that they will not come up in 25 throws. Consequently, he who bets on throwing sixes in 24 throws, does so with disadvantage; but if he lays an even bet that they will come up in 25 throws, the advantage is in his favour.

3d. *What probability is there of throwing any determinate doublet, for example two threes, in one throw with two or more dice?*

To determine this question, we must first observe, that he who undertakes to throw two threes with two dice, has only one favourable chance, in the 36 chances or combinations given by two dice; and it thence follows that he ought to bet no more than 1 to 35.

In the case of three dice, it will be found that he ought to bet no more than 16 to 200; for the number of chances or combinations possible with three dice is 216. But when it is proposed to throw two threes with three dice, they may come up 16 different ways; for in the 36 combinations of the two dice A and B, all those in which one 3 only is found, as 1, 3; 3, 1, &c., being 10 in number, by combining with the side marked 3 of the die C, give two threes. Besides the combination 3, 3 of the dice A and B, by combining with one of the six faces of the third C, will give two threes. Here then we have 16 ways of throwing two threes with three dice, which give 16 favourable chances in 216. Consequently, the probability of throwing two threes with three dice is $\frac{16}{216}$; and no more ought to be betted on the success of that event than 16 to 200, or 2 to 25.

If the probability of throwing two threes with four dice be required, we shall find that it is expressed by $\frac{171}{1296}$; for, of the 1296 combinations of the faces of four dice, there are 150 which give two threes, 20 that give 3, and one that gives 4, making altogether 171 throws, which give 2 or 3 or 4 threes. Consequently, no more than 171 to 1125, or about 1 to $6\frac{3}{5}$, ought to be betted on throwing, at least, once threes with four dice.

In the last place, if the probability of throwing any doublet, at one throw, with two or more dice, be required, it may be easily determined by the preceding method of calculation; for if an indeterminate doublet be proposed, it is evident that the probability is six times as great as when an assigned doublet is proposed; and therefore we have only to multiply the probabilities already found by 6. The probability therefore with two dice, will be $\frac{6}{36}$ or $\frac{1}{6}$; with three dice, $\frac{96}{216}$ or $\frac{4}{9}$; with four dice, $\frac{1026}{1296}$ or $\frac{19}{24}$, &c. So that there is an advantage in taking an even bet to throw at least one doublet with four dice.

This property is not true when the number of dice exceeds *three*. The probability of an assigned doublet with four dice is $\frac{171}{1296}$, which multiplied by 6, and added to the

* Let n be the exponent of that power of $\frac{35}{36}$ which is equal to $\frac{1}{2}$; that is to say, let $\frac{35^n}{36^n}$ be equal to $\frac{1}{2}$. As the unknown quantity n is in the exponent, it must be disengaged from it, which may be done by means of logarithms. For $\frac{35^n}{36^n} = \frac{1}{2}$, by taking the logarithms we shall have n log. 35 $-$ n log. 36 $=$ log. $\frac{1}{2}$, or $= -$ log. 2; for log. $\frac{1}{2} = -$ log. 2. Hence n log. 35 $-$ n log. 36 $= -$ log. 2, or log. 2 $= n$ log. 36 $-$ n log. 35. Therefore, $n = \frac{\text{log. 2.}}{\text{log. 36} - \text{log. 35}}$. Which gives $n = 24{\cdot}605$, or $24\frac{6}{10}$ nearly.

probability of a different face coming up with each die, viz. to $\frac{6}{6} \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6}$ or $\frac{360}{1296}$, gives $\frac{1386}{1296}$, being 90 chances more than there is in all the four dice, which is impossible.

The probability of an assigned doublet with four dice, viz. $\frac{171}{1296}$, includes the probability of some other doublet; for *aces*, *twos*, or any other doublet may turn up at the same throw, which cannot happen with two dice or with three; so that the multiplier 6 will not answer to the probability of an indeterminate doublet, when there are more than three dice.

In such cases it is the safest and easiest way to find the probability of the reverse problem—of not throwing doublets—and then subtracting that probability from *unity* or *certainty*, the remainder is the probability for doublets.

PROBLEM III.

Two persons sit down to play for a certain sum of money; and agree that he who first gets three games shall be the winner. One of them has got two games, and the other one; but being unwilling to continue their play, they resolve to divide the stake: how much of it ought each person to receive?

This problem is one of the first that engaged the attention of Pascal, when he began to study the calculation of probabilities. It was resolved by Fermat, a celebrated geometrician, to whom he proposed it, by a different method, viz. that of combinations: we shall here give both.

It is evident that each of the players, in depositing his money, resigns all right to it; but, in return, each has a right to what chance may give him; consequently when they give over playing, the stake ought to be divided in proportion to the probability each had of winning the whole sum had they continued.

Case 1st. This proportion may be determined by the following reasoning. Since the first player wants one game to be out, and the second two, it may be readily perceived, that if they continue their play, and if the second should win one game, he would want, in the same manner as the first, one game to be out; and in that case, the two players being equally advanced, their hopes or chances of winning would be equal. This being supposed, they would have an equal right to the stake, and consequently each ought to have an equal share of it.

It is evident therefore, that if the first should win the game about to be played, the whole money deposited would belong to him; and that if he lost it he would have a right only to the half. But the one case being as probable as the other, the first has a right to the half of both these sums taken together. But together they make $\frac{3}{2}$, the half of which is $\frac{3}{4}$; and this is the share of the stake belonging to the first player; consequently that belonging to the second is only $\frac{1}{4}$.

Case 2nd. The solution of the first case will enable us to solve the second, in which we suppose that the first player wants one game to be out, and the second three; for if the first should win one game, he would be entitled to the whole stake, and if he lost one game, by which means the second would want only two games to be out, $\frac{3}{4}$ of the money would belong to the former, as the parties would then be in the situation alluded to in the preceding case. But as both these events are equally probable, the first ought to have the half of the two sums taken together, or the half of $\frac{7}{4}$, that is $\frac{7}{8}$: the remainder $\frac{1}{8}$ will therefore be what belongs to the second.

Case 3rd. It will be found, by the like reasoning, if we suppose two games wanting to the first player, and three to the second, that on ceasing to play, they ought to divide the stake in such a manner that the former may have $\frac{11}{16}$ and the latter $\frac{5}{16}$.

Case 4th. If four games were to be played, and if the first wanted only two games, and the second four, the money ought to be divided in such a manner that the former should have $\frac{13}{16}$ and the latter $\frac{3}{16}$.

But we may dispense with the above reasoning, and employ the following general rule, deduced from it, which is to be applied by means of the arithmetical triangle. Enter that diagonal of the arithmetical triangle, the order of which is equal to the number of the games wanting to both players. As this number in the first case is 3, we must enter the third diagonal of the triangle; then because the first player wants only one game, we must take the first number of that diagonal; but because two are wanting to the second, we must take the sum of the two first numbers, which will give 3. These two numbers therefore, 1 and 3, will indicate, that the stake ought to be divided in the same proportion: consequently the first player ought to have $\frac{3}{4}$, and the second $\frac{1}{4}$.

As this rule may be easily applied to every other case whatever, we shall enlarge no farther on the subject.

The second method of resolving problems of this kind, which is that of combinations, is as follows:

To resolve, for example, the fourth case, where, according to the supposition, the first player wants two games to be out, and the second four, so that together they want six games; take unity from that sum, and because 5 remain, we shall suppose the five similar letters *a a a a a*, favourable to the first player, and the five following, *b b b b b*, favourable to the second. These letters must be combined, as in the following table, where, of the 32 combinations which they form, the first 26, towards the left, where *a* occurs at least twice, will indicate the number of chances which the first has of winning; and the last 6, towards the right, in which *a* never occurs oftener than once, will indicate those favourable to the second.

a a a a a	*a a a b b*	*a a b b b*	*a b b b b*
a a a a b	*a a b b a*	*a b b b a*	*b b b b a*
a a a b a	*a b b a a*	*b b b a a*	*b a b b b*
a a b a a	*b b a a a*	*a b a b b*	*b b a b b*
a b a a a	*a a b a b*	*a b b a b*	*b b b a b*
b a a a a	*a b a a b*	*b b a a b*	*b b b b b*
	b a a a b	*b a a b b*	
	b a a b a	*b a b b a*	
	b a b a a	*b b a b a*	
	a b a b a	*b a b a b*	

The expectation therefore of the first player, will be to that of the second, as 26 to 6, or as 13 to 3.

In like manner, to resolve the case where the first player is supposed to have won three games, and the other none, as he must win who first gets four games, the number of the games wanting to both will be 5, which being diminished by unity, will give 4. We must then examine in how many different ways the letters *a* and *b* can be combined four and four, which will be found to be 16, viz.:

a a a a	*a a b b*	*a b b b*	
a a a b	*a b a b*	*b a b b*	*b b b b*
a a b a	*b a a b*	*b b a b*	
a b a a	*a b b a*	*b b b a*	
b a a a	*b a b a*		
	b b a a		

But, of these 16 combinations, it is evident there are 15 where *a* is found at least once; which indicates that there are 15 combinations or chances favourable to the first player, and one favourable to the second. Consequently they ought to divide the stake in the ratio of 15 to 1, or the former ought to have $\frac{15}{16}$ of it, and the latter $\frac{1}{16}$.

PROBLEM IV.

Of the Genoese Lottery.

All persons are acquainted with the nature of lotteries, a kind of institution which originated in Italy, and which was afterwards introduced into other countries of Europe. It took its rise at Genoa, where it had long been customary to choose annually by ballot five members of the senate, which was composed of 90 persons, in order to form a particular council. Some idle persons took this opportunity of laying bets, that the lot would fall on such or such senators. The government then seeing with what eagerness people interested themselves in these bets, conceived the idea of establishing a lottery on the same principle; which was attended with so great success, that all the cities of Italy wished to participate in it, and sent large sums of money to Genoa for that purpose. The same motive, and that no doubt of increasing the revenues of the church, induced the pope to establish one of the same kind at Rome, the inhabitants of which became so fond of this species of gambling, that they often deprived themselves and their families of the necessaries of life, that they might have money to lay out in the lottery. Many of them also indulged in every kind of foolery that credulity or superstition could inspire, in order to obtain fortunate numbers.

The analysis of this kind of lottery is reduced to the solution of the following problem.

Ninety numbers being given, five of which are to be drawn by chance; it is required to determine what probability there is that among these five, there will be one, two, three, four, or five numbers, which any one has chosen from among the 90?

It may be readily seen, that if one determinate number only were proposed, and that if no more than one number were to be drawn from the wheel, the adventurer would have only one favourable chance in the 90; but as five numbers are drawn from the wheel, this quintuples the chance favourable to the adventurer, so that he has five favourable chances in the ninety. His probability therefore of winning, is $\frac{1}{18}$; and, to play an equal game, the stakes ought to be in the same ratio, or, what amounts to the same thing, the proprietor of the lottery ought to reimburse the price of the ticket 18 times.

To determine what probability there is, that two numbers selected will both come up, we must first find how many combinations may be produced by 90 numbers, taken two and two. In treating on combinations we have already shewn, that in this case they amount to 4005; but as five numbers are drawn from the wheel, and as these five numbers, combined together two and two, give 10 twos, it thence results that, in these 4005 chances, there are only 10 favourable to the adventurer. The probability therefore, that the two numbers selected may be among those drawn from the wheel, will be expressed by $\frac{10}{4005}$ or $\frac{1}{400\frac{1}{2}}$. For this reason the proprietor of the lottery ought to give the adventurer, in case he should win, 400½ times the price of the ticket.

To determine what probability there is, that three numbers selected will come up among the five drawn from the wheel, we must find how many ways 90 numbers can be combined three and three, or how many threes they make. These combinations amount to 117480; but as the five numbers drawn from the wheel form 10 threes, the adventurer has 10 favourable chances in 117480, and the probability in his favour is $\frac{10}{117480}$ or $\frac{1}{11748}$. To risk his money therefore on equal terms, the prize ought to be 11748 times the price of the ticket.

In the last place, it will be found that in 511038 chances, there is only one favourable to the person who should bet that 4 determinate numbers will come up; and 1

in 43949268 favourable to the person who should bet that five determinate numbers will be the five drawn; consequently, in the last case, to risk his money on equal terms, according to mathematical strictness, the adventurer, should he be successful, ought to receive nearly 44 millions of times the money which he lays out.

PROBLEM V.

A and B playing at piquet: A is first in hand, and has no ace: what probability is there that he will get one, or two, or three, or four?

It is well known that at this game 12 cards are dealt to each of the players, and that 8 remain in the pack, of which the first takes 5, and the last 3. This being premised, it will be found that A's chance to have any one ace is . . . $\frac{455}{969}$
to have two . . . . $\frac{270}{969}$
to have three . . . . $\frac{30}{969}$
to have four . . . . $\frac{1}{969}$
the sum of all these is $\frac{756}{969}$, which is equal to $\frac{252}{323}$.

Hence it follows, that the probability of his having an ace among the five cards he has to take in, is $\frac{252}{323}$, the difference between which numbers is 71, so that one may bet 252 to 71 that A will take in some of the aces. But let us suppose that A is last in hand; in that case it is required how much he may bet that he will have at least one ace among his three cards?

The probability of A having an ace among his three cards is . . . $\frac{8}{19}$ or $\frac{120}{285}$
of having two it is . . . . $\frac{24}{285}$
of having three . . . . $\frac{1}{285}$
the sum of all which is $\frac{145}{285}$ or $\frac{29}{57}$.

Consequently, the probability that he will have either one, or two, or three indeterminately, is $\frac{29}{57}$. A may therefore take an equal bet with advantage, that he will have one of the aces, for the ratio of the stakes would be 29 to 28.

PROBLEM VI.

At the game of whist, what probability is there, that the four honours will not be in the hands of any two partners?

De Moivre, in his Doctrine of Chances, shews that the chance is nearly 27 to 2 that the partners, one of whom deals, will not have the four honours.

That it is about 23 to 1 that the other two partners will not have them.

That it is nearly 8 to 1 that they will not be found on any one side.

That one may bet about 13 to 7, without disadvantage, that the partners who are first in hand will not count honours.

That about 20 to 7 may be betted, that the other two will not count them.

And, in the last place, that it is 25 to 16, that one of the two sides will count honours, or that they will not be equally divided.

PROBLEM VII.

Of the game of the American Savages.

We are told by Beron de la Hontan, in his *Voyages en Canada*, that the Indians play at the following game: they have 8 nuts, black on the one side, and white on the other: these they throw into the air, and if it happens, when they fall to the ground, that the black are odd, the player wins the stake; if they are all black, or all white, he wins the double; but if there are an equal number of each, he loses.

M. de Montmort, who analysed this game, finds, that he who tosses up the nuts, has

an advantage, which may be estimated at $\frac{1}{43}$; and that to render the game equal, he ought to deposit 22 when his adversary stakes 21.

PROBLEM VIII.

Of the game of Backgammon.

The game of backgammon is one of those where the spirit of combination is displayed in a very striking manner, and where it is of great utility to know, at every throw, what may be hoped or feared from the succeeding throws, whether your own or those of your adversary. The chances in this game, like those in others, may be appreciated mathematically; but we shall here confine ourselves to a small number of examples, selected from those easiest to be comprehended.

I. *A, being at play at backgammon, is obliged to make a blot; now his throw is such, that he can make it either where his adversary B may take it up with a single ace, or where he can take it up by throwing seven in any manner: the question is, where should he make the blot?*

As the number of chances for throwing one ace or more, is 11, and the number of chances for throwing seven in any manner, are but 6, it will be safest to make the blot where it may be taken up by throwing 7.

II. *Whether it is safer to make a blot, at backgammon, where it may be taken up by an ace, or where it may be taken up by a tré?*

The number of chances for throwing one ace or more, and those for throwing one tré or more, are each 11; but there are 2 chances for throwing deux ace, or 3; it will therefore be safer to make the blot where it can be taken up only by an ace.

The following table will shew the chances of taking up a single blot however situated.

No. of points to hit.	Chances.	Total chances.
1	11	11
2	11 + 1	12
3	11 + 2	13
4	11 + 3	14
5	11 + 4	15
6	11 + 5	16

No. of points to hit.	Chances.	Total chances.
7	6	6
8	5	5
9	4	4
10	3	3
11	2	2
12	1	1

Hence, if a blot is liable to be hit by any one face of the die, the mean probability of hitting it will be $\frac{11+16}{2 \times 36} = \frac{27}{72} = \frac{3}{8}$ nearly.

III. *If two blots be made at backgammon, so as to be hit by two different faces of the die, what is the probability of hitting one or both of them?*

By the first table it will appear, that the probability of throwing one or more, of any two given faces, is $\frac{20}{36}$. But besides this, one or both the blots may be at length hit by the two dice, and the probability in this case will be different, according to the number of points that will hit them, as in the following table:

Faces to hit.	Chances.	Total chances.
1.2	20 + 1	21
1.3	20 + 2	22
1.4	20 + 3	23
1.5	20 + 4	24
1.6	20 + 5	25
2.3	20 + 1 + 2	23
2.4	20 + 1 + 3	24
2.5	20 + 1 + 4	25

Faces to hit.	Chances.	Total chances
2.6	20 + 1 + 5	26
3.4	20 + 2 + 3	25
3.5	20 + 2 + 4	26
3.6	20 + 2 + 5	27
4.5	20 + 3 + 4	27
4.6	20 + 3 + 5	28
5.6	20 + 4 + 5	29

Hence the probability of hitting two such blots, will be at a medium $\frac{21+29}{2\times 36}=\frac{25}{36}$.

IV. *If there be three blots, so situated as to be hit by three different faces; the probability of hitting one or more of them is required?*

The first table will give the probability of hitting one or more of the blots with a single face or faces; but besides this, there will be the probability of hitting one or more of the blots with two dice, the least of which will be when the given faces are 1, 2, 3, which have $1+2=3$ such chances, and the greatest when the given faces are 4, 5, 6, which have $3+4+5=12$ such chances; the medium of these, viz. $\frac{3+12}{2}=\frac{15}{2}$, being added to 27, will make the whole probability about $27+\frac{15}{2}=\frac{69}{2}$, which divided by the common denominator 36, becomes $\frac{69}{72}=\frac{23}{24}$.

Hence, if a player at backgammon makes 3 blots, which are severally within the reach of being hit by a single face of the die, it is almost a certainty that one of them at least will be hit.

PROBLEM IX.

A mountebank at a country fair amused the populace with the following game: he had 6 dice, each of which was marked only on one face, the first with 1, the second with 2, and so on to the sixth, which was marked 6; the person who played gave him a certain sum of money, and he engaged to return it a hundred-fold, if in throwing these six dice, the six marked faces should come up only once in 20 throws. If the adventurer lost, the mountebank offered a new chance on the following conditions: to deposit a sum equal to the former, and to receive both the stakes in case he should bring all the blank faces in 3 successive throws.

Those unacquainted with the method to be pursued in order to resolve such problems, are liable to reason in an erroneous manner on dice of this kind; for observing that there are five times as many blank as marked faces, they thence conclude that it is 5 to 1 that the person who throws them will not bring any point. They are however mistaken, as the probability, on the contrary, is near 2 to 1 that they will not come up all blanks.

If we take only one die, it is evident that it is 5 to 1 that the person who holds it will throw a blank; but if we add a second die, it may be readily seen, that the marked face of the first may combine with each of the blank faces of the second, and the marked face of the second with each of the blank faces of the first; and, in the last place, the marked face of the one with the marked face of the other: consequently, of 36 combinations of the faces of these two dice, there are 11 in which there is at least one marked face. But, as we have already observed, this number 11 is the

difference of the square of 6, the number of the faces of one die, and of the square of the same number diminished by unity, that is to say of 5.

If a third die be added, we shall find, by the like analysis, that of the 216 combinations of three dice, there are 91 in which there is at least one marked face; and 91 is the difference of the cube of 6 or 216, and the cube of 5 or 125; the result will be the same in regard to the more complex cases; and hence we may conclude that, of the 46656 combinations of the faces of the 6 dice in question, there will be 31031 in which there is at least one marked face, and 15625 in which all the faces are blank; consequently the chance is 2 to 1 that some point at least will be thrown; whereas, by the above reasoning, it would appear that 5 to 1 might be betted on the contrary being the case.

This example may serve to shew how diffident we ought to be in regard to the ideas which occur on the first consideration of subjects of this kind; and it may be added that, in this case, our reasoning is confirmed by experience. But to return to the problem; it is evident that, of the 46656 combinations of the faces of 6 dice, there is only one which gives the 6 marked faces uppermost; the probability therefore of throwing them at one throw, is expressed by $\frac{1}{46656}$; and as the adventurer was allowed 20 throws, the probability of his succeeding was only $\frac{20}{46656}$, which is nearly equal to $\frac{1}{2332}$. To play an equal game therefore, the mountebank should have engaged to return 2332 times the money. But he offered only 100 times the stake, that is, about the 23d part of what he ought to have offered, to give an equal chance, and consequently he had an advantage of 22 to 1.

The chance offered to those who might lose was a mere deception; for the proposer artfully availed himself of that propensity which every man, who had not sufficiently examined the subject, would have to adopt the false reasoning above mentioned; and the adventurer would have the less hesitation to accept the offer as it would seem that he might bet 5 to 1 on bringing blanks every throw; whereas it is 2 to 1 that the contrary will happen. But the chance of not bringing blanks in one throw, being to that of bringing them, as 2 to 1; it thence follows, that the probability of not bringing them three times successively, is to that of bringing them, as 8 to 1. To play an equal game therefore, the mountebank ought to have staked 7 to 1; consequently, in the chance which he gave to the loser, in a game where he had an advantage of 22 to 1, he had still an advantage of 7 to 1.

PROBLEM X.

In how many throws with six dice, marked on all their faces, may a person engage, for an even bet, to throw 1, 2, 3, 4, 5, 6.

We have just seen that there are 46655 chances to 1, that a person will not throw these 6 points with dice marked only on one of their faces; but the case is very different with 6 dice marked on all their faces; and to prove it, we need only observe that the point 1, for example, may be thrown by each of the dice, as well as the 2, 3, &c.; which renders the probability of these six points, 1, 2, 3, &c. coming up, much greater.

But to analyse the problem more accurately, we shall observe, that there are 2 ways of throwing 1, 2, with two dice; viz. 1 with the die A, and 2 with the die B; or 1 with the die B, and 2 with A. If it were proposed to throw 1, 2, 3 with 3 dice; of the whole of the combinations of the faces of 3 dice, there are 6 which give the points 1, 2, 3; for 1 may be thrown with the die A, 2 with B, and 3 with C; or 1 with A, 2 with C, and 3 with B; or 1 with B, 2 with A, and 3 with C; or 1 with B, 2 with C, and 3 with A; or 1 with C, 2 with A, and 3 with B; or 1 with C, 2 with B, and 3 with A.

It hence appears that, to find the number of ways in which 1, 2, 3 can be thrown with 3 dice, 1, 2, 3 must be multiplied together. In like manner, to find the number of ways in which 1, 2, 3, 4 can be thrown with 4 dice, we must multiply together 1, 2, 3, 4, which will give 24; and, in the last place, to find in how many ways 1, 2, 3, 4, 5, 6 can be thrown with 6 dice, we must multiply together these six numbers, the product of which will be 720.

If the number 46656, which is the combinations of the faces of 6 dice, be divided by 720, we shall have $64\frac{4}{5}$ for the chances to 1, that these points will not come up at one throw; consequently a person may undertake, for an even bet, to bring them in 65 throws; and one may bet more than 2 to 1 that they will come up in 130 throws. In the last place, as the dice may be thrown 130 times and more, in a quarter of an hour, a person may with advantage bet more than 2 to 1, that they will come up in the course of that time.

He therefore who engages, for an even bet, to throw these points in a quarter of an hour, undertakes what is highly advantageous to himself, and equally disadvantageous to his adversary.

PROBLEM XI.

A certain person proposed to play with 7 dice, marked on all their faces, on the following conditions: he who held the dice was to gain as many crowns as he brought sixes; but if he brought none, he was to pay to his adversary as many crowns as there were dice, that is 7. What was the ratio of their chances?

To resolve this problem, we must analyze it in order. Let us suppose then, that there is only one die; in this case it is evident, that as there is only 1 chance in favour of him who holds the die, and 5 against him, the ratio of the stakes ought to be that of 1 to 5. If the first therefore gave a crown every time he did not throw 6, and received only the same sum when a 6 came up, he would play a very unequal game.

Let us now suppose 2 dice. In the 36 combinations, of which the faces of 2 dice are susceptible, there are 25 which give no 6; 10 which give 1, and 1 which gives 2. He therefore who holds the dice, has only 11 chances in his favour, 10 of which may each make him gain a crown, and the remaining 1 make him gain two. His chance then of winning, according to the general rule, will be $\frac{10}{36} + \frac{2}{36}$; and because, if the 25 chances which do not give a 6 should take place, he would be obliged to pay 2 crowns, the chance of his adversary will be $\frac{50}{36}$. Consequently the chance of winning will be to that of losing as $\frac{12}{36}$ to $\frac{50}{36}$, or 12 to 50, or less than 1 to 4.

To determine, in the more complex cases, the chances which give no 6, those which give one, those which give two, &c. it must be observed, that they are always expressed by the different terms of the power of $5 + 1$, the exponent of which is equal to the number of the dice. Thus when there is only one die, the number $5 + 1$ expresses, by its first term, that there are five chances without a 6, and one which gives a 6; if there be two dice, as the product of $5 + 1$ by $5 + 1$, or the square of $5 + 1$, is $25 + 10 + 1$, the first term 25 indicates that there are 25 chances, in the 36, which give no 6; 10 which give one, and 1 which gives two.

In like manner, as the cube of $5 + 1$ is $125 + 75 + 15 + 1$, it denotes that, in the 216 combinations of the faces of six dice, there are 125 in which there is no 6; 75 in which there is one; 15 in which there are two, and 1 where there are three.

The fourth power of $5 + 1$ being $625 + 500 + 150 + 20 + 1$, it indicates, in the same manner, that in the 1296 combinations of the faces of four dice, there are

625 without a 6; 500 which give 6; 150 which give two, 20 which give three, and only 1 that gives four.

We shall pass over the intermediate cases, and proceed to that where 7 dice are employed. In this case then it will be found, that the 7th power of $5 + 1$ is $78125 + 109375 + 65625 + 21875 + 4375 + 525 + 35 + 1 = 279936$. In the 279936 combinations of the faces of 7 dice, there are 78125 which give no 6; 109375 where there is one; 65625 where there are two; 21875 where there are three, &c. But as he who holds the dice would have to pay 7 crowns for each of the first 78125 chances, should they take place, we must consequently, according to the general rule, multiply that number by 7, and divide the product by the sum of all the chances, in order to obtain the chance against him, which is $= \frac{546875}{279936}$. To find the favourable chance, we must multiply each of the other terms by the number of the sixes it presents; add together the different products, and divide the sum by the whole of the chances, or 279936; in this manner we shall have, for the chance in favour of the person who holds the dice, $\frac{325592}{279936}$. His chance of winning, therefore, is to that of losing, as 325592 to 54687; that is to say, he plays a disadvantageous game, or it is 54 to 32, or 27 to 16, or more than 3 to 2, that he will lose.

By a like process it may be found, in the case of eight dice, that the chance of the person who holds them, is to that of his adversary, as 2259488 to 3125000, which is nearly as 3 to 4.

If there were nine dice, the chance of the person who holds them, would be to that of his adversary, nearly as 151 to 175, or nearly 25 to 29.

If there were ten dice, the chance of the former to that of the latter, would be as 101176960 to 97656250, that is to say, nearly as 101 to $97\frac{5}{10}$. The advantage then begins to be in favour of the former, only when the number of the dice is 10; and, to play an equal game, a less number ought not to be employed.

CHAPTER X.

ARITHMETICAL AMUSEMENTS IN DIVINATION AND COMBINATIONS.

PROBLEM I.

To tell the Number thought of by a person.

I. Desire the person, who has thought of a number, to triple it, and to take the exact half of that triple, if it be even, or the greater half if it be odd. Then desire him to triple that half, and ask him how many times it contains 9; for the number thought of will contain the double of that number of nines, and one more if it be odd.

Thus, if 5 has been the number thought of; its triple will be 15, which cannot be divided by 2 without a remainder. The greater half of 15 is 8; and if this half be multiplied by 3, we shall have 24, which contains 9 twice: the number thought of will therefore be $4 + 1$, that is to say 5.

Proof.—If the number be an even one, it may be represented by $2x$, and if an odd one by $2x + 1$. Then in the case of an even number $\frac{2x}{2} \times 3 \times 3$ represents the operations which the person thinking of a number is requested to perform upon it. The result is $9x$, the ninth part of which doubled is $2x$, the number thought of. In the case of the odd number $\frac{2x+1}{2} \times 3 \times 3 = 9x + 4\frac{1}{2}$, which contains 9, x times, and $2x + 1$ is the number thought of.

In the same way may each of the following methods be shewn to be true.

II. Bid the person multiply the number thought of by itself; then desire him to add unity to the number thought of, and to multiply that sum also by itself; in the last place ask him to tell the difference of these two products, which will certainly be an odd number, and the least half of it will be the number required.

Let the number thought of, for example, be 10, which multiplied by itself gives 100; in the next place 10 increased by 1 is 11, which multiplied by itself makes 121, and the difference of these two squares is 21, the least half of which, being 10, is the number thought of.

This operation might be varied in the second step, by desiring the person to multiply the number by itself, after it has been diminished by unity, and then to tell the difference of the two squares; the greater half of which will be the number thought of.

Thus, in the preceding example, the square of the number thought of is 100, and that of the same number less unity is 81; the difference of these is 19, the greater half of which, or 10, is the number thought of.

III.—Desire the person to take 1 from the number thought of, and to double the remainder; then bid him take 1 from this double, and add to it the number thought of. Having asked the number arising from this addition, add 3 to it, and the third of the sum will be the number required.

Let the number thought of be 5; if one be taken from it there will remain 4, the double of which, 8, being diminished by 1, and the remainder, 7, being increased by 5, the number thought of, the result will be 12: if to this we add 3, we shall have 15, the third part of which, 5, will be the number required.

Remark.—This method may be varied a great many ways; for instead of doubling the number thought of, after unity has been deducted from it, the person may be desired to triple it; then after he has been desired to subtract unity from that triple, and to add the number thought of, he must add 4 to it, and the $\frac{1}{4}$ of the sum arising from these operations will be the number required.

Let the number required be x: if unity be subtracted from it the remainder will be $x-1$; multiply this remainder by any number whatever, n, and the product will be $nx-n$; again subtract unity, and we shall have for remainder $nx-n-1$; if x, the number thought of, be then added, the sum will be $(n+1)x-n-1$; and if to this sum we add the above multiplier increased by unity, that is to say 3, if the first remainder was doubled, 4 if it was tripled, &c., the result will be $(n+1)x$; which being divided by the same number, the quotient will be x, the number required.

Unity, instead of being subtracted from the number thought of, might be added to it; and then, instead of adding, at the end of the operation, the multiplier increased by unity, it ought to be subtracted, after which the remainder may be divided as above.

Let the number thought of, for example, be 7; if unity be added, the sum will be 8, and this sum tripled will give 24; if 1 be still added, we shall have 25, and this sum increased by 7 will make 32; from which if 4 be deducted, because the number thought of was tripled after unity had been added, we shall have 28; one fourth of which will be the number required.

IV.—Desire the person to add 1 to the triple of the number thought of, and to multiply the sum by 3; then bid him add to this product the number thought of, and the result will be a sum, from which if 3 be subtracted, the remainder will be ten times the number required. If 3 therefore be taken from the last sum, and if the cipher on the right be cut off from the remainder, the other figure will indicate the number sought.

Let the number thought of be 6; the triple of which is 18, and if unity be added it makes 19; the triple of this last number is 57, and if 6 be added it makes 63, from which if 3 be subtracted the remainder will be 60: now if the cipher on the right be cut off, the remaining figure 6 will be the number required.

Remark.—If 1 were subtracted from thrice the number thought of, the remainder tripled, and the number thought of again added, it would be necessary, after the person had told the result, which would always terminate with 7, to add 3 instead of subtracting it, as in the above operation; and the sum would then be the decuple of the number thought of.

To the preceding methods, given by Montucla, of telling the number of which a person has thought, may be added another ingenious one, by means of the annexed columns of numbers, which are thus prepared.

Having entered the geometrical series, 1, 2, 4, 8, 16, 32, as the top series of the six columns, the other numbers in each column downward are produced by this rule.

To the first number add successively a unit as often as is denoted by one less than the first number; and then, to the last of these, add a number which is one more than the top number, and so on till the columns are filled up.

Now having prepared the columns, it may be as well, for the sake of secresy, to have them on different slips of paper.

Request a person to think of any number not greater than the highest contained in the columns, in the present case 63, and desire him to point out all the columns in which it is contained, or shewing each column separately, ask, Is the number there? Then, recollecting that the numbers at the top are 1, 2, 4, 8, 16, 32, add together in your mind the figures of this series at the tops of all the columns containing the number thought of, and the sum of these numbers will be the number required.

I.	II.	III.	IV.	V.	VI.
1	2	4	8	16	32
3	3	5	9	17	33
5	6	6	10	18	34
7	7	7	11	19	35
9	10	12	12	20	36
11	11	13	13	21	37
13	14	14	14	22	38
15	15	15	15	23	39
17	18	20	24	24	40
19	19	21	25	25	41
21	22	22	26	26	42
23	23	23	27	27	43
25	26	28	28	28	44
27	27	29	29	29	45
29	30	30	30	30	46
31	31	31	31	31	47
33	34	36	40	48	48
35	35	37	41	49	49
37	38	38	42	50	50
39	39	39	43	51	51
41	42	44	44	52	52
43	43	45	45	53	53
45	46	46	46	54	54
47	47	47	47	55	55
49	50	52	56	56	56
51	51	53	57	57	57
53	54	54	58	58	58
55	55	55	59	59	59
57	58	60	60	60	60
59	59	61	61	61	61
61	62	62	62	62	62
63	63	63	63	63	63

Thus for example, if the person says his number is in the 2nd, 5th, and 6th columns, $2+16+32$, or 50, is the number. If he says it is in the 1st, 2nd, 4th, and 6th columns, $1+2+8+32$, or 43 is the number.

The problem may be varied by requesting the person who thinks of a number to give you those columns only which do not contain it, and you will then discover it by subtracting the sum of the top numbers from the highest number, 63. Thus, if the number is not in the 2nd, 5th, nor 6th column, it must be $63-50$, or 13.

PROBLEM II.

To tell two or more numbers which a person has thought of.

I.—When each of the numbers thought of does not exceed 9, they may be easily found in the following manner:

Having made the person add 1 to the double of the first number thought of, desire him to multiply the whole by 5, and to add to the product the second number. If there be a third, make him double this first sum and add 1 to it; after which desire him to multiply the new sum by 5, and to add to it the third number. If there be a fourth, you must proceed in the same manner, desiring him to double the preceding sum; to add to it unity; to multiply by 5, and then to add the fourth number, and so on.

Then ask the number arising from the addition of the last number thought of, and if there were two numbers, subtract 5 from it; if three, 55; if four, 555; and so on; for the remainder will be composed of figures of which the first on the left will be the first number thought of, the next the second, and so of the rest.

Suppose the numbers thought of to be 3, 4, 6: by adding 1 to 6, the double of the first, we have 7, which being multiplied by 5, gives 35; if 4, the second number thought of, be then added, we shall have 39, which doubled gives 78, and if we add 1, and multiply 79, the sum, by 5, the result will be 395. In the last place, if we add 6, the third number thought of, the sum will be 401; and if 55 be deducted from it, we shall have for remainder 346; the figures of which, 3, 4, 6, indicate in order the three numbers thought of.

One method we shall here omit, as we shall have occasion to employ it in another amusement of the same kind, called the game of the ring.

II.—If one or more of the numbers thought of are greater than 9, two cases must be distinguished: 1st, that where the number of the numbers thought of is odd; 2d, that where it is even.

In the first case, desire the person to tell the sums of the first and the second; of the second and the third; of the third and the fourth, &c., as far as the last, and then the sum of the first and the last. Having written down these sums in order, add together all those the places of which are odd, as the first, the third, the fifth, &c.; make another sum of all those the places of which are even, as the second, the fourth, the sixth, &c.; subtract this sum from the former, and the remainder will be the double of the first number.

Let us suppose, for example, that the five following numbers are thought of, viz.: 3, 7, 13, 17, 20, which, when added two and two, as above, give 10, 20, 30, 37, 23: the sum of the first and third and fifth is 63; and that of the second and fourth is 57: if 57 be subtracted from 63, the remainder 6 will be the double of the first number 3. Now if 3 be taken from 10, the first of the sums, the remainder 7 will be the second number; and, by proceeding in the same manner, we may find all the rest.

In the second case, that is to say, when the number of the numbers thought of is even; ask, and write down as above, the sum of the first and the second; that of the second and third; and so on as before; but instead of the sum of the first and the last, take that of the second and the last; then add together those which stand in the even places, and form them into a new sum apart; add also those in the odd places, the first excepted, and subtract this sum from the former: the remainder will be the double of the second number; and if the second number thus found be subtracted from the sum of the first and second, the remainder will be the first number; if it be taken from that of the second and third, it will give the third; and so of the rest.

Let the numbers thought of be, for example, 3, 7, 13, 17: the sums formed as above are 10, 20, 30, 24: the sum of the second and fourth is 44, from which if 30, the third sum, be subtracted, the remainder will be 14, the double of 7, the second number. The first therefore is 3, the third 13, and the fourth 17.

PROBLEM III.

A person having in one hand an even number of shillings, and in the other an odd, to tell in which hand he has the even number.

Desire the person to multiply the number in the right hand by any even number whatever, such as 2; and that in the left by an odd number, as 3; then bid him add together the two products, and if the whole sum be odd, the even number of shillings will be in the right hand, and the odd number in the left; if the sum be even, the contrary will be the case.

Let us suppose, for example, that the person has 8 shillings in his right hand, and 7 in his left; 8 multiplied by 2 gives 16, and 7 multiplied by 3 gives 21; the sum of which, 37, is an odd number.

If the number in the right hand were 9, and that in the left 8, we should have $9 \times 2 = 18$, and $8 \times 3 = 24$; the sum of which two products is 42, an even number.

Investigation.—Let x represent the number in the left, and y that in the right hand, and let $2n$ and $2m + 1$ represent any even and odd numbers. Then $2ny + \overline{2m + 1}\,.\,x = 2\,.\,\overline{ny + mx} + x$ is the sum of the products directed to be taken. Now $2\,.\,\overline{ny + mx}$ is necessarily even. Therefore when the whole product is even, x, the remaining term, is also even; and when odd, x is odd, which is the rule.

PROBLEM IV.

A person having in one hand a piece of gold, and in the other a piece of silver, to tell in which hand he has the gold, and in which the silver.

For this purpose, some value, represented by an even number, such as 8, must be assigned to the gold, and a value represented by an odd number, such as 3, must be assigned to the silver: after which the operation is exactly the same as in the preceding example.

Remarks.—I. To conceal the artifice better, it will be sufficient to ask whether the sum of the two products can be halved without a remainder; for, in that case, the total will be even, and in the contrary case odd.

II. It may be readily seen that the pieces, instead of being in the two hands of the same person, may be supposed to be in the hands of two persons, one of whom has the even number, or piece of gold, and the other the odd number, or piece of silver. The same operations may then be performed in regard to these two persons as are performed in regard to the two hands of the same person, calling the one privately the right, and the other the left.

PROBLEM V.

The Game of the Ring.

This game is nothing else than an application of one of the methods employed to tell several numbers thought of, and should be performed in a company not exceeding 9, in order that it may be less complex. Desire any one of the company to take a ring, and to put it on any joint of whatever finger he may think proper. The question then is to tell what person has the ring, and on what hand, what finger, and what joint.

For this purpose, call the first person 1, the second 2, the third 3, and so on; also

call the right hand 1, and the left 2: the first finger of the hand, that is to say the thumb, must be denoted by 1, the second by 2, and so on to the little finger; and the first joint of each finger, or that next the extremity, must be called 1, the second 2, and the third 3.

Let is now suppose that the fifth person has taken the ring, and put it on the first joint of the fourth finger of his left hand. To resolve the problem, nothing is necessary but to discover these numbers 5, 2, 4, 1, which may be done in the following manner.

Desire some one to double the first number 5, which will give 10, and to subtract 1 from it; desire him to multiply 9, the remainder, by 5 which will give 45; to this product bid him add the second number 2, which will make 47, and then 5 which will make 52: desire him to double this number, and the result will be 104, and to subtract 1, which will leave 103. Desire him to multiply this remainder by 5, which will give 515, and to add to the product the third number 4, or that expressing the finger, which will give 519: then bid him add 5, which will make 524, and from 1048, the double of this sum, let him subtract 1, which will leave 1047: then desire him to multiply this remainder by 5, which will give 5235, and to add to this product 1, the fourth number, or that expressing the joint, which will make 5236; in the last place bid him again add 5, and the sum will be 5241, the figures of which will indicate, in order, the person who has the ring, and the hand, finger, and joint, on which it was put.

It is evident, that all these operations amount, in reality, to nothing else than multiplying by 10, the number which expresses the person; then adding that which expresses the hand; multiplying again by 10, and so on.* But as this artifice is too easily detected, it might be better to employ the method taught in Prob. II. No. 1, to discover any number of numbers thought of at pleasure; for, on account of the number which must be subtracted, the operation will be more difficult to be comprehended.

The problem might be proposed in the following manner, and be resolved by the same process.

Three or more persons having each selected a card, the number of the spots of which does not exceed 9, to tell the number of the spots of each.

Desire the first person to add 1 to double the number of the spots of his card; to multiply the sum by 5, and to add to the product the spots of the card of the second person: then desire him to double that sum; to add unity to it, to multiply the whole by 5, and to add to this product the spots of the card of the third person: by subtracting from the last result 55, if the number of the persons be 3; 555, if it be 4; 5555, if it be 5, the figures which compose the remainder will indicate, in order, the spots of the cards selected by each person.

This process may be demonstrated with as much ease as the former; let the numbers to be guessed, less than 10, be x, y, z: we confine ourselves to three, for the sake of brevity. If 1 be added to the double of the first number, we shall have $2x + 1$, and multiplying by 5, the product will be $10x + 5$; if the second number y be

* For the satisfaction and information of the reader, we shall here give the following demonstration. Let the four numbers to be guessed be x, y, z, u; according to the above method, we must double x, which will give $2x$; if 1 be then subtracted we shall have $2x - 1$, and multiplying by 5, the result will be $10x - 5$. If y, the second number, be added, we shall have $10x - 5 + y$, and 5 added to this sum will make $10x + y$, which being doubled will give $20x + 2y$; if 1 be subtracted, there will remain $20x + 2y - 1$, which multiplied by 5 will give $100x + 10y - 5$; to this product if the third number z, and 5 be added, the sum will be $100x + 10y + z$; and if unity be taken from the double of this sum, the result will be $200x + 20y + 2z - 1$; if we then multiply by 5, we shall have for product $1000x + 100y + 10z - 5$; and by adding 5 and the last number, u, the sum will be $1000x + 100y + 10z + u$. If x, y, z, u represent numbers, below 10, as 5, 2, 4, 1, the sum will be $5000 + 200 + 40 + 1$, or 5241. If the numbers were 9, 6, 5, 4, the sum for the same reason, would be 9654; which is a demonstration of the process above indicated.

F 2

added, the sum will be $10x + 5 + y$, and 1 added to the double will make $20x + 10 + 2y + 1$, which multiplied by 5 gives $100x + 50 + 10y + 5$; if we then add the third number z, we shall have $100x + 50 + 10y + 5 + z$, or $100x + 10y + z + 55$: if x, y, z are, for example, 5, 6, 7, this expression will be $567 + 55$, or 612. From this last sum therefore, if we deduct 55, the remainder will be 567, which indicates in order the three numbers to be guessed.

For the sake of brevity, we shall not give any other example, as the reader may recur to that before given in Prob. II.

PROBLEM VI.

To guess the number of spots on any card, which a person has drawn from a whole pack.

Take a whole pack, consisting of 52 cards, and desire some person in company to draw out any one at pleasure, without shewing it. Having assigned to the different cards their usual value, according to their spots, call the knave 11, the queen 12, and the king 13. Then add the spots of the first card to those of the second; the last sum to the spots of the third, and so on, always rejecting 13, and keeping the remainder to add to the following card. It may be readily seen that it is needless to reckon the kings, which are counted 13. If any spots remain at the last card, subtract them from 13, and the remainder will indicate the spots of the card that has been drawn; if the remainder be 11, it has been a knave; if 12 it has been a queen; but if nothing remains, it has been a king. The colour of the king may be known by examining which one among the cards is wanting.

If you are desirous of employing only 32 cards, the number used at present for piquet, when the cards are added as above directed, reject all the tens; then add 4 to the spots of the last card, and a sum will be obtained, which taken from 10, if it be less, or from 20 if it exceeds 10, the remainder will be the number of the card that has been drawn; so that if 2 remains, it has been a knave, if 3 a queen, if 4 a king, and so on.

If the pack be incomplete, attention must be paid to those deficient, in order that the number of the spots of all the cards wanting may be added to the last sum, after as many tens as possible have been subtracted from it; and the sum arising from this addition must, as before, be taken from 10 or 20, according as it is greater or less than 10. It is evident that by again looking at the cards, the one which has been drawn may be discovered.

The demonstration of this rule is as follows: since, in a complete pack of cards, there are 13 of each suit, the values of which are 1, 2, 3, &c., to 13, the sum of all the spots of each suit, calling the knave 11, the queen 12, and the king 13, is seven times 13 or 91, which is a multiple of 13; consequently the quadruple of this sum is a multiple of 13 also: if the spots then of all the cards be added together, always rejecting 13, we must at last find the remainder equal to nothing. It is therefore evident that if a card, the spots of which are less than 13, has been drawn from the pack, the difference between these spots and 13 will be what is wanting to complete that number: if at the end then, instead of reaching 13, we reach only 10, for example, it is evident that the card wanting is a three; and if we reach 13, it is also evident that the card wanting is one of those equivalent to 13, or a king.

If two cards have been drawn from the pack, we may tell, in like manner, the number of spots which they contain both together: that is, how much is wanting to reach 13, or that deficiency increased by 13; and to know which two, nothing is necessary but to count privately how many times 13 has been com-

pleted, for with the whole of the cards it ought to be counted 28 times: if it be counted therefore only 27 times, with a remainder, as 7 for example, the spots of the two cards drawn amount together to 6: if 13 be counted only 26 times, with the same remainder, it may be concluded that the two cards formed together 13+6, or 19.

The demonstration of the rule given when the same number of cards is used, as that employed for the game of piquet, viz. 32, calling the ace 1, the knave 2, the queen 3, the king 4, and assigning to the other cards the value of their spots, is attended with as little difficulty; for in each suit there are 44 spots, making altogether 176, which, as well as 44, is a multiple of 11; we may therefore always count to 11, rejecting 11, and the number wanting to reach 11, will be the value of the card which has been drawn.

But the same number 176, if 4 were added to it, would be a multiple of 10 or of 20; and hence a demonstration also of the method which has been taught.

PROBLEM VII.

A person having an equal number of counters, or pieces of money, in each hand, to find how many he has altogether.

Desire the person to convey any number, as 4, for example, from the one hand to the other, and then ask him how many times the less number is contained in the greater. Let us suppose that he says the one is triple of the other; and in this case multiply 4, the number of the counters conveyed from one hand into the other, by 3, and add to the product the same number 4, which will make 16. In the last place, from the number 3 subtract unity, and if 16 be divided by 2, the remainder, the quotient 8 will be the number contained in each hand, and consequently the whole number is 16.

Let us now suppose that when 4 counters are conveyed from one hand to the other, the less number is contained in the greater $2\frac{1}{3}$ times: in this case we must, as before, multiply 4 by $2\frac{1}{3}$, which will give $9\frac{1}{3}$; to which if four be added, we shall have $13\frac{1}{3}$, or $\frac{40}{3}$; if unity be then taken from $2\frac{1}{3}$, the remainder will be $1\frac{1}{3}$, or $\frac{4}{3}$, by which if $\frac{40}{3}$ be divided, the quotient, 10, will be the number of counters in each hand, as may be easily proved on trial.

Proof.—Let x be the number in each hand, a the number transferred from the one hand to the other, and $n =$ the multiple which the sum is of the remainder. The $x + a = n \,.\, \overline{x - a}$, or $\overline{n - 1} \,.\, x = \overline{n + 1} \,.\, a$; whence $x = \frac{n+1}{n-1} \,.\, a$; a general rule.

PROBLEM VIII.

Several cards being presented, in succession, to several persons, that they may each choose one at pleasure; to guess that which each has thought of.

Shew as many cards to each person as there are persons to choose; that is to say, 3 to each if there are 3 persons. When the first has thought of one, lay aside the three cards in which he has made his choice. Present the same number to the second person, to think of one, and lay aside the three cards in the like manner. Having done the same in regard to the third person, spread out the three first cards with their faces upwards, and place above them the next three cards, and above these the last three, that all the cards may thus be disposed in three heaps, each consisting of three cards. Then ask each person in which heap the card is which he thought of, and when this is known it will be easy to tell these cards, for that of the first person will be the first in the heap

to which it belongs; that of the second will be the second of the next heap, and that of the third will be the third of the last heap.

PROBLEM IX.

Three cards being presented to three persons, to guess that which each has chosen.

As it is necessary that the cards presented to the three persons should be distinguished, we shall call the first A, the second B, and the third C; but the three persons may be at liberty to choose any of them at pleasure. This choice, which is susceptible of six different varieties, having been made, give to the first person 12 counters, to the second 24, and to the third 36: then desire the first person to add together the half of the counters of the person who has chosen the card A, the third of those of the person who has chosen B, and the fourth part of those of the person who has chosen C, and ask the sum, which must be either 23 or 24; 25 or 27; 28 or 29, as in the following table:

First.	Second.	Third.	Sums.
12	24	36	
A	B	C	23
A	C	B	24
B	A	C	25
C	A	B	27
B	C	A	28
C	B	A	29

This table shews, that if the sum be 25, for example, the first person must have chosen the card B, the second the card A, and the third the card C; and that if it be 28, the first person must have chosen the card B, the second the card C, and the third the card A; and so of the rest.

PROBLEM X.

A person having drawn, from a complete pack of fifty-two cards, one, two, three, four, or more cards, to guess the whole number of the spots which they contain.

Assume any number whatever, such as 15, for example, greater than the number of the spots of the highest card, counting the knave 11, the queen 12, and the king 13, and desire the person to add as many cards from the pack, to the first card he has chosen, as will make up 15, counting the spots of that card; let him do the same thing in regard to the second, the third, the fourth, &c.; and then desire him to tell how many cards remain in the pack. When this is done, proceed as follows:

Multiply the above number 15, or any other that may have been assumed, by the number of cards drawn from the pack, which we shall here suppose to be 3; to the product, 45, add the number of these cards, which will give 48; subtract the 48 from 52, and take the remainder 4 from the cards left in the pack: the result will be the number of spots required.

Let us suppose, for example, that the person has drawn from the pack a 7, a 10, and a knave, which is equal to 11: to make up the number 15 with a 7, eight cards will be required; to make up the same number with a 10, will require five; and with the knave, which is equal to 11, four will be necessary. The sum of these three numbers, with the 3 cards, makes 20, and consequently 32 cards remain in the pack. To find the sum of the numbers 7, 10, 11, multiply 15 by 3, which will give 45; and if the number of the cards drawn from the pack be added, the sum will be 48, which taken from 52, leaves 4. If 4 then be subtracted from 32, the remainder, 28, will be the sum of the spots

contained on the three cards drawn from the pack, as may be easily proved by trial.

Another Example.—Let us suppose two cards only drawn from the pack, a 4 and a king, equal to 13; if cards be added to these to make up 15, there will remain in the pack 37 cards.

If 15 be multiplied by 2, the product will be 30, to which if 2, the number of the cards drawn from the pack, be added, we shall have 32; and if 32 be taken from 52, the remainder will be 20. In the last place, if 20 be subtracted from 37, the number of the cards left in the pack, the remainder, 17, will be the number of the spots of the 2 cards drawn from the pack.

Remarks.—I. If 4 or 5 cards are drawn from the pack, it may sometimes happen that a sufficient number will not be left to make up the number 15; but even in this case the operation may be still performed. For example, if 5 cards, the spots contained on which are 1, 2, 3, 4, 5, have been drawn; to complete with each of these cards the number 15 would require, together with the 5 cards, at least 65; but as there are only 52, there are consequently 13 too few. He who counts the pack must therefore say that 13 are wanting.

On the other hand, he who undertakes to tell the number of the spots, must multiply 15 by 5, which makes 75; and to this if 5, the number of the cards, be added, it will give 80; that is to say, 28 more than 52: if 13 then be subtracted from 28, the remainder 15 will be the number of the spots contained on these 5 cards.

But if we suppose that the cards left in the pack are, for example 22, which would be the case if the five cards drawn were the 8, 9, 10, knave $= 11$, and queen $= 12$, it would be necessary to add these 22 to the excess of 5 times $15 + 5$, over 52, that is to say to 28, and we should have 50 for the spots of these 5 cards, which is indeed the exact number of them.

II. If the pack consists not of 52 cards, but of 40, for example, there will still be no difference in the operation: the number of the cards, which remain of these 40, must be taken from the sum produced by multiplying the made up number by that of the cards drawn, and adding to the product the number of these cards.

Let us suppose, for example, that the cards drawn are 9, 10, 11, that the number to be made up is 12, and that the cards left in the pack are 31. Then $12 \times 3 = 36$, and 3 added for the 3 cards, makes 39, which subtracted from 40 leaves 1. If 1 then be taken from 31, the remainder 30 will be the number of the spots required.

III. Different numbers to be made up with the spots of each card chosen might be assumed; but the case would still be the same, only that it would be necessary to add these three numbers to that of the cards, instead of multiplying the same number by the number of cards drawn, and then adding the number of the cards. In this there is so little difficulty, that an example is not necessary.

IV. The demonstration of this method, which some of our readers perhaps may be desirous of seeing, is exceedingly simple, and is as follows. Let a be the number of cards in the pack, c the number to be made up by adding cards to the spots of each card drawn, and b the cards left in the pack; let x, y, z express the spots of the cards, which we shall here suppose to be 3, and we shall then have, for the number of the cards drawn, $c - x + c - y + c - z + 3$; which with the cards left in the pack b, must be equal to the whole pack. Then $3c + 3 - x - y - z + b = a$, or $x + y + z = 3c + 3 + b - a$, or $= b - (a - 3c - 3)$. But $x + y + z$ is the whole number of the spots; b is the number of cards left in the pack, and $a - 3c - 3$ is the whole number of cards in the pack, less the product of the number

to be completed by the number of the cards drawn, minus that number. Therefore, &c.

PROBLEM XI.

Three things being privately distributed to three persons; to guess that which each has got.

Let the three things be a ring, a shilling, and a glove. Call the ring A, the shilling E, and the glove I; and in your own mind distinguish the persons by calling them first, second, and third. Then take twenty-four counters, and give one of them to the first person, two to the second, and three to the third. Place the remaining 18 on the table, and then retire, that the three persons may distribute among themselves the three things proposed, without your observing them. When the distribution has been made, desire the person who has the ring to take from the 18 remaining counters as many as he has already; the one who has the shilling to take twice as many as he has already, and the person who has the glove to take four times as many; according to the above supposition then, the first person has taken 1, the second 4, and the third 12; consequently one counter only remains on the table. When this is done, you may return, and by the number left can discover what thing each has got, by employing the following words:

1 2 3 5 6 7
Par fer César jadis devint si grand prince.

To make use of these words, you must recollect that in all cases there can remain only 1 counter, or 2, 3, 5, 6, or 7, and never 4: it must be likewise observed that each syllable contains one of the vowels which we have made to represent the three things proposed, and that the above line must be considered as consisting only of six words: the first syllable of each word must also be supposed to represent the first person, and the second syllable the second. This being comprehended, if there remains only 1 counter you must employ the first word, or rather the two first syllables, *par fer*, the first of which, that containing A, shews that the first person has the ring, represented by A; and the second syllable, that containing E, shews that the second person has the shilling, represented by E; from which you may easily conclude that the third person has the glove. If two counters remain, you must take the second word *César*, the first syllable of which, containing E, will shew that the first person has the shilling, represented by E, and the second syllable, containing A, will indicate that the second person has the ring, represented by A: you may then easily conclude that the third person has the glove. In general, whatever number of counters remain, that word of the verse which is pointed out by the same number must be employed.

Remarks.—Instead of the above French verse, the following Latin one might be used.

1 2 3 5 6 7
Salve certa animæ semita vita quies.

This problem might be proposed in a manner somewhat different, and might be applied to more than three persons: those who are desirous of farther information on the subject, may consult Bachet in the 25th of his "Problèmes plaisants et delectables."

PROBLEM XII.

Several numbers being disposed in a circular form, according to their natural series, to tell that which any one has thought of.

The first ten cards of any suit, disposed in a circular form, as seen in the figure below, may be employed with great convenience for performing what is announced in this problem. The ace is here represented by the letter A annexed to 1, and the ten by the letter K joined to 10.

	2	3	4	
	B	C	D	
1 A				E 5
10 K				F 6
	I	H	G	
	9	8	7	

Having desired the person who has thought of a number or card to touch also any other number or card, bid him add to the number of the card touched the number of the cards employed, which in this case is 10. Then desire him to count that sum in an order contrary to that of the natural numbers, beginning at the card he touched, and assigning to that card the number of the one which he thought of; for by counting in this manner, he will end at the number or card which he thought of, and consequently you will easily know it.

Thus, for example, if the person has thought of the number 3, marked C, and has touched 6, marked F; if 10 be added to 6, it will make 16; if 16 be then counted* from F, the number touched, towards E, D, C, B, A, and so on in the retrograde order, counting 3, the number thought of, on F, 4 on E, 5 on D, 6 on C, and so round to 16, the number 16 will terminate on C, and shew that the person thought of 3, which corresponds to C.

Remarks.—1. A greater or less number of cards may be employed at pleasure. If there are 15 or 8 cards, 15 or 8 must be added to the number of the card touched.

2. To conceal the artifice better, you may invert the cards, so as to prevent the spots from being seen; but you must remember the natural series of the cards, and the place of the first number, or the ace, that you may know the number of the card touched, in order to find the one to which the person ought to count.

PROBLEM XIII.

Two persons agree to take alternately numbers less than a given number, for example 11, and to add them together till one of them has reached a certain sum, such as 100; by what means can one of them infallibly attain to that number before the other?

The whole artifice of this problem consists in immediately making choice of certain numbers, which we shall here point out. Subtract 11, for example, from 100, the number to be reached, as many times as possible, and the remainders will be 89, 78, 67, 56, 45, 34, 23, 12, and 1, which must be remembered; for he who by adding his number less than 11, to the sum of the preceding, shall count one of these numbers before his adversary, will infallibly win, without the other being able to prevent him.

These numbers may be found also, with still greater ease, by dividing 100

* It is to be observed that the person must not count this sum aloud, but privately in his own mind.

by 11, and adding 11 continually to 1, the remainder, which will give 1, 12, 23, 34, &c.

Let us suppose, for example, that the first person, who knows the game, takes 1 for his number: it is evident that his adversary, as he must count less than 11, can at most reach 11 by adding 10 to it. The first will then take 1, which will make 12; if the second takes 8, which will make 20, the first will take 3, which will make 23; and proceeding in this manner successively he will first reach 34, 45, 56, 67, 78, 89. When he attains to the last number it will be impossible for the second to prevent him from getting first to 100; for whatever number the second takes, he can attain only to 99, after which the first may say, "and 1 makes 100." If the second takes 1 after 89, it will make 90; and his adversary may finish by saying, "and 10 make 100."

It is evident that when two persons are equally well acquainted with the game, he who begins must necessarily win.

But if the one knows the game and the other does not, the latter, though first, may not win; for he will think it highly advantageous to take the greatest number possible, that is to say 10; and in that case the other, acquainted with the nicety of the game, will take 2, which with 10 will make 12; one of the numbers he ought to secure. But he may even neglect this advantage, and take only 1 to make 11; for the first will probably still take 10, which will make 21, and the second may then take 2, which will make 23; he may then wait a little longer to get hold of some of the following numbers 34, 45, 56, &c.

If the first is desirous to win, the least number proposed must not be a measure of the greater; for in that case the first would have no infallible rule to direct him in his operations. For example, if 10, which measures 100, were assumed, instead of 11, by subtracting 10 from 100 as many times as possible, we should have the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, the first of which, 10, could not be taken by the first; for being obliged to employ a number less than 10, if the second were as well acquainted with the game, he might take the complement to 10; and would thus have an infallible rule for winning.

PROBLEM XIV.

Sixteen counters being disposed in two rows, to find that which a person has thought of.

The counters being arranged in two rows, as A and B, desire the person to think of one, and to observe well in which row it is.

A	B	C	B	D	E	B	F	H	B	I
o	o	o	o	o	o	o	o	o	o	*
o	o	o	o	o	*	o	o	o	o	o
o	o	*	o	o	o	o	o	o	o	o
o	o	o	o	o	o	o	o	o	o	o
*	o		o			o			o	
o	o		o			o			o	
o	o		o			o			o	
o	o		o			o			o	

Let us suppose that the counter thought of is in the row A; take up that whole row, in the order in which it stands, and dispose it in two rows C and D, on the right and left of the row B; but in arranging them, take care that the first of the row A may be the first of the row C; the second of the row A the first of the row D; the third of the row A, the second of the row C, and so on; then ask again in which of the vertical rows, C or D, the counter thought of is. Suppose it to be in C: take up that row as well as the row D, putting the last at the end of the first, without deranging the order of the counters, and, observing the rule already

given, form them into two other rows, as seen at E and F; then ask, as before, in which row the counter thought of is. Let us suppose it to be in E: take up this row, and the row F, as above directed, and form them into two new rows, on the right and left of B. After these operations, the counter thought of must be the first of one of the perpendicular rows, H and I; if you therefore ask in which row it is, you may easily point it out; and as it is here supposed that each of the counters has some distinguishing mark, you may desire them to be mixed together, and still be able to tell the number thought of, by observing the mark.

It may be readily seen that, instead of counters, cards may be employed; and when you have discovered, by the above means, the one thought of, you may cause them to be mixed, which will better conceal the artifice.

Remark.—If a greater number of counters or cards, arranged in two vertical rows, be supposed, the counter or card thought of will not necessarily be the first in the row to which it belongs, after the third transposition: if there be 32 counters or cards, four transpositions will be necessary; if there are 64, five; and so on, before it can be said, with confidence, that the counter or card thought of occupies the first place in its row; for if this counter or card were at the bottom of the perpendicular row A, supposing 16 counters in each row, or 32 altogether, it would not arrive at the first place till after four transpositions: if there were 64, or 32 in each row, it would require five; and so on, as may be easily proved by trial.

PROBLEM XV.

A certain number of cards being shewn to a person, to guess that which he has thought of.

To perform this trick, the number of the cards must be divisible by 3; and to do it with more convenience, the number must be odd.

The first condition, at least, being supposed, desire the person to think of a card; then place the cards on the table with their faces downward; and, taking them up in order, arrange them in three heaps, with their faces upward, and in such a manner that the first card of the packet shall be the first of the first heap; the second the first of the second, and the third the first of the third; the fourth the second of the first, and so on. When the heaps are completed, ask the person in which heap is the card thought of, and when told, place that containing the card thought of in the middle; then turning up the packet, form three heaps, as before, and again ask in which is the card thought of. Place the heap containing the card thought of still in the middle, and, having formed three new heaps, ask which of them contains the card thought of. When this is known, place it as before between the other two; and again form three heaps, asking the same question. Then take up the heaps for the last time; put that containing the card thought of in the middle, and placing the packet on the table, with the faces of the cards downward, turn up the cards till you count half the number of those contained in the packet; 12 for example, if there be 24, in which case the 12th card will be the one the person thought of.

If the number of the cards be, at the same time, odd, and divisible by 3, as 15, 21, 27, &c., the trick will become much easier; for the card thought of will always be that in the middle of the heap in which it is found the third time; so that it may be easily distinguished without counting the cards; nothing will be necessary for this purpose, but to remember, while you are forming the heaps the third time, the card which is the middle one of each. Let us suppose, for example, that the middle card of the first heap is the ace of hearts; that of the second the king of hearts, and that of the third the knave of spades; it is evident, if you are told that the heap containing the required card is the third, that this card must be the knave of spades. You may therefore cause the cards to be shuffled, without touching them any more,

and then, looking them over for the sake of form, may name the knave of spades when it occurs.

PROBLEM XVI.

Fifteen Christians and fifteen Turks being at sea in the same vessel, a dreadful storm came on, which obliged them to throw all their merchandise overboard; this however not being sufficient to lighten the ship, the captain informed them that there was no possibility of its being saved, unless half the passengers were thrown overboard also. Having therefore caused them all to arrange themselves in a row, by counting from 9 to 9, and throwing every ninth person into the sea, beginning again at the first of the row when it had been counted to the end, it was found that after fifteen persons had been thrown overboard, the fifteen Christians remained. How did the captain arrange these thirty persons so as to save the Christians?

The method of arranging the thirty persons may be deduced from these two French verses:

Mort, tu ne failliras pas
En me livrant le trepas.

Or from the following Latin one, which is not so bad of its kind:

Populeam virgam mater regina ferebat.

Attention must be paid to the vowels A, E, I, O, U, contained in the syllables of these verses; observing that A is equal to 1, E to 2, I to 3, O to 4, and U to 5. You must begin then by arranging 4 Christians together, because the vowel in the first syllable is O; then five Turks, because the vowel in the second syllable is U; and so on to the end. By proceeding in this manner, it will be found, taking every ninth person circularly, that is to say, beginning at the first of the row, after it is ended, that the lot will fall entirely on the Turks.

The solution of this problem may be easily extended still farther. Let it be required, for example, to make the lot fall upon 10 persons in 40, counting from 12 to 12. Arrange 40 ciphers in a circular form, as below;

```
              † †   †   †
      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0                                  0
  1 0                                   0
    0                                   0 †
     0                                 0 †
      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          † † †                   †
```

Then, beginning at the first, mark every twelfth one with a cross; continue in this manner, taking care to pass over those already crossed, still proceeding circularly, till the required number of places has been marked; if you then count the places of the marked ciphers, those on which the lot falls will be easily known: in the present case they are the 7th, the 8th, the 10th, the 12th, the 21st, the 22d, the 24th, the 34th, the 35th, and the 36th.

A captain, obliged to decimate his company, might employ this expedient, to make the lot fall upon those most culpable.

It is related that Josephus, the historian, saved his life by means of this expedient. Having fled for shelter to a cavern, with forty other Jews, after Jotapat had been taken by the Romans, his companions resolved to kill each other rather than surrender. Josephus tried to dissuade them from their horrid purpose, but not being able to succeed, he pretended to coincide with their wishes, and retaining the authority he had over them as their chief, to avoid the disorder which would necessarily be

the consequence of this cruel execution, if they should kill each other at random, he prevailed on them to arrange themselves in order, and, beginning to count from one end to a certain number, to put to death the person on whom that number should fall, until there remained only one, who should kill himself. Having all agreed to this proposal, Josephus arranged them in such a manner, and placed himself in such a position, that when the slaughter had been continued to the end, he remained with only one more person, whom he persuaded to live.

Such is the story related of Josephus by Hegesippus; but we are far from warranting the truth of it. However, by applying to this case the method above indicated, and supposing that every third person was to be killed, it will be found that the two last places on which the lot fell were the 16th and 31st; so that Josephus must have placed himself in one of these, and the person he was desirous of saving in the other.

PROBLEM XVII.

A man has a wolf, a goat, and a cabbage, to carry over a river; but being obliged to transport them one by one on account of the smallness of the boat, in what manner is this to be done, that the wolf may not be left with the goat, nor the goat with the cabbage?

He must first carry over the goat, and then return for the wolf; when he carries over the wolf, he must take back with him the goat, and leave it, in order to carry over the cabbage; he may then return and carry over the goat. By these means, the wolf will never be left with the goat, nor the goat with the cabbage, but when the boatman is present.

PROBLEM XVIII.

Three jealous husbands, with their wives, having to cross a river at a ferry, find a boat without a boatman; but the boat is so small that it can contain no more than two of them at once. How can these six persons cross the river, two and two, so that none of the women shall be left in company with any of the men, unless when her husband is present?

The solution of this problem is contained in the two following Latin distichs:

> It duplex mulier, redit una, vehitque manentem,
> Itque una; utuntur tunc duo puppe viri.
> Par vadit et rèdeunt bini, mulierque sororem
> Advehit; ad propriam fine maritus abit.

That is: "Two women cross first, and one of them, rowing back the boat, carries over the third woman. One of the three women then returns with the boat, and remaining, suffers the two men, whose wives have crossed, to go over in the boat. One of the men then carries back his wife, and leaving her on the bank, rows over the third man. In the last place, the woman who had crossed enters the boat, and returning twice, carries over the other two women."

This question is proposed also under the title of the three masters and the three valets. The masters agree very well, and the valets also; but none of the masters can endure the valets of the other two; so that if any one of them were left with any of the other two valets, in the absence of his master, he would infallibly cane him.

PROBLEM XIX.

In what manner can counters be disposed in the eight external cells of a square, so that there may be always 9 in each row, and yet the whole number shall vary from 20 to 32?

Ozanam proposed this problem in the following manner, with a view no doubt to excite the curiosity of his readers:

A certain convent consisted of nine cells, one of which in the middle was occupied by a blind abbess, and the rest by her nuns. The good abbess, to assure herself that the nuns did not violate their vows, visited all the cells, and finding 3 nuns in each, which made 9 in every row, retired to rest. Four nuns however went out, and the abbess returning at midnight to count them, still found 9 in each row, and therefore retired as before. The 4 nuns then came back, each with a gallant, and the abbess on paying them another visit, having again counted 9 persons in each row, entertained no suspicion of what had taken place. But 4 more men were introduced, and the abbess again counting 9 persons in each row, retired in the full persuasion that no one had either gone out or come in. How was all this possible?

This problem may be easily solved by inspecting the four following figures: the first of which represents the original disposition of the counters in the cells of the square; the second that of the same counters when 4 are taken away; the third the manner in which they must be disposed when these 4 are brought back with 4 others; and the fourth that of the same counters with the addition of 4 more. It is here evident that there are always 9 in each external row; and yet, in the first case, the whole number is 24, while in the second it is 20, in the third 28, and in the fourth 32.

I.

3	3	3
3		3
3		3

II.

4	1	4
1		1
4	1	4

III.

2	5	2
5		5
2	5	2

IV.

1	7	1
7		7
1	7	1

It would seem that Ozanam had not observed that these variations might have been carried still farther: that four men more might have been introduced into the convent, without the abbess perceiving it; and that all the men might have afterwards gone out with six nuns, so as to leave only 18, instead of the 24 who were in the cells at first. The possibility of this will appear by inspecting the two following figures.

V.

0	9	0
9		9
0	9	0

VI.

5	0	4
0		0
4	0	5

It is almost needless to explain in what manner the illusion of the good abbess arose. It is because the numbers in the angular cells of the square were counted twice; these cells being common to two rows. The more therefore the angular cells are filled, by emptying those in the middle of each band, these double enumerations become greater; on which account the number, though diminished, appears always to be the same; and the contrary is the case in proportion as the middle cells are filled by emptying the angular ones, which renders it necessary to add some units to have 9 in each band.

PROBLEM XX.

A gentleman has a bottle, containing 8 pints, of choice wine, and wishes to make a present of one half of it to a friend; but as he has nothing to measure it, except two other bottles, one capable of containing 5 and the other 3 pints, how must he manage, so as to put exactly 4 pints into the bottle capable of containing 5?

To enable us to resolve this problem we shall call the bottle containing the 8 pints, A; that of 5 pints, B; and that of 3 pints, C; supposing that there are 8 pints of wine in the bottle A, and that the other two are empty, as seen at D. Having filled the bottle B with wine from the bottle A, in which there will remain no more than 3 pints, as seen at E, fill the bottle C from B, and consequently there will remain only 2 pints in the latter, as seen at F: then pour the wine of C into A, which will thus contain 6 pints, as seen at G, and pour the two pints of B into C, as seen at H. In the last place, having filled the bottle B from the bottle A, in which there will remain only 1 pint, as seen at I, fill up C from B, in which there will remain 4 pints, as seen at K; and thus the problem is solved.

	8	5	3
	A	B	C
D	8	0	0
E	3	5	0
F	3	2	3
G	6	2	0
H	6	0	2
I	1	5	2
K	1	4	3

Remark.—If you are desirous of making the four pints of wine remain in the bottle A, which we have supposed to be filled with 8 pints, instead of remaining in the bottle B, fill the bottle C with wine from the bottle A, in which there will remain only 5 pints, as seen at D; and pour 3 pints of C into B, which will consequently contain 3 pints, as seen at E: having then filled C from A, in which there will remain no more than 2 pints, as seen at F; fill up B from C, which will thus contain only 1 pint, as seen at G. In the last place, having poured the wine of the bottle B into the bottle A, which will thus have 7 pints, as seen at H; pour the pint of wine which is in C into B, consequently the latter will contain 1 pint, as seen at I; and then fill up C from A, in which there will remain only 4 pints, as was proposed, and as seen at K.

	8	5	3
	A	B	C
	8	0	0
D	5	0	3
E	5	3	0
F	2	3	3
G	2	5	1
H	7	0	1
I	7	1	0
K	4	1	3

PROBLEM XXI.

A gentleman has a bottle containing 12 pints of wine, 6 of which he is desirous of giving to a friend; but as he has nothing to measure it, except two other bottles, one of 7 pints, and the other of 5, how must he manage, to have the 6 pints in the bottle capable of containing 7 pints?

This problem is of the same nature as the preceding, and may be solved in the like manner. Let T represent the twelve-pint, S the seven-pint, and F the five-pint bottle. The bottle T is full, and the other two, S and F, are empty, as seen at G. Fill the bottle F with wine from T, so that T shall contain only 7 pints, as seen at H; then pour into S the wine contained in F, which will remain empty, and the bottle S will contain 5 pints, as seen at I; having filled F from T, the latter will contain only 2 pints, the bottle S will contain 5, and the bottle F will be full, as seen at K; in the next place fill S from F, and T will still contain only 2 pints, while S contains 7, and F 3, as seen at L; then empty S into T, and F into S, by which means T will contain 9 pints, and S 3, F remaining empty, as seen at M: fill F from T, and pour from F into S as much as will fill it, so that there will then be 4 pints in T, 7 pints in S, and 1 pint in F, as seen at N: pour the 7 pints from S into T, and the pint contained in F into S, after which T will contain 11 pints, S 1, and F will be empty, as seen at O. In the last place, having filled the five-pint bottle F from the bottle T, and poured

	12	7	5
	T	S	F
G	12	0	0
H	7	0	5
I	7	5	0
K	2	5	5
L	2	7	3
M	9	3	0
N	4	7	1
O	11	1	0
P	6	6	0

these 5 pints from F into S, which already contains 1, it will be found that T contains 6 pints, and that S contains 6 also; so that the desired result has been obtained.

PROBLÉM XXII.

To make the knight move into all the squares of the chess board, in succession, without passing twice over the same.

As the reader perhaps may be unacquainted with the movement of the knight at the game of chess, we shall here explain it. The knight being placed in the square A, cannot move into any of those immediately surrounding it, as 1, 2, 3, 4, 5, 6, 7, 8; nor into the squares 9, 10, 11, 12, which are directly above or below, and on each side of it; nor into the squares 13, 14, 15, 16, which are in the diagonals; but only into one of those which, in the annexed figure, are empty.

13		10		14
	1	2	3	
9	8	A	4	11
	7	6	5	
16		12		15

Several eminent men have amused themselves with this problem, such as Montmort, Demoivre, and Mairan, and each of these has given a solution of it. In those of the two former, the knight is supposed to be placed at first in one of the angular squares of the chess board; in that of the third, he is supposed to begin to move from one of the four central squares; but in our opinion it was not known, till within these few years, that placing the knight in any square whatever, he may be made to traverse the whole chess board, and even in such a manner that, without returning the same way, he shall pass a second time over the board under the like conditions. For this last solution we are indebted to M. W——, a captain in the Kinski regiment of dragoons, in the imperial service.

We shall here give four tables, representing these four solutions, with an explanation and some remarks.

I. *M. Montmort.*

1	38	31	44	3	46	29	42
32	35	2	39	30	43	4	47
37	8	33	26	45	6	41	28
34	25	36	7	40	27	48	5
9	60	17	56	11	52	19	50
24	57	10	63	18	49	12	53
61	16	59	22	55	14	51	20
58	23	62	15	64	21	54	13

II. *Demoivre.*

34	49	22	11	36	39	24	1
21	10	35	50	23	12	37	40
48	33	62	57	38	25	2	13
9	20	51	54	63	60	41	26
32	47	58	61	56	53	14	3
19	8	55	52	59	64	27	22
46	31	6	17	44	29	4	15
7	18	45	30	5	16	43	28

III. *M. Mairan.*

40	9	26	53	42	7	64	29
25	52	41	8	27	30	43	6
10	39	24	57	54	63	28	31
23	56	51	60	1	44	5	62
50	11	38	55	58	61	32	45
37	22	59	48	19	2	15	4
12	49	20	35	14	17	46	33
21	36	13	18	47	34	3	16

IV. *M. W——.*

25	22	37	8	35	20	47	6
38	9	24	21	52	7	34	19
23	26	11	36	59	48	5	46
10	39	62	51	56	53	18	33
27	12	55	58	49	60	45	4
40	63	50	61	54	57	32	17
13	28	1	42	15	30	3	44
64	41	14	29	2	43	16	31

Of these four ways of resolving the problem, that of Demoivre is doubtless the easiest to be remembered; for the principle of his method consists in filling up, as much as possible, the two exterior bands, which form as it were a border, and not entering the third, till there is no other method of moving the knight from the place where he is, to one of the two first, a rule which in the clearest manner subjects the movement of the knight to a certain necessary progress, from his first step to the 50th, and beyond it; for from the cell marked 50 there is no choice in placing him, except on those marked 51 and 63; but the cell 51 being nearer the band, ought to be preferred, and then the movement must necessarily be through 52, 53, 54, 55, 56, 57, 58, 59, 60, 61. When he arrives at 61, it is a matter of indifference whether he be placed in the cell marked 64, for he may thence proceed to the last but one 63, and end at 62; or be placed in 62, to proceed to 63, and end at 64. It may therefore be said that the movement of the knight in this solution is almost constrained.

The case is not the same with the fourth, which it is difficult to practise in any other manner than from memory; but it is attended with one very great advantage, which is, that you may begin, as already said, at any cell at pleasure; because the author took the trouble to bring the knight at the conclusion to a place from which he can pass into the first. His movement therefore is in some measure circular, and interminable, by adhering to the condition of not passing twice over the same cell, till after 64 steps.

It may be readily seen, that to make the knight perform this movement without confusion, the cell he has quitted must be marked at each step. For this purpose a counter may be placed in each cell, and removed as the knight passes over it; or, what will be still better, a counter may be placed in each cell when he has passed it.

PROBLEM XXIII.

To distribute among 3 persons, 21 casks of wine, 7 of them full, 7 of them empty, and 7 half full, so that each shall have the same quantity of wine, and the same number of casks.

This problem admits of two solutions, which may be clearly comprehended by means of the two following tables.

	Persons.	full casks.	empty.	half full.
I.	1st	2	2	3
	2d	2	2	3
	3d	3	3	1

Persons.		full casks.	empty.	half full.
II.	1st	3	3	1
	2d	3	3	1
	3d	1	1	5

It is evident that, in these two combinations, each person will have 7 casks, and 3½ casks of wine.

But it may be easily seen that the whole number of the casks must be divisible by the number of persons, otherwise the thing required would be impossible.

It will be found, in like manner, that if 24 casks were to be divided among 3 persons, under the same conditions, we should have three different solutions, as follow:

Persons.		full casks.	empty.	half full.
I.	1st	3	3	2
	2d	3	3	2
	3d	2	2	4
II.	1st	2	2	4
	2d	2	2	4
	3d	4	4	0
III.	1st	1	1	6
	2d	3	3	2
	3d	4	4	0

If there should be 27 casks to be divided, there would be three solutions also:

Persons.		full casks.	empty.	half full.
I.	1st	3	3	3
	2d	3	3	3
	3d	3	3	3
II.	1st	1	1	7
	2d	4	4	1
	3d	4	4	1
III.	1st	2	2	5
	2d	3	3	3
	3d	4	4	1

CHAPTER XI.

CONTAINING SOME CURIOUS ARITHMETICAL PROBLEMS.

PROBLEM I.

A gentleman, in his will, gave orders that his property should be divided among his children in the following manner:—The eldest to take from the whole £1000, and the 7th part of what remained; the second £2000, and the 7th part of the remainder; the third £3000, and the 7th part of what was left; and so on to the last, always increasing by £1000. The children having followed the disposition of the testator, it was found that they had each got an equal portion: how many children were there, what was the father's property, and to how much did the share of each child amount?

It will be found by analysis, that the father's property was £36000; that there were 6 children, and that the share of each was £6000.

Thus, if the first takes £1000, the remainder of the property will be £35000, the 7th part of which £5000, together with £1000, makes £6000. The remainder, after deducting the first child's portion, is £30000, from which if the second takes £2000, the remainder will be £28000, but the 7th part of this sum is £4000, which if added to the above £2000, will make £6000, and so on.

PROBLEM II.

A gentleman meeting a certain number of beggars, and being desirous to distribute among them all the money he had about him, finds that if he gave sixpence to each he would have 2s. too little; but that by giving each a groat, he would have 2s. 8d. over: how many beggars were there, and what sum had the gentleman in his pocket?

There were 28 beggars, and the gentleman had in his pocket 12 shillings; for if 28 be multiplied by 6, the product will be 168, from which if 2 shillings or 24 pence be subtracted, as he wanted 24 pence to be able to give each sixpence, the remainder will be 144 pence = 12 shillings; but by giving each of the beggars 4 pence, he had occasion only for 112 pence, or 4 times 28; consequently he had 32 pence, or 2s. 8d. remaining.

PROBLEM III.

A gentleman purchased for £110. a lot of wine, consisting of 100 bottles of Burgundy, and 80 of Champagne; and another purchased at the same price, for the sum of £95, 85 bottles of the former, and 70 of the latter: what was the price of each kind of wine?

It will be found that the Burgundy cost 10s. per bottle, and the Champagne 15s. as may be easily proved.

PROBLEM IV.

A gentleman, on his death-bed, gave orders in his will, that if his lady, who was then pregnant, brought forth a son, he should inherit two thirds of his property, and the widow the other third; but that if she brought forth a daughter, the mother should inherit two thirds, and the daughter one third; the lady however was delivered of two children, a boy and a girl, what was the portion of each?

The only difficulty in this problem is to discover, in what manner the testator would have disposed of his property, had he foreseen that his lady would have been delivered of two children. It has generally been explained in the following manner: As the testator desired that if his wife brought forth a boy, the latter should have two thirds of his property, and the mother the other third, it hence follows that his intention was to give the son a portion double to that of the mother; and as he gave orders that in case she brought forth a daughter, the mother was to have two thirds of the property, and the daughter the other, there is reason to conclude that he intended the portion of the mother to be double that of the daughter. Consequently, to combine these two conditions, the property must be divided in such a manner, that the son may have twice as much as the mother, and the mother twice as much as the daughter. If we therefore suppose the property to be £30000, the share of the son will be £17142$\frac{6}{7}$, that of the mother £8571$\frac{3}{7}$, and that of the daughter £4285$\frac{5}{7}$.

As a supplement to this problem, another is differently proposed. In case the mother should be delivered of two sons and a daughter, in what manner must the property be divided?

In our opinion, no other answer can be given to this question, than what would be given by the gentlemen of the bar; that the will, in such a case, would be void; for a child having been omitted in the will, all the laws with which we are acquainted would pronounce its nullity: 1st, because the law is precise; and 2d, because it is impossible to determine what would have been the disposition of the testator, had he had two sons, or had he foreseen that his wife would be delivered of two.

PROBLEM V.

A brazen lion, placed in the middle of a reservoir, throws out water from its mouth, its eyes, and its right foot. When the water flows from its mouth alone, it fills the reservoir in 6 hours; from the right eye it fills it in 2 days; from the left eye in 3, and from the foot in 4. In what time will the bason be filled by the water flowing from all these apertures at once?

To solve this problem it must be observed, that as the lion, when it throws the water from its mouth, fills the bason in 6 hours, it can fill $\frac{1}{6}$ of it in an hour; and that as it fills it in 2 days when it throws the water from its right eye, it can fill $\frac{1}{48}$ of it in an hour. It will be found, in like manner, that it can fill $\frac{1}{72}$ of it in an hour when the water flows from its left eye, and $\frac{1}{96}$ when it flows from its foot. By throwing the waters from all these apertures at once, it furnishes in an hour $\frac{1}{6} + \frac{1}{48} + \frac{1}{72} + \frac{1}{96}$, and these fractions added together are equal to $\frac{61}{288}$. We must therefore make the following proportion: If $\frac{61}{288}$ are filled in one hour or 60 minutes, how many minutes will the whole bason, or $\frac{288}{288}$, require: or as 61 is to 288, so is 1 hour, to the answer, which will be 4h. 43m. $16\frac{44}{61}$ seconds.

PROBLEM VI.

A mule and an ass travelling together, the ass began to complain that her burthen was too heavy. "Lazy animal," said the mule, "you have little reason to complain; for if I take one of your bags, I shall have twice as many as you, and if I give you one of mine we shall then have only an equal number." With how many bags was each loaded?

This problem, which is one of those commonly proposed to beginners in Algebra, is taken from a collection of Greek epigrams, known under the name of the Anthology; and has been translated, almost literally, into Latin as follows:

Una cum mulo vinum portabat asella,
Atque suo graviter sub pondere pressa gemebat.
Talibus at dictis mox increpat ipse gementem:
Mater, quid luges, teneræ de more puellæ?
Dupla tuis, si des mensuram, pondera gesto:
At si mensuram accipias, æqualia porto.
Dic mihi mensuras, sapiens geometer, istas?

The analysis of this problem has also been expressed in indifferent Latin verses, which we shall here give, to gratify the reader's curiosity.

Unam asina accipiens, amittens mulam et unam
Si fiant æqui, certe utrique ante duobus
Distabant a se. Accipiat si mulus at unam,
Amittatque asina unam, tunc distantia fiet
Inter eos quatuor. Muli at cum pondera dupla
Sint asinæ, huic simplex, mulo est distantia dupla.
Ergo habet hæc quatuor tantum, mulusque habet octo,
Unam asinæ si addas, si reddat mulus et unam,
Mensuras quinque hæc et septem mulus habebunt.

That is: "As the mule and the ass will both have equal burthens when the former gives one of his measures to the latter, it is evident that the difference between the measures which they carry is equal to 2. Now if the mule receives one from the ass, the difference will be 4; but in that case the mule will have double the number of measures that the ass has; consequently the mule will have 8, and the ass 4. If the mule then gives one to the ass, the latter will have 5 and the former 7:" these were the number of the measures with which each was loaded, and which solve the problem.

This problem, which might be expressed in a great variety of forms, is not the only

one furnished by the Greek Anthology. The following are a few more, translated into the Latin verse by Bachet de Meziriac, who inserted them in a note to one of the problems of Diophantus.

I.

Aurea mala ferunt Charites, æqualia cuique
Mala insunt calatho; Musarum bis obvia turba
Mala petunt, Charites cunctis æqualia donant;
Tunc æqualia tres contingit habere, novemque:
Dic quantum dederint numerus sit ut omnibus idem?

That is: "The three Graces, carrying each an equal number of oranges, were met by the nine Muses, who asked for some of them; and each Grace having given to each Muse the same number, it was then found that they had all equal shares: How many had the Graces at first?"

The least number which will answer this problem is 12; for if we suppose that each Grace gave one to each Muse, the latter would each have three; and there would remain 3 to each Grace.

The numbers 24, 36, 48, &c., will also answer the question; and after the distribution is made, each of the Graces and Muses will have 6, or 9, or 12, &c.

II.

Dic, Heliconiadum decus O sublime Sororum,
Pythagora, tua quot tyrones tecta frequentent,
Qui, sub te, sophiæ sudant in agone magistro?
Dicam; tuque animo mea dicta, Polycrates, hauri;
Dimidia horum pars præclara mathemata discit,
Quarta immortalem naturam nosse laborat;
Septima, sed tacite, sedet atque audita revolvit;
Tres sunt fœminei sexus.

"Tell me, illustrious Pythagoras, how many pupils frequent thy school? One half, replied the philosopher, study mathematics, one fourth natural philosophy, one seventh observe silence, and there are three females besides."

The question here is, to find a number, the $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{7}$ of which $+ 3$, shall be equal to that number. It may be easily replied that this number is 28.

III.

Dic quota nunc hora est? Superest tantum ecce diei
Quantum bis gemini exacta de luce trientes.

"A person being asked what o'clock it was, replied, the hours of the day which remain, are equal to $\frac{4}{3}$ of those elapsed."

If we divide the day, as the ancients did, into 12 equal portions, the question will be to divide that number into two such parts, that $\frac{4}{3}$ of the first may be equal to the second; in this case the result will be $5\frac{1}{7}$ for the number of the hours elapsed; and consequently for the remainder of the day $6\frac{6}{7}$ hours.

IV.

Hic Diophantus habet tumulum, qui tempora vitæ
Illius mira denotat arte tibi.
Egit sextantem juvenis, lanugine malas
Vestire hinc cœpit parte duodecima.
Septante uxori post hæc sociatur et anno
Formosus quinto nascitur inde puer.
Semissem ætatis postquam attigit ille paternæ,
Infelix subita morte peremptus obit.
Quatuor æstates genitor lugere superstes
Cogitur, hinc annos illius assequere.

"This is the epitaph of the celebrated mathematician Diophantus. It tells us that Diophantus passed the sixth part of his life in childhood, and the twelfth part in the

state of youth; that after a seventh part of his life and five years more were elapsed, he had a son, who died when he had attained to half the age of his father, and that the latter survived him only four years."

To resolve this problem, we must find a number, the $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{7}$, and $\frac{1}{2}$ of which $+ 5 + 4$, shall be equal to the number itself. This number is 84.

PROBLEM VII.

The sum of £500 having been divided among four persons, it was found that the shares of the first two amounted to £285; those of the second and third to £220; those of the third and fourth to £215; and that the share of the first was to that of the last as 4 to 3. What was the share of each?

The solution of this problem is exceedingly easy. The first had £160, the second £125, the third £95, and the fourth £120.

It is to be observed, that without the last-mentioned condition, or a fourth one of some kind or other, the problem would be indeterminate; that is to say, would be susceptible of a great many answers: the last condition however limits it to one only.

PROBLEM VIII.

A labourer hired himself to a gentleman on the following conditions: for every day he worked he was to receive 2s. 6d.; but for every day he remained idle he was to forfeit 1s. 3d.: after 40 days' service he had to receive £2. 15s. How many days did he work, and how wany remain idle?

He worked 28 days of the 40, and remained idle 12.

PROBLEM IX.

A bill of exchange, of £2000 was paid with half-guineas and crowns; and the number of the pieces of money amounted to 4700. How many of each sort were employed?

There were 3000 half guineas and 1700 crowns.

The solution of this and that of the preceding problem are left as exercises for the young student.

PROBLEM X.

A gentleman, having lost his purse, could not tell the exact sum it contained, but recollected that when he counted the pieces two by two, or three by three, or five by five, there always remained one; and that when he counted them seven by seven, there remained nothing. What was the number of pieces in his purse?

It may be readily seen that, to solve this problem, nothing is necessary but to find a number which when divided by 7 shall leave no remainder; and which when divided by 2, 3, 5, shall always leave 1. Several methods may be employed for this purpose; but the simplest is as follows:

Since nothing remains when the pieces are counted seven by seven, the number of them is evidently some multiple of 7; and since 1 remains when they are counted two by two, the number must be an odd multiple: it must therefore be some of the series 7, 21, 35, 49, 63, 77, 91, 105, &c.

This number also, when divided by 3, must leave unity; but in the above series, 7, 49, and 91, which increase arithmetically, their difference being 42, are the only numbers that have the above property. It appears likewise, that if 91 be divided by 5, there will remain 1; and we may thence conclude that the first number which

answers the question is 91: for it is a multiple of 7, and being divided by 2, 3, or 5, the remainder is alwys 1.

Several more numbers, which answer this question, may be found by the following means: continue the above progression, in this manner: 7, 49, 91, 133, 175, 217, 259, 301, until you find another term divisible by 5, that leaves unity; this term will be 301, and will also answer the conditions of the problem; but the difference between it and 91 is 210, from which it may be concluded, that if we form the progression:

91, 301, 511, 721, 931, 1141, &c.

all these numbers will answer the conditions of the problem also.

It would therefore be still uncertain what money was in the purse, unless the owner could tell nearly the sum it contained. Thus, for example, if he should say that there were about 500 pieces in it, we might easily tell him that the number was 511.

Let us now suppose that the owner had said, that when he counted the pieces two by two there remained 1; that when he counted them three by three there remained 2; four by four, 3; five by five, 4; six by six, 5; and, in the last place, that when he counted them seven by seven, nothing remained.

It is here evident that the number, as before, must be an odd multiple of 7, and consequently one of the series 7, 21, 35, 49, 63, 77, 91, 105, &c. But the numbers 35 and 77, of this series, answer the condition of leaving 2 as a remainder when divided by 3, and their difference is 42. For this reason we must form a new arithmetical progression, the difference of which is 42, viz.

35, 77, 119, 161, 203, 245, 287, &c.

We must then seek for two numbers in it, which when divided by 4 shall leave 3 as remainder. Of this kind are the numbers 35, 119, 203, 287; and therefore we must form a new progression, the difference of the terms of which is 84.

35, 119, 203, 287, 371, 455, 539, 623, &c.

In this new progression we must seek for two terms, which when divided by 5, shall leave 4; and it will be readily seen that these numbers are 119 and 539, the difference of which is 420. A series of terms therefore which answer all the conditions of the problem except 1, is

119, 539, 959, 1379, 1799, 2219, 2639, &c.

But the last condition of the problem is, that the required number, when divided by 6, leaves 5 as remainder. This property belongs to 119, 959, 1799, &c., always adding 840; consequently the number sought is one of those in that progression. For this reason, as soon as we know nearly within what limits it is contained, we shall be able to determine it.

If the owner therefore of the purse had said, that it contained about 100 pieces, the number required would be 119; if he had said there were nearly 1000, it would be 959, &c.

Remark.—The solution of this problem, according to the method taught by Ozanam, would be imperfect. For after finding the smallest number which answers the conditions of the problem, viz. 119, he would merely say, that to obtain the other numbers which answer them, the numbers 2, 3, 4, 5, 6, 7, ought to be successively multiplied together, and their product 5040 added to 119, the first number found: this would give the number 5159, which would answer the proposed conditions also. But it may be readily seen, that there are several other numbers, between 119 and 5159, which answer these conditions, viz. 959, 1799, 2639, 3479, 4319.

In treating of chronology, we shall give the solution of another problem of the same kind; viz., To find any year of the Julian period, the golden number, cycle of the sun, and cycle of indiction, for that year, being given.

PROBLEM XI.

A sum of money, placed out at a certain interest, increased in 8 months to £3616. 13s. 4d.; and in two years and a half it amounted to £3937. 10s. What was the original capital, and at what rate of interest was it placed out?

That young algebraists may have an opportunity of exercising their own ingenuity, we shall here give the answer only of this problem. By employing the proper means of analysis, they will find that if $r =$ the interest of one pound for a year, a the first amount and b the second, that $r = \frac{6 . \overline{b-a}}{15\,a - 4\,b}$, which, with the given value of a and b, gives $r = \frac{1}{20}$th of a pound, or the rate £5. per cent. per annum; and hence the capital is easily found to be £3500.

PROBLEM XII.

Three women went to market to sell eggs; the first of whom sold 10, the second 25, and the third 30, all at the same price. As they were returning, they began to reckon how much money they carried back, and it was found that each had the same sum. How many eggs did they sell, and at what price?

It is evident that, to make what is announced in this problem possible, these women must have sold their eggs at two different times, and at different prices; for if the one who had the least number of eggs sold a very small number at the lowest price, and the remainder at the highest, while the one who had the greatest number sold the greater part at the lowest price, and could sell only a very small number at the highest, it may be easily seen that they might have got equal sums of money.

The question then is to divide each of the numbers 10, 25, 30, into two such parts, that if the first part of each be multiplied by the first price, and the second by the second, the sum of the two products shall be equal.

This problem is indeterminate, and susceptible of ten different solutions. It is, in the first place, necessary that the difference of the prices of the first and the second sale shall be an exact divisor of 15, 20, 5, the differences of the three numbers given; but the least divisor of these three numbers is 5, and for this reason the prices must be 6 and 1, or 7 and 2, or 8 and 3, &c.

If we suppose the two prices to be 6 and 1, we shall have seven different solutions, as seen in the following table:

	Women.	1st sale.	2d sale.	Total amount.
I.	1st.	4 eggs at 6d.	6 at 1d.	30
	2d.	1	24	30
	3d.	0	30	30
II.	1st.	5	5	35
	2d.	2	23	35
	3d.	1	29	35
III.	1st.	6	4	40
	2d.	3	22	40
	3d.	2	28	40
IV.	1st.	7	3	45
	2d.	4	21	45
	3d.	3	27	45
V.	1st.	8	2	50
	2d.	5	20	50
	3d.	4	26	50
VI.	1st.	9	1	55
	2d.	6	19	55
	3d.	5	25	55
VII.	1st.	10	0	60
	2d.	7	18	60
	3d.	6	24	60

If we suppose the two prices to be 7 and 2, we shall have also the three following solutions:

	Women.	1st sale.	2d sale.	Total amount.
I.	1st.	8 eggs at 7*d.*	2 at 2*d.*	60
	2d.	2	23	60
	3d.	0	30	60
II.	1st.	9	1	65
	2d.	3	22	65
	3d.	1	29	65
III.	1st.	10	0	70
	2d.	4	21	70
	3d.	2	28	70

It would be needless to try 8 and 3, or any other number, as no solution could be obtained from them, for reasons which will be seen hereafter.

Remarks.—We are told by M. de Lagny, in the second part of his "Arithmetique Universelle," p. 456, that this question is susceptible of no more than six solutions; but the author is here mistaken, for we have pointed out ten. As it may afford pleasure to those who are studying algebra, to be made acquainted with the method employed for obtaining them, we think it our duty here to give it.

We shall call the price at which the women sold the first time u; and that at which they sold the second time p.

If x then be the number of the eggs sold by the first woman, at the price u, the number of those sold at the price p will be $10 - x$; the money arising from the first sale will be $x u$, that of the second will be $10 p - p x$, and the sum total will be $x u + 10p - p x$. If z be the number of eggs sold by the second woman, at the first sale, we shall have $u z$ for the money arising from the first sale, and $25 p - p z$ for that arising from the second, making together $z u + 25 p - p z$.

In like manner, if y represent the number of eggs sold, the first time, by the third woman, we shall have $u y$ for the money arising from the first sale, $30 p - p y$ for that of the second, and for the total of the two sales $u y + 30 p - p y$. But, by the supposition, these three sums are equal; consequently $x u + 10 p - p x = z u + 25 p - p z = u y + 30 p - p y$, from which we obtain the three following new equations:

$$x u - p x = z u - p z + 15 p$$
$$x u - p x = u y - p y + 20 p$$
$$z u - p z = u y - p y + 5 p$$

And dividing the whole by $u - p$, we have these three others, viz.,

$$x = z + \frac{15 p}{u - p}$$
$$x = y + \frac{20 p}{u - p}$$
$$z = y + \frac{5 p}{u - p}$$

from which it may be concluded, that $u - p$ must be a divisor of 15, 20, and 5, otherwise $\frac{15 p}{u - p}$, $\frac{20 p}{u - p}$, $\frac{5 p}{u - p}$, would not be integral numbers, which it is necessary they should be. But the only number which is a divisor of 15, 20, and 5, is 5, which shews that the prices of the two sales could be only 5 and 0, 6 and 1, 7 and 2, 8 and 3, &c.

It may be easily seen, that the supposition of 5 and 0 will not answer the conditions, since in that case there would have been only one sale.

We must therefore try the second supposition, 6 and 1, viz. $u = 6$ and $p = 1$, which gives for the two last equations, $x = y + 4$, $z = y + 1$.

But we have here three unknown quantities, and only two equations; for which reason one of these unknown quantities must be assumed at pleasure. Let us take y, and first suppose it $= 0$.

This will give $x = 4$, and $z = 1$, and we shall have the first solution, which shews that the first woman sold the first time 4 eggs at 6 pence each, and consequently, the second time 6 at 1 penny each; while the second sold 1 the first time at 6 pence, and the other 24 at 1 penny each, and the third sold all her eggs at the second price: they would then all have 30 pence each.

By making $y = 1$, we shall have the second solution.

By making $y = 2$, we shall have the third.

By making $y = 3$, we shall have the fourth.

By making $y = 4$, we shall have the fifth.

By making $y = 5$, we shall have the sixth.

By making $y = 6$, we shall have the seventh.

We cannot suppose y to be greater than 6, because then we should have $x = 10$; which is impossible, as the first woman has only 10 eggs to sell.

We must therefore proceed to the following supposition, viz. $u = 7$, and $p = 2$, which gives two equations, $x = y + 8$; $z = y + 2$.

If y here be first made $= 0$, we shall have $x = 8$, and $z = 2$, which gives the eighth solution.

By making $y = 1$, we shall have the ninth.

By making $y = 2$, we shall have the tenth.

But y cannot be made greater, for then x would be greater than 10, which is impossible.

It would be useless also to try for the values of u and p, 8 and 3; for these would necessarily give to x a value greater than 10, which cannot be the case.

We may therefore rest assured that the problem is susceptible of no more solutions than the ten above-mentioned.

PROBLEM XIII.

To find the number and the ratio of the weights with which any number of pounds, from unity to a given number, can be weighed in the simplest manner.

Though this problem on the first view seems to belong to mechanics, it may be readily seen that it is only an arithmetical question: for, to solve it, nothing is necessary but to find a series of numbers beginning with unity, which, added or subtracted from each other in every way possible, shall form all the numbers from unity to the greatest proposed.

It may be solved two ways; either by addition alone, or by addition combined with subtraction. In the first case, the series of weights which answers the problem, is that of the numbers increasing in double progression; in the second, it is that of those in the triple progression.

Thus, for example, with weights of 1 pound, 2 pounds, 4 pounds, 8 pounds, and 16 pounds, we can weigh any number of pounds up to 31: for, with 2 and 1 we can form 3 pounds; with 4 and 1, 5 pounds; with 4 and 2, 6 pounds; with 4, 2, and 1, 7 pounds, &c With the addition of a weight of 32 pounds, we can weigh as far as 63 pounds; and so on, doubling the last weight, and deducting from that double unity.

But by employing weights in the triple progression, 1, 3, 9, 27, 81, all weights from 1 pound to 121 can be weighed with them: for, with the second less the first, that is to say, putting the first into one scale and the second into the other, we can make 2 pounds; by putting both in the same scale, we can form 4 pounds; by putting 9 on the one side and 3 and 1 on the other, 5 pounds; by 9 on the one side and

3 on the other, 6 pounds; by 9 and 1 on the one side and 3 on the other, 7 pounds; and so on.

It is here evident that this last method is the simplest, being that which requires the least number of different weights.

Both these progressions are more advantageous than any of the arithmetical ones; as will appear on trial; for if the increasing arithmetical weights, 1, 2, 3, 4, &c. were employed, 15 would be necessary to weigh 120 pounds; to weigh 121 with weights in the increasing progression 1, 3, 5, 7, &c., 11 would be required. No other progression would make up all the weights possible, from 1 pound to the greatest resulting from the whole of the weights. The triple proportion therefore is the most convenient of all.

The solution of this problem may be of the greatest utility in commerce, and the ordinary concerns of life, as it affords the means of weighing any weight whatever with the least possible number of different weights.

PROBLEM XIV.

A country woman carrying eggs to a garrison, where she had three guards to pass, sold at the first, half the number she had and half an egg more; at the second, the half of what remained and half an egg more; and at the third, the half of the remainder and half an egg more: when she arrived at the market place, she had three dozen still to sell. How was this possible without breaking any of the eggs?

It would appear, on the first view, that this problem is impossible; for how can half an egg be sold without breaking any? The possibility of it however will be evident when it is considered, that by taking the greater half of an odd number, we take the exact half $+ \frac{1}{2}$. It will be found therefore that the woman, before she passed the last guard, had 73 eggs remaining, for by selling 37 of them at that guard, which is the half $+ \frac{1}{2}$, she would have 36 remaining. In like manner, before she came to the second guard she had 147; and before she came to the first, 295.

This problem might be proposed also in a different manner, as follows:

PROBLEM XV.

A gentleman went out with a certain number of guineas, in order to purchase necessaries at different shops. At the first he expended half his guineas and half a guinea more; at the second, half the remainder and half a guinea more; and so at the third. When he returned he found that he had laid out all his money, without having received any change. How was this possible?

He had 7 guineas, and at the first shop expended 4, at the second 2, and at the third 1; for 4 is the half of 7 and $\frac{1}{2}$ more; the remainder being 3, its half is $1\frac{1}{2}$, and $\frac{1}{2}$ more makes 2; but 2 taken from 3 leaves 1, the half of which is $\frac{1}{2}$, and $\frac{1}{2}$ makes 1; consequently nothing more remains.

Remark.—If the number of places at which the gentleman expended all his money were greater, nothing would be necessary but to raise 2 to such a power, that the exponent should be equal to the number of places, and to diminish it by unity. Thus, if there were 4, as the fourth power of 2 is 16, the required number would be 15; if there were 5, the fifth power of 2 being 32, the required number would be 31.

PROBLEM XVI.

Three persons have each such a number of crowns, that if the first gives to the other two as many as they each have; and if the second and third do the same; they will then all have an equal number, namely 8. How many has each?

The first has 13, the second 7, and the third 4; as may be easily proved, by distributing the crowns of each as announced in the problem.

PROBLEM XVII.

A wine merchant who has only two sorts of wine, one of which he sells at 10*s*., *and the other at* 5*s*. *per bottle, being asked for some at* 8*s*. *per bottle, wishes to know how many bottles of each kind he must mix together, to form wine worth* 8*s*. *per bottle?*

The difference between the highest price, 10*s*., and the mean price required, is 2; and that between the mean price and the lowest is 3; which shews that he must take 3 bottles of the wine at the highest price, and two of that at the lowest. This mixture will form 5 bottles, worth 8*s*. each.

In problems of this kind, in general, as the difference between the highest price and the mean price, is to the difference between the mean price and the lowest, so is the number of measures at the lowest price, to that of those at the highest, which must be mixed together to have a similar measure at the mean price.

PROBLEM XVIII.

A gentleman is desirous of sinking £100,000, which together with the interest is to become extinct at the end of 20 years, on condition of receiving a certain annuity during that time. What sum must the gentleman receive annually, supposing interest to be at the rate of five per cent.?

The sum which the gentleman ought to receive annnally is £8014. 19*s*. 2*d*. 1·7*f*.

If the number of years were small, for example 5, this problem might be resolved, without algebra, by the retrograde method, and false position; for if we suppose the sum, which at the last year exhausts the capital and interest, to be £10,000, we shall find that the capital alone at the commencement of that year was £9523$\frac{17}{21}$; and if we add the £10,000, which were paid at the end of the year preceding the last, the sum, £19523$\frac{17}{21}$. will be the capital increased with the interest of the 4th year; consequently the capital at the beginning of that year was only £18594$\frac{46}{441}$; whence it follows, that before the payment, at the end of the third year, the sum was £28594$\frac{46}{441}$, which represented the capital increased with the interest of the third year. By thus going back to the commencement of the first year, the original capital will be found to be £43294. 15*s*. 4*d*. We must then make the following proportion: As this capital is to the supposed sum of £10000, so is the sum to be sunk, on the above conditions, to the annuity, or sum to be received every year.

But it may be readily perceived, that in the case of 20 or 30 years, this method would require very long calculations, which are greatly shortened by algebra.*

PROBLEM XIX.

What is the interest with which any capital whatever would be increased, at the end of a year, if the interest due at every instant of the year were itself to become capital and to bear interest?

This problem, to be well understood, has need of explanation. A person might place out his money under this condition, that the interest due at the end of a month, which at the interest of 5 per cent. would make a 60th of the capital, should be added to the capital, and bear interest the following month at the same rate; that at the

* If a be the capital, m the interest, and n the number of years; the annuity or sum to be received every year, will be $\dfrac{a \times (1 + \frac{1}{m})^n}{m \times (1 + \frac{1}{m})^n - m}$, which in the case of 20 years, and allowing interest to be at 5 per cent., (m being then = 20) will be found = $a \times \frac{2 \cdot 6584}{33 \cdot 1680}$.

expiration of this month, the interest of the above sum, which would be a 60th + $\frac{1}{[illegible]}$ of the original capital, should be still added to the capital, increased by the interest of the first month, and bear interest the following month, and so on to the end of the year.

What is done here in regard to a month, might be done in regard to a day, an hour, a minute, or even a second, which may be considered as a part of the day infinitely small: the question then is to know, what in this case would be the interest produced by the capital at the end of the year, the interest of the first second being at the rate of five per cent. or $\frac{1}{20}$th.

It might be supposed, on the first view, that this compound and super-compound interest would greatly increase the 5 per cent., and yet it will be found that it produces an increase scarcely sensible; for if the capital be 1, the same capital increased with simple interest, at five per cent., will be $1 + \frac{1}{20}$, or $1 + \frac{50}{1000}$, and when increased with the interest accumulated every second, it will be $1\frac{51}{1000}$, or rather, when expressed more exactly, $1\frac{05127}{100000}$.

PROBLEM XX.

A dishonest butler, every time he went into his master's cellar, stole a pint from a particular cask, which contained 100 pints, and supplied its place by an equal quantity of water. At the end of 30 days, the theft being discovered, the butler was discharged. Of what quantity of wine did he rob his master, and how much remained in the cask?

It may be readily seen that the quantity of wine which the butler stole did not amount to 30 pints; for the second time that he drew a pint from the cask, taking the hundredth part of what it contained, it had already in it a pint of water, and as he each day substituted for the liquor he stole a pint of water, he every day took less than a pint of wine. To resolve, therefore, the problem, nothing is necessary but to determine in what progression the wine which he every day stole decreased.

For this purpose, we must first observe, that after the first pint of wine was drawn, there remained in the cask no more than 99 pints, and the pint of water which had been added. When a pint therefore was drawn from the mixture, it was only $\frac{99}{100}$ of a pint of wine; but before the pint was drawn, the cask contained 99 pints of wine; consequently, after it was drawn, there remained 99 pints $- \frac{99}{100}$, that is to say $\frac{9801}{100}$, or 98 pints $+ \frac{1}{100}$. When the third pint was drawn, the wine contained in it would be only $\frac{98}{100} + \frac{1}{10000}$, which being taken from the quantity of wine in the cask, viz. $98\frac{1}{100}$ pints, would leave $\frac{970299}{10000}$, or 97 pints $+ \frac{299}{10000}$.

It must here be remarked, that $\frac{9801}{100}$ is the square of 99 divided by 100; and that $\frac{970299}{10000}$ is the cube of 99 divided by the square of 100, and so on; consequently, when the second pint is drawn, the wine remaining will be the square of 99 divided by the first power of 100; after the third, it will be the cube of 99 divided by the square of 100, &c. Whence it follows, that after the 30th pint is drawn, the quantity of wine remaining will be the 30th power of 99 divided by the 29th power of 100. But it may be found, by logarithms, that this quantity is $73\frac{97}{100}$, consequently the quantity of wine stolen is 26. 3.*

* If the usual method of calculation were employed, it would be necessary to find the 30th power 99, which would contain not less than 59 figures, and to divide it by unity followed by 58 ciphers; whereas if logarithms be used, nothing is necessary but to multiply the logarithm of 99 by 30, which will give 598690560, and to subtract the product of the logarithm of 100 multiplied by 29, which is 580000000. The remainder, 18690560, is the logarithm of the required quantity; which, in the tables, will be found to be nearly $73\frac{97}{100}$.

PROBLEM XXI.

A and B can perform a certain piece of work in 8 days, A and C can do it in 9 days, and B and C in 10 days; how many days will each of them require to perform the same work, when they labour separately?

A will perform it in $14\frac{34}{49}$ days; B in $17\frac{23}{41}$ days; and C in $23\frac{7}{31}$ days.

PROBLEM XXII.

An Englishman owes a Frenchman £1. 11s.; but has no other money to pay his debt than seven shillings pieces, and the Frenchman has only French crowns, valued at five shillings. How many seven shillings pieces must the Englishman give to the Frenchman, and how many crowns must the latter give to the former, that the difference shall be equal to 31 shillings in favour of the Frenchman, so that the debt may be paid?

The simplest numbers that answer this question, are 8 seven shillings pieces, and 5 crowns; for 8 seven shillings pieces make 56 shillings, and 5 crowns make 25; consequently their difference, of which the Frenchman has the advantage in this kind of exchange, is 31 shillings.

This problem is susceptible of an infinite number of solutions; for it will be found that the same result may be obtained with 13 seven shillings pieces and 12 crowns; 18 seven shillings pieces and 19 crowns; always increasing the number of seven shillings pieces by 5, and that of the crowns by 7.

Remark.—For the sake of young algebraists, we shall here give the analytical solution of this problem. Let x represent the number of the seven shillings pieces, and y that of the crowns; $7x$ then will be the sum given by the Englishman, and that given by the Frenchman will be $= 5y$. But as the difference of these two sums must be equal to 31, we shall have $7x - 5y = 31$ shillings; consequently $7x = 31 + 5y$, and $x = \frac{31+5y}{7} = 4 + \frac{3+5y}{7}$ shillings. But x is a whole number, and 4 being one also, $\frac{3+5y}{7}$ must be a whole number, and three times that quantity, which is $\frac{9+15y}{7} = 1 + 2y + \frac{2+y}{7}$, must also be a whole number. Consequently $\frac{2+y}{7}$ must be a whole number. Put it equal to u, then $y = 7u - 2$, and x, which is equal to $\frac{31+5y}{7} = 5u + 3$. If $u = 1$, then $y = 5$, and $x = 8$. If $u = 2$, then $y = 12$, and $x = 13$. If $u = 3$, then $y = 19$, and $x = 18$, &c.

CHAPTER XII.

OF MAGIC SQUARES.

THE name Magic Square, is given to a square divided into several other small equal squares or cells, filled up with the terms of any progression of numbers, but generally an arithmetical one, in such a manner, that those in each band, whether horizontal, or vertical, or diagonal, shall always form the same sum.

There are also squares in which the product of all the terms in each horizontal, or vertical, or diagonal band, is always the same. We shall speak of these also, but in a cursory manner, because they are attended with as little difficulty as the former.

These squares have been called *magic squares*, because the ancients ascribed to them great virtues; and because this disposition of numbers formed the bases and principle of many of their talismans.

According to this idea, a square of one cell, filled up with unity, was the symbol of the deity, on account of the unity and immutability of God; for they remarked that this square was by its nature unique and immutable; the product of unity by itself being always unity.

The square of the root 2 was the symbol of imperfect matter, both on account of the four elements, and of the impossibility of arranging this square magically, as will be shewn hereafter.

A square of 9 cells was assigned or consecrated to Saturn; that of 16 to Jupiter; that of 25 to Mars; that of 36 to the Sun; that of 49 to Venus; that of 64 to Mercury; and that of 81, or nine on each side, to the Moon.

Those who can find any relation between the planets and such an arrangement of numbers, must no doubt have minds strongly tinctured with superstition; but such was the tone of the mysterious philosophy of Jamblichus, Porphyry, and their disciples. Modern mathematicians, while they amuse themselves with these arrangements, which require a pretty extensive knowledge of combination, attach to them no more importance than they really deserve.

Magic squares are divided into even and odd. The former are those the roots of which are even numbers, as 2, 4, 6, 8, &c.; the latter of those the roots of which are odd, and which, by a necessary consequence, have an odd number of cells; such are the squares of 3, 5, 7, 9, &c. As the arrangement of the latter is much easier than that of the former, we shall first treat of them.

SECTION I.

Of Odd Magic Squares.

There are several rules for the construction of these squares; but, in our opinion, the simplest and most convenient, is that which, according to M. de la Loubere, is employed by the Indians of Surat, among whom magic squares seem to be held in as much estimation as they were formerly among the ancient visionaries before mentioned.

We shall here suppose an odd square, the root of which is 5, and that it is required to fill it up with the first 25 of the natural numbers. In this case, begin by placing unity in the middle cell of the horizontal band at the top; then proceed from left to right, ascending diagonally, and when you go beyond the square, transport the next number 2 to the lowest cell of that vertical band to which it belongs; set 3 in the next cell, ascending diagonally from left to right, and as 4 would go beyond the square, transport it to the most distant cell of the horizontal band to which it belongs; set 5 in the next cell, ascending diagonally from left to right, and as the following cell, where 6 would fall, is already occupied by 1, place 6 immediately below 5; place 7 and 8 in the two next cells, ascending diagonally, as seen in the figure; and then, in consequence of the first rule of transposition, set 9 at the bottom of the last vertical band; then 10, in consequence of the second, in the last cell on the left of the second horizontal band; then 11 below it, according to the third rule: after which continue

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

to fill up the diagonal with the numbers 12, 13, 14, 15; and as you can ascend no farther, place the following number 16 below 15; if you then proceed in the same manner, the remaining cells of the square may be filled up without any difficulty, as seen in the above figure.

The following are the squares of 3 and 7 filled up by the same method; and as these examples will be sufficient to exercise such of our readers as have a taste for amusements of this kind, we shall proceed to a few general remarks on the properties of a square arranged according to this principle.

8	1	6
3	5	7
4	9	2

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	45	3	12
22	31	40	49	2	11	20

1st. According to this disposition, the most regular of all, the middle number of the progression occupies the centre, as 5 in the square of 9 cells, 13 in that of 25, and 25 in that of 49; but this is not necessary in the arrangement of all magic squares.

2d. In each of the diagonals, the numbers which occupy the cells equally distant from the centre, are double that in the centre: thus $30 + 20 = 47 + 3 = 28 + 22 = 24 + 26$, &c., are double the central number 25.

3d. The case is the same with the cells centrally opposite, that is to say, those similarly situated in regard to the centre, but in opposite directions both laterally and perpendicularly: thus 31 and 19 are cells centrally opposite, and the case is the same in regard to 48 and 2, 13 and 37, 14 and 36, 32 and 18. But it happens that, according to this magic arrangement, those cells opposite in this manner, are always double the central number, being equal to 50, as may be easily proved.

4th. It may be readily seen, that it is not necessary that the progression to be arranged magically, should be that of the natural numbers 1, 2, 3, 4, &c.: any arithmetical progression whatever, 3, 6, 9, 12, &c., or 4, 7, 10, 13, 16, &c., may be arranged in the same manner.

5th. Nor is it necessary that the progression should be continued: it may be disjunct, and the rule is as follows. If the numbers of the progression, arranged according to their natural order in the cells of the square, exhibit in every direction, vertical and horizontal, an arithmetical progression, they are susceptible of being arranged magically in the same square, and by the same process. Let us take, for example, the series of numbers 1, 2, 3, 4, 5; 7, 8, 9, 10, 11; 13, 14, 15, 16, 17; 19, 20, 21, 22, 23; 25, 26, 27, 28, 29: as these, when arranged in the cells of a square, every where exhibit an arithmetical progression, they may be arranged magically; and indeed, according to the above rule, they may be formed into the annexed magic square.

1	2	3	4	5
7	8	9	10	11
13	14	15	16	17
19	20	21	22	23
25	26	27	28	29

Moscopulus, a modern Greek author, and Bachet have also invented magic squares. But their methods are inferior to one contrived by M. Poignard, and im-

proved by M. de la Hire. Of this method we now proceed to give a short account.

Let it be required to fill up a square having an odd root, such as 5. Having constructed the square of cells, place in the first horizontal row at the top, the five first numbers of the natural progression, in any order, at pleasure, which we shall here suppose to be 1, 3, 5, 2, 4; then make choice of a number, which is prime to the root 5, and which when diminished by unity does not measure it: let this number be 3; and for that reason begin with the third figure of the series, and count from it to fill up the second horizontal band 5, 2, 4, 1, 3; then begin again by the next third figure, including the 5, that is to say by 4, which will give for the third band 4, 1, 3, 5, 2; by following the same process, we shall then have the series of numbers 3, 5, 2, 4, 1, to fill up the fourth band: continue in this manner, always beginning at the third figure, the preceding included, until the whole square is filled up. This square will be one of the components of the required square, and will be magic; for the sum of each band, whether horizontal, or vertical, or diagonal, is the same, as the five numbers of the progression are contained in each without the same figure being ever repeated.

1	3	5	2	4
5	2	4	1	3
4	1	3	5	2
3	5	2	4	1
2	4	1	3	5

Now construct a second geometrical square of 25 cells, in the first band of which inscribe the multiples of the root 5, beginning with a cipher, viz. 0, 5, 10, 15, 20, and in any order at pleasure, such for example as 5, 0, 15, 10, 20: then fill up the square according to the same principle as before, taking care not to assume the same number in the series always to begin with. Thus, for example, as in the former square, the third figure in the series was taken, in the present one the fourth must be assumed; and thus we shall have a square of the multiples, as seen in the annexed figure. This is the second component of the required magic square, and is itself magic, since the sum of each band in every direction is the same.

5	0	15	10	20
10	20	5	0	15
0	15	10	20	5
20	5	0	15	10
15	10	20	5	0

Now to obtain the magic square required, nothing is necessary but to inscribe, in a third square of 25 cells, the sum of the numbers found in the corresponding cells of the preceding two; for example $5 + 1$, or 6, in the first on the left, at the top of the required square; $0 + 3$, or 3 in the second, and so on; by these means we shall have the annexed square of 25 cells, which will necessarily be magic.

6	3	20	12	24
15	22	9	1	18
4	16	13	25	7
23	10	2	19	11
17	14	21	8	5

By these means, any of the numbers may be made to fall in any cells at pleasure; for example 1 in the central cell; nothing is necessary for this purpose, but to fill up the middle band with the series of numbers in such a manner that 1 may be in the centre, as seen in the annexed figure; and then to fill up the rest of the square according to the above principles, beginning at the highest band, when the lowest has been filled up.

2	1	3	4	5
3	4	5	2	1
5	2	1	3	4
1	3	4	5	2
4	5	2	1	3

To form the second square, place a cipher in the centre, as seen in the annexed figure, and fill up the remaining cells in the same manner as before, taking care not to assume the same quantities as in the former for beginning the bands.

20	5	10	0	15
0	15	20	5	10
5	10	0	15	20
15	20	5	10	0
10	0	15	20	5

In the last place, form a third square by adding together the numbers in the similar cells, and you will have the annexed square, where 1 will necessarily occupy the centre.

22	6	13	4	20
3	19	25	7	11
10	12	1	18	24
16	23	9	15	2
24	5	17	21	8

Remarks.—I. We must here observe, that when the number of the root is not prime; that is, if it be 9, 15, 21, &c., it is impossible to avoid a repetition of some of the numbers, at least in one of the diagonals; but in that case it must be arranged in such a manner, that the number repeated in that diagonal shall be the middle one of the progression; for example 5, if the root of the square be 9; 8 if it be 15; and as the square formed of the multiples will be liable to the same accident, care must be taken, in filling them up, that the opposite diagonal shall contain the mean multiple between 0 and the greatest; for example 36 if the root be 9; 105 if it be 15.

II. The same thing may be done also in squares which have a prime number for their root. By way of example we shall here form a magic square of the first two of the following ones:

1

1	2	5	4	3
2	5	4	3	1
5	4	3	1	2
4	3	1	2	5
3	1	2	5	4

2

10	0	5	15	20
20	10	0	5	15
15	20	10	0	5
5	15	20	10	0
0	5	15	20	10

3

11	2	10	19	23
22	15	4	8	16
20	24	13	1	7
9	18	21	12	5
3	6	17	25	14

in the first of which 3 is repeated in the diagonal descending from right to left, and in the second 10 is repeated in the diagonal descending from left to right. This however does not prevent the third square, formed by their addition, from being magic.

SECTION II.

Of Even Magic Squares.

The construction of these squares is attended with more difficulty than that of the odd squares, and the degree of difficulty is different, according as they are evenly even, or oddly even: for this reason we must divide them into two classes.

Squares evenly even, are those the root of which when halved is even, or can be divided by 4 without remainder; of this kind are the squares of 4, 8, 12, &c. The oddly even are those the root of which when halved gives an odd number; as those of 6, 10, 14, &c.

As the ancients have left us no general rule on this subject, but only some examples of even squares magically arranged, we shall here give the best methods invented by the moderns, and shall begin with squares evenly even.

Let us suppose then that the annexed square A B C D is to be filled up magically, with the first 16 of the natural numbers: fill up first the two diagonals; and for that purpose begin to count the natural numbers, in order, 1, 2, 3, 4, &c., on the cells of the first horizontal band from left to right; then proceed to the second band, and when you come to the cells belonging to the diagonals, inscribe the numbers counted as you fall upon them; by which means you will have the arrangement represented in the annexed figure.

A B

1			4
	6	7	
	10	11	
13			16

C D

When the diagonals have been thus filled, to fill up the cells which remain vacant, begin to count the same numbers, proceeding from the angle D in the cells of the lower band, going from right to left, and then in the next above it; and when any cells are found empty, fill them up with the numbers that belong to them; in this manner you will have the square 16 filled up magically, as seen in the annexed figure, and the sum of each band and each diagonal will be 34.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Rule for Squares evenly even.

Having given, according to M. de la Hire, a very general rule for odd squares, which is capable of producing a great number of variations, we shall do the same in regard to even squares; especially as it will equally serve for evenly even and oddly even magic squares. It is as follows:

Let it be required, for example, to fill up magically a square of 8 cells on each side.

For this purpose, arrange, in the first horizontal band in a square of that kind, the first eight numbers of the arithmetical progression, but in such a manner, that those equally distant from the middle shall form the same sum; viz. that of the root augmented by unity, which in this case is 9; the second band must be the inverse of the first; the third must be like the first; the fourth like the second, and so on alternately, till the half of the square is filled up; after which the other half may be formed by merely reversing the first, as may be seen in the above figure. This will be the first primitive square.

1	6	5	2	7	4	3	8
8	3	4	7	2	5	6	1
1	6	5	2	7	4	3	8
8	3	4	7	2	5	6	1
8	3	4	7	2	5	6	1
1	6	5	2	7	4	3	8
8	3	4	7	2	5	6	1
1	6	5	2	7	4	3	8

To form the second, fill it up according to the same principle with the multiples of the root, beginning with 0, that is to say, 0, 8, 16, 24, 32, 40, 48, 56; taking care that the extremes shall always make 56, but instead of arranging these numbers in a horizontal direction, they must be arranged vertically, as in the following figure.

48	8	48	8	8	48	8	48
16	40	16	40	40	16	40	16
32	24	32	24	24	32	24	32
0	56	0	56	56	0	56	0
56	0	56	0	0	56	0	56
24	32	24	32	32	24	32	24
40	16	40	16	16	40	16	40
8	48	8	48	48	8	48	8

49	14	53	10	15	52	11	56
24	43	20	47	42	21	46	17
33	30	37	26	31	36	27	40
8	59	4	63	58	5	62	1
64	3	60	7	2	61	6	57
25	38	29	34	39	28	35	32
48	19	44	23	12	45	22	41
9	54	13	50	55	12	51	16

When this is done, add together the similar cells of the two squares, and you will have a square of 8 on each side, as in the last figure above.

Without enlarging farther on squares evenly even, we shall give the simplest method of constructing squares oddly even.

Method for Squares oddly even.

We shall take, by way of example, the square of the root 6. To fill it up, inscribe in it the first six numbers of the arithmetical progression, 1, 2, 3, &c., according to the above method; which will give the first primitive square, as in the annexed figure.

5	6	3	4	1	2
2	1	4	3	6	5
5	6	3	4	1	2
5	6	3	4	1	2
2	1	4	3	6	5
5	6	3	4	1	2

The second must be formed by filling up the cells in a vertical direction, according to the same principle, with the multiples of the root, beginning at 0, viz. 0, 6, 12, 18, 24, 30.

24	6	24	24	6	24
0	30	0	0	30	0
12	18	12	12	18	12
18	12	18	18	12	18
30	0	30	30	0	30
6	24	6	6	24	6

The similar cells of the two squares if then added, will form a third square, which will require only a few corrections to be magic. This third square is as here annexed.

				A			
	29	12	27	28	7	26	
	2	31	4	3	36	5	
	17	24	15	16	19	14	
C	23	18	21	22	13	20	D
	32	1	34	33	6	35	
	11	30	9	10	25	8	
				B			

To render the square magic, leaving the corners fixed, transpose the other numbers of the upper horizontal band, and of the first vertical one on the left, by reversing all the remainder of the band; writing 7, 28, 27, 12, instead of 12, 27, &c., and in the vertical one, 32, 23, 17, and 2, from the top downwards, instead of 2, 17, &c.

It will be necessary also to exchange the numbers in the two cells of the middle of the second horizontal band at the top, of the lowest of the second vertical band on the left, and of the last on the right. The numbers in the cells A and B must also be exchanged, as well as those in C and D; by which means we shall have the square corrected and magically arranged.

29	7	28	9	12	26
32	31	3	4	36	5
23	18	15	16	19	20
14	24	21	22	13	17
2	1	34	33	6	35
11	25	10	27	30	8

SECTION III.

Of Magic Squares with Borders.

Modern arithmeticians have added a new difficulty to the subject of magic squares, by proposing not only to arrange magically in a square a progression of numbers, but by requiring that this square, when lessened by a band on each side, or two or three bands, &c., shall still remain magic; or a magic square being given, to add to it a border of one or more bands, in such a manner, that the enlarged square thence resulting shall be still magic.

To give an example of this construction, let it be required to form a magic square of the root 6, and to fill it up with the natural numbers, from 1 to 36. The first even magic square possible being that of 4 on each side, we shall first arrange it magically, filling it up with the mean terms of the progression, to the number 16, and reserving the first and the last 10 for the border. For the interior square therefore we shall take the numbers 11, 12, &c., as far as 26 inclusively, and shall give them any magic disposition whatever: there will then remain the numbers 1, 2, &c., as far as 10, and 27 as far as 36, for the border.

To dispose these numbers in the border, first place the numbers 1, 6, 31, 36, in the four corners, and in such a manner that diagonally they shall make 37. As each band must make 111, it will be necessary to place in the first band four such numbers, that their sum shall be 104; and as their complements to 37 must be found in the lowest, where there is already 67, it will be necessary that they should together make 44: there are several combinations of these numbers, four and four, which can make 104, and their complements 44; but it is necessary at the same time that four of those remaining should make 79, to fill up the first vertical band, while their complements make 69 to complete the last. This double condition limits the combination to 35, 34, 30, 5, which may be placed in the first band in any order whatever, provided their complements be placed below each of them in the last band; and the four numbers requisite to fill up the first vertical band will be 33, 28, 10, 8, which may be arranged any how at pleasure, provided the complement of each be placed opposite to it in the corresponding cell on the other side.

1	35	34	5	30	6
33	11	25	24	14	4
28	22	16	17	19	9
8	18	20	21	15	29
10	23	13	12	26	27
31		3	32	7	36

It is not absolutely necessary that 1, 6, 31, 36 should be placed in the four corners of the square: if we suppose them to be filled up, in the same order, with 2, 7, 30, 35, it would be then necessary that the four first numbers should make 102, and their complements 46, while the four last make 79, and their complements 69: but it is found that the first four numbers are 36, 31, 27, 8, and the second 34, 32, 9, 4. The first being arranged any how in the four empty cells of the first band, and their complements below, the second must be arranged in the cells of the first vertical band, and their complements each at the extremity of the same horizontal band; by which means we shall have the new square with a border, as seen above.

2	36	31	27	8	7
34	11	25	24	14	3
32	22	16	17	19	5
9	18	20	21	15	28
4	23	13	12	26	33
30	1	6	10	29	35

If it were required to form a bordered square of the root 8; it would be necessary to reserve for the interior square of 36 cells, the 36 mean numbers of the progression; and they might be formed into a bordered square around the magic square of 16 cells; with the 28 remaining numbers, we might then form a border to the square of 36 cells, &c.

Hence it appears, in what manner we might form a magic square, which when successively lessened by one, two, or three bands, shall still remain magic.

SECTION IV.

Of another kind of Magic Square in Compartments.

Another property, of which most magic squares are susceptible, is, that they are not only magic when entire, but that when divided into those squares into which they can be resolved, these portions of the original square are themselves magic. A square of 8 cells on a side, for example, formed of four squares, each having 4 for its root, being proposed, it is required that not only the square of 64 shall be disposed magically, but each of those of 16, and that the latter even, however arranged, shall still compose a magic square.

What is here required, is easy; and this is even the simplest method of all for constructing squares that are evenly even, as will appear from what follows.

To construct a square of 64, in this manner, take the first 8 numbers of the natural progression, from 1 to 64, and the 8 last, and arrange them magically in a square of 16 cells; do the same thing with the 8 terms which follow, the first 8 and the 8 which precede the last 8, and by these means you will have a second magic square; form a similar square of the 8 following numbers with their corresponding ones, and another with the 16 means: the result will be four squares of 16 cells, the numbers in which will be equal when added together, either in bands or diagonally; for they will every where be 130. It is therefore evident, that if these squares be arranged side by side, in any order whatever, the square resulting from them will be magic, and the sum in every direction will be 260.

1	63	62	4	9	55	54	12
60	6	7	57	52	14	15	49
8	58	59	5	16	50	51	13
61	3	2	64	53	11	10	56
17	47	46	20	25	39	38	28
44	22	23	41	36	30	31	33
24	42	43	21	32	34	35	29
45	19	18	48	37	27	26	40

SECTION V.

Of the Variations of Magic Squares.

The square having 3 for its root is suceptible of no variation: whatever method may be employed, or whatever arrangement may be given to the numbers of the progression from 1 to 9, the same square will always arise, except that it will be inverted, or turned from left to right, which is not a variation. But this is not the case with the square having 4 for its root, or that of 16 cells: this being susceptible of at least 880 variations, which M. Frenicle has given in his Treatise on Magic Squares.

The square of 5 is susceptible of, at least, 57600 different combinations; for according to the process of M. de la Hire, the 5 first numbers may be arranged 120 different ways in the first band of the first primitive square; and as they may be afterwards arranged in the lower bands, beginning again by two different quantities, this will make 240 variations, at least, in the primitive square; which, combined with the 240 of the second, form 57600 variations in the square of 5. But there are doubtless a great many more, for a bordered square of 5 cannot be reduced to the method of M. de la Hire; but one bordered square of 5, the corners remaining fixed, as well as the interior square of 3, may experience 36 variations. What a number therefore of other variations must be produced by changing the interior square and the angles!

A bordered square of 6, when once constructed, the corners remaining fixed, and the interior square being composed of the same numbers, may be varied 4055040 different ways; for the interior square may be varied and differently transposed in the centre 7040 ways: each of the horizontal bands at top and at bottom, the extremities remaining fixed, may be varied 24 ways; for there are four pairs of numbers susceptible of changing their place, which may be combined 24 ways; and there are also four pairs in the vertical bands between the corners. The number of the combinations, therefore, is the product of 7040 by 576, the square of 24, which gives 4055040 variations. But the corners may be varied, as well as the numbers assumed to form the interior square; and it hence follows, that the wnole number of the variations of a square of 6, while it still remains bordered, is equal to several millions of times the former.

The square of 7, by M. de la Hire's method alone, may be varied 406425600 different ways.

These variations, however numerous, ought to excite no surprise; for the number of dispositions, magic or not magic, of 49 numbers, for example, forms one of 62 figures, of which the preceding is, as we may say, but a part infinitely small.

SECTION VI.

Of Geometrical Magic Squares.

We have already observed, in the beginning of this chapter, that numbers in geometrical progression might be arranged in the cells of a square, and in such a manner, that the product of these numbers, in each band, whether vertical, or horizontal, or diagonal, shall always be the same.

To construct a square of this kind, the same principles must be followed as in the construction of other magic squares; and this may be easily demonstrated from the property of logarithms. Without enlarging further therefore on this subject, we shall confine ourselves to giving one example; it is that of the 9 first terms of the double geometric progression, 1, 2, 4, 8, &c. arranged in a square of 3 cells on each side. The product is evidently the same in every direction, viz. 4096.

128	1	32
4	16	64
8	256	2

Remarks.—The ingenious Dr. Franklin, it seems, carried this curious speculation further than any of his predecessors in the same way. He constructed both a magic square of squares, and a magic circle of circles. The magic square of squares is formed by dividing a great square into 256 little squares, in which all the numbers from 1 to 256, or the square of 16, are placed in 16 columns, which may be taken either horizontally or vertically. Their chief properties are as follow:

1. The sum of the 16 numbers in each column or row, vertical or horizontal, is 2056.

2. Every half column, vertical and horizontal, makes 1028, or just one half of the same sum 2056.

3. Half a diagonal ascending, added to half a diagonald descending, makes also the same sum 2056; taking these half diagonals from the ends of any side of the square to the middle of it; and so reckoning them either upward or downward; or sideways from right to left, or from left to right.

4. The same with all the parallels to the half diagonals, as many as can be drawn in the great square: for any two of them being directed upward and downward, from the place where they begin, to that where they end, their sums still make the same 2056. Also the same holds true downward and upward; as well as if taken sideways to the middle, and back to the same side again.

5. The four corner numbers in the great square added to the four central numbers in it, make 1028, the half sum of any vertical or horizontal column which contains 16 numbers; and also equal to half a diagonal or its parallel.

6. If a square hole, equal in breadth to four of the little squares or cells, be cut in a paper, through which any of the 16 little cells in the great square may be seen, and the paper be laid upon the great square; the sum of all the 16 numbers seen through the hole, is always equal to 2056, the sum of the 16 numbers in any horizontal or vertical column.

The magic circle of circles is composed of a series of numbers, from 12 to 75 inclusive, divided into 8 concentric circular spaces, and ranged in 8 radii of numbers, with the number 12 in the centre; which number, like the centre, is common to all these circular spaces, and to all the radii.

The numbers are so placed, that 1st, the sum of all those in either of the concentric circular spaces above mentioned, together with the central number 12, amount to 360, the same as the number of degrees in a circle.

2. The numbers in each radius also, together with the central number 12, make just 360.

3. The numbers in half of any of the above circular spaces, taken either above or below the double horizontal line, with half the central number 12, make just 180, or half the degrees in a circle.

4. If any four adjoining numbers be taken, as if in a square, in the radial divisions of these circular spaces, the sum of these, with half the central number, make also the same 180.

5. There are also included four sets of other circular spaces, bounded by circles that are excentric with regard to the common centre; each of these sets containing five spaces. For distinction, these circles are drawn with different marks, some dotted, others by short unconnected lines, &c.; or still better with inks of divers colours, as blue, red, green, yellow.

These sets of excentric circular spaces intersect those of the concentric, and each other; and yet, the numbers contained in each of the excentric spaces, taken all around through any of the 20, which are excentric, make the same sum as those in the concentric, namely 360, when the central number 12 is added.

Their halves also, taken above or below the double horizontal line, with half the central number, make up 180.

It is observable, that there is not one of the numbers, but what belongs at least to two of the circular spaces; some to three, some to four, some to five; and yet they are all so placed, as never to break the required number 360, in any of the 28 circular spaces within the primative circle.

CHAPTER XIII.

POLITICAL ARITHMETIC.

Since politicians have acquired juster ideas respecting what constitutes the real strength of states, various researches have been made in regard to the number of the inhabitants in different countries, in order to ascertain their population. Besides, as almost all governments have been under the necessity of making loans for the most part on annuities, they have naturally been induced to examine according to what progression mankind die, that the interest of these loans may be proportioned to the probability of the annuities becoming extinct. These calculations have been distinguished by the name of political arithmetic, and as it exhibits several curious facts, whether considered in a political or a philosophical point of view, we have thought it our duty to give it a place heie.

SECTION I.

Of the Proportion between the Males and the Females.

Many people imagine that the number of the females born exceeds that of the males; but it has long since been proved that the contrary is the case. More boys than girls are born every year; and since the year 1631, a small interval excepted, we have a register of births, in regard to sex, and it has never been observed that the number of the females born ever equalled that of the males. It is found, by taking a mean or average term in a greater number of years, that the number of the males born is to that of the females, as 18 to 17. This proportion is nearly that which prevails throughout all France; but, to whatever reason owing, it seems at Paris to be as 27 to 26.

This kind of phenomenon is observed, not only in England and in France, but in every other country. We may be convinced of the truth of it by inspecting the monthly and other periodical publications, which at the commencement of every year give a table of the births that have taken place in most of the capital cities of Europe: it may there be seen that the number of the males born, always exceeds that of the females; and consequently it may be considered as a general law of nature.

We may here observe a striking instance of the wisdom of Providence, which has thus provided for the preservation of the human race. Men, in consequence of the active life for which they are naturally destined by their strength and their courage, are exposed to more dangers than the female sex; war, long sea voyages, employments laborious or prejudicial to health, and dissipation, carry off great numbers of the males; and it thence results, that if the number born of the latter did not exceed that of the females, the males would rapidly decrease, and soon become extinct.

SECTION II.

Of the Mortality of the Human Race, according to the Different Ages.

In this respect, there is apparently a considerable difference between large towns

and the country; but this arises from the women in town rarely suckling their own children; and consequently the greater part of their children being put out to nurse in the country, as it is in the period of childhood that the greatest mortality prevails, it becomes most apparent in the country. To make an exact calculation, it ought to be founded on the deaths which happen in the towns, as well as in the country; and this M. Dupré de St. Maur has endeavoured to do, by comparing the registers of three parishes in Paris, and twelve in the country.

According to the observations of this author, in 23994 deaths, 6454 of them were those of children not a year old; and carrying his researches on this subject as far as possible, he concludes, that of 24000 children born, the numbers who attain to different ages are as follow:

Ages.	Number.	Ages.	Number.	Ages.	Number.
2	17540	30	9544	90	103
3	15162	35	8770	91	71
4	14177	40	7929	92	63
5	13477	45	7008	93	47
6	12968	50	6197	94	40
7	12562	55	5375	95	33
8	12255	60	4564	96	23
9	12015	65	3450	97	18
10	11861	70	2544	98	16
15	11405	75	1507	99	8
20	10909	80	807	100	6 or 7
25	10259	85	291		

Such then is the condition of the human species, that of 24000 children born, scarcely one half attain to the age of 9; and that two thirds are in their grave before the age of 40; about a sixth only remain at the expiration of 62 years; a tenth after 70; a hundredth part after 86; about a thousandth part attain to the age of 96; and six or seven individuals to that of 100.

We must however observe, that the authors who have treated on this subject, differ from each other. According to the table of M. de Parcieux, for example, the half of the children born do not die before 31 years are completed; but according to M. Dupré de St. Maur they are cut off before the commencement of the ninth year. This difference arises from the table of M. de Parcieux having been formed from lists of annuitants, who are always select subjects: for a father never thinks of purchasing an annuity on the life of a child who is sickly, or has a bad constitution. The laws of mortality in these cases therefore are different; and if the one is a general and common law, the other is that which public administrators, who grant annuities, ought to consult with great care, that they may not make loans too burthensome.

SECTION III.

Of the Vitality of the Human Species, according to the Different Ages, or Medium of Life.

When a child is born, to what age may a person bet, on equal terms, that it will attain? Or if the child has already attained to a certain age, how many years is it probable it will still live? These are two questions, the solution of which is not only curious, but important.

We shall here give two tables on this subject; one by M. Dupré de St. Maur, and the other by M. Parcieux; and add to them a few general observations.

TIME TO LIVE.

	M. de St. Maur.		M. de Parcieux.	
AGE.	YEARS.	MONTHS.	YEARS.	MONTHS.
0	8			
1	33		41	9
2	38		42	8
3	40		43	6
4	41		44	2
5	41	6	44	5
6	42		44	3
7	42	3	44	
8	41	6	43	9
9	40	10	43	3
10	40	2	42	8
20	33	5	36	3
30	28		30	6
40	22	1	25	6
50	16	7	19	5
60	11	1	14	11
70	6	2	9	2
75	4	6	6	10
80	3	7	5	
85	3		3	4
90	2		2	2
95		5		6
96		4		5
97		3		4
98		2		3
99		1		2
100		½		1

Two observations here occur, in regard to these tables. The first is respecting the difference between them. In that of M. Parcieux, the time assigned to each age to live, is more considerable, and the reason has been already mentioned. We have even suppressed the first year in the table of M. Parcieux, because the difference was by far too great, and in our opinion it arose from two causes. 1st. No one ever thinks of purchasing an annuity for a child in its first year, until the goodness of its constitution has been fully ascertained. 2d. It is not at the birth of a child, but in the course of the first year, towards the middle or end, that such a measure is hazarded; for as annuities remain sometimes several months, and even a whole year, to be filled up, people are not under the necessity of sinking money on so young a life, and have time during the course of several months to acquire some certainty respecting the constitution of the subject. In our opinion, therefore, the 34 years of vitality, assigned by M. de Percieux to a child just born, ought to be considered as applicable to a child from 6 to 9 months old, and more; but it is during the first months of the first year that the life of a child is most uncertain, and that the greatest number die.

The second observation, which is common to both tables, is, that vitality, exceedingly weak at the moment of birth, goes on increasing after that period, till it comes to another, at which it is the greatest; for the chance is less than 3 to 1 that a new born child will attain to the end of its first year,* and one may take an even bet that

* According to the principles explained in treating of probabilities, the probability of a child newly born being alive at the end of a year, is to that of its dying before that period, as the number of the children alive at the end of a year, is to the number of those dead; that is to say, as 17540 to 6400; which is somewhat less than the ratio of 3 to 1. In the other cases the calculation is the same. Take the number of those who have died in the course of the year, and divide by it the number of those alive; this will express what may be betted to 1, that the person who has completed that year will complete another.

it has only 8 years to live; but when it has attained to the commencement of the second year, one may bet 6 to 1 that it will attain to the third; and it is an even chance that it will live 33 years. In a word, it is seen by the table of M. Dupré de St. Maur, that it is towards the age of 10 years, or betwen 10 and 15, that life is most secure. At that period, one may take an even bet that the child will live 43 years; and it is 125 to 1 that it will live a year, or 25 to 1 that it will live five years. Beyond that period the probability of living a year longer decreases. At the age of 20, for example, it is somewhat less than 16 to 1, that the person will not die within the five following years. When a person has reached his sixtieth year, it is no more than 3¼ to 1 that he will attain to the beginning of his sixty-fifth year.

SECTION IV.

Of the number of Men of different ages in a given number.

It may be deduced from the preceding observations, that when the inhabitants of a country amount to a million, the number of those of the different ages will be as follows:

Between 0 and 1 year complete		38740
1 and	5	119460
5	10	99230
10	15	94530
15	20	88674
20	25	82380
25	30	77650
30	35	71665
35	40	64205
40	45	57230
45	50	50605
50	55	43940
55	60	37110
60	65	28690
65	70	21305
70	75	13195
75	80	7065
80	85	2880
85	90	1025
90	95	335
95	100	82
Above 100 years		3 or 4
	Total..	1,000,000

Thus in a country peopled with a million of inhabitants, there are about 573460 between the age of 15 and 60; and as nearly one half of them are men, this number of inhabitants could, on any emergency, furnish 250,000 men capable of bearing arms, even if an allowance be made for the sick, the lame, &c., who may be supposed to be among that number.

SECTION V.

Of the Proportion of the Births and Deaths to the whole Number of the Inhabitants of a Country.—The consequences thence resulting.

As it would be difficult to number the inhabitants of a country, and much more to repeat the enumeration as often as it might be necessary to ascertain the population, means have been devised for accomplishing the same object, by determining the proportion which the births and deaths bear to the whole number of the inhabitants; for as registers of births and deaths are regularly kept in all the civilized countries of Europe, we may judge, by comparing them, whether the population has increased or decreased; and in the latter case can examine the causes which have produced the diminution.

It is deduced, for example, from Dr. Halley's tables of the state of the populations of Breslaw, about the year 1690, that among 34000 inhabitants, there took place, every year on an average, 1238 births; which gives the proportion of the former to the latter as 27½ to 1. In regard to cities, such as Breslaw, where there is no great influx of strangers, we may therefore adopt it as a rule, to multiply the births by 27½ in order to find the number of the inhabitants.

There appeared in 1766, a very interesting work on this subject, entitled, "Recherches sur la Population des la Généralités d'Auvergne, de Lyon, de Rouen, et de quelques Provinces et Villes du Royaume," &c., by M. Messance. By an enumeration of the inhabitants of seventeen small towns or villages in the generality of Auvergne, compared with the average number of births in the same places, the author shews that the number of births is to that of the inhabitants, as 1 to 24½, $\frac{1}{10}$ $\frac{1}{80}$: a similar enumeration, in twenty-eight small towns or villages of the generality of Lyons, gave the ratio of 1 to 23¾; and by another made in five small towns or villages of the generality of Rouen, it appeared that the ratio was as 1 to 27½ and $\frac{1}{20}$. But as these three generalities comprehend a very mountainous district, such as Auvergne, another which is moderately so, as the generality of Lyons, and a third which consists almost entirely of plains or cultivated hills, as the generality of Rouen, there is reason to conclude that these three united afford a good representation of the average state of the kingdom; combining therefore the above proportions, which gives that of 1 to 25½, this will give the proportion of births to the number of the inhabitants, for the whole kingdom, without including the great cities: so that for two births in the year we shall have 51 inhabitants.

But as, in towns of any magnitude, there are several classes of citizens who spend their lives in celibacy, and who contribute either nothing or very little to the population, it is evident that this proportion, between the births and effective inhabitants, must be greater. M. Messance says, he ascertained, by several comparisons, that the ratio nearest the truth, in this case, is 1 to 28, and that this is the proportion which ought to be employed in deducing, from the number of births, the number of the inhabitants of a town of the second order, such as Lyons, Rouen, &c.; which agrees pretty well with what Dr. Halley found in regard to the city of Breslaw.

In the last place, for cities of the first class, or the capitals of states, such as Paris, London, Amsterdam, &c., where a great many strangers, attracted either by pleasure or business, are mixed with the inhabitants, and where great luxury prevails, which increases the number of those who live in voluntary celibacy, it is very probable that the above ratio must be raised, and that it ought to be carried to 30 or 31.

M. Kerseboom, in his book entitled, "Essai de Calcul politique, concernant la quantité des habitans des Provinces de Hollande et de Westfriesland," &c., printed at the Hague in 1748, has endeavoured to shew that to obtain the number of the inhabitants in Holland, the number of the births ought to be multiplied by 35. If this be the case, there is reason to conclude that marriages are less fruitful or less numerous in Holland than in France; and this difference may be founded on physical causes.

If these calculations be applied to determining the population of great cities, it will be seen that the opinions entertained in general on this subject, are erroneous; for it is commonly said that Paris contains a million of inhabitants; but the number of births there, taking one year with another, never exceeds 19500, which, multiplied by 30, gives 585000 inhabitants; if we employ as multiplier the number 31, we shall have 604500, and this is certainly the utmost extent of the population of Paris.

SECTION VI.

Of some other Proportions between the Inhabitants of a Country.

We shall present to the reader a few more short observations in regard to population. The book, which we quoted in the preceding paragraph, shall still serve us as a guide.

By combining together the three generalities above mentioned, it is found,

1st. That the number of the inhabitants of a country, is to that of the families, as 1000 to 222½; so that 2000 inhabitants give in common 445 families, and con-

sequently 4$\frac{1}{2}$ heads on an average for each, or 9 persons for two families. In this respect, those of Auvergne are the most numerous; those of the Lyonnois, are next; and those of the generality of Rouen are the least numerous. By taking a mean, it is found also, that in 25 families, there is one where there are six or more children.

2d. The number of male children born exceeds, as has been said, that of females; and this excess continues till a certain age; for example, the number of boys of the age of fourteen, or below, is greater than that of the females of the same age, and in the ratio of 30 to 29. The whole number of the females, however, exceeds that of the males, in the ratio of about 18 to 19. We here see the effect of the great consumption of men, occasioned by war, navigation, laborious employments, and debauchery.

3d. It is found that there are three marriages annually among 337 inhabitants; so that 112 inhabitants produce one marriage.

4th. The proportion of married men or widowers, to married women or widows, is nearly as 125 to 140; and the whole number of this class of society, is to the whole of the inhabitants, as 265 to 631, or as 53 to 126.

5th. According to King and Kerseboom, the number of widowers is to that of widows, as 1 to 3 nearly; so that there are three widows for one widower. This at least is deduced from the enumerations made in Holland and in England. But is the case the same in France? It is to be regretted, that the above-mentioned author did not make researches on this subject. In our opinion, however, this proportion is pretty near the truth, and it will excite no astonishment when it is considered that the greater part of the men marry late, in comparison of the women.

6th. If the above proportion between widowers and widows be admitted, it thence follows, that among 631 inhabitants there are 118 married couples, 7 or 8 widowers, and 21 or 22 widows; the remainder are composed of children, people in a state of celibacy, servants, or passengers.

7th. It thence results also, that 1870 married couples give annually 357 children; for a city of 10000 inhabitants would contain that number of married couples, and give 357 annual births. Five married couples therefore, of all ages, produce annually one birth.

8th. The number of servants is to the whole number of the inhabitants, as 136 to 1535 nearly; which is somewhat more than the eleventh part, and less than the tenth.

The number of male servants is nearly equal to that of the female, being in the ratio of 67 to 69; but it is very probable that in large cities, where a great deal of luxury prevails, the proportion is different.

9th. The number of ecclesiastics of both sexes, that is to say, secular as well as regular, comprehending the nuns, is to the inhabitants of the above three generalities, as 1 to 112 nearly: this is contrary to the common opinion, which supposes the proportion to be much greater.

10th. By dividing the territory of these three generalities among their inhabitants, it is found, that the square league would contain 864; but the square league contains 6400 acres; each man therefore, on an average, would have 7$\frac{4}{10}$ acres, and each family being composed, one with another, of 4$\frac{1}{2}$ heads, 33$\frac{1}{3}$ acres would fall to the share of each family. But it is to be observed that the generality of Rouen, considered alone, is much more populous, since it contains 1264 inhabitants for each square league, which gives to each head no more than 5 acres.

11th. It appears by the same enumerations, that a very sensible increase in the population has taken place since the beginning of the last century. It is indeed found, that the annual number of the births has been augmented; and by comparing the present period with the commencement of the last century, there is reason to

conclude, that the number of the inhabitants is now greater than what it was at the beginning of the century, in the ratio of 1456 to 1350; which makes less than a twelfth, and more than a thirteenth, of increase. This is doubtless owing to the great extent to which agriculture and commerce have been carried, and to the cessation of those wars which so long exhausted the interior of France. The wound given to the nation by the revocation of the edict of Nantes seems healed, and even more; but had it not been for that event, France, in all probability, would contain a sixth more of inhabitants than it did at the commencement of the 18th century; for the number who expatriated in consequence of that revocation, amounted perhaps to a twelfth part of the whole people.

SECTION VII.

Some Questions which depend on the preceding Observations.

The following are some of those questions, in the solution of which the preceding observations may be employed: we shall not explain the principles on which each is resolved; but shall merely confine ourselves to referring to them sometimes, that we may leave to the reader the pleasure of exercising his own ingenuity.

1st. *The age of a man being given, that of 30 for example, what probability is there that he will be living at the end of a determinate number of years, such as* 15.

Seek in the table of the second section for the given age of the person, viz. 30 years, and write down the number opposite to it, which is 11405; then take from the same table the number opposite to 45, which is 7008, and form these two numbers into a fraction, having the latter for its numerator, and the former as its denominator; this fraction will express the probability of a person of 30 years of age living 15 years, or attaining to the age of 45.

The demonstration of this rule is obvious to every one who understands the theory of probabilities.

2d. *A young man 20 years of age borrows £1000, to be paid, capital and interest, when he attains to the age of 25; but in case he dies before that period, the debt to become extinct. What sum ought he to engage to pay, on attaining to the age of 25?*

It is evident that if it were certain he would not die before the age of 25, the sum to be then paid would be the capital increased by five years' interest, which we here suppose to be simple interest: the sum therefore which in that case he ought to engage to pay, on attaining to the age of 25, would be £1250. But this sum must be increased, in proportion to the danger of the debtor dying in the course of these five years, or in the inverse ratio of the probability of his being alive when they are expired. As this probability is expressed by the fraction $\frac{10259}{10909}$, we must multiply the above sum by this fraction inverted, or by $\frac{10909}{10259}$, which will give £1329. 3*s*. 11*d*., that is to say, £79. 3*s*. 11*d*. for the risk of losing the debt, which certainly cannot be considered as usury.

3d. *A state or an individual having occasion to raise money on annuities, what interest ought to be given for the different ages, legal interest being at the rate of 5 per cent.?*

The vulgar, who are accustomed to burthensome loans, entertain no doubt that 10 per cent. is a great deal for any age below 50, and that such a method of borrowing cannot be advantageous to the state. But this is a great mistake; for it will be found by calculation, employing the before-mentioned data, according to the table of M. de Parcieux, that 10 per cent. cannot be allowed before the age of 56. According to the same table, no more than $6\frac{1}{4}$ can be given at the age of 20; $6\frac{1}{2}$ at 25; $6\frac{3}{4}$ at 30; $7\frac{1}{3}$ at 40; $8\frac{1}{2}$ at 50; 10 at 56; $11\frac{1}{10}$ at 60; $16\frac{2}{3}$ at 70; $27\frac{1}{3}$ at 80; $39\frac{1}{10}$ at 85.

It is also a very great error to believe, that on account of the number of persons who lay out money on these loans made by governments, they are soon freed from a part of the annuities by the death of a part of the annuitants. The slowness of the increase of annuities in tontines, is a sufficient proof of the falsity of this idea; besides, the great number of the persons is the very cause why the extinction of the annuities takes place according to the laws of probability above explained. A fortunate circumstance, at the end of some years, may free an individual from an annuity established on the life of a man aged 30; but if this annuity were divided among 300 persons of nearly the same age, it is certain that he would not be freed from this burthen before the expiration of about 65 years; and at the end of 32 or 33 years one half of the annuitants would still be living. This M. de Parcieux has shewn, in the clearest manner, by examining the lists of different tontines.

4th. *Legal interest being at 5 per cent.; at what rate of interest may an annuity be granted on the lives of two persons, whose ages are given, and payable till the death of the last survivor?*

5th. *What interest may be allowed on a capital, sunk for an annuity on the lives of two persons, whose ages are given, and payable only while both the annuitants are living?*

6th. *A certain person, whose age is given, has an annuity, secured on the public funds, of £1000; but being in want of money, he is desirous to sell it. How much is it worth?*

7th. *A, aged 20, and B, aged 50, agree to purchase, on their joint lives, an annuity of £1000, to be equally divided between them, during their lives, with a reversion to the survivor. How much ought each of them to contribute towards the purchase money?*

8th. *How much ought each to contribute, supposing it stipulated between them, that B, the eldest, should enjoy the whole till his death?*

9th. *Legal interest being at 5 per cent., what is the worth of an annuity of £100, on the lives of three persons, whose ages are given, and payable till the death of the last survivor?*

10th. *An annuity is purchased for the life of a child, of 3 years of age, on this condition, that the annuity at the end of each year is to be added to the purchase money, till the annuity equals the capital sunk. At what age will the annuity be due, legal interest being at 5 per cent.?*

Many people imagine that a capital can be deposited in the bank of Venice on this condition, that nothing is received for the first 10 years, but after that period the annuitant receives an annuity equal to the capital. This however is entirely groundless, as has been shewn by M. de Parcieux in his "Addition à l'Essai sur les Probabilités de la durée de la Vie Humaine," published in 1760; for it is there shewn, by a calculation, the demonstration of which is evident, that if £100, for example, were sunk on the life of a child 3 years of age, it could not begin to enjoy an annuity of £100 till it had attained to the age of 45 or 46.

The table of M. de Parcieux presents, on this subject, two things very curious. For example, on the above supposition, if the increase of the annuity were not stopped till the end of 54 years, the person ought to receive £205 per annum during the remainder of his life; if it were not stopped till 58 years, he ought to receive till the time of his death £300; and by stopping it only at 75 years, he would be entitled to £2900 per annum: in the last place, if the arrears due each year were left, on the like conditions, to accumulate till the 94th year, the annuity for the remainder of the person's life ought to be £134069. 19*s.* 2*d.*, a sum which must appear prodigious.

But it may seem astonishing that M. de Parcieux should begin his calculations only at the age of 3 years. It is very true that people do not venture capitals in the

purchase of annuities on the lives of new-born children; but if ever such an establishment existed at Venice, it is evident that it must have been only on the supposition of the money being risked on the life of a child just born, because great mortality takes place during the first year. For this reason we have examined what would be the result of such a supposition; and we have found that, if the sum of £100 were sunk, on the above conditions, on the life of a child just born, it ought, according to the table of M. Dupré de St. Maur, to procure it an annuity of £10. 15s.; that this sum sunk in like manner, at 8 per cent., at the end of the first year, by adding the annuity, would give at the end of the second year £11. 11*s*. 7*d*. These £11. 11*s*. 7*d*. sunk at $6\frac{5}{16}$ per cent., which is the interest that might be allowed at the commencement of the third year, would, at the end of the third, or the commencement of the first fourth, amount to £12. 5*s*. 1*d*., and by a calculation similar to that of M. de Parcieux, it will be found, that the annuity would be increased to £100 at about the age of 36; which is still very far distant from what is commonly believed.

If legal interest be supposed to be 10 per cent., as it was in the 16th century, it will be found, that it would be only about the 26th year that a person could receive an annuity equal to the capital sunk at the time of his birth.

Those who are desirous of farther information on this subject, may consult Demoivre's Essay upon Annuities on Lives, and M. de Parcieux's "Essai sur les Probabilités de la durée de la Vie Humaine," and Dr. Price on Reversionary Payments. The other authors who have treated mathematically on these matters, are Dr. Halley, Sir William Petty, Major Graunt, King, Davenant, Simpson, Maseres; and among the Dutch, the celebrated John de Wit, grand pensionary of Holland, M. Kerseboom, Struyk, &c.

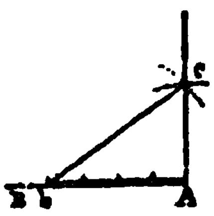
PART SECOND.

CONTAINING A SERIES OF GEOMETRICAL PROBLEMS AND QUESTIONS, CALCULATED FOR EXERCISE AND AMUSEMENT.

PROBLEM I.

From the extremity of a given right line to raise a perpendicular, without continuing the line, and even without changing the opening of the compass if necessary.

Fig. 1.



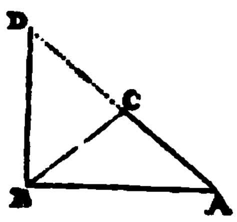
I. LET A B (Fig. 1.) be the given straight line, and A the extremity, from which it is required to raise a perpendicular, without prolonging it.

From A towards B assume 5 equal parts at pleasure; and extending the compasses from A, so as to include 3 of these parts, describe the arc of a circle; then from *b*, the extremity of the fourth part, with an opening equal to the 5 parts, describe another; these two arcs will necessarily cut each other in a certain point C, from which if a straight line, as C A, be drawn, it will be perpendicular to A B.

For the square of C A, which is 9, added to the square of A *b*, which is 16, are together equal to 25, the square C *b*: the triangle C A *b* is therefore rightangled at A.

We might assume also, for the radius of the arc to be described from the point A, a line equal to 5 parts; for the base 12, and for the other radius 13; because 5, 12, and 13, form a rightangled triangle. Indeed, all the rightangled triangles in numbers, of which there are a great variety, may be employed in the solution of this problem.

Fig. 2.

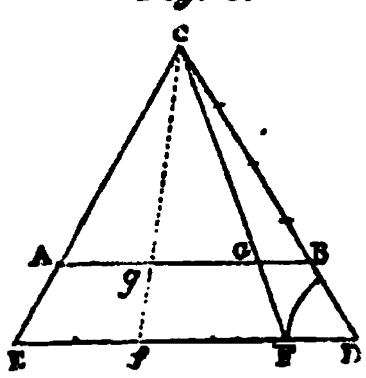


II. On any part whatever of the given line A B (Fig. 2.), describe an isosceles triangle A C B, that is, so that the sides A C, C B shall be equal; and continue A C to D, so that C D shall be equal to C B; if a line be then drawn from D to B, it will be perpendicular to A B. The demonstration of this is so easy that it requires no illustration.

PROBLEM II.

To divide a given straight line into any number of equal parts, at pleasure, without repeated trials.

Fig. 3.



Let it be proposed, for example, to divide the line A B (Fig. 3.) into 5 equal parts. Make this given line the base of an equilateral triangle A B C; and from the point C, in the side C B, continued if necessary, set off 5 equal parts, which we shall suppose to terminate at D, and make C E equal to C D; then make D F, for example, equal to one of the five parts of C D; and draw C F, which will intersect A B in G: it is evident that B G will be the fifth part of A B.

If D *f* were equal to $\frac{2}{5}$ of C D, by drawing C *f* we should have *g*, as the point of intersection of C *f* and A B, which would give B *g* equal to $\frac{2}{5}$ of A B. And so on.

PROBLEM III.

Without any other instrument than a few pegs and a rod, to perform on the ground the greater part of the operations of geometry.

It is well known that most geometrical operations may be performed in the fields,

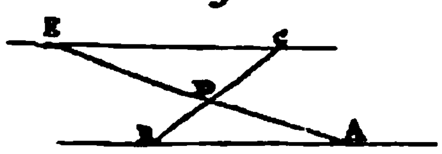
by means of the graphometer: and it would even seem that this instrument is absolutely necessary in practical geometry.

A geometrician however may happen to be unprovided with such an instrument, and even destitute of the means of procuring one. We shall suppose him in the woods of America, with nothing but a knife to cut a few pegs, and a long stick to serve him as a measure: he has several geometrical operations to perform, and even inaccessible heights to measure; how must he proceed to accomplish what is here proposed?

We shall suppose also, that the reader is acquainted with the method of tracing out a straight line on the ground, between two given points; and in what manner it may be indefinitely continued on either side, &c. This being premised, we shall now proceed to give a few of those elementary problems of geometry, required to be performed, without employing any other line than a straight one, and even excluding the use of a cord, with which the arc of a circle may be described.

1st. *Through a given point to draw a straight line, parallel to a given straight line.*

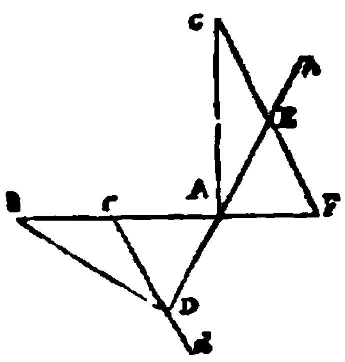
Fig. 4.



Let A B (Fig. 4.) be the straight line, and C the point through which it is required to draw a straight line parallel to A B. From the point C draw the line C B, to any point in A B, and divide C B into two equal parts in D; in this point D fix a peg, and from any point A in the given straight line, draw, through D, an indefinite line A D E, and make D E equal to A D: if a straight line be then drawn through the points C and E, it will be parallel to A B.

2d. *From a given point in a given straight line, to raise a perpendicular.*

Fig. 5.

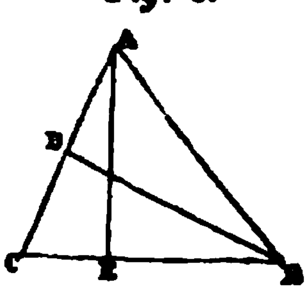


Divide the given line A B (Fig. 5.) into two equal parts, A C and C B; and from the point C draw, any how at pleasure, the line C *d*; make C D equal to C A; draw D A *h*, and make A E equal to A C, and A F equal to A D; through the points E and F draw the line F E G; and if E G be made equal to E F, we shall have the point G, which with the point A will determine the position of the perpendicular A G.

For the sides A D and A C of the triangle C A D, being respectively equal to the sides A F and A E of the triangle E A F, these two triangles are equal; and, in the triangle D C A, the sides C D and C A being equal, the sides E A and E F of the other will be equal also: the angle E F A therefore will be equal to E A F, and consequently to C A D. But in the triangle F G A, the side F G is equal to A B, for F G by construction is the double of F E, and F E or A E is equal to A C, which is the half of A B: the triangles F A G and A D B then are equal; since the sides F G, F A, are equal to the sides A B, A D, and the included angles equal: the angle F A G will therefore be equal to A D B; but the latter is a right angle, because the lines C B, C D, C A, being equal, the point D is in the circumference of a semicircle, described on the diameter A B. The angle F A G then is a right angle, and G A is perpendicular to A B.

3d. *From a given point* A, *to draw a straight line perpendicular to a given straight line.*

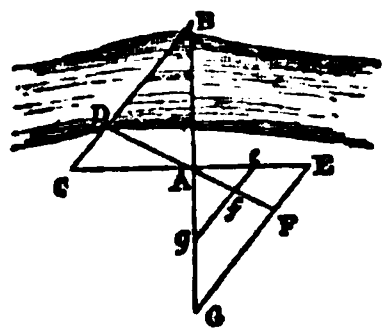
Fig. 6.



Assume any point B (Fig. 6.) in the indefinite line B C; and, having measured the distance B A, make B C equal to B A; draw C A, which must be measured also, and then form this proportion: as B C is to C D, the half of A C, so is A C to a fourth proportional, which will be C E; if C E be then made equal to this fourth proportional, we shall have the point E, from which if the line A E be drawn through A, it will be the perpendicular required.

4th. To measure a distance A B, *accessible only at one of its extremities, as the breadth of a river or ditch, &c.*

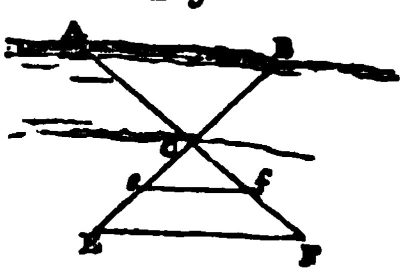
Fig. 7.



First fix a peg at A (Fig. 7.); then another in any point C, assumed at pleasure, and a third at D, in the straight line between the points B and C; continue the lines C A and D A indefinitely beyond A, and make the lines A E and A F respectively equal to A C and A D; in the last place, fix a peg at G, in such a manner as to be in a straight line with A and B, and also with F and E: the distance A G will then be equal to A B.

If it be found impossible to proceed far enough from the line A B towards E or F, we may take in A E or A F, only the half or the third of A C and A D, for example A *e*, A *f*; if a peg be then fixed in *g*, so as to fall in the continuation of both the lines B A and *e f*, we shall have A *g* equal to the half or the third of A B respectively.

Fig. 8.



Now let the distance A B (Fig. 8.) be inaccessible throughout. The solution of this case may be easily deduced from that of the former: for having fixed a peg in C, and having continued by a series of pegs the lines B C and A C, if the parts C E and C F be, by the above means, made respectively equal to B C and C A, or the half or the third of these lines, it may be readily seen that the line which joins the points E and F, will be equal to the line required, or to the half or third of it; and that in either case it will be parallel to it, which resolves the problem, to draw a line parallel to an inaccessible line.

These examples are sufficient to shew in what manner a person, who has only a slight knowledge of geometry, may execute the greater part of geometrical operations, without any other instruments than those which might be procured in a wood by means of a knife. It must indeed be allowed that one can never be in such circumstances, unless on some very extraordinary occasion; but, however, it may afford satisfaction to those who have a turn for geometry, to know in what manner they might proceed, if ever such a case should happen.

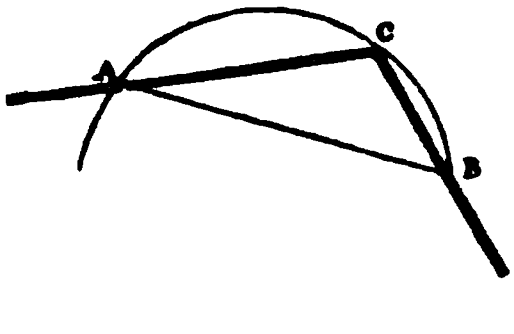
It is remarkable, that it is not perhaps possible to resolve in this manner, that is to say, without employing the arc of a circle, the very simple problem, and one of the first in the elements of geometry, viz., to describe an equilateral triangle. We have often attempted it, but without success, while trying how far we could proceed in geometry by the means of straight lines only.

PROBLEM IV.

To describe a circle, or any determinate arc of a circle, without knowing the centre, and without compasses.

To those who are little acquainted with geometry, this will appear to be a sort of paradox; but it may be easily explained by that proposition, in which it is demonstrated, that the angles whose summits touch the circumference, and whose sides pass through the extremity of the chord, are equal.

Fig. 9.

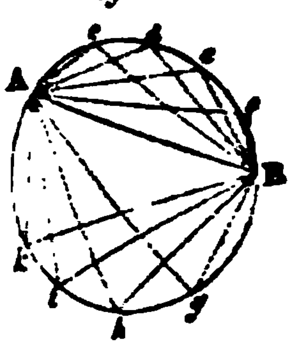


Let A, C, B, (Fig. 9.) be three points in the required circle or arc; having drawn the lines A C and C B, make an angle equal to A C B of any solid substance, and fix two pegs in A and B; if the sides of the determinate angle be then made to slide between these pegs, the vertex or summit will describe the circumference of the circle. So that if the summit or vertex be furnished with a spike or pencil, it will trace out, as it revolves between A and B, the required arc.

If another angle of the like kind were constructed, forming the supplement of A C B to two right angles, and if it were made to revolve with its sides always touching the points A and B, but with its summit in a direction opposite to C, it would describe the other segment of the circle, which with the arc A C B would make up the whole circle.

It may sometimes happen that it is necessary to describe, through two given points, the arc of a determinate circle, the centre of which is at a great distance, or inaccessible on account of some particular causes. Should it be required, for example, to describe on the ground a circle, or the arc of a circle, with a radius equal to 2 or 3 or 4 hundred yards; it may be readily seen that it would be impracticable to do it by means of a cord; the mode of operation therefore must be as follows. In A and B, (Fig. 10.) the extremities of that line which we here suppose to be the chord of the required arc, the amplitude or subtending angle of which is known, fix two pegs, and then find out, by means of a graphometer or plane table, any point c, in such a position, that A c and B c shall form an angle, A c B, equal to the given angle, and in that point fix a peg; then find out another point d, so situated that A d and B d shall form an angle, A d B, equal to the former; if the points f and e be found in like manner, it is evident that the points c, d, e, and f will be in the arc of a circle capable of containing the given angle. If the points g, h, i, k, be then found, on the other side of A B, so situated, that the angle A g B, or A h B, &c. shall be the supplement of the former, the points c, d, e. f, g, h, i, k, will evidently be all in a circle.

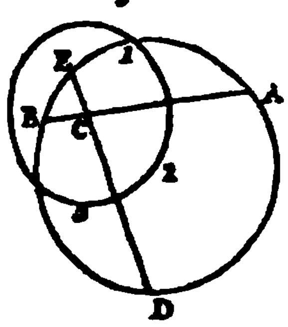
Fig. 10.



PROBLEM V.

Three points, not in the same straight line, being given, to describe a circle which shall pass through them.

Fig. 11.



Let the three points be those marked 1, 2, 3, (Fig. 11.): from one of them as a centre, that for example marked 2, and with any radius at pleasure, describe a circle; and from one of the other two points, 1 for example, assumed as a centre, make with the same radius two intersections in the circumference of the first circle, as at A and B; draw the line A B, and assuming the third point 3 as a centre, make with the same radius two more intersections in the circumference of the first circle, as D and E: if D E be then drawn, it will cut the former line A B in the point C, which will be the centre of the circle required. If a circle therefore be described from this point as a centre, through one of the given points, its circumference will pass through the other two.

It may be readily seen that this construction is the same, in principle, as the common one, taught by Euclid and all other elementary writers; for it is evident that the the lines 1 A, 2 A, 1 B, 2 B, are equal to each other; consequently the line A B is perpendicular to that which would join the points 1 and 2, or to the chord 1 2 of the required circle; hence it follows, that the centre of the circle is in the line A B: for the same reason this centre is in the line D E, and therefore it is in the point where they intersect each other.

If the three given points were in a straight line, the lines A B and D E would become parallel, and consequently there would be no intersection.

PROBLEM VI.

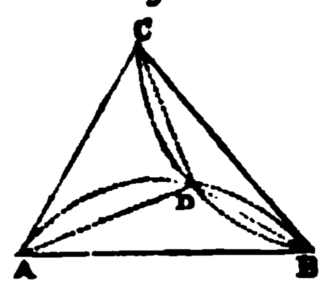
An engineer, employed in a survey, observed from a certain point the three angles formed by three objects, the positions of which he had before determined: it is required to determine the position of that point, without any farther operation.

This problem, reduced to an enunciation purely geometrical, might be proposed in

the following manner: a triangle, the sides and angles of which are known, being given, to determine a point from which, if three lines be drawn to the three angles, they shall form with each other given angles.

In this problem there are a great number of cases; for either the three angles, under which the distances of the three given points are perceived, occupy the whole extent of the horizon, that is to say are equal to four right angles, or occupy only the half, or less than the half. In the first case, it is evident that the required point is situated within the given triangle; in the second, it is situated in one of the sides; and in the third, it is without. But for the sake of brevity we shall here confine ourselves to the first case.

Fig. 12.



Let it be required then to determine, between the points A, B, C, (Fig. 12) the distances of which are given, a point D so situated, that the angle ADB shall be equal to 160 degrees, CDB to 130°, and CDA to 70°. On the side AB describe an arc of a circle capable of containing an angle of 160°; and on the side BC another capable of containing an angle of 130°; the point where they intersect each other will be the point required.

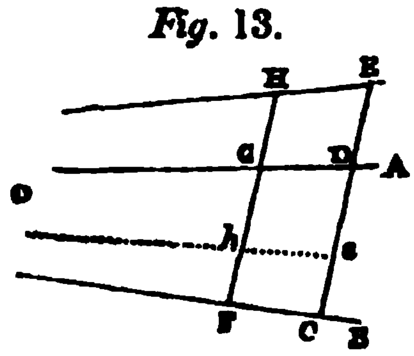
For it is evident that this point is in the circumference of the arc described on the side AB, and capable of containing an angle of 160°; because from all the points of that arc, and of no other, the distance AB is seen under the angle of 160°. In like manner the point D must be found in the arc described on the side BC, and capable of containing an angle of 130°; consequently it must be in the place where they intersect each other, and no where else.

Remark.—On this construction, a trigonometrical solution may be founded, to determine in numbers the distance between D and the points A, B, and C; but we shall leave this to the ingenuity of the reader.

PROBLEM VII.

If two lines meet in an inaccessible point, or a point which cannot be observed, it is proposed to draw, from a given point, a line tending to the inaccessible point.

Fig. 13.



Let the unknown and inaccessible point be O (Fig. 13.), the lines tending to it AO and BO; and let E be the point from which it is required to draw a straight line tending towards O.

Through the point E draw any straight line EC, intersecting AO and BO in the points D and C; and through any point F, assumed at pleasure, draw FG parallel to it; then make this proportion: as CD is to DE, so is FG to GH; if the indefinite line HE be then drawn through the points E and H, it will be the line required.

Or if the given point be *e*, make this proportion, as CD is to C*e*, so is FG to F*h*; the line *eh* will be that required.

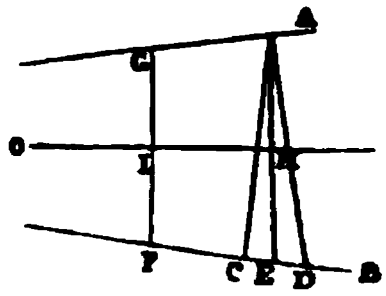
The demonstration of this problem will be easy to those who know, that, in any triangle, if lines be drawn parallel to the base, all those drawn from the vertex of the triangle, will divide them proportionally.

PROBLEM VIII.

The same supposition being made; to cut off two equal portions from the lines BO *and* AO (*Fig.* 14.)

From the point A, draw AC perpendicular to BO, and AD perpendicular to AO; if the angle CAD be then divided into two equal parts by the line AE, meeting BO in E, this line will cut off from BO and AO the two equal parts, AO and EO.

Fig. 14.



This may be easily demonstrated, by shewing that, in consequence of this construction, the angle O A E becomes equal to O E A. But the angle O A E is equal to the angle O A C plus C A E; and the angle O E A is equal to O D A or O A C plus E A D, or E A C, which is equal to it; the angle O A E then is equal to O E A, and the triangle O A E is isosceles, therefore, &c.

PROBLEM IX.

The same supposition still made; to divide the angle A O E *into two equal parts, (See last figure.)*

Construct the same figure as in the preceding problem; then between the two given lines draw any line F G, parallel to the line A E; and divide the lines A E and F G into two equal parts in H and I: the line H I will divide the angle A O E into two equal parts. The demonstration of this is so easy that it requires no illustration.

These problems, as may be readily seen, contain operations of practical geometry of great utility in certain cases; such, for example, as when it is necessary to cut roads through a forest, or when it is required to make them tend to, or end at, a common centre.

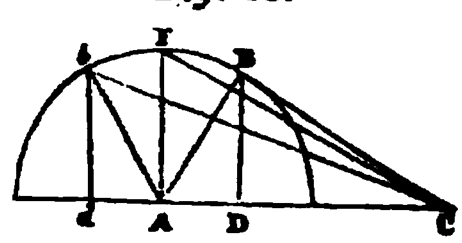
PROBLEM X.

Two sides of a triangle, and the included angle, being given; to find its area.

Multiply one of the sides by half the other, and the product by the sine of the included angle: this new product will be the area.

It may be easily demonstrated, that the area of every triangle is equal to half the rectangle of any two of its sides, multiplied by the sine of the included angle.

Fig. 15.



Let A B C (Fig. 15.) be a triangle, having an acute angle at A; produce A C towards *d*, and from A as a centre, with the distance A B, describe the semicircle B F *b*; then from the point A, draw F A perpendicular to A C; and from the point B, draw B D also perpendicular to A C.

It is here evident that the two triangles F A C and B A C are respectively to each other as A F is to B D; that is to say, as radius is to the sine of the angle B A C, or as unity is to the number which expresses that sine; the triangle F A C then being equal to half the rectangle of F A by A C, the other will be equal to that half rectangle multiplied by the sine of the angle B A C.

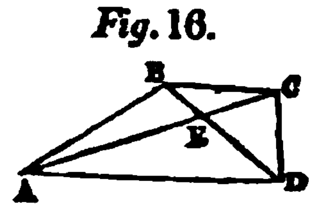
This property enables us to avoid that tedious process, necessary to be employed in order to find out the measure of the perpendicular let fall from the extremity of one of the known sides on the other, that the latter side may be then multiplied by the half of this perpendicular.

Thus for example, let the two sides A B and A C be respectively equal to 24 and 63 yards; and let the included angle be 45°. The product of 63 by 12 is 756, and the sine of 45° is 0·70710; if 756 therefore be multiplied by 0·70710, according to the method of decimal fractions, the product will be 534$\frac{56}{100}$.

PROBLEM XI.

To find the superficial content of any trapezium or quadrilateral figure, without knowing its sides.

Fig. 16.

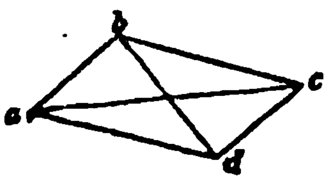


The solution of this problem is a consequence of the preceding. Let the given trapezium be A B C D (Fig. 16.); measure the diagonals A C and B D, as well as the angle which they make at the point where they intersect each other in E; if these diagonals be then multiplied together, and half their product by the sine of the above angle, the last product will be the area. This method is far shorter

than if we should reduce the trapezium to triangles, in order to find the area of each of them.

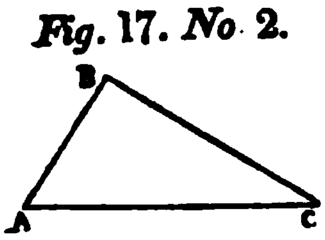
Corollaries.—A very curious theorem, which no author has before remarked, may be deduced from this problem. It is as follows: If two quadrilateral figures have their diagonals equal, and intersecting each other at the same angle, whatever may be their difference in other respects, these quadrilateral figures will be equal as to their area.

Fig. 17. *No.* 1.



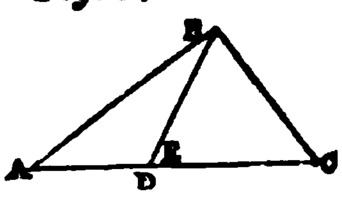
1st. Thus, the quadrilateral A B C D (see last figure) is equal to the parallelogram *a b c d* (Fig. 17. No. 1.), which has its diagonals equal to those of A B C D, and inclined toward each other at the same angle.

Fig. 17. *No* 2.



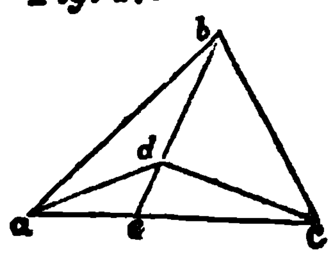
2d. The same quadrilateral A B C D, is equal to the triangle B A C (Fig. 17. No, 2.), formed by the two lines A C and A B, equal to the diagonals A C, D B, and inclined at the same angle.

Fig. 17. *No.* 3.



3d. The same quadrilateral will be equal also to the triangle A B C (Fig. 17. No. 3.), if the lines A C and D B of that triangle, are equal to the diagonals of the quadrilateral, and equally inclined.

Fig. 17. *No.* 4.

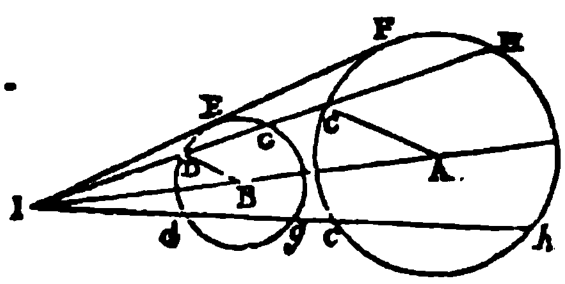


4th. In the last place, this same quadrilateral A B C D (Fig. 16.), will be equal to the quadrilateral *a b c d* (Fig. 17. No. 4.) the diagonals of which do not intersect each other, if *a c* and *d b* are equal to A C and D B, and if the angle *b e c* is equal to the angle B E C.

PROBLEM XII.

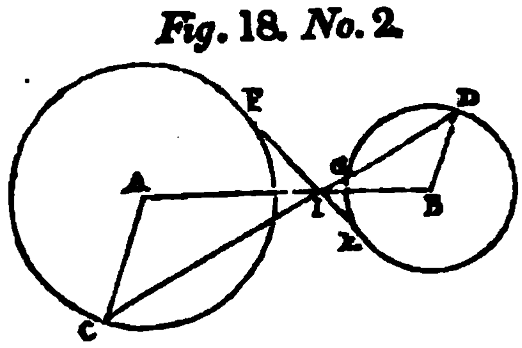
Two circles, not entirely comprehended one within the other, being given; to find a point from which, if a tangent be drawn to the one, it shall be a tangent also to the other.

Fig. 18. *No.* 1.



Through the centres A and B (Fig. 18. No. 1.), of the two circles, draw the indefinite straight line A B I: then from the centre A draw any radius A C, and through the centre B draw the radius B D parallel to it. If the points C and D be joined by the line C D, it will meet A B in I, which will be the point required; that is to say, if I E be drawn from the point I, a tangent to one of the circles, it will be a tangent also to the other.

Fig. 18. *No.* 2.



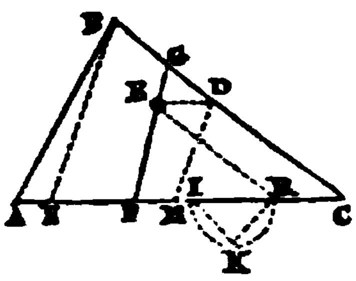
When the circles do not cut each other, the point I (Fig. 18. No. 2.) may happen to fall between them. To find it, in that case, nothing is necessary but to draw the radius B D parallel to A C, and in a direction opposite to that of Fig. 18. No. 1. A B and C D will intersect each other in the point I, which will have the same property as the former.

Remark.—We cannot here help observing, that if any secant whatever, as I D H or I *d h* (Fig. 18. No. 1.), be drawn from the point I, through the two circles, the rectangle of I D and I H, or of I *d* and I *h*, will be always the same, that is, equal to the rectangle of the two tangents I E and I F. In like manner, the rectangle of I C and I G, or of I *c* and I *g*, will be equal to the rectangle of the same tangents. This is a very remarkable extension of the well known property of the circle, by which the rectangle of the two segments I D and I G is equal to the square of the tangent I E.

PROBLEM XIII.

A gentleman, at his death, left two children, to whom he bequeathed a triangular field, to be divided equally between them; in the field is a well, which serves for watering it; and as it is necessary that the line of division should pass through this well, in what manner must it be drawn, so as to intersect the well, and divide the field, at the same time, into two equal parts.

Fig. 19.

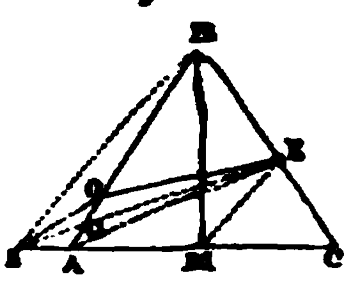


Let the given triangle be ABC (Fig. 19.), and the given point be E. From the point E draw the lines ED and ER, parallel to the base and the side BC respectively, and meeting them in D and R; let the base AC be divided into two equal parts in M; and having drawn the line DM from the point D, draw BN parallel to it, and divide CN into two equal parts, in I; on IR describe the semi-circle IKR, in which apply RK = RC; and, having drawn IK, if IF be made equal to it, the points F and E will determine the line FEG.

Remark.—It is evident that CI must be at least double of CR; otherwise CR could not be applied in the semicircle described on IR, which would render the problem impossible.

In numbers.—Let AB = 48 fathoms, BC = 42, AC = 30, CD = 18, and DE or CR = 6; consequently CM will be = 15. But CD : CM :: CB : CN, that is to say 18 : 15 :: 42 : 35; hence it follows that CN = 35, and CI = $17\frac{1}{2}$; and as CR is equal to 6, we shall have IR = $11\frac{1}{2}$. But the triangle IKR being right-angled, IK = $\sqrt{IR^2 - RK^2} = \sqrt{132\frac{1}{4} - 36} = \sqrt{96\frac{1}{4}}$, or $\frac{81}{100}$ fathoms, which gives CF = $27\frac{31}{100}$ fathoms.

Fig. 20.

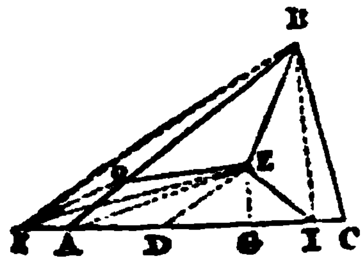


The demonstration of this construction is too prolix to be given in this work; and there are even a variety of cases which it would be tedious to explain. We shall therefore confine ourselves to one of the simplest; that is, where the point E is in one of the sides (Fig. 20.)

The construction in this case is exceedingly easy; for having divided AC into two equal parts in M, and drawn EM, and BN parallel to it; if the point N falls within the triangle, by drawing the line EN, the problem will be solved; but if the point N falls without the triangle, it will be necessary to draw the line AE; then NO parallel to it, through the point N, and OE through the point O: the last line, OE, will solve the problem.

For because EM is parallel to BN, the triangle MBE = MNE; and if the triangle CME be added to each, we shall have the triangles CBM and CEN equal to each other. But the triangle CBM is the half of the triangle ABC, because AM = MC; consequently CEN is the half of ABC also. In like manner, because EA is parallel to NO, the triangles ANE and AOE are equal; and therefore if the triangle AGE, which is common to both, be taken away, the triangle ANG will be equal to GOE; hence it follows, that if we add to the space CAGE the triangle GOE, we shall have the space CAOE = the triangle CEN, which we have already shewn to be equal to the half of ABC.

Fig. 21.



But if the gentleman had left the field to be divided equally among three children, by lines proceeding from the given point E (Fig. 21.); if we suppose one line of division EB already drawn, it would be necessary to proceed as follows:

Divide the base AC into three equal parts; and let the points of division be D and G; draw the line ED, and BF parallel to it; then draw the line EF from the point E, and if the point F does not fall without the triangle, the trapezium BEFAB will be one of the thirds required.

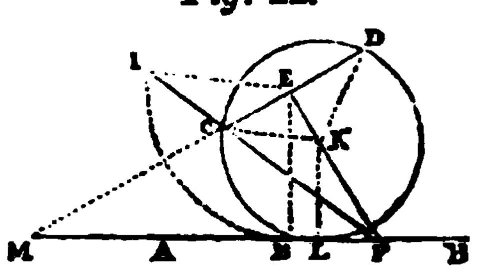
But if the point F falls without the triangle, we must proceed as above directed; that is to say, the line E A must be drawn towards the angle A, and F O parallel to it from the point F, as far as the side A B, which it meets, we shall suppose in O; the line E O will give the triangle B O E, equal to the third of the triangle proposed. B E I C B, the other third, may be found in like manner; consequently the remainder of the figure will be a third also. The three lines therefore, E O, E I, and E B, which proceed from the point E, will divide the proposed triangle into three equal parts.

By the same method a triangle might be divided into 4, or 5, or 6, &c. equal parts, by lines all proceeding from a given point; and this point may be assumed even without the triangle.

PROBLEM XIV.

Two points and a straight line, not passing through them, being given; to describe a circle which shall touch the straight line, and pass through the two given points.

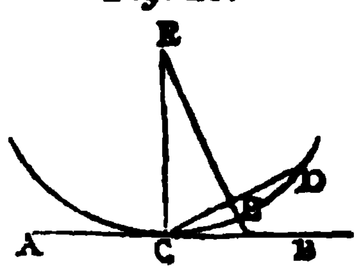
Fig. 22.



Let the given line be A B (Fig. 22.) and the given points C and D. Join these two points, and on the middle of the line C D raise the perpendicular E F, meeting the given straight line in F; and on the same line let fall the perpendicular E H; draw F C, and from the point E, with the radius E H, describe a circle intersecting F C, continued, in I; draw I E, and through the point C draw C K parallel to it: the point K will be the centre, and K C the radius of the circle required.

For if the perpendicular K L be let fall from the point K, on the line A B, it will be equal to K C, which is equal to K D. But F E is to F K, as E H is to K L, and as E I to K C; therefore E H is to K L as E I to K C; and consequently, as E I is equal to E H, K L will be equal to K C: therefore, &c.

Fig. 23.



It may be readily seen, that if the given line passed through one of the given points, the centre of the required circle would be in the point K (Fig. 23.) where C K, drawn perpendicular to A B, intersects E K, which is perpendicular to C D, and divides it into two equal parts in E.

In the first case, the problem might be resolved in a different manner, viz., by continuing the line C D (Fig. 22.) till it meets A B in M; then taking a mean proportional between M C and M D, and making M L equal to it; if a circle were then described through the points C, D, L, it would be the one required. But this solution would be attended with difficulty, if the point M were at a great distance, whereas in the former case this is a matter of indifference.

PROBLEM XV.

Two lines A B *and* C D (*Fig.* 24.) *with a point* E *between them, being given; to describe a circle which shall pass through this point, and touch the two lines.*

Fig 24.

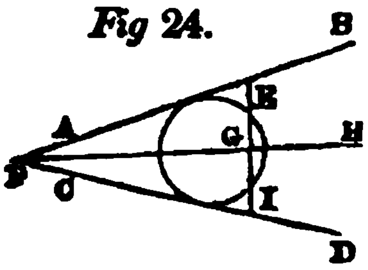
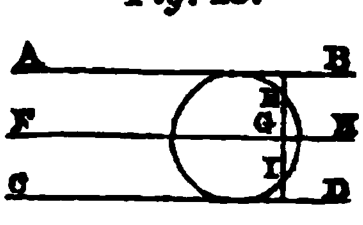


Fig. 25.



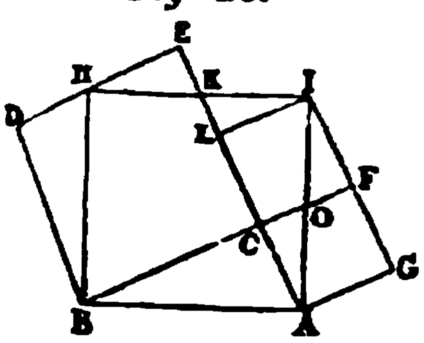
If the two lines meet, as at F, draw the line F H dividing the angle B F D into two equal parts; or, if they are parallel, draw one, such as F H (Fig. 25.) equally distant from both; then from the point E draw E G I, perpendicular to F H, and make G I equal to G E; the points I E will be so situated that if a circle, touching one of the given lines, be described through them, it will touch the other given line also; which reduces this problem to the preceding one.

THEOREM I.

Various demonstrations of the forty-seventh proposition of the first book of Euclid, by the mere transposition of parts.

The beauty of this elementary proposition, and the difficulty beginners often find to comprehend the demonstration, have induced some geometricians to invent others of a simpler nature. Some of these are very ingenious, and worthy of notice, because it can be seen on the first view, that the square of the hypothenuse is composed of the same parts as the squares of the two sides; with this difference only, that they are differently arranged. Some of these demonstrations are as follow:

Fig 26.



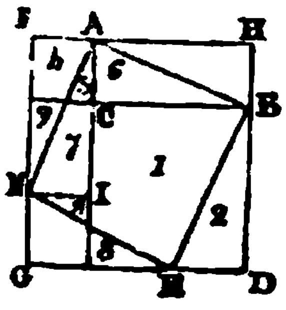
1st. Describe the right angled triangle A B C (Fig. 26.), and on the two sides of it, A C and B C, construct the two squares C G and C D. On the base A B raise the two perpendiculars A I and B H, the former meeting G F, continued, in I, and the latter meeting E D in H; and then draw I H. It may, in the first place, be easily demonstrated that A I and B H are equal to A B; so that A I H B is the square of the base A B; for it may be readily seen that the triangle B H D is equal and similar to the triangle B A C, as well as the triangle I G A; so that B H and A I are each equal to A B.

It may be shewn, with equal ease, that the small triangle K E H is equal to I F O: and lastly, that the triangle I K L is equal to A O C.

But the constituent parts of the two squares are, the quadrilateral C B H K, the triangle B D H, the triangle K H E, the quadrilateral G A O F, and the triangle A C O, which we shall shew to be the same that compose the square A B H I; for the quadrilateral C B H K is common, and the triangle B H D is equal to B C A, and may be substituted for it, and transposed into its place. In like manner, we may conceive the triangle A C O transposed into I K L; there will then remain, in the square of the hypothenuse, the vacuity I L A, and we shall have, to fill it up, the quadrilateral F O A G, with the triangle K E H: let the triangle K E H be transposed into O F I, which is equal to it, and it will complete the triangle I A G, which is equal and similar to I A L; hence it follows that the square of the hypothenuse is composed of the same parts as the squares of the other two sides.

We may therefore cut these parts from a piece of card, and first compose the two squares of the two sides, and then that of the hypothenuse, which will form a sort of amusement in combination.

Fig. 27.



2d. The second method, which is nearly the same as the preceding, will appear perhaps a little more evident. Let C D and C F (Fig. 27) be the squares of the two sides, which contain the right angle of the triangle A C B: having continued F A until A H is equal C B, on the side F H construct the square F H D G: and on A B, the hypothenuse, the square A E. It may be easily proved that the angles E and N will be in the sides of the former, and that A H, B D, E G, N F will be all equal, as well as F A, B H, D E, G N.

But it may be readily seen that, by drawing the line N I parallel to F H, the two squares C D and C F will be composed of the parts 1, 2, 3, 4, 5; and the square A E is composed of the parts 1, 5, 6, 7, 8. But the parts 1 and 5 are common, and the parts 6 and 2 are evidently equal: it remains then that the parts 4 and 3 should be equal to the parts 7 and 8. But this is also evident; for the part 3 is equal to 9, and the part 8 to 5, consequently the parts 4 and 3, or 4 and 9, are equal to the parts 7 and 8, or 7 and 5, since the rectangle F I is divided into two

equal parts by the diagonal. The squares of the sides then are composed of the same parts as the square of the hypothenuse, and consequently they are equal.

3d. Retaining the same construction, it is evident that the square F D is equal to the squares of the two sides A C and C B of the right-angled triangle A C B, plus the two equal rectangles C G and C H. But the square A E, of the hypothenuse, is equal to the same square less the four equal triangles A B H, B E D, E G N, N F A, which taken together are equal to the two rectangles above mentioned, since each of the triangles is the half of one of the rectangles. The quantity by which the square F D exceeds the squares of the sides of the right-angled triangle A C B, is the same as that by which it exceeds the square of the hypothenuse: these squares and that of the hypothenuse are therefore equal; for quantities which are less than a third by an equal quantity, are themselves equal.

We shall now give a few propositions which are only generalizations of the forty-seventh of the first book of Euclid, and from which that celebrated proposition is deduced as a simple corollary.

THEOREM II.

*If a square be described on each of the sides of any triangle A B C (Fig. 28 and 29.); and if a perpendicular B D be let fall from one of the angles, as B, on the opposite side A C; if the lines B E and B F be drawn in such a manner that the angles A E B and C F B shall be equal to the angle B; and lastly if E I and F L be drawn parallel to C G, the side of the square, the square of A B will be equal to the rectangle A I, and the square of B C to the rectangle C L; consequently the sum of the squares on A B and B C will be equal to the square of the base less the rectangle E L, if the angle B be obtuse, and plus the same rectangle if the angle B be acute.**

Fig. 28.

The demonstration of this theorem is as follows: the triangle A E B is similar to the triangle A B C, because the angle A is common, and the angle A E B equal to the angle A B C; consequently A C : A B :: A B : A E, whence it follows that the rectangle of A C × A E, or of A E × A H, which is the same since A H = A C, is equal to the square of A B.

In like manner it may be proved that the square of B C is equal to the rectangle C L.

But it may be readily seen, that if the angle B be obtuse, the line B E will fall between the points A and D, and the line B F between C and D; the contrary of which is the case if the angle B be acute; and that these two lines are confounded with, or coincide with, the perpendicular B D, when the angle B is a right one.

Fig. 29.

In the first case then it is evident, that the sum of the squares of the sides, is less than the square of the base by the rectangle E L.

And in the second case, that they exceed it by the rectangle E L.

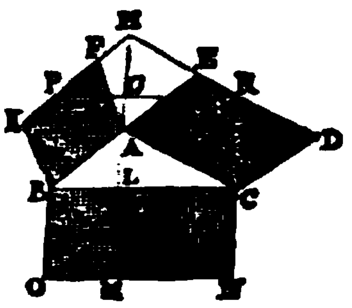
Lastly, that if the triangle be right-angled at B; as the rectangle E L vanishes, the sum of the squares of the sides is equal to the square of the base; which is a very ingenious generalization of the celebrated theorem of Pythagoras.

* For this ingenious theorem, from which is deduced the famous problem of the right-angled triangle, we are indebted to Clairaut, junior, who published it at the age of sixteen, in a small work printed in 1731. This young man would certainly have trodden in the steps of his brother, had he not been cut off by a premature death.

THEOREM III.

Let A B C (Fig. 30.) be a triangle, and let any parallelogram C E be described on the side A C, and any parallelogram B F on the side A D; continue the sides D E and K F till they meet in the point H, from which draw the straight line H A L, and make L M equal to H A; if the parallelogram C O be then completed on the base B C, by drawing B O or C N parallel to L M, this parallelogram will be equal to the two C E and B F.

30.



Continue O B and N C till they meet the sides of the parallelograms B F and C E, in P and R, and draw P R.

Then since C R and H A are parallel, and comprehended between the same parallels, viz. C A and D H, they are equal; consequently C R is equal to L M. In like manner it may be demonstrated that B P is equal to L M. C R and B P therefore are equal, and the figure B P R C is a parallelogram equal to B N.

Now it is evident that the parallelogram R L, on the base R C, is equal to the parallelogram R C A H, because it is on the same base and between the same parallels; and for the same reason the parallelogram A C D E $=$ A C R H; consequently the parallelogram A C D E $=$ R C L G.

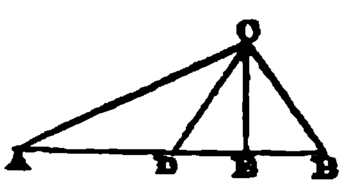
It may be demonstrated, in like manner, that the parallelogram B K F A $=$ B P G L; consequently the two parallelograms C E, B F, are together equal to B P R C, or to B C N O, which is equal to it.

Corollary.—The reader, if in the least acquainted with geometry, may readily see that this very ingenious proposition is only a generalization of the celebrated proposition by which it is proved, that in every right-angled triangle, the squares of the two sides, containing the right angle, are equal to that of the hypothenuse. For if we suppose that the triangle B A C is right-angled at A, and that the two parallelograms C E and B F are the two squares, it may be easily conceived that the third parallelogram B N will be also a square, viz. that of the hypothenuse; in consequence of the preceding demonstration then, these two first squares will be equal to the third. This theorem is extracted from Pappus Alexandrinus.

THEOREM IV.

If the base of a triangle be bisected, the squares of the other two sides are equal to twice the square of half the base, and twice the square of the line joining the middle point of the base to the vertical angle.

Fig. 31.

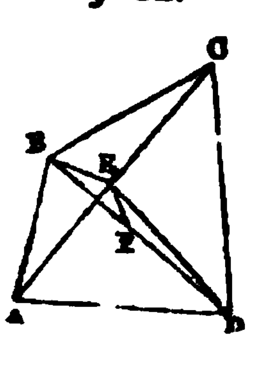


For let D (Fig. 31.) be the middle of the base, and C E a perpendicular from the vertex C on the base A B. Then $AC^2 = AD^2 + DC^2 + 2\,AD\,.\,DE$ and $BC^2 = BD^2 + DC^2 - 2\,BD\,.\,DE$ or $BC^2 = AD^2 + DC^2 - 2\,AD\,.\,DE$. Whence by adding we have $AC^2 + BC^2 = 2\,AD^2 + 2\,DC^2$.

THEOREM V.

In every quadrilateral figure whatever, the sum of the squares of the four sides, is equal to the sum of the squares of the two diagonals, plus four times the square of the line which joins the middle of these diagonals.

Fig. 32.



Let A B C D (Fig. 32,) be a quadrilateral figure, the two diagonals of which are A C and B D; and let us suppose them divided each into two equal parts in E and F, and that the straight line E F has been drawn. It may be demonstrated, that the squares of the four sides, taken together, are equal to the squares of the two diagonals, plus four times the square of E F.

It is said that we are indebted to Euler for this elegant and very curious problem, which may be easily demonstrated by the aid of the preceding theorem.

For join E to B and D. Then $AB^2 + BC^2 = 2AE^2 + 2EB^2$, and $AD^2 + DC^2 = 2AE^2 + 2ED^2$. Therefore $AB^2 + BC^2 + AD^2 + DC^2 = 4AE^2 + 2EB^2 + 2ED^2$. But $BE^2 + ED^2 = 2BF^2 + 2FE^2$; therefore $2BE^2 + 2ED^2 = 4BF^2 + 4FE^2$. Therefore $AB^2 + BC^2 + CD^2 + AD^2 = 4BE^2 + 4BF^2 + 4FE^2 = AC^2 + BD^2 + 4FE^2$.

Corollary.—If the quadrilateral is a parallelogram, then E F coincide, and the proposition shews that the square of the sides of a parallelogram are together equal to the squares of the diagonals.

The preceding theorem, therefore, is only a particular case of the present one.

PROBLEM XVI.

The three sides of a rectilineal triangle being given; to determine its superficial content, without measuring the perpendicular let fall from one of the angles on the opposite side.

From half the sum of the three sides subtract each of the three sides separately; multiply the three remainders together, and the product by the half sum of the sides; the square root of the last product is the area required.

Let the three sides, for example, be 50, 120, and 150 yards; the half sum of which is 160; the first difference is 110, the second 40, and the third 10: the product of these four numbers is 7040000, the square root of which is 2653 and $\frac{3}{10}$ nearly, which is the area.

It may be easily shewn, that the usual method, that is to say, by finding the perpendicular let fall from one of the angles on the opposite side, would require a much more tedious calculation.

Remark.—By this method we have a very easy rule for finding the radius of the circle inscribed in a triangle, the three sides of which are given: nothing is necessary but to multiply together the difference between each side and the half sum; to divide the product by this half sum, and to extract the square root of the quotient: the result will be the radius required.

Thus, in the above example, the product of the differences is 44000; which divided by 160, gives 275; the square root of this quotient $16\frac{58}{100}$, is the radius of the circle inscribed in the given triangle.

PROBLEM XVII.

In surveying the side of a hill, ought its real surface to be measured, or only the space occupied by its horizontal projection?

It may be easily proved that, in this case, the horizontal projection or base only ought to be measured; for the object of surveying is nothing else than to determine the quantity of any kind of production that land is capable of producing, or the number of the buildings that can be erected on it. But it is evident that as trees and plants always rise in a direction perpendicular to the horizon, an inclined plane can contain no more than the horizontal one which corresponds to it as its base. In like manner, no more buildings can be raised on inclined ground, than on its horizontal projection; because the walls of an edifice must always be vertical: a little more care only is required in building on such ground than on horizontal.

Another reason is, that inclined ground, compared with the horizontal ground in the neighbourhood, contains less vegetable earth or mould, as part of it is always carried away by the rains, and deposited on the lower grounds; consequently it is not capable of supplying nourishment to such a quantity of productions as the other.

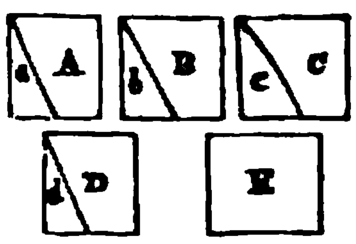
It is therefore evident that the horizontal surface only, and not the real or inclined surface, ought to be measured, unless these considerations are thought to be of little value in adjusting the price.

Remark.—It is in topographical descriptions of mountainous countries chiefly, that care should be taken to reduce the whole to a horizontal plane; for if we suppose that a country has been surveyed, and that, in measuring the sides of pretty steep mountains, the real and not the horizontal distances of places have been taken, it will be impossible, in constructing a map, to make the measures agree. This indeed would be the same thing as if one should attempt to transfer to the plane or base of a pyramid, the triangles which form its inclined sides: for if one of the triangles were laid down on it, all the rest would be falsely represented.

PROBLEM XVIII.

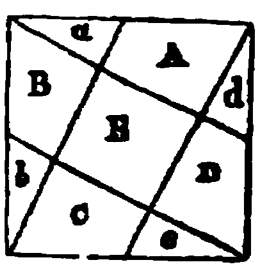
To form one square of five equal squares.

Fig. 33. *No.* 1.



Divide one side of each of four of the squares, as A, B, C, D, (Fig. 33. No. 1.) into two equal parts, and from one of the angles adjacent to the opposite side draw a straight line to the point of division; then cut these four squares in the direction of that line, by which means each of them will be divided into a trapezium and a triangle.

Fig. 33 *No.* 2.



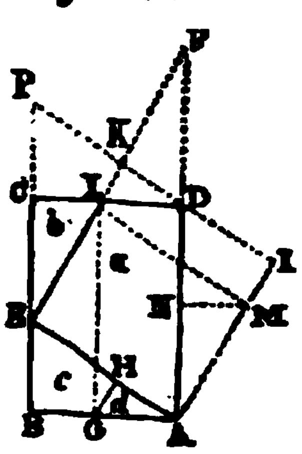
Lastly, arrange these four trapeziums and these four triangles around the whole square E, as seen Fig. 33. No. 2; and you will have a square evidently equal to the five squares given.

Remark.—By means of the solution to the following problem, one square may be formed of any number of squares at pleasure; for any number of squares may be transformed into an oblong, and we shall shew, in the next problem, how an oblong may be resolved into several parts, susceptible of being arranged in such a manner as to form a square.

PROBLEM XIX.

Any rectangle whatever being given; to convert it, by a simple transposition of parts, into a square.

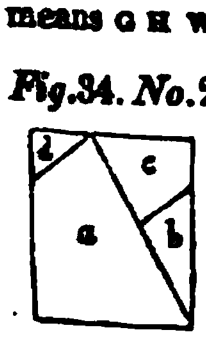
Fig. 34. *No.* 1.



Let the given rectangle be ABCD (Fig.34. No.1.) To cut it into several parts susceptible of being arranged in a square, first find the geometric mean proportional between the sides BA and AD; make AE equal to that mean proportional, and draw EF perpendicular to AE. EF will cut AD in the point F, which will either fall beyond D, in regard to the point A, or on the point D itself, or between D and A: this forms three cases, the last of which subdivides itself into two, but if one of them be well understood, there will be no difficulty in the rest.

Case 1st. In the first place then, let the point F be beyond D, as seen (Fig. 34. No. 1.) As the line EF will intersect CD in the point L, make AG equal to DL, and draw GH perpendicular to AE, by which means GH will cut off from the triangle ABE, the small triangle AGH.

*Fig.*34. *No.*2.

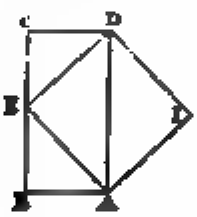


Then cut the given rectangle AC into four parts, according to the lines AE, EL, and GH, and the result will be the trapezium AELD, the triangle ECL, the trapezium GBEH, and the small triangle AGH, which we shall respectively denote by the letters *a*, *b*, *c*, *d*; lastly, arrange these four parts as seen Fig. 34, No. 2, and you will have a perfect square.

The demonstration may be easily found, by considering, in Fig. 34, No. 1, the square constructed on A E, viz. A E K I; but it is first necessary to shew, that if A I be drawn parallel to E F, and K I, through the point D, parallel to A E, the rectangle A E K I, thence resulting, will be a square. Now this is easy; for if I K be continued till it meet B C produced in P, we shall evidently have the rectangle A E K I equal to the parallelogram A E P D, which is equal to the rectangle A B C D, or that of A B and A D; hence it follows that A E into A I is equal to A B × A D. But the square of A E is equal to A B into A D, consequently A E into A I is the same thing as the square of A E.

This being demonstrated, draw L G parallel to A D, and L M parallel to A E; then, from the points M and G, to A D and A E, draw the perpendiculars M N and G H. It is here evident that the triangle A M N is equal and similar to E L C: in like manner the triangle A G H is equal and similar to D L K; and the trapezium B E H G is equal and similar to N D I M, for B E is equal and parallel to D N, E G to M N, D I to E H, and M I to G H. The four parts A E L D, E C L, B E H G, A G H, which compose the rectangle A C, are therefore equal to the four A E L D, A M N, N D I M, and D L K, which compose the square A E K I, or its equal, that of the same figure, No. 2, &c.

Fig. 34. No. 3.



Case 2d. If the point F falls on the point D, the solution of the problem will be exceedingly easy; for in that case the triangle *d* vanishes, since D L vanishes; the square equal to the rectangle, therefore, will be composed of the rightangled isosceles triangle A E D (Fig. 34. No. 3.), and the other two right-angled and isosceles triangles A B E and C D E, equal to each other, and to the half of the former; consequently these parts may be arranged in a square without any difficulty. This case indeed can never exist but when the side A B is exactly the half of A D: the rectangle A C is then composed of two equal squares. But the manner in which two equal squares may be fomed into one is well known.

Fig. 35. No. 1.

Case 3d. Let us now suppose, that the point F falls between A and D (Fig. 35. No. 1.), but in such a manner that F D is less than E B. In this case make E G equal to F D, and draw G H perpendicular to A E; by which means the rectangle A C will be divided into four parts, viz., the triangle A E F, the trapezium C D F E, the trapezium A B G H, and the triangle E G H; which we shall distinguish by the letters *a*, *b*, *c*, *d*. If these four parts be arranged as seen Fig. 35. No. 2, we shall have a perfect square, as may be easily demonstrated.

Fig. 35. No. 2.

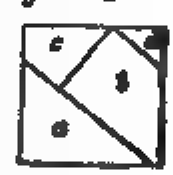
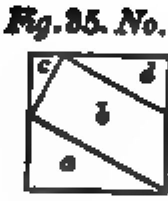


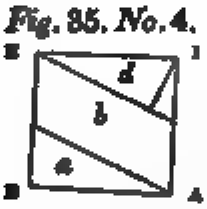
Fig. 35. No. 3.



If F D be exactly equal to B E, it is evident, that instead of the trapezium A B G H, we should have a triangle A B *h*; so that the square to be formed would consist of three triangles, and a trapezium E C D F, as seen Fig. 35. No. 3.

If F D exceeds E B, and is exactly equal to A F, draw D P parallel to E F, and if the rectangle be cut according to the lines A E, E F, and F D, there will be formed three triangles and a parallelogram E D, which if arranged as seen Fig. 35. No. 4, will compose the square A I K B.

Fig. 35. No. 4.



Lastly, we may suppose the height A D of the given rectangle to be such, that having the general construction described in the first part of this problem, the line F D exceeds the line A F, or is any multiple of it, with or without a remainder. In that case, to resolve the problem, set off the line A F as many times as possible on F D. For the sake of simplification, we shall here suppose that the former is contained in the latter only once, with the remainder L D. Draw L M parallel to E F, and by these means we shall have the parallelogram L M E F,

which may be placed in F A N O; then make E G equal to D L, and draw G H perpendicular to A E; cut the rectangle A B C D according to the lines A E, E F, M L, and G H, into five parts, viz., the triangle A E F, the parallelogram F L M F, the trapeziums L D C M, A H G B, and the triangle G H E; which we shall distinguish by the letters *a*, *b*, *c*, *d*, *e*; these five parts can be arranged into a perfect square, as A I K E, which is composed of the triangle *a*, the parallelogram *b*, the trapeziums *c* and *d*, and the small triangle *e*.

If A F were contained twice in F D, six parts would be requisite; two of them parallelograms as *b*.

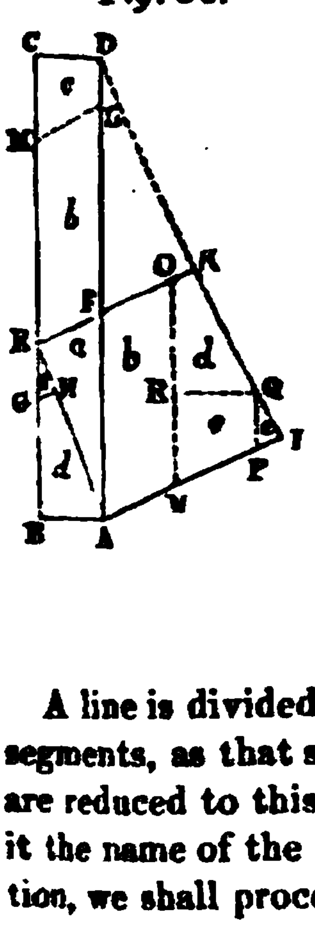
By a sort of retrograde progress, the following problem may be resolved.

PROBLEM XX.

To cut a given square into 4, or 5, or 6, &c. dissimilar parts, which can be arranged so as to form a rectangle.

Let it be required, for example, to divide the square A E K I (Fig. 35. No. 1.), into four parts susceptible of such an arrangement. On the side E K assume E F greater than the half of it, and draw A F; make A O equal to E F, and draw O M parallel to A F; lastly, from the point M, where O M meets I K, draw M N perpendicular to A F; the four parts required will be the triangles A E F, O M I, and the two trapeziums A O M N, M N F K, which may be arranged in such a manner, as to form the rectangle A B C D. To those who have comprehended the solution of the preceding problem, this will appear evident.

Fig. 36.



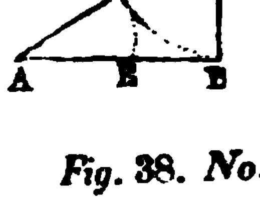
If five parts be required, assume E F (Fig. 36.), of such a length, that it may be contained in E K twice, with a remainder; let these parts of the line E K be E F and F O, and let the remainder be O K; draw A F, and, making A N and N P each equal to E F, draw N O and P Q parallel to A F, the latter of which will meet the side K I in Q; from this point draw Q R perpendicular to N O; and we shall have two triangles, a parallelogram, and two trapeziums, which are evidently susceptible of being formed into an oblong such as A B C D; since they are the same parts into which that oblong might be divided, in order to form, by their transposition, the square A E K I; therefore, &c.

PROBLEM XXI.

To divide a line in extreme and mean ratio.

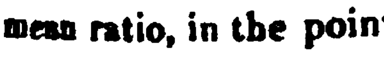
A line is divided in extreme and mean ratio, when the whole line is to one of the segments, as that segment is to the other. As a great many geometrical problems are reduced to this division, some of the geometricians of the sixteenth century gave it the name of the *divine section*. But without adopting so emphatical a denomination, we shall proceed to the solution of the problem.

Fig. 37.



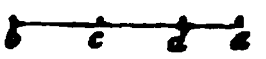
Let the line, to be divided in extreme and mean ratio, be A B (Fig 37.) From its extremity B raise the perpendicular B C, and make it equal to the half of A B; draw A C, and make C D equal to C B; if A E be then made equal to the remainder A D, the line A B will be divided as required, and we shall have this ratio; A B is to A E as A E is to E B.

Fig. 38. *No.* 1.



Remarks.—The line *a b* (Fig. 38. No. 1.) being divided in extreme and mean ratio, if its greater segment be added to it, we shall have the line *b c*, also divided in extreme and mean ratio, in the point *a*; so that *b c* will be to *b a* as *b a* is to *a c*.

Fig. 38. *No.* 2.

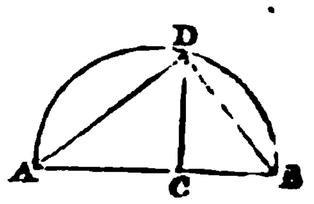


2d. The line *b a* (Fig. 38. No. 2.), being divided, in the same manner in *c*, if *c d* be made equal to the small segment *b c*, *c a* will then be divided in the same manner; that is to say, *c a* will be to *c d* as *c d* to *d a*.

PROBLEM XXII.

On a given base to describe a right angled triangle, the three sides of which shall be in continued proportion.

Fig. 39.

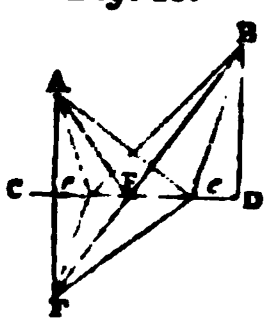


On the given base A B (Fig. 39.) describe a semicircle; divide A B in extreme and mean ratio in C, and raise the perpendicular C D till it meet the semicircle in D; then draw the lines A D and D B: the triangle A B D will be the one required, and A B will have the same ratio to A D as A D has to D B, as might be easily demonstrated.

PROBLEM XXIII.

Two men, who run equally well, propose for a bet to start from A, *and to try who shall first reach* B, *after touching the wall* C D, (*Fig.* 40.) *What course must be pursued in order to win?*

Fig. 40.



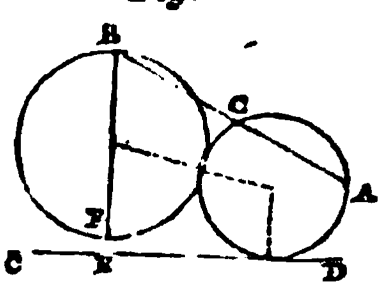
It may be readily seen that, to determine the course to be pursued in order to win, it will be necessary to determine the position of the lines A E and E B, of such a nature, that their sum shall be less than that of all the others, as A *e*, *e* B, &c. But it may be demonstrated that this sum is the least possible, when the angle A E C is equal to the angle B E D. For let us suppose A C drawn perpendicular to C D, and continued till C F be equal to A C, and that E F and E B have been drawn; in this case the angles A E C and C E F will be equal. But A E C is equal to B E D by the supposition, consequently the angles C E F and B E D will be equal also; and it thence follows, that, as C D is a straight line, F E B will likewise be one. But B E F is equal to B E and E A taken together, as B *e* and *e* F are to B *e* and *e* A; the course B E A therefore will be shorter than any other B *e* A, for the same reason that B F is shorter than the lines B *e* and *e* F.

To find then the point E, we must draw A C and B D perpendicular to the line C D, and then divide C D in E, in such a manner, that C E shall be to E D as C A to D B.

PROBLEM XXIV.

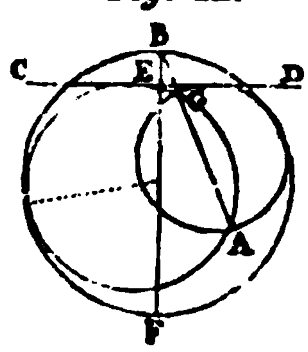
A point, a circle, and a straight line, being given in position, to describe a circle which shall pass through the given point, and touch the circle and straight line.

Fig. 41.



Through the centre of the given circle draw B E (Fig. 41.) perpendicular to the given straight line, and let it cut the circle in B and F; draw also B A to the given point A, and take B G a fourth proportional to B A, B E, B F; if a circle be then described through the points A and G, touching the line C D, it will touch also the given circle.

Fig. 42.



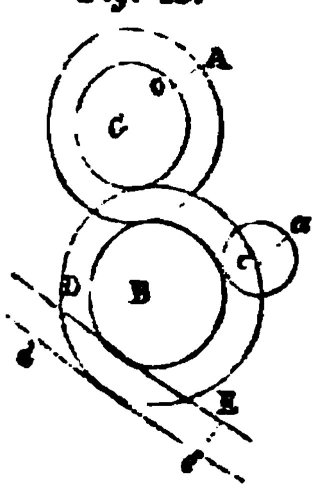
If the point A be within the circle, the construction will be the same: in this case it is evident that the line which ought to be touched by the required circle, must enter the given circle also; and there are even two circles which will resolve the problem, as may be seen in Fig. 42.

PROBLEM XXV.

Two circles and a straight line being given; to describe a circle which shall touch them all.

This problem is evidently susceptible of several cases; for the circle which touches the straight line may inclose both the other circles, or only one of them, or may leave them both without it; but, for the sake of brevity, we shall confine ourselves to the last case, and leave the rest to the sagacity of our readers, who, when they comprehend this solution, will find no difficulty to resolve the rest.

Fig. 43.

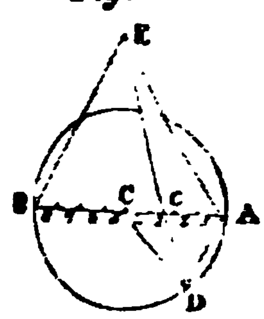


Let there be given two circles, whose radii are C A and *c a* (Fig. 43.), and let the line D E be given in position. In the present case, on the radius C A make A O equal *c a*, and with the radius C O describe a new circle; draw also beyond D E the line *d e* parallel to D E, and distant from it by a quantity equal to *c a*; then, by the preceding problem, describe a circle through *c*, which shall touch the circle having for its radius C O, and also the straight line *d e*; let the centre of this circle be B; if its radius be diminished by the quantity A O or *c a*, the circle described with this new radius will evidently be a tangent to the two given circles, as well as to the straight line D E.

PROBLEM XXVI.

Of inscribing regular polygons in the circle.

Fig. 44.



The following general method of inscribing regular polygons in the circle, is given in various books of practical geometry. On the diameter A B (Fig. 44.) of the given circle, describe an equilateral triangle; and divide this diameter into as many equal parts as the required polygon is intended to have sides; then from E, the summit of the triangle, draw through *c*, the extremity of the second division, the line E *c*; and continue it till it meet the circumference of the circle in D: the chord A D, they say, will be the side of the required polygon to be inscribed.

We have noticed this method merely to say that it is erroneous, and could be invented only by a person ignorant of geometry, or else intended only as near the truth. For it may be easily demonstrated that it is false, even when employed for finding the simplest polygons, such for example as the octagon. It will be found indeed, by trigonometrical calculation, that the angle D C A, which ought to be 45°, is 48° 14′; whence it follows, that the chord A D is not the side of the inscribed octagon.

None of the regular polygons can be inscribed geometrically and without trial, by means of a rule and compasses, except the triangle, and those polygons deduced from it, by doubling the number of sides, as the hexagon, the dodecagon, &c.

The square and those polygons deduced from it in like manner, as the octagon, the sedecagon, &c.

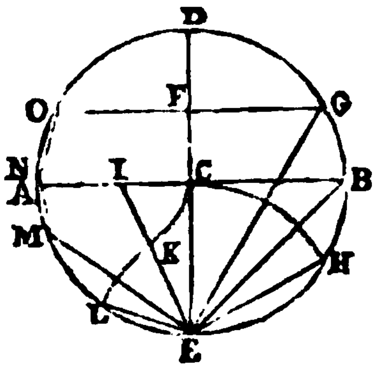
The pentagon and those deduced from it, as the decagon, and the eikosiagon, &c.

The pentedecagon and its derivatives, as the polygon of thirty sides, &c.

The rest, such as the heptagon, enneagon, endecagon, &c., cannot be described by means of the rule and compasses alone, without trial; and all those who have attempted this method, have failed or have produced ridiculous paralogisms.

The following in a few words, is the method of describing geometrically in a circle, the five primitive polygons, which may be inscribed with the rule and compasses.

Fig. 45.



Divide the circle A D B E (Fig. 45.) into four equal parts, by the two diameters A B and D E, intersecting each other at right angles; then divide the radius C D into two equal parts in F, and draw O F G parallel to A B: the line E G will be the side of the inscribed equilateral triangle, as well as G O and O E.

The line E B, as every one knows, will be the side of the square.

If E H be made equal to the radius, it is in like manner evident, that it will be the side of the hexagon.

Divide the radius A C into two equal parts in I, and draw E I; make I K equal to I C, and the chord E L equal to the remainder E K: E L will be the side of the decagon; and by making the arc L M equal to the arc E L, we shall have the chord E M for the side of the pentagon.

Then divide the arc O M, which is the difference between the arc of the pentagon and that of the triangle, into two equal parts in N, and draw the straight line O N, which will be the side of the pentadecagon, or polygon of 15 sides.

Remark.—The heptagon is susceptible of a construction, not geometrical, but approximated, which is pretty near the truth, and which on that account deserves to be known; it is as follows: First describe an equilateral triangle, or at least determine the side of one, the half of which will be the side nearly of the insusceptible heptagon. It will be found indeed by calculation, that the side of the triangle, radius being unity, will be equal to 0·86602, the half of which is 0·43301, and the side of the heptagon is 0·43387; the difference therefore between it and half the side of triangle, is less than a thousandth part. Whenever then the thousandth part of the radius of the given circle is an insensible quantity, the above construction will approach very near to the truth.

It is much to be wished that methods of construction equally simple, and as near the truth, could be discovered for all other polygons; which indeed is not possible

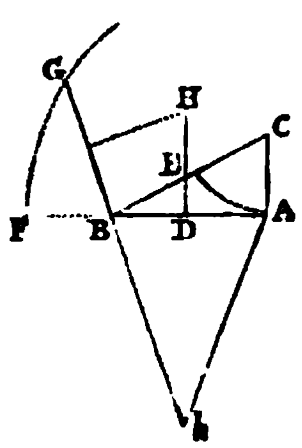
PROBLEM XXVII.

The side of a polygon of a given number of sides being known; to find the centre of the circumscriptible circle.

This problem is, in some measure, the reverse of the former, and may be easily solved for the same polygons.

We shall say nothing of the triangle, the square, and the hexagon, because those who are acquainted with the first elements of geometry, know how to find the centre of an equilateral triangle and a square, and that the side of the hexagon is equal to the radius of the circumscriptible circle.

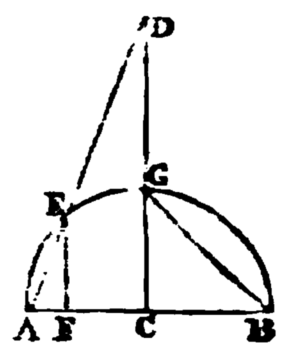
Fig. 46.



We shall begin therefore with the *pentagon.*—Let A B (Fig. 46.) be the side of the pentagon: at the extremity of which raise the perpendicular A C, equal to ½ A B; draw B C, and cut off from it C E = A C, and make B F = B E; then with the centre A, and the radius A F, describe an arc of a circle, and from the point B, with the radius B A, describe another intersecting the former in G: the line B G will be the position of the second side of the pentagon, and the two perpendiculars on the middle of the sides A B and B G will give, by their intersection, he position of the centre H.

For the octagon.—Let A B (Fig. 47.) be the given side; on this line describe a semicircle, and raise the radius C G perpendi-

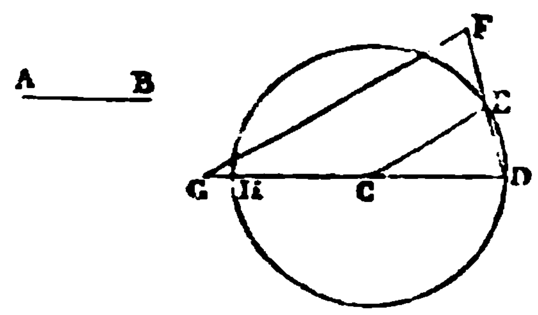
Fig. 47.



cular, and indefinitely continued; draw the side of the square B G, and make C F equal to the half of B G; draw F E perpendicular to the diameter, and through the point E, where it cuts the semicircle, draw A E, which will meet C G continued in D: this point D will be the centre of the circle required.

For the decagon.—If A B (Fig. 46.) be the given side, find, as if a pentagon were to be constructed, the line B F, and from the points A and B with the radius A F, describe the isosceles triangle A *h* B: the point *h* will be the centre of the decagon.

Fig. 48



For the dodecagon, and any other polygons whatever.—Let the line given for the side of the polygon be A B (Fig. 48.) With any radius whatever C D describe a circle, and inscribe in it the required dodecagon or polygon, the side of which we shall suppose to be D E: continue D E to F, if A B exceeds D E, so that D F shall be equal to A B, and then draw C E, and its parallel F G: the point where the latter meets the diameter D H continued, will evidently be the centre of the circle, in which the required polygon is inscriptible.

Though we have given particular methods for the pentagon, octagon, and decagon, it is evident that the last method may be applied equally to them all.

We shall conclude this article, on polygons, with the two useful tables, one of which contains the sides of the polygons, the radius of the circle being given, and the other the length of the radius, the side of the polygon being known. If the radius of the circle then be expressed by 100000, the side of the inscribed triangle will be

within an unit of	173205	that of the decagon	61803
that of the square........	141421	that of the endecagon	56347
that of the pentagon	117557	that of the dodecagon	51763
that of the hexagon	100000	that of the tredecagon....	47844
that of the heptagon	86777	that of the tesseradecagon.	44503
that of the octagon	76536	that of the quindecagon ..	41582
that of the enneagon	68404		

On the other hand, if the side of the polygon be 100000, the radius of the circle will be, in the case of the triangle

in the case of the triangle	57735	of the decagon	161804
of the square............	70710	of the endecagon	177470
of the pentagon	85065	of the dodecagon	193188
of the hexagon	100000	of the tredecagon	209012
of the heptagon...........	115237	of the tesseradecagon	224703
of the octagon	130657	of the quindecagon	240488
of the enneagon	146190		

PROBLEM XXVIII.

Method of forming the different regular bodies.

It was long ago demonstrated in geometry, that there can be only five bodies terminated by regular figures, all equal to each other, and forming with one another equal angles. These bodies are:

The tetraedron, which is formed by four equilateral triangles.

The cube, or hexaedron, formed of six equal squares.

The octaedron, formed of eight equal equilateral triangles.

The dodecaedron, formed of twelve equal pentagons.

The icosaedron, formed of twenty equilateral triangles.

Two methods may be employed to form any one of these regular bodies. The first is, to construct a sphere, and then to cut off the excess, so that the remainder

shall form the regular body required; the other, which resembles the process used in stone-cutting, consists in first tracing out on a plane, made at hazard, one of the faces of the body to be formed, and then cutting out the adjacent faces, under the determinate angles.

To resolve then the problem in question, we shall first answer the following questions.

1st. The diameter of a sphere being given, to find the sides of the faces of each of the regular bodies.

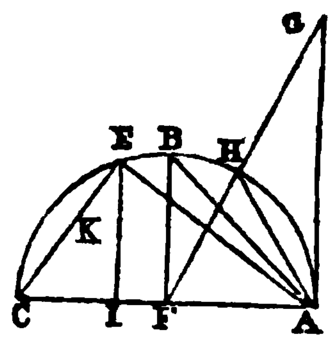
2d. To find the diameters of the less circles of that sphere, in which the faces of each of these bodies are inscriptible.

3d. To determine the opening of the compasses, with which each of these circles may be described on the surface of the same sphere.

4th. To determine the angles which the contiguous faces form with each other, in their common intersection.

1st. *A sphere being given; to find the sides of the faces of each of the five regular bodies.*

Fig. 49.



Let A B C (Fig. 49.) be the half of a great circle of the given sphere, and A C one of its diameters. Divide A C into three equal parts, and let A I be two thirds; draw E I perpendicular to the diameter, cutting the circle in E, and join A E: this line will be one of the faces of the tetraedron, and C E will be that of the cube or hexaedron.

Then, through the centre F, draw the radius F B, perpendicular to A C, cutting the circle in B, and join A B: this line A B will be the side of the octaedron inscribed in the same sphere.

The side of the dodecaedron will be found, by dividing E C, the side the hexaedron, in mean and extreme ratio, and taking for the side of the dodecaedron the larger segment C K.

Lastly, from A, the extremity of the diameter, draw the perpendicular A G, equal to A C, and from the centre F drawn the line F G, intersecting the circle in H; A H will be the side of the icosaedron.

The radius of the circle being 10000, the side of the tetraedron will be found, by calculation, to be equal to 16329; that of the hexaedron or cube, 11546; that of the octaedron, 14142; that of the dodecaedron, 77136; and that of the icosaedron, 10514.

2d. *To find the radius of the lesser circle of the sphere, in which the face of the proposed regular body is inscriptible.*

The method of determining the radius of the circle circumscriptible to the triangle, the square, and the pentagon, which are the only faces of the regular bodies, has been shewn already, and consequently the problem is thus solved.

To express them in numbers, as we know that when the side of the equilateral triangle is 10000, the radius of the circumscribing circle is 5773, therefore, as the side of the tetraedron is 16329, nothing is necessary but to say, As 10000 is to 5773, so is 16329 to a fourth proportional, which will be 9426.

It will be found, in like manner, that the radius of the lesser circle, in which the octaedron can be inscribed, is 8164.

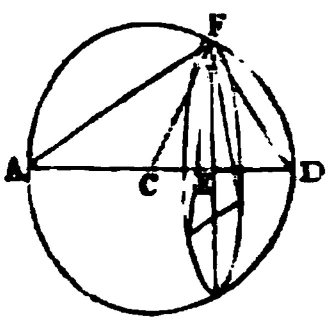
And it will be found also, that the radius of the circle in which the face of the icosaedron can be inscribed, is 6070.

The side of the square being 10000, the radius for the circumscribing circle, as is well known, is 7071; which will give for the radius of the face of the hexaedron, 8164.

Lastly, the side of the pentagon being 10000, we shall have for the radius of the circumscribing circle 8506, which will give for the radius of the face of the dodecaedron, 6070.

3d. *To determine the opening of the compasses, with which the circle, capable of receiving the face of the regular body, ought to be described on the sphere.*

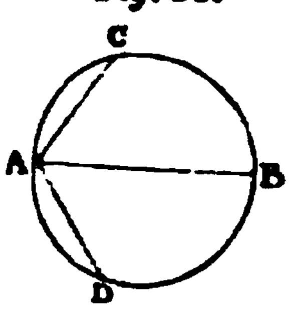
Fig. 50.



This is very easy; for if E F (Fig. 50.) be the radius of the lesser circle of the sphere, capable of receiving the given face, it is evident that F D is the opening of the compasses proper for describing this circle on the surface of the sphere. But E F is the sine of the angle F C D, which will consequently be given; and F D is the double of the sine of half this first angle: F D therefore may be found by seeking in the tables for the angle F C D, then halving it, afterwards seeking for the sine of that half, and then doubling this sine. This operation will give the value of F D, which in the case of the tatraedron will be 11742; in those of the hexaedron and octaedron, 9192; and in those of the dodecaedron and icosaedron, 6408.

4th. *To find the angle formed by the faces of the different regular bodies.*

Fig. 51.



Describe a circle (Fig. 51.) as large as possible, and determine in it the side of the regular body required; if a perpendicular be then let fall from the centre on this side, it will be the diameter of a second circle, which must also be described. We shall here suppose that this diameter is A B.

Describe then, on the side of the regular body found, the proper polygon, or at least find the centre of the circumscribing circle, and from this centre let fall a perpendicular on the side which has been found; in the second circle already mentioned, make the lines A D and A C equal to this perpendicular, and the angle D A C will be equal to the angle required.

It will be found, by calculation, that this angle, for the tetraedron, is 70° 32′; for the hexaedron, 90°; this is evident because the faces of the cube are perpendicular to each other; for the octaedron, 109° 28′; for the dodecaedron, 116° 34′; and for the icosaedron, 138° 12′.

We shall here collect all these dimensions in the following table, where we suppose the radius of the sphere to be 10000 parts.

Names of the regular bodies.	Sides of the faces.	Radii of the circumscribing circles.	Distances from the poles.	Angles of the contiguous faces.
Tetraedron	16329	9426	11742	70° 32′
Hexaedron	11540	8164	9192	90 00
Octaedron	14142	8164	9192	109 28
Dodecaedron	77336	6070	6408	116 34
Icosaedron	10514	6070	6408	138 10

It will now be easy to trace out, by either of the above methods, any required regular body whatever.

First method.—Let it be required, for example, to form a dodecaedron from a sphere. First describe a circle of a diameter equal to that of the sphere, and determine in it the side of the dodecaedron, or the side of the pentagon, which is one of its faces; also the radius of the circle in which this pentagon can be inscribed, and the

opening of the compasses proper for describing it on the sphere; which may be easily done by the precepts before given.

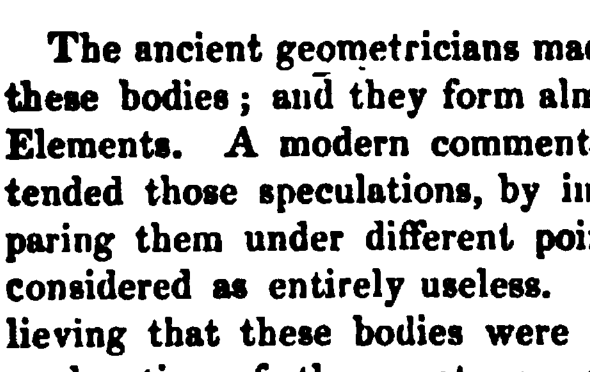
Or, if we suppose the radius of the proposed sphere to be 10000 parts, take upon a scale 6408 of these parts, and with this opening of the compasses describe, on the surface of the sphere, a circle on the circumference of which the five angles of the inscriptible pentagon may be determined; from two neighbouring points describe, with the same opening of the compasses, two arcs, the intersection of which will be the pole of a new circle, equal to the former: continue in this manner, from every two points, and you will have the five poles of the five faces, which rest on the first. In like manner, you may easily determine the other poles, the last of which, if the operation be exact, ought to be diametrically opposite to the first. Lastly, from these twelve poles, describe two equal circles, which will both be cut into five equal parts, and these will determine twelve segments of a sphere, which being cut off, will give the twelve faces of the dodecaedron required.

Second method.—Having first found out, on the proposed block, a plane face, describe on it the polygon belonging to the regular body required; then cut out, on each side of this polygon, a new plane, inclined according to the proper angle, as determined in the above table, or which has been traced out by means of the geometrical construction before given, and you will thus obtain so many plane faces, on which new polygons, having one side common with the first polygons, must be described. If the same thing be done on these polygons, you will at length arrive at the last, which, if the operation has been exactly performed, must be perfectly equal to the first.

5th. *To form the same bodies of a piece of card.*

If you are desirous of forming these bodies of a piece of card or stiff paper, the following method will be the most convenient.

Fig. 52.



First trace out on the card all the faces of the required body, viz. four triangles for the tetraedron, as seen Fig. 52, No. 1, six squares for the cube, as No. 2, eight equilateral triangles for the octaedron, No. 3, twelve pentagons for the dodecaedron, No. 4, and twenty equilateral triangles for the icosaedron, No. 5. If you then cut the edges, it will be easy to fold up the faces so as to join, and if they be then glued together, you will have the regular body complete.

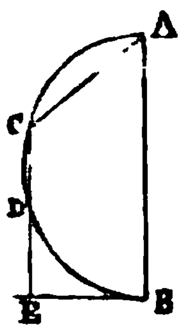
The ancient geometricians made a great many geometrical speculations respecting these bodies; and they form almost the whole subject of the last books of Euclid's Elements. A modern commentator on Euclid, M. de Foix Candalle, has even extended those speculations, by inscribing these bodies within each other, and comparing them under different points of view; but, at present, such researches are considered as entirely useless. They were suggested to the ancient, by their believing that these bodies were endowed with mysterious properties, on which the explanation of the most secret phenonomena of nature depended. With these bodies they compared the celestial orbs, &c. But since rational philosophy has begun to prevail among mankind, the pretended energy of numbers, and that of the regular bodies of nature, have been consigned to oblivion, along with the other visions, which were in vogue during the infancy of philosophy, and the reign of Platonism. For this reason, we shall say nothing farther of these speculations, and confine ourselves to a very curious problem, respecting the cube or hexaedron.

PROBLEM XXIX.

To cut a hole in a cube, through which another cube of the same size shall be able to pass.

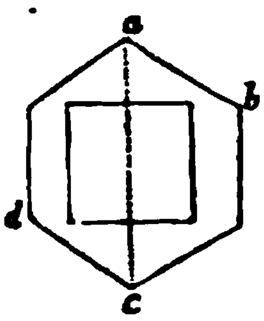
If we conceive a cube raised on one of its angles, in such a manner, that the diagonal passing through that angle shall be perpendicular to the plane which it touches; and if we suppose a perpendicular let fall on that plane from each of the elevated angles, the projection thence resulting will be a regular hexagon, each side and each radius of which may be found in the following manner.

Fig. 53.



On the vertical line A B (Fig. 53.) equal to the diagonal of the cube, the square of which is triple to that of the cube, describe a semicircle, and make A C equal to the side of the cube; from the point C let fall, on the horizontal tangent of the circle in B, the perpendicular C E, B E will be the side and the radius of the required hexagon *a b c d*, Fig. 54.

Fig. 54.



When this operation is finished, describe on its hexagonal projection, and around the same centre, the square which forms the projection of the given cube placed on one of its bases, so that one of sides shall be parallel, and the other perpendicular to the diameter *a c*: it may be demonstrated, that this square can be contained within the hexagon, in such a manner, as not to touch with its angles any of the sides: a square hole therefore, equal to one of the bases of the cube, may be made in it, in a direction parallel to one of its diagonals, without destroying the continuity of any side; and consequently another cube of equal size may pass through it, provided it be made to move in the direction of the diagonal of the former.

PROBLEM XXX.

With one sweep of the compasses, and without altering the opening, or changing the centre, to describe an oval.

This problem, as is the case with others of a similar kind, is a mere deception; for it is not specified on what kind of surface the required curve ought to be described. Those to whom this problem is proposed, will think of a plane surface, and therefore will consider it impossible, as it really is; while indeed the surface meant is a curved one, on which it may be easily performed.

If a sheet of paper be spread round on a cylindric surface, and if a circle be described upon it with a pair of compasses, assuming any point whatever as a centre, it is evident that, when the sheet of paper is extended on a plane surface, we shall have an oval figure, the shortest diameter of which will be in the direction corresponding to that of the axis of the cylinder.

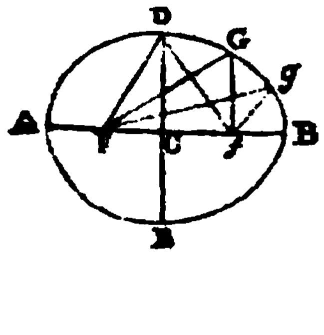
We should however be deceived, were we take this curve for the real ellipsis, so well known to geometricians. The method of describing the latter is as follows.

PROBLEM XXXI.

To describe a true oval or ellipsis geometrically.

The geometrical oval is a curve with two unequal axes, and having in its greater axis two points so situated, that if lines be drawn to these two points, from each point of the circumference, the sum of these two lines will always be the same.

Fig. 55.



Let A B (Fig. 55.) then be the greater axis of the ellipsis to be described; and let D E, intersecting it at right angles, and dividing it into two equal parts, be the lesser axis, which is also divided into two equal parts in C; from the point D as a

centre, with a radius equal to A C, describe an arc of a circle, cutting the greater axis in F and f: these two points are what are called the foci: fix in each of these a pin, or, if you operate on the ground, a very straight peg; then take a thread, or a chord if you mean to describe the figure on the ground, having its two ends tied together, and in length equal to the line A B, plus the distance F f; place it round the pins or pegs F f; then stretch it as seen at F G f, and with a pencil, or sharp pointed instrument, make it move round from B, through D, A, and E, till it return again to B: the curve described by the pencil on paper, or on the ground by any sharp instrument, during a whole revolution, will be the curve required.

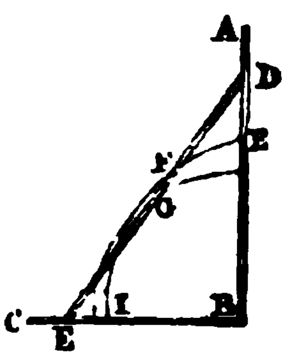
This ellipsis is called the Gardener's Oval; because when gardeners describe that figure, they use this method.

It is here seen that the geometric ellipsis, or oval, is, as we may say, a circle with two centres; for in the circle the distance from the centre to any point of the circumference, and from that point back to the centre, is always equal to the same sum, viz. the diameter. In the ellipsis, where there are two centres, the distance from one of them to any point of the circumference, and from that point to the other centre, is always equal to the same sum, or to the greater diameter.

A circle therefore is nothing else than an ellipsis, the two foci of which, by continually approaching, have at length been united and confounded with each other.

Another method of describing an ellipsis, which may be also used sometimes, is as follows.

Fig. 56.



Let A B C (Fig. 56.) be a square, and B H and B I the two semi-axes of the ellipsis to be described. Provide a rule, such as E D, equal to the sum of these two lines, and having taken E F equal to B H, fix in the point F, by some mechanism which may be easily invented, a pencil or piece of chalk, capable of tracing out a line upon paper; then make this rule turn in the given right angle, in such a manner, that its two extremities shall always touch the sides of that angle, and during this movement the pencil fixed in F will describe a real geometrical ellipsis.

It may be readily seen, that if the pencil or chalk were fixed in the point G, which divides D E into two equal parts, the curve described would be a circle.

Remark.—Another sort of oval, very much used by architects and engineers, when they intend to form a flat or an acute arch, is called by the French workmen *anses de paniers*. It consists of several arcs of circles having different radii, which mutually touch each other, and which represent pretty nearly a geometrical ellipsis. But it has one fault, which is, that however well these arcs touch each other, a nice eye will always observe at the place of junction an inequality, which is the effect of the sudden transition of one curve to another that is larger. For this reason, any arch which rises on its pier without an impost, seems to form an inequality, though the arch at its junction with the pier may touch it exactly.

This inconvenience however is compensated by one advantage, which is, that for the *voussoirs* of the arch, there is no need but of two *panneaux*, or model boards, if the quarter of the oval be formed of two arcs, or of three if it be formed of three; whereas, if it were a real ellipsis, it would have occasion for as many *panneaux* as *voussoirs*. If any one however should have the courage, and it would require no small degree of it, to surmount this difficulty, we entertain no doubt that the real ellipsis would have more beauty than this bastard kind of it.

PROBLEM XXXII.

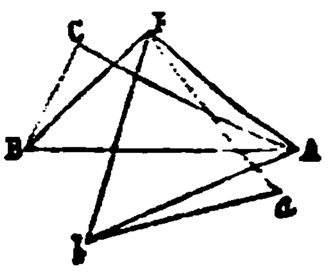
On a given base, to describe an infinite number of triangles, in which the sum of the two sides, standing on the base, shall be always the same.

This is only a corollary to the preceding problem. For on a given base let there be described an ellipsis, having the two extremities of that base as its foci: all the points of the ellipsis will be the summits of as many triangles on the given base F G f, F g f, (Fig. 55.), and the sum of their sides will be the same; consequently they will all have the same perimeter, and the greatest triangle will be that which has its two sides equal; for it is that which has the summit at the most elevated part of the ellipsis.

THEOREM VI.

Of all the isoperimetric figures, or figures having the same perimeter, and a determinate number of sides, the greatest is that which has all its sides and all its angles equal.

Fig. 57.

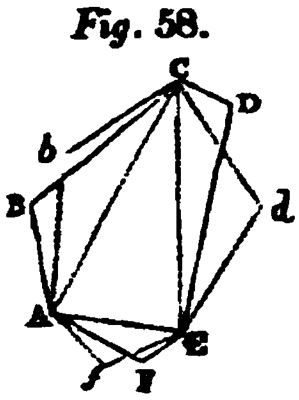


We shall first demonstrate this theorem in regard to triangles. Let A C B (Fig. 57.) then be a triangle on the base A B, the sides of which A C and C B are unequal. We have already shewn, that if there be constructed a triangle A F B, the equal sides of which A F and F B are together equal to A C and C B, the triangle A F B will be greater than A C B.

For the same reason, if there be constructed, on A F as a base, the triangle A b F, the sides of which, A b and b F, are equal to each other, and together equal to A B and B F, the triangle A b F will be greater than A F B. In like manner, if we suppose F a and a b equal, and their sum equal to F A and A B, the latter triangle F a b will be still greater than A F B, which has the same perimeter, &c. But it may be readily seen by this operation, that the three sides of a triangle always approximate towards equality, and that, by conceiving it continued *ad infinitum*, the triangle would at length become equilateral, and consequently the equilateral triangle will be the greatest of all.

For example, if the three sides of the first triangle be 12, 13, 5, the sides of the second will be 12, 9, 9; those of the third 9, $10\frac{1}{2}$, $10\frac{1}{2}$; those of the fourth $10\frac{1}{2}$, $9\frac{3}{4}$, $9\frac{3}{4}$; those of the fifth $9\frac{3}{4}$, $10\frac{1}{8}$, $10\frac{1}{8}$; those of the sixth $10\frac{1}{8}$, $9\frac{15}{16}$, $9\frac{15}{16}$; those of the seventh $9\frac{15}{16}$, $10\frac{1}{32}$, $10\frac{1}{32}$, and so on; by which it is seen that the difference always decreases; so that at last the three sides become 10, 10, 10, and the triangle will then be the greatest of all.

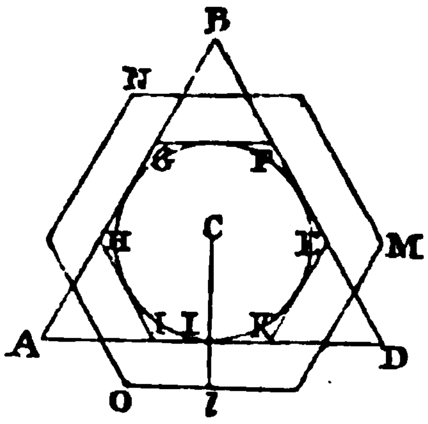
Fig. 58.



If we now take a rectilineal polygon, such as A B C D E F (Fig. 58.), all the sides of which are unequal: draw the lines A C, C E, and E A. By what has been already shewn it will be seen, that if an isosceles triangle A b C be described on A C, in such a manner, that A b and b C shall be together equal to A B and B C, the polygon, though of the same perimeter, will become greater by the excess of the triangle A b C above A B C. If the same thing be done all around, the surface of the polygon will be continually augmented; all its sides and its angles will more and more approach to equality; and consequently the greatest of all will be that which has all its sides and angles equal.

We shall now demonstrate, that, of two regular polygons, having the same perimeter, the greater is that which has the greatest number of sides. For this purpose let any polygon, an equilateral triangle for example, be circumscribed round a circle, and let E F H I (Fig. 59.), be an hexagon circumscribed about the same circle: it is evident that the perimeter of the latter will be less than that of the triangle; for the

Fig. 59.



parts F E, G H, and I K, are common, and the side G F is less than F B plus B G, &c.; a hexagon, concentric to the former, and equal in perimeter to the triangle, which we here suppose to be M N O, will therefore be without the hexagon K F H; consequently the perpendicular C l will be greater than C L. But as the triangle has the same perimeter as the hexagon M N O, their areas will be as the perpendiculars C L, C l, let fall from the centre of the circle; and therefore the hexagon, having the same perimeter as the triangle, will be the greater.

What has been demonstrated in regard to a triangle and hexagon of the same perimeter, is evidently applicable to any other two polygons, one of which has a number of sides double to that of the other; consequently the more sides a polygon of a determinate perimeter has, the greater is its area.

Remarks.— 1st. This leads us to a consequence much celebrated in geometry, which is: that of all the figures, having the same perimeter, the circle is of the greatest capacity; for a circle is only a polygon of an infinite number of sides, or, to use a more geometrical expression, is the last of the polygons resulting from their sides being continually doubled; consequently it is the greatest of all.

2d. We shall here remark also, that if upon any determinate base, and with a determinate perimeter, there be described several figures, the greatest will be that which has the greatest number of sides, beside the base, and which approaches nearest to regularity; hence it follows, that if it be required to describe, with a determinate length, on a given base, the greatest figure, that figure will be the segment of a circle, viz. a segment having that base for its chord, and for its arc the given length.

All these things may be demonstrated by a mechanical consideration. For let us suppose a vessel, the sides of which are flexible, and that any liquor is poured into it; the sides it is certain will arrange themselves in such a manner as to contain the greatest quantity possible. On the other hand, it is well known that the vessel will assume the cylindric form; that is to say its base and the sections parallel to the base will be circular; hence it follows that, of all figures having the same perimeter, the circle is that which comprehends the greatest area.

By means of the above observations it will be easy to solve the following questions.

I.—*A has a field 500 poles in circumference, which is square; B has one of the same circumference which is an oblong, and proposes to A an exchange. Ought the latter to accept the offer?*

It is easy to answer that he ought not; and A would sustain more loss by the exchange the greater the inequality is between the sides of the field belonging to B. This inequality might even be such, that the latter field would be only the half, or the fourth, or the tenth part of that of A. For let us suppose the field of A to be 100 poles on each side; and that the field of B is a rectangle, one side of which is 190 poles, and the other 10, by which means it will have the same perimeter as the other; it will however be found that the surface of the latter will be only 1900 square poles, while that of the former will be 10000. If one side of the field belonging to B were 195 poles, and the other 5, which would still make the perimeter 400 poles; its surface would be only 975 poles, which is not even a tenth part of that of the field belonging to A.

II.—*A farmer borrowed a sack of wheat, measuring 4 feet in length, and 6 feet in circumference; for which he returned two sacks of the same length, and each 3 feet in circumference: did he return the same quantity of wheat?*

He returned only half the quantity; for two equal circles, having the same perimeter, taken together, as a third, do not contain the same area; the area of both is only the half of the third, each of them being but a fourth of it.

III.—*A green-grocer purchased for a certain sum, as many heads of asparagus as could be contained in a string a foot in length; being desirous to purchase double that quantity, he returned next day to the market, with a string of twice the length, and offered to double the price of the former quantity, for as many as it would contain. Was his offer reasonable?*

No—the man was in an error to imagine that a string of twice the length would contain only double the quantity of what he purchased the preceding day; for a circle which has its circumference double to that of another, has its diameter double also. But the area of a circle, the diameter of which is double to that of another, is equal to four times the area of the other.

Remark.—It remains for us to observe here that as the circle of all the figures having an equal perimeter, is the greatest; the sphere among the solids is that which contains the greatest volume. Thus, if it were required to make a vessel of a determinate capacity, but in such a manner as to save the materials as much as possible, it ought to be in the form of a sphere. But this will be better illustrated by the following problem.

PROBLEM XXXIII.

A gentleman wishes to have a silver vessel of a cylindric form, open at the top, capable of containing a cubic foot of liquor; but being desirous to save the material as much as possible, requests to know the proper dimensions of the vessel.

If we suppose that the vessel ought to be a line in thickness, for example, it is evident that the quantity of the matter will be proportional to the surface. The question then is: Of all the cylinders, capable of containing a cubic foot, to determine that which shall have the least surface, exclusive of the top.

It will be found that the diameter of the base ought to be 16 inches 4 lines; and the height 8 inches $2\frac{8}{15}$ lines, which is the ratio of nearly 2 to 1 between the diameter and the height.

If it were required to have the vessel in the form of a cask, close at both ends, the question would be: To find a cylinder which shall have its whole surface, comprehending the two bases, greater than that of any other of the same capacity. In this case the diameter of the base ought to be 13 inches, and the height 12 inches $5\frac{2}{3}$ line.

PROBLEM XXXIV.

On the form in which the Bees construct their Combs.

The ancients admired bees on account of the hexagonal form of their combs. They observed that, of all the regular figures which can be united, without leaving any vacuum, the hexagon approaches nearest to the circle, and with the same capacity has the least perimeter; whence they inferred that this animal was endowed with a sort of instinct, which made it choose this figure as that which, containing the same quantity of honey, would require the least wax to construct the comb; for it appears that bees do not prepare wax on its own account, but in order to construct their combs destined to be the repositories of their honey, and receptacles for their young.

This however is far from being the principal wonder in regard to the labour of bees, if we can give the name of wonder to a labour blindly determined by a peculiar organization; for it may be remarked, in the first place, that it is not absolutely wonderful that small animals, all endowed with the same activity and the same force, pressing outwards, from within, small cells all arranged close to each other,

and all equally flexible, should give them, by a sort of mechanical necessity, a hexagonal form. If we suppose indeed a multitude of circles, or small cylinders, highly flexible and somewhat extensible, close to each other, and that forces acting internally, and all equal, tend to make heir sides approach each other, by filling up the vacuities left between them, the first form they will assume will be the hexagonal; after which all these forces remaining in equilibrium, nothing will tend to change that form.

However, not to deprive the bees of the admiration which they have excited in the above respect, we shall remark that this is not the manner in which they labour. They do not first make circular cells, and then transform them into hexagons by extending them in concert. The cells, which terminate an imperfect comb, are composed of equal planes inclined to each other, nearly in that angle which the hexagonal form requires. But let us proceed to another singularity, still more wonderful, in regard to the labour of bees.

This singularity consists in the manner in which the bottom of their cells is formed. We must not indeed imagine that they are all uniformly terminated by a plane perpendicular to their axes; there is a method of terminating them which employs less wax, and even the least possible, still leaving to the cells the same capacity; and it is this method which these insects adopt, and which they execute with great precision.

To execute this disposition, it is necessary, in the first place, that the two rows of cells, of which it is well known a comb consists, and which stand back to back, should not be arranged so as to make their axes correspond, but in such a manner that the axis of the one may be in a line with the common juncture of three posterior. As is seen Fig. 60, where the hexagon described with black lines corresponds with the three formed of dotted lines, which represent the plane of the posterior cells; and it is thus that the cells of bees are arranged, to suit the disposition of their common bottoms.

Fig. 60.

Fig. 61.

In the second place, to give an idea of this disposition, let us suppose an hexagonal prism, the upper base of which is the hexagon A B C D E F (Fig. 61.) with a triangle A E C inscribed in it. Let the axis F O be continued to S, and through the point S and the side A C let a plane pass, which shall cut off from the prism the angle B, so as to form a rhomboidal face A S C T; such is one of the bottoms of the cell of a comb: if two other similar planes be made to pass through S and the sides A E and E C, they will form the other two; so that the bottom is terminated by a triangular pyramid.

It may be readily seen, that wherever the point S may be situated, as the pyramid A C O S is always equal to A C B T, and as the case is the same with the rest, the capacity of the cell will not vary, whatever be the inclination of that part of the bottom turned towards A C. But the case is different with the surface where there is such an inclination, that the whole surface of the prism and of its bottom will be less than with any other inclination. It has been found, by the researches of geometricians, that, for this purpose, the angle formed by the bottom with the axis ought to be $54^\circ\ 44'$; from which there results the smaller angle of the rhombus A T C or A S C, equal to $70^\circ\ 32'$, and the other S A T or S C T of $109^\circ\ 28'$.

But this is exactly the inclination of the sides of the parallelogram, formed by each of the three inclined planes of the bottom of the cells of a comb, as appears by the measurement of a great many of these cells. Hence there is reason to conclude, that bees construct the bottom of their cells in the most advantageous form, so as to have the least surface possible, and in such a manner indeed, as can be de-

termined only by modern geometry.* Who can have given to these insects, so contemptible, not in the eyes of the philosopher, who never despises the least of the works of the Deity, but in the eyes of the vulgar, that wonderful instinct, which directs them to perform so perfect a work, but the supreme Geometrician, of whom Plato said, what is verified more and more as we become acquainted with the works of nature, that he does every thing *numero, pondere, et mensura.*

PROBLEM XXXV.

What is the greatest polygon that can be formed of given lines?

It may be demonstrated that the greatest polygon that can be formed with given lines, is that about which a circle can be circumscribed.

But it may be still asked, whether there be any particular order, in regard to the sides, capable of giving a greater polygon than any other arrangement. We can answer that there is not; and that, whatever be the arrangement, if the polygon can be inscribed in a circle, it will be always the same; for it may be easily demonstrated, that whatever be this order, the size of the circle will not vary; the polygon will always be composed of the same triangles, having their summits at its centre: the only difference will be, that they will be differently arranged.

PROBLEM XXXVI.*

What is the largest triangle that can be inscribed in a circle; and what is the least that can be circumscribed about it?

The triangle required in both these cases is the equilateral.

The case is the same with the other polygons. The greatest quadrilateral figure that can be inscribed in the circle, is the square; this figure also is the least of all those that can be circumscribed about a circle.

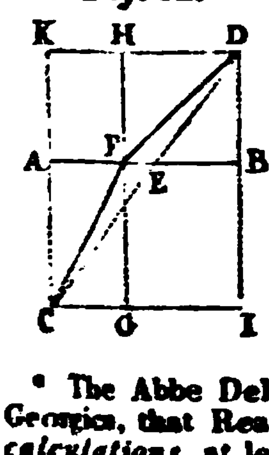
The regular pentagon is likewise the greatest of all the five-sided figures that can be inscribed in the circle; and the same figure is the least of all the pentagons that can be circumscribed about the circle. And so on.

PROBLEM XXXVII.

A B (*Fig.* 62.) *is the line of separation between two plains; one of which* A C I B *consists of soft sand, in which a vigorous horse can scarcely advance at the rate of a league per hour; the other* A B D K *is covered with fine turf, where the same horse, without much fatigue, can proceed at the rate of a league in half an hour; the two places* C *and* D *are given in position, that is to say the distance* C A *and* D B *of each from the line of boundary* A B, *as well as the position and length of* A B, *are known; now if a traveller has to go from* D *to* C, *what route must he pursue, so as to employ the least time possible on his journey?*

Most people, judging of this question according to common ideas, would imagine that the route to be pursued by the traveller, ought to be the straight line. In this however they would be deceived, as may be easily shewn; for if the straight line C E D be drawn, it may be readily conceived that it will be gaining an advantage to perform, in the first plain, where it is difficult to travel, the part of the journey C F, which is somewhat shorter than C E; and to perform in the second, where it is much easier to travel, the part F D, longer than D E, that is to say, than the space which would be passed over by going directly from C to D; so that less time would really be employed to go from C to D, by C F and F D, than by C E and E D, though the road by the latter is shorter.

Fig. 62.



* The Abbe Delisle says improperly, in the notes to the fourth book of his Translation of the Georgics, that Reaumer, having proposed this problem to Koenig, the latter, *after a great many calculations*, at length found the angle of the inclination of the planes which form the bottom of these cells. Nothing however is easier than the solution of this problem by means of fluxions: two lines of calculation are sufficient, and a solution may even be given without that assistance.

This indeed may be demonstrated by calculation. For if H G be drawn perpendicular to A B, through the point F, it will be found that one can go from C to D, in the least time possible, when the sines of the angles C F G and D F H are to each other respectively in the inverse ratio of the velocity with which the traveller can pass over the planes A C I B and A B D K, that is to say, in the present case, as 1 to 2; and therefore the sine of the angle C F G, ought to be half only of that of the angle D F H.

PROBLEM XXXVIII.

On a given base to describe an infinite number of triangles, in such a manner, that the sum of the squares of the sides shall be constantly the same, and equal to a given square.

Fig. 63.

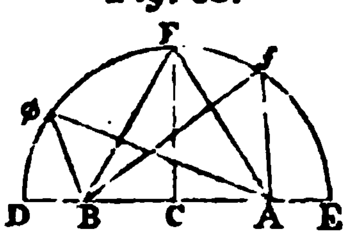
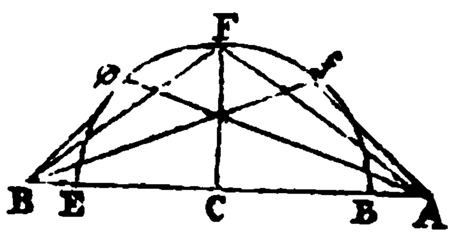


Fig. 64.



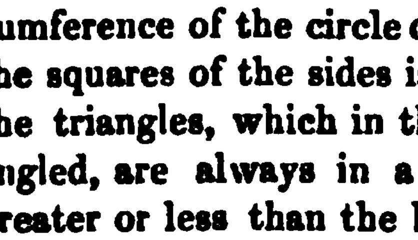
Let A B (Fig. 63 and 64.) be the given base, which must be divided into two equal parts in C; then from the points A and B, with a radius equal to half the diagonal of the given square, describe an isosceles triangle, of which F is the vertex; draw C F, and from the point C, with the radius C F, describe a semicircle on A B, produced if necessary: all the triangles having A B for their base, and whose vertices are at F, *f*, φ, in the circumference of the circle, will be of such a nature, that the sum of the squares of their sides will be equal to the square given.

Remark.—Every one knows that when the sum of the squares of the sides is equal to the square of the base, the triangle is right-angled, and has its vertex in the circumference of the circle described on that base. Here it is seen, that if the sum of the squares of the sides is greater or less than the square of the base, the vertices of the triangles, which in this first case are acute-angled, and in the second obtuse-angled, are always in a semicircle also, having the same centre, but on a diameter greater or less than the base of the triangle; which is a very ingenious generalization of the well known property of the right angled triangle.

PROBLEM XXXIX.

On a given base, to describe an infinite number of triangles, in such a manner, that the ratio of the two sides, on that base, shall be constantly the same.

Fig. 65.



Divide the given base A B (Fig. 65.) in such a manner in D, that A D may be to D B, in the given ratio, which we shall here suppose to be as 2 to 1. Then say, as the difference between A D and D B is to D B, so is A B to B E; and if A D exceeds D B, B E must be taken in the direction A B E; then divide D E into two equal parts in C, and from C as a centre, with the radius C D or C E, describe a semicircle on the diameter D E: all the triangles, as A F B, A *f* B, A φ B, &c., having the same base A B, and their vertices F, *f*, φ, in the circumference of this semicircle, will be of such a nature, that their sides A F, F B; A *f*, *f* B; A φ, φ B, will be in the same ratio, viz., that of A D to D B, or of A E to E B, which is the same thing.

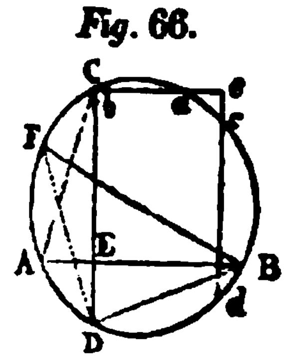
But the centre C will be found much easier by the following construction: on A D describe the equilateral triangle A G D, and on D B, the equilateral triangle D H B; through their summits, G and H, draw a straight line, which being continued will cut the continuation of A B in the point C, and this point will be the centre required.

THEOREM VII.

In a circle, if two chords, as A B *and* C D *(Fig. 66.) intersect each other at right angles; the sum of the squares of their segments,* C E, A E, E D, *and* E B, *will always be equal to the square of the diameter.*

The demonstration of this curious and elegant theorem, is exceedingly easy; for it may be readily seen, if the lines B D and A C be drawn, that their two squares are together equal to the squares of the four segments in question.

Fig. 66.



Moreover, by making the arc F C equal to A D, we shall have the arc F D equal to A C, and consequently the angle F D C equal to A C E, which is itself equal to A B D; the angle F D B therefore will be a right angle, since it is equal to E D B and D B E, which together make a right angle; hence the squares of F D and D B are equal to the square of the hypothenuse F B, which is the diameter.

It must here be remarked, that the result would be the same, if we suppose the point *e*, where the chords meet, to be without the circle; in that case the four squares, viz. those of *e a*, *e b*, *e c*, and *e d*, would still be together equal to the square of the diameter.

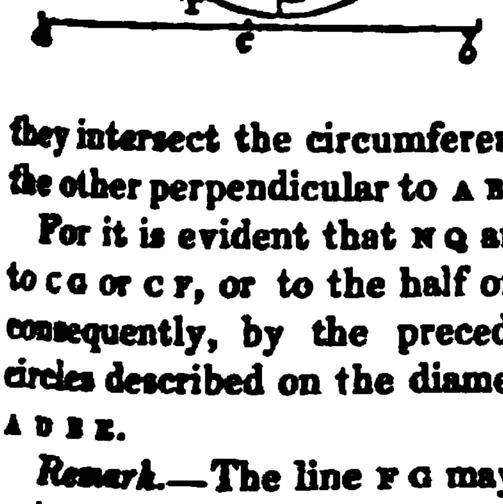
Remark.—Circles being to each other as the squares of their diameters; it is evident that if on E A, E B, E C, and E D, as diameters, four circles be described, these circles will be together equal to the circle A C B D. And they will also be proportional; for we know that B E is to E C as E D is to E A. But if four magnitudes are proportional, their squares are so also. Moreover, it is evident that whatever be the position of these two chords, their sum will always be equal, at the most, to two diameters if they both pass through the centre; or at least to one, if one of them passes through the centre, and the other almost at the distance of a radius. By means of this theorem, therefore, it will be easy to solve the following problem.

PROBLEM XL.

To find four proportional circles, which taken together shall be equal to a given circle, and which shall be of such a nature, that the sum of their diameters shall be equal to a given line.

It is evident, for the above reasons, that the given line must be less than twice the diameter of the given circle, and greater than once that diameter, or, which is the same thing, that the half of this line must be less than the diameter of the given circle, and greater than its radius.

Fig. 67.

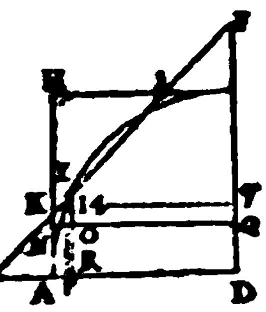


This being premised; let the given line, the sum of the diameters of the required circles, be *a b* (Fig. 67.) the half of which is *a c*; let A D B E be the given circle, the two diameters of which are A B and D E, perpendicular to each other. On the radii C A and C E continued, make the lines C F and C G equal to *a c*, and draw F G, which will necessarily intersect C H, the square of the radius of the circle. In the part I K of that line comprehended within the square, assume any point L, from which draw the lines L M *q*, and L N *r*, the one parallel and the other perpendicular to the diameter A B; through the points M and N, where they intersect the circumference of the circle, draw M R and N Q, the one parallel and the other perpendicular to A B: the chords N S and M T will be the two chords required.

For it is evident that N Q and M R are equal to L *q*, and L *r*, which are together equal to C G or C F, or to the half of *a b*; the whole chords then are together equal to *a b*; consequently, by the preceding theorem, they solve the problem, and the four circles described on the diameters N O, O M, O S, and O T, will be equal to the circle A D B E.

Remark.—The line F G may happen only to touch the circle; in which case any point, except the point of contact, will equally solve the problem

Fig. 68.



But if F G intersect the circle, as seen Fig. 68, the point L must be assumed in that part of the line I K, which is without the circle, as seen in the same figure.

This solution is much better than that given by M. Ozanam; for he tells us to take on *a c* (Fig. 67.) a portion less than the radius, and to set it off from C to *q*: then to draw the lines *q* M and M R, and to set off the remainder of *a c* from C to *r*; but it is necessary that the point *r* should fall beyond R, otherwise the two semi-chords would not intersect each other. In the last place, according to the magnitude of *a c*, in regard to the radius, there is a certain magnitude which must not be exceeded, and which M. Ozanam does not determine: this therefore renders the solution defective.

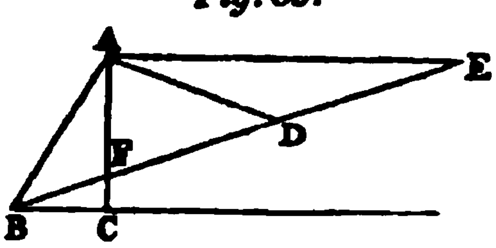
PROBLEM XLI.

Of the Trisection and Multisection of an Angle.

This problem is celebrated on account of the fruitless attempts made, from time to time, to resolve it geometrically, by the help of a rule and compasses, and of the paralogisms and false constructions given by pretended geometricians. But it is now demonstrated, that the solution of it depends on a geometry superior to the elementary, and that it cannot be effected by any construction in which a rule and compasses only, or the circle and straight line, are employed, except in a very few cases; such as those where the arc which measures the proposed angle is a whole circle, or a half, a fourth, or a fifth part of one. None therefore but people ignorant of the mathematics attempt at present to solve this problem by the common geometry.

But though it cannot be solved by the rule and compasses alone, without repeated trials, there are some mechanical constructions or methods, which, on account of their simplicity, deserve to be known. They are as follow:

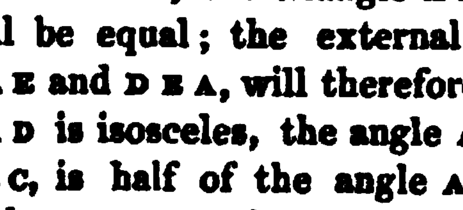
Fig. 69.



Let it be proposed, for example, to divide the angle A B C (Fig. 69.) into three equal parts. From the point A let fall, on the other side of the angle, the perpendicular A C, and through the same point A draw the indefinite straight line A E parallel to B C; if from the point B you then draw to A E a line B E, in such a manner, that the part F E, intercepted between the lines A C and A E, shall be equal to twice the line A B, which may be done very easily by repeated trial, you will have the angle F B C equal to the third part of A B C.

If F E indeed be divided into two equal parts in D, and if A D be drawn; as the triangle F A E is right-angled, D will be the centre of the circle passing through the points F, A, E; consequently D A, D E, and D F will be equal to each other, and to the line A B; the triangle A D E then will be isosceles, and the angles D A E and D E A will be equal; the external angle A D F, which is equal to the two interior ones D A E and D E A, will therefore be the double of each of them. But as the triangle B A D is isosceles, the angle A B D is equal to A D B, and the angle A E D, or its equal F B C, is half of the angle A B D; consequently the angle A B C is divided by B E, in such a manner, that the angle E B C is the third part of it.

Fig. 70.



Another method.—Let the given angle be A C B (Fig. 70.): from the vertex of it as a centre, describe a circle, and continue the radius B C indefinitely to E; then draw the line A E in such a manner, that the part D E, intercepted between B E and the circumference of the circle, shall be equal to the radius B C; if C H be then drawn through the centre C, parallel to A E, the angle B C H will be the third part of the given angle A C B.

If the radius C D be drawn, it may be readily seen that the angle H C A is equal, on account of the parallel lines, to C A D or C D A. But the latter is equal to the angles D C E and D E C, or to the double of one of them; since C D and D E are equal by construction; and as the angle H C B is equal to D C E or D E C, the angle A C H is the double of H C B, and consequently A C B is the triple of H C B.

PROBLEM XLII.

The Duplication of the Cube.

To double a rectilineal surface, or any curve whatever, as the circle, square, triangle, &c., is easy; that is to say, one of these figures being given, it is easy to construct a similar one, which shall be the double or any multiple of it whatever, or which shall be in any given ratio to it at pleasure: nothing is necessary for this purpose, but to find the mean geometrical proportional between one of the sides of the given figure, and the line which is to that side in the given ratio, this mean will be the side homologous to that of the given figure. Thus, to describe a circle double of another, a mean proportional must be found between the diameter of the former and the double of that diameter; this proportional will be the diameter of the double circle, &c. The case is the same with every other ratio.

All this belongs to the elements of geometry. But to construct a double solid figure, or a figure in a given ratio to another similar figure, is a much more difficult problem, which cannot be solved by means of the circle and straight line, or of the rule and compasses, unless a method of repeated trial, which geometry rejects, be employed. This at present is clearly demonstrated; but the demonstration is not susceptible of being comprehended by every one.

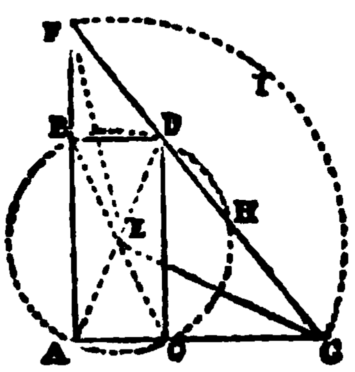
Respecting the origin of this problem, a very curious circumstance is related. During the plague at Athens, which made a dreadful havoc in that city, some persons being sent to Delphos to consult Apollo, the deity promised to put an end to the destructive scourge, when an altar, double to that which had been erected to him, should be constructed. The artists who were immediately dispatched to double the altar, thought they had nothing to do, in order to comply with the demand of the oracle, but to double its dimensions. By these means it was made octuple; but the god, being a better geometrician, wanted it only double. As the plague still continued, the Athenians dispatched new deputies, who received for answer, that the altar was more than double. It was then thought proper to have recourse to the geometricians, who endeavoured to find out a solution of the problem. There is reason to think that the god was satisfied with an approximation, or mechanical solution; had he required more, the situation of the people of Athens would have deserved pity indeed.

There was no necessity for introducing a deity into this business. What is more natural to geometricians than to try to double a solid, and the cube in particular, after having found the method of doubling the square and other surfaces? This is the progress of the human mind in geometry.

Geometricians soon observed that, as the duplication of any surface consists in finding a geometrical mean between two lines, one of which is the double of the other, the duplication of the cube, or of any solid whatever, consists in finding the first of two continued mean proportionals between the same lines. We are indebted for this remark to Hippocrates of Chios, who from being a wine merchant, ruined by shipwreck or the officers of the excise at Athens, became a geometrician. Since that time, all the efforts of geometricians have been confined merely to the finding of two continued geometrical mean proportionals between two given lines, and these two problems, viz., that of the duplication of the cube, or, more generally, of the construction of a cube in a given ratio to another, and that of the two continued mean proportionals, have become synonymous.

The different methods of solving this problem, some of which require repeated trial, and some no other instruments than a rule and compasses, are as follow:

Fig. 71.

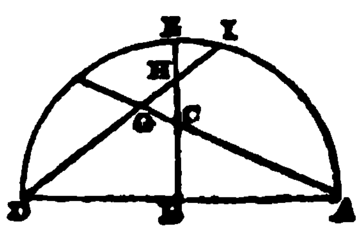


1st. Let the two lines, between which it is required to find two mean proportionals, be A B and A C (Fig. 71.) Form of them the rectangle B A C D, and continue the sides A B and A C indefinitely; draw the two diagonals of the rectangle intersecting each other in E; and we shall then have the solution of the problem, if the line F D G terminated by the sides of the right angle F A G, be drawn through the point D, in such a manner, that the points G and F shall be equally distant from the point E; for in that case the lines A B, C G, B F, and A C, will be continued in proportion.

Or, with E as a centre, describe an arc of a circle, as F I G, in such a manner, that by drawing the line F G, it shall pass through the point D: we shall then have a solution of the problem.

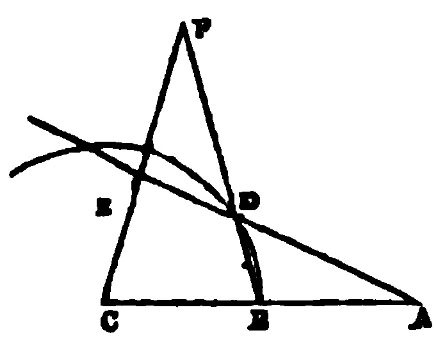
Another method is as follows: Circumscribe a circle about the rectangle B A C D; then through the point D, draw the line F G, in such a manner, that the segments F D and G H shall be equal: the lines C G and B F will be continued mean proportionals between A B and A C.

Fig. 72.



2d. Form a right angle of the two given lines A B and B C (Fig. 72.); and having continued B C and A B indefinitely, from the point B as a centre, describe the semicircle D E A; draw also the line A C, and in the continuation of it find a point G of such a nature, that by drawing the line D G H I, the segments G H and H I shall be equal to each other: the line B H will be the first of the two means.

Fig. 73.



3d. Let C A (Fig. 73.) be the first of the given lines: from the point C, with the radius C B, equal to the half of C A, describe a circle, and in this circle make the chord B D equal to the second of the given lines, which must be continued indefinitely; draw the indefinite line A D E, and from the point C draw the line C E F, in such a manner, that the part E F, intercepted with the angle E D F, shall be equal to C B; the line D F will then be the first of the required mean proportionals, and C E will be the second. This construction is that of Sir Isaac Newton.

PROBLEM XLIII.

An angle, which is not an exact portion of the circumference, being given, to find its value with great accuracy, by means of a pair of compasses only.

From the vertex of the given angle, with as great a radius as possible, describe a circle, and mark its principal points of division, as the half, third, fourth, fifth, sixth, eighth, twelfth, and fifteenth parts of the circumference; then by means of the compasses take the chord of the given arc, and set it off along the circumference, from a determinate point, going round it once, twice, thrice, &c.; and counting the number of times that the chord is applied to the circumference, until you fall exactly on one of the points of division, which cannot fail to be the case after a certain number of revolutions, unless the given arc be incommensurable to the circumference; then examine what the point of division is, or how many and what aliquot parts of the circumference it is distant from the first point; add the number of degrees which it gives to the product of 360 degrees multiplied by the complete number of turns made with the compasses, and divide the sum by the number of times

that the compasses were applied to the circumference: the quotient will be the number of degrees, minutes, and seconds, required.

Let us suppose, for example, that the compasses, with an opening equal to the chord of the given arc, have been applied to the circumference seventeen times, and that after four complete revolutions they have coincided exactly on the second division of the circle divided into five equal parts. The fifth part of the circumference is 72°, and two fifths are 144°; if 144 then be added to the product of 360° by 4, which is the number of the complete revolutions, and if the sum 1584° be divided by 17, the quotient 93° 10′ 35″ will be the value of the required arc.

PROBLEM XLIV.

A straight line being given; to find, by an easy operation, and without a scale, to a thousandth, ten thousandth, hundred thousandth, &c. part, nearly, its proportion to another.

Let the first or least of these lines be called A, and second B.

Take with a pair of compasses the extent of the line A, and set it off as many times as possible on B: we shall here suppose that A is contained in the latter three times, with a remainder.

Take this remainder in the compasses, and set it off, in like manner, on the line B, as often as possible: we shall suppose that it is contained in it seven times, with a remainder.

Take the second remainder, and perform the same operation on the line B, in which we shall suppose it to be contained 13 times, with a remainder; and, in the last place, let us suppose that this third remainder is contained in B exactly 24 times.

Then form the following series of fractions; $\frac{1}{3}$, $\frac{1}{3 \cdot 7}$, $\frac{1}{3 \cdot 7 \cdot 13}$, $\frac{1}{3 \cdot 7 \cdot 13 \cdot 24}$, and reduce them to decimal fractions, which will be 0·333333, 0·047619, 0·003663, 0·000152. The given line is in decimals equal to the first of these fractions, minus the second, plus the third, minus the fourth, which gives 0·289225, without the error of one of these parts entirely, that is to say of a millionth part.

It may be easily seen that no scale, however small the divisions, could give so approximate a ratio; and even if we suppose such a scale to exist, there would still remain an uncertainty in regard to the division on which the extremity of the given line would fall; whereas, a line applied with the compasses along a greater one, can never leave any uncertainty in regard to the number of times it is contained in it, with or without a remainder.

If the above fractions be added in the usual manner, we shall find that the given line is equal to [illegible] of the second.

PROBLEM XLV.

To make the same body pass through a square hole, a round hole, and an elliptical hole.

We give a place to this pretended problem, merely because it is found in all the Mathematical Recreations hitherto published; for nothing is easier to those who are in the least acquainted with the simplest geometrical bodies.

Provide a right cylinder, and suppose it to be cut through its axis; this section will be a square or a rectangle; if cut through a plane perpendicular to the axis, the section will be a circle; and if cut obliquely to that axis, the section will be an ellipsis. Consequently, if three holes, the first equal to this rectangle, the second to the circle, and the third to the ellipsis, be cut in a piece of wood or pasteboard, it is evident that the cylinder may be made to pass through the first of these holes, by moving it in a direction perpendicular to its axis; it will also pass through the circular hole when moved in the direction of its axis; and through the elliptical hole, when held with the proper degree of obliquity; in all these cases it will exactly touch

the edges of the hole, so that if the hole were smaller it would be impossible to make it pass through it.

This problem might be solved by means of other bodies; but it is so simple that nothing farther needs be said on the subject.

PROBLEM XLVI.

To measure the circle, that is to say, to find a rectilineal space equal to the circle, or more generally, to find a straight line equal to the circumference of the circle, or to a given arc of that circumference.

We are far from pretending to give an exact and perfect solution of this problem: it is more than probable that it will ever baffle the efforts of the human mind; but it is allowed in geometry, that when a problem cannot be completely solved, it is some merit to approach near to it, and the more so when the unknown quantity is circumscribed within the nearest limits. But though geometricians despair of ever being able to find the exact measure of the circle, they have accomplished things highly worthy of notice; for they have found means to approach so near to it that even if the radius of a circle were equal to the distance between the sun and the first of the fixed stars, it is certain that its circumference might be found from the radius, without the error of a hair's breadth. This is doubtless more than sufficient to answer the nicest purposes in the arts; but it must be allowed that it would give great pleasure to a geometrical genius, to be able to tell exactly the measure of the circle; that is to say, to know it with the same precision that we know, for example, that a parabolic segment is equal to two thirds of a parallelogram having the same base and the same altitude.

I.—*The diameter of a circle being given; to find, in approximate numbers, the circumference; or vice versa.*

When moderate exactness only is required, we may employ the proportion of Archimedes, who has demonstrated that the diameter is to the circumference nearly as 1 to $3\frac{1}{7}$, or as 7 to 22.

If we therefore make this proportion: as 7 is to 22, so is a given diameter to a fourth term; or if we triple the diameter and add to it a seventh, we shall have the circumference very nearly.

The circumference of a circle, the diameter of which is equal to 100 feet, will be found therefore to be 314 feet 3 inches $5\frac{1}{7}$ lines: the error in this case is about 1 inch 6 lines.

If we are desirous of approaching still nearer to the truth, we must employ the proportion of Metius, which is that of 113 to 355: we must therefore say as 113 to 355, so is the given diameter to the required circumference. The same diameter as before being supposed, we shall find the circumference to be 314 feet, 1 inch, 10 [illegible]/113 lines; the difference between which and the real circumference is less than a line.

If still greater exactness be required, we have only to employ the proportion of 10000000000 to 31415926535; the error in this case, if the circumference were a great circle, such as the equator of the earth, would be, at most, half a line.

To find the diameter, the circumference being given, it is evident that the inverse proportion must be employed. We must therefore say as 22 is to 7, or as 355 to 113, or as 314159 is to 100000, or as 31415926535 to 10000000000, so is the given circumference to a fourth term, which will be the diameter required.

II.—*The diameter of a circle being given; to find the area.*

Archimedes has demonstrated that a circle is equal to the rectangle of half the radius by the circumference. Find therefore the circumference, by the preceding paragraph, and multiply it by half the radius, or the fourth part of the diameter: the product will be the area of the circle, and the more exact the nearer to the truth the circumference has been found.

By employing the proportion of Archimedes, the error, in a circle of 100 feet diameter, will be about $3\frac{1}{4}$ square feet.

That of Metius would give an error less than 25 square inches, or about a sixth of a square foot. As the circle in question would contain about 7854 square feet, the error, at most, would be only one 47124th part of the whole area.

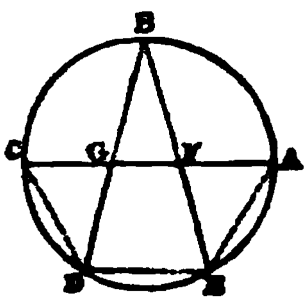
But the area of a circle may be found, without determining the circumference; for it follows, from the proportion of Archimedes, that the square of the diameter is to the area, as 14 to 11; from that of Metius, that it is as 452 to 355; from the proportion of 100000 to 314159, that is as 100000 to 78539, or with still greater exactness as 1000000 to 785398.

The area of the circle therefore will be found by making this proportion, as 14 is to 11, or as 452 to 355, or as 1000000 is to 785398, so is the square of the given diameter, to a fourth proportional, which, if the last proportion has been employed, will be very near the truth.

III —*Geometrical constructions for making a square very nearly equal to a given circle, or a straight line equal to a given circular circumference.*

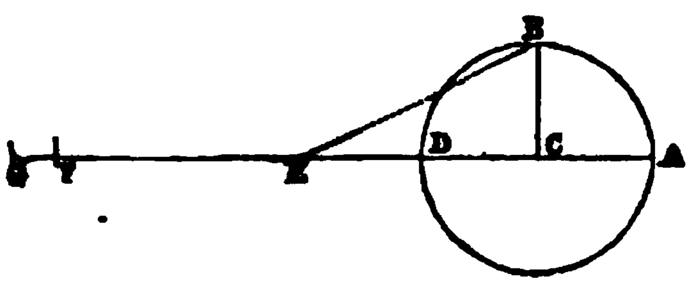
Having shewn some methods for finding numerically, and very near the truth, the proportion between a circle and the square of its diameter, we shall now give some geometrical constructions, exceedingly simple and ingenious, for accomplishing the same object.

Fig. 74.



1st. Let B A D C (Fig. 74.) be a circle, of which A C is the diameter, and A B a quadrant; let A E, E D, and D C be chords equal to the radius; from the point B, draw to the points E and D, the lines B E and B D, intersecting the diameter in F and G; the sum of the lines B F and F G will be equal to the quadrant of the circle, within a five thousandth part.

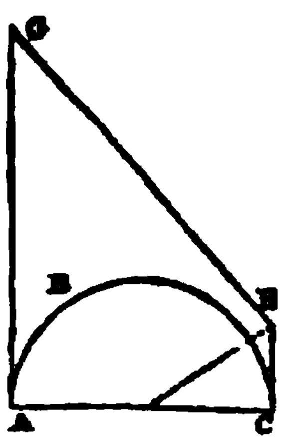
Fig. 75.



2d. Let A D (Fig. 75.) be the diameter of the circle, C the centre, and C B the radius perpendicular to that diameter. In A D continued, make D E equal to the radius; then draw B E, and in A E continued make E F equal to it; if to this line E F, its fifth part F G be added, the whole line A G will be equal nearly within a 17000th part to the circumference described with the radius C A.

For if D A be supposed equal to 100000, A G will be found equal to 314153, with less than an unit of error: but the circumference corresponding to this diameter is, with the difference of nearly an unit, 314159; the error therefore at most is $\frac{6}{100000}$ of the diameter, or about the 17000th part.

Fig. 76.



3d. If the semicircle A B C (Fig. 76.) be given; from the extremities A and C of its diameter, raise two perpendiculars, one of them C E equal to the tangent of 30°, and the other A G equal to three times the radius; if the line G E be then drawn, it will be equal to the semi-circumference of the circle, within a hundred thousandth part nearly.

For it will be found by this construction, the radius being supposed to be 100000, that the line E G, within a unit nearly, is equal to 314162, and the semi-circumference would be, with the difference of nearly an unit, 314159; the error therefore is about $\frac{3}{100000}$ of the radius, or less than a hundred thousandth part of the circumference.

Fig. 77.

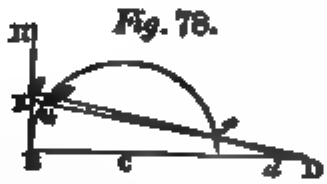
4th. Let A (Fig. 77.) be the centre of the given circle, and D E and C B its two diameters, perpendicular to each other. On any radius, such as A D, make A F equal to half the side E C of the inscribed square; draw B F I indefinitely, and to the point H, draw F H dividing A C in extreme and mean ratio, A H being the lesser segment; if C I be drawn parallel to F H through the point C, the square B L K I, constructed on B I, will be nearly equal to the circle of which B C is the diameter.

For it will be found by calculation, that B H and B F are respectively equal to 69098 and 61237, the radius being 100000; B I therefore will be found equal to 88623, the square of which is 78540, &c., the square of the diameter being 100000, &c., while the circle is 78539, &c.

5th. Inscribe in the given circle a square, and to three times the diameter add a fifth part of the side of the square; the result will be a line which will differ from the circumference by about a 17000th part only.

IV.—*Several methods for making, either numerically or geometrically, and very near the truth, a straight line equal to the given arc of a circle.*

Fig. 78.



1st. Let the given arc, which ought never to exceed 30°, be B G (Fig. 78). To obtain the length of it very nearly in a straight line, draw B H perpendicular to the diameter A B, and continue the diameter to D, so that A D shall be equal to the radius; if D E be then drawn, it will cut off from B H the line B E somewhat less, but very nearly equal to the arc B G.

But if the line $d f g e$, be drawn in such a manner, that the segment $d f$, intercepted between the circle and the diameter continued, shall be equal to the radius, the straight line B e will then be somewhat greater than the arc B g; but very near it, if the arc does not exceed 30 degrees.

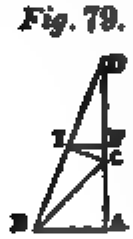
For this theorem we are indebted to Snellius; but it was first demonstrated by Huygens. We shall shew hereafter, that it is very useful in trigonometry.

2d. It has been demonstrated also by Huygens, that twice the chord of half an arc, plus the third of the difference between that sum and the chord of the whole arc, is nearly equal to the arc itself, when it does not exceed 30°.

For if we suppose the arc to be 30°, the chord will be 25882 parts, the diameter being 100000; that of half the same arc, or of 15°, will 13053, the double of which is 26106; if from this we subtract 25882, the difference will be 224, the third of which, 74⅔, added to 26106, will give 26180⅔ for the arc of 30°. Twelve times this arc ought to give the whole circumference; but 26180⅔ multiplied by 12, is equal to 314168, and the circumference is 314159, the difference therefore is only the nine hundred thousandth part of the radius.

It being remembered that a circle is a polygon, whose sides are indefinitely small and infinite in number, the following is a simple method of arriving approximately at the ratio of the circumference to the diameter.

Fig. 79.



Let A B be the semi-side of a regular polygon, C its centre; along A C produced take C D = C B; then the isosceles triangle B C D will have the angle D = ½ B C A, and the perpendiculars C I and I F on B D and A D respectively, will give the middle points I and F of those lines. Since then I F = ½ B A, and D = to ½ B C A, I F is the semi-side of a regular polygon of the same perimeter as the first, but having twice the number of sides.

Calling the radii of the circumscribed and inscribed circles of the first polygon, viz. BC and CA, R, r, and those of the second, DI and DF, R′, r', we have

$$\text{DF} = \tfrac{1}{2}(\text{DC} + \text{CA}) = \tfrac{1}{2}(\text{BC} + \text{CA}), \text{ or } r' = \tfrac{1}{2}(R + r);$$

and by the rectangular triangle CID, we have

$$\text{DI} = \sqrt{\text{DF} \times \text{DC}}, \text{ or } R' = \sqrt{R\,r'}$$

Of the four radii, R, r, R′, r', therefore, the first two being known, the last two may be deduced from them.

Another polygon of the same perimeter as the second, but of twice the number of sides, will have for its radii R″, r'', and these have the same relation to R′, r', that R′, r', have to R, r; we shall then form an infinite series,

$$r, R;\ r', R';\ r'', R'';\ r''' \ R''' \ldots;$$

in which each term r is half the sum, and each term R the square root of the product of the two terms which precede it.

Now in a hexagon the side is equal to the radius of the circumscribing circle; if then we call this radius 1, the perimeter of the figure will be 6, and CA, the radius of the inscribed circle, $= \sqrt{\text{BC}^2 - \text{BA}^2} = \sqrt{\tfrac{3}{4}} =$ 0·866025. And from these values of R and r we deduce successively the following results.

r = 0·866025	0·949469	0·954588	0·954908
R = 1·000000	0·957662	0·955100	0·954940
r = 0·933013	0·963566	0·954844*	0·954924
R = 0·965925	0 95561	0·954972	0·954932, &c.

Here r, r', r''.... are readily found, and the calculations of R, R′, R″.... are very rapid by logarithms; and moreover when r and R agree in the first half of the figures, as at the place we have marked with an asterisk, R as well as r may be found by taking half the sum of the two preceding terms.

We arrive at last to R $= r =$ ·954929; or the radius of the circumscribing = radius of the inscribed circle, therefore these circles coincide with each other, and with the polygon which lies between them. Then ·954929 is the radius of a circle whose perimeter is 6, and the proportion

$$2 \times \cdot954929 \text{ or } 1\cdot909858 : 1 :: 6 : 3\cdot14159$$

gives 3·14159 for the circumference of a circle whose diameter is 1. By this method we can carry the approximation to any point we wish.

In the Mathematical Tracts of Dr. Charles Hutton, several series are investigated for computing the circumference of a circle from its diameter.

The following is better adapted to computation than any other that has yet been discovered.

Let A be an arc of 45°, to radius *unity*. Then

$$A = \begin{cases} \dfrac{4}{5} \times [\,1 + \dfrac{4}{3\cdot10} + \dfrac{8\,\alpha}{5\cdot10} + \dfrac{12\,\beta}{7\cdot10} + \dfrac{16\,\gamma}{9\cdot10} + \&\text{c.}\,] \\[2ex] -\dfrac{7}{50} \times [\,1 + \dfrac{4}{3\cdot100} + \dfrac{8\,\alpha}{5\cdot100} + \dfrac{12\,\beta}{7\cdot100} + \dfrac{16\,\gamma}{9\cdot100} + \&\text{c.}\,] \end{cases}$$

Where α, β, γ, &c. denote the preceding terms in each series.

Remark.—As we promised to give a short account of the different attempts made respecting the quadrature of the circle, we shall here discharge our promise. What we are going to say on the subject, is only an abstract from a very curious work, published by Jombert in 1754.*

It will first be proper to divide those who have employed themselves on this problem, into two classes. The first, consisting of able geometricians, were not led away by illusions. Being aware of the difficulty or impossibility of the problem,

* The author of that curious little work was Montucla himself.—*Note by Dr. Hutton.*

they confined themselves merely to the finding out methods of approximation more and more exact; and their researches have often terminated in discoveries in almost every part of geometry.

The other class consists of those who, though scarcely acquainted with the elements of geometry, and scarcely knowing on what principles the problem depends, have made every effort to solve it, by accumulating paralogisms on paralogisms. Like the unfortunate Ixion, condemned to roll up a heavy burden eternally without being able to bring it to the place of its destination, we find them twisting and turning the circle in every direction, without advancing one step further. When a geometrician has convinced them of an error in their pretended demonstrations, we see them returning a few days after, with the same demonstration in a new form, but equally contemptible. Very often they do not hesitate to contest the best established truths in the elements of geometry; and, in general, sensible of the weakness of their knowledge in this department of science, they consider themselves as specially illuminated by Heaven to reveal truths to mankind, the discovery of which it has withheld from the learned, in order to confer the honour of it on idiots. Such is the ridiculous but real picture of this sort of men. It may be readily conceived that in the short history we are about to give of the quadrature of the circle, we shall not be so unjust towards the eminent geometricians, as to couple them with such visionaries. The singular flights of the latter will only furnish us, towards the end of this article, with matter for an amusing addition to it.

Geometry had scarcely been introduced among the Greeks, when the quadrature or measure of the circle began to give employment to all those who possessed a mathematical genius. Anaxagoras, it is said, exercised himself upon it while in prison; but with what success we are not informed.

The question had been already become celebrated in the time of Aristophanes, and perhaps had made some geometrician lose his senses; for in order to ridicule the celebrated Meto, that comic writer introduces him on the stage, promising to square the circle.

Hippocrates of Chios certainly made it an object of his research: for it could be only by endeavouring to square the circle that he discovered his famous lunules. Some even ascribe to him a certain combination of lunules, from which, as they pretend, he deduced the quadrature of the circle; but in our opinion without any foundation; for as he held a distinguished place among the geometricians of his time, he could not be a dupe to the paralogism of a school-boy: his object was only to shew, that if the lunule described on the side of an inscribed hexagon, could be made equal to a rectilineal space, the quadrature of the circle could be thence deduced; and in this he was perfectly right.

It is very probable that geometricians were not long ignorant that the circle is equal to the rectangle of half the circumference by the radius. Before the time of Plato, geometry had been enriched with more difficult discoveries, yet this truth is first found in the writings of Archimedes. Something more however was necessary: the proportion between the circumference and the diameter, or the radius, remained to be determined; and this discovery occasioned, no doubt, many a sleepless night to that profound geometrician. Not being able to succeed with geometrical precision, he had recourse to approximation, and found, by calculating the length of an inscribed polygon of 96 sides, and that of a similar one circumscribed, that the diameter being 1, the circumference would be more than $3\frac{10}{71}$, and less than $3\frac{10}{70}$, or $3\frac{1}{7}$. For he shewed that the inscribed polygon is somewhat less than $3\frac{11}{70}$, and that the circumscribed is somewhat greater than $3\frac{11}{70}$.

Since that time, if great exactness be not required, to find the ratio of the diameter to the circumference, the proportion of 1 to $3\frac{1}{7}$, or of 7 to 22, is employed; that is to

say, the diameter is tripled, and one seventh of it is added: this seventh is never neglected, but by the most ignorant workmen.

This object, we know, engaged the attention of several more of the ancient geometricians; among whom were Apollonius, and one Philo of Gadara; but the exactest approximations which they found have not reached us.

The first of the modern geometricians, who made any additions to what the ancients had transmitted to us, respecting the measure of the circle, was Peter Metius, a geometrician of the Netherlands, who lived about the end of the sixteenth century. Being employed in refuting the pretended quadrature of one Simon à Quercu; he found this very remarkable proportion, which approaches exceedingly near to the truth between the diameter and the circumference, viz. as 113 to 355. The error is scarcely the ten millionth part of the circumference.

After him, or about the same time, Vieta, a celebrated French analyst and geometrician, expressed the ratio of the circumference to the radius by the proportion of 10000000000 to 31415926535, and shewed that the latter number was too small, but that if its last figure were augmented by only one unit, it would be too great. About the same period also Adrian Romanus, a geometrician of the Netherlands, carried this approximation to 16 figures; but all these were far exceeded by Ludolph van Ceulen, a native of the Netherlands likewise, who carried this proportion to 35 figures, and shewed that, if the diameter be unity followed by 35 ciphers, the circumference will be greater than 314159265358979323846264338327950288, and less than 314159265358979323846264338327950289. He was so proud of this labour, which however required less sagacity than patience, that, like Archimedes, he requested it might be inscribed on his tomb-stone: his desire was complied with, and this singular monument is still to be seen, it is said, in one of the towns of Flanders.

Willebrord Snell, another countryman of Metius, made several important additions to what had been done on this subject, in his book entitled "Cyclometria." He discovered the method of expressing, by a very approximate proportion and an exceedingly simple calculation, the magnitude of any arc whatever; and he made use of it to verify the calculation of van Ceulen, which he found to be correct. He then calculated a series of polygons, both inscribed and circumscribed, always doubling the number of sides, from the decagon to that of 5242880 sides; so that when a proportion between the diameter and circumference of the circle pretended to be exact is proposed, one may refute it by means of this table, and shew which is the circumscribed polygon greater than the supposed value of the circumference, and what circumscribed polygon it surpasses; in either case this will serve to prove the falsity of the pretended rectification of the circular circumference.

The celebrated Huygens, when very young, enriched the theory of the measure of the circle with a great many new theorems. He combated also the pretended quadrature of the circle, which Father Gregory St. Vincent, a jesuit of the Netherlands, announced as discovered, and requiring only a few calculations, which he dexterously forgot to make. Gregory St. Vincent, however, was an able geometrician; he wrote an answer to Huygens, and the latter replied; some of Gregory's pupils entered the lists also; and another jesuit, a geometrician, combated on the same side. But it is certain, whatever Father Castel may have said, that Gregory was mistaken, and that his large work, which contains some very ingenious things, ended with an error, or something unintelligible. As he pretended to have found the quadrature of the circle, why did he not perform those calculations which are necessary to express it numerically? But this was never done, either by him, or by any of his pupils, who carried on the dispute with a great deal of asperity.

James Gregory, a celebrated geometrician in Scotland, undertook, in 1668, to

demonstrate the absolute impossibility of the quadrature of the circle. This he did by a very ingenious method of reasoning, which deserves perhaps to be better examined. However it did not meet with the approbation of Huygens, and this produced a very warm dispute between these two geometricians. But, it must be confessed, that Gregory gave several very ingenious methods for approaching nearer to the measure of the circle, and even to that of the hyperbola.

The higher geometry supplies us with a great number of different methods for finding, by approximation, the measure of the circle, and the greater part of them are easier than the preceding: but this is not a proper place for entering into an explanation of them. We shall content ourselves with observing, that by means of these methods the approximation of Ludolph van Ceulen has been carried as far as 127 figures or decimals. Sharp, an English geometrician, first carried it to 74 figures; Mr. Machin extended it to a hundred, and M. de Lagny continued it to 127: it is as follows. If the diameter be unity, followed by 127 ciphers, the circumference will be greater than 31415926535897932384626433832795028841971-693993751058209749445923078174062962089986280348253421170679821480865132-723066470938446, and less than the same number, when the last figure is increased only by unity. The error therefore is less than a portion of the diameter expressed by unity, divided by unity followed by 127 ciphers. If we suppose a circle, the diameter of which is a thousand millions of times greater than the distance of the sun from the earth, the error in the circumference would be a thousand millions of times less than the thickness of a hair.

It is even possible to go still further; and Euler has pointed out the method, in the Transactions of the Imperial Academy of Sciences at Petersburgh; but it must be confessed that it would be superfluous labour.

We cannot conclude better this short history of the quadrature of the circle, than by an account, which will no doubt amuse some of our readers, of those who have miscarried in their attempts to solve this problem, or who have fallen into ridiculous errors on the subject

The first, among the moderns, who pretended to have found the quadrature of the circle, was Cardinal de Cusa. One of his methods was, to roll a circle or cylinder over a plane, till the point which first touched it should touch it again; and he then endeavoured, by a train of reasoning, which displayed nothing geometrical, to determine the length of the line thus described. He was refuted by Regiomantanus, in 1464 or 1465

After him, that is to say, about the middle of the sixteenth century, Orontius Finæus, though professor royal of the mathematics, rendered himself famous by his paralogisms, not only in regard to the quadrature of the circle, but also in regard to the trisection of an angle and the duplication of the cube. Peter Nonius, however, a Portuguese geometrician, and J. Borelli his former pupil, clearly exposed the fallacy of his reasoning. The same Orontius Finæus published also a work on Gnomonics, which is nothing but a series of paralogisms.

We are astonished to find the celebrated Joseph Scaliger fall, soon after, into the same error. As he had no great esteem for geometricians, he was desirous to shew them the superiority of a man of letters, in solving, by way of amusement, what had so long puzzled them: he attempted the quadrature of the circle, and seriously imagined he had discovered it, by giving, as the measure of it, a quantity which was only a little less than the inscribed dodecagon. It was therefore no great difficulty for Vieta, Clavius, and others, to refute his reasoning: this threw him into a violent passion; and according to the practice of that period, exposed the latter in particular to a great many epithets not very decent, while it confirmed Scaliger more and more in his opinion, that geometricians were destitute of common sense.

We are sorry to include, among this class, the celebrated Danish astronomer, Longomontanus, who pretended to prove that the diameter of a circle is to the circumference, exactly as 100000 is to 314185. Soon after, the famous Hobbes imagined also that he had found the quadrature of the circle; and being refuted by Dr. Wallis, he undertook to prove that the whole system of geometry before taught was nothing but a series of paralogisms. This forms the subject of a work entitled, "De Ratiociniis et Fastu Geometrarum."

Olivier de Serres, the agriculturist, by weighing a circle and a triangle, equal to the equilateral triangle inscribed, believed he had found that the circle was exactly the double of it. This weak man did not see that this double is exactly the hexagon inscribed in the same circle.

A. M. Dethlef Cluver pretended, in 1695, to have squared the circle; and he reduced the problem to one much easier, which he announced in the following manner: "Invenire mundum Menti Divinæ analogum." He unsquared the parabola, and endeavoured to prove that Archimedes had been deceived in regard to the measure of that figure.

Leibnitz endeavoured to engage him in a dispute with M. Nieuwentyt, who then started a great many difficulties against these new calculations; but the attempt did not succeed.

Though these ridiculous attempts, as appears, ought to have prevented others, men were seen, and are still seen daily, falling into errors of the like kind. About 30 years ago, a M. Liger pretended that he had found out the quadrature of the circle, by demonstrating that the square root of 24 was the same as that of 25; and that of 50 the same as that of 49: this he demonstrated, according to his own terms, not by geometrical reasoning, which he abhorred, but by mechanism combined with figures.

A certain Sieur T. de N—— found out something not less curious, viz., that curves ought not to be measured by comparing them with straight lines, but by comparing them with curves. This being once demonstrated, the quadrature of the circle is merely children's amusement.

M. Clerget made another discovery no less interesting, viz., that the circle is a polygon of a determinate number of sides; and he thence deduced, which is very curious, the magnitude of the point where two unequal spheres touch each other. He demonstrated also the impossibility of the motion of the earth. No one before him had been able to suspect the least affinity between these questions.

But what shall we say of the complex calculations of the late M. Basselin, a professor in a university, who, after as much labour almost as Van Ceulen, found a proportion between the diameter and circumference beyond the limits even of Archimedes? This weak man, who had so happily discovered the quadrature of the circle, was ignorant, till some days before his death, that Archimedes had squared the parabola. He proposed also, had he recovered from his malady, to examine the process of Archimedes, being fully convinced that the geometrician of Syracuse had been deceived.

But if these men incurred only ridicule, and ridicule confined to the circle of a small number of geometricians, we are now going to introduce one to whom the ambition of squaring the circle cost much dearer. We allude to the Sieur Mathulon, who, from being a manufacturer of stuffs at Lyons, commenced geometrician and mechanist; but with less success than Hippocrates of Chios, who, from being a wine merchant at Athens, became an illustrious geometrician. Sieur Mathulon, about forty years ago, deposited the sum of 1000 crowns at Lyons, and having announced to geometricians and mechanists the discovery of the quadrature of the circle and perpetual motion, declared he would give the above sum to the person who should prove that he was in an error. M. Nicole, of the Academy of Sciences, proved that

his knowledge of geometry was very limited; that his pretended quadrature was a mere paralogism; and demanded the 1000 crowns, which were adjudged to him. The Sieur Mathulon demurred, and maintained that he ought to prove also the falsity of his perpetual motion; but he lost his suit, and M. Nicole gave up the 1000 crowns to the general hospital at Lyons, to which they were delivered.

Had the Chatelet of Paris been equally severe, a similar folly would have cost much more to a man of some property, who, about thirty years ago, announced the quadrature of the circle; defied the whole world to refute him; and at last, by way of challenge, deposited 10000 livres to be adjudged to the person who should prove that he was mistaken. It is impossible, without lamenting the weakness of the human mind, to see this grand discovery reduced to dividing a circle into four equal parts, by perpendicular diameters, turning these quadrants with their four angles outwards, so as to form a square, and then pretending that this square is equal to the circle. According to the principles of this pretended mathematician, for two figures to be equal it is not necessary that they should touch each other throughout their whole extent; it is sufficient that they touch, or can touch. Thus the square is not only equal to the inscribed circle, but even to any figure included in the circle, the salient angles of which touch the circumference.

It would not have been difficult to shew to any other person than the author, that this was absolute nonsense. Three persons appeared as claimants of the 10000 livres; the cause was tried at the Chatelet, but this tribunal was of opinion that a man's fortune ought not to suffer for the errors of his judgment, when these errors are not prejudicial to society. On the other hand, the king decreed that the bet should be considered as void; and that both parties should take back their money. The author extorted from the Academy of Sciences a sentence, by which he was desired to study the elements of geometry; but he was still convinced that future ages would blush for the injustice done to him by that in which he lived. Before we conclude this article, we must say a few words respecting M. le Rohberger de Vausenville, who in a work called "Consultation sur la Quadrature du Circle," asks geometricians, whether the quadrature of the circle would not be found, if means could be devised for determining the centre of gravity of a sector of a circle, in common parts of the radius and the circumference of the same circle. We do not rightly understand what the author means by common parts of the radius and the circumference. If he means those parts of the radius in which it is usual to express the circumference—as when it is said that if the radius be 100, the circumference will be 314—we can answer, in the name of all geometricians, that the quadrature of the circle would, in that case, be found. We will even not hesitate to tell him, that in whatever manner he determines, in the axis of a sector or arc of a circle, its centre of gravity, provided that in this determination the arc itself is not employed as one of the data, he will have solved this famous problem; for who does not know that the distance of the centre of gravity of the semi-circumference, for example, from the centre, forms a third proportional to the fourth part of the circle and the radius? But this determination of the centre of gravity of the sector, or arc of a circle, is a discovery rather to be wished than hoped for.

M. de Vausenville had no need to challenge, either individually or in general, all the geometricians of Europe, and even those of Turkey and Africa, where the meaning of the words centre of gravity is certainly not known; and he had still less occasion to inform them that if they did not refute him he would consider their silence as a sign of their defeat, and that his quadrature was acknowledged as resting on a solid foundation. This bravado will certainly excite neither the Eulers, the d'Alemberts, nor the Bernouillis, &c., to attack his quadrature. Either M. de Vausenville is right, and in that case mathematicians will acknowledge his discovery, and bestow on him every just praise; or his pretended quadrature is a mere paralo-

gism, and of course it will meet with as little attention as that of Henry Sullamar, a real Bedlamite, who found it in the number 666, inscribed on the forehead of the beast in the Revelations; or those of many others which deserve, in like manner, to be consigned to oblivion.

PROBLEM XLVII.

Of the length of the Elliptical Circumference.

We have spoken in a pretty full manner of the circular circumference, the exact determination of which in length would give the quadrature of the circle. But no author, as far as we know, has said any thing satisfactory, or useful in a practical view, respecting the circumference of the ellipse. It is however necessary, in many cases, and even in practical geometry, to know the length of that curve; in the higher geometry there are also a great many problems the solution of which depends on the same knowledge: a few observations therefore on this subject may be of utility.

Some authors, who have written on practical geometry, are of opinion that the circumference of an ellipse is an arithmetical mean between the circumference of the circles described on the two axes as diameters; but this is a mistake; and had they possessed a little more of the spirit of geometry, they would have readily perceived it; for it may be easily demonstrated that this is false in an ellipse much elongated, as in that which has the greater axis 20, and the lesser 2. The circumference of this ellipse indeed will certainly be greater than 40, while the mean proportional between the circumferences of the circles described on its axes, as diameters, will be only $34\frac{1}{2}$.

The rectification of the elliptical circumference is a problem which is almost the same, in regard to the quadrature of the circle, as the latter is to a common problem in geometry. John Bernouilli is the only person who has given a method susceptible of being reduced to practice for measuring the length of the elliptic line. He shews indeed, in an excellent memoir, published in his works, how to determine the circular circumferences which are limits alternately less and greater than the circumference of a given ellipse: and by this method we have calculated the following table. We have supposed a series of ellipses, one half of the common greater axis of which is 10 parts, while the half of the less axis becomes successively 1, 2, 3, &c., as far as 10, the last value given by a circle; and we have found that the length of the circumferences of the ellipse is as here expressed.

Common length of the greater axis, 20.*

Lesser axis.	Length of the elliptic circumference.	Length of the mean circumference of the circles described on the greater and less axis.
2	40·63245	34·5579
4	42·01968	37·6990
6	43·68526	40·8406
8	46·02506	43·9822
10	48·44215	47·1238
12	51·05407	50·2654
14	53·82377	53·4070
16	56·72739	56·5486
18	59·81022	59·6902
20	62·83185	62·83185

* Sir Jonas Moore also calculated a like table for the elliptic circumferences, extending ten

It here appears, that the circumference of the circle, which forms a mean between those described on the greater and less axis, is always less than the elliptic line, and the more sensibly so, the more the ellipsis differs from a circle: in the first of the above ellipsis the error is the 7th part.

By the help of this table all the mean lengths of the ellipsis between the preceding may be calculated: nothing is necessary but to take the proportional parts.

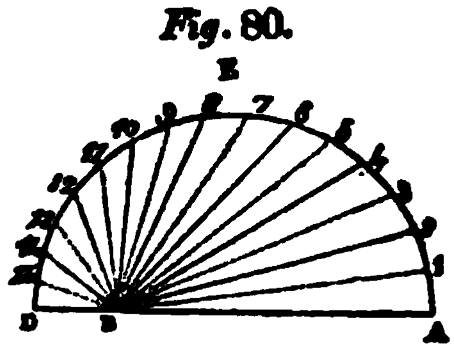
Let us suppose, for example, that the greater axis of a semi-ellipse is 20 feet, and that the half of its less axis is $7\frac{1}{2}$ feet; it is evident that, in this case, the whole of the less axis will be 15 feet. This ellipsis then will hold a mean place between that in which half the less axis is [illegible] of the greater, and another in which the less axis is [illegible]. But by dividing the difference between the lengths of these two ellipses into two equal parts, it will be found, without any considerable error, that the length of the circumference of the mean ellipse will be 55·27558 parts, the axis being 20; consequently the half of the proposed ellipsis having its transverse diameter equal to 20 feet and its conjugate to $7\frac{1}{2}$, will be 27 feet 6 inches and 8 lines: the error being scarcely a line.

PROBLEM XLVIII.

To describe geometrically a circle, the circumference of which shall approach very near to that of a given ellipse.

It is to Mr. John Bernouilli also that we are indebted for this simple and ingenious method of describing a circle isoperimetrous to a given ellipse. As it may serve as a supplement to what we have said of the rectification of the ellipse, we shall here give it a place.

Fig. 80.



Form the two axes of the given ellipsis into one straight line, as A D (Fig. 80.), in which A B is equal to the greater axis, and B D to the less: let this line A B be the diameter of a semicircle A E D, which must be divided into 4, 8, 16, or 32 parts, &c. at pleasure, and according as greater precision may be required. We shall here suppose the number of equal parts to be 16. From the point B draw to each point of division straight lines; then take the 16th part of the sum of all these lines B A, B 1, B 2, B 3, &c., as far as B D inclusively; and if with the line hence arising as radius, a circle be described, you will have a circular circumference so nearly equal to that of the given ellipse, that it will not differ from it one hundred thousandth part, even in the most unfavourable case, such as that, for example, where the ratio of the axes of the ellipse is as 10 to 1.

It may be readily seen, that if the semicircle had been divided into 8 parts, it would have been necessary to take only the 8th part of the sum of all the lines drawn to the points of division, including the points D and A.

If this operation were performed with a circle of a foot radius, the precision of the result would approach very near the truth; and by means of a geometric scale, with exceedingly minute divisions, a very satisfactory numerical approximation might be found without calculation.*

times as far, that is, to a hundred different ellipses, the conjugate axes gradually increasing from 1 to 100, which is the constant transverse. The numbers indeed are set down to four decimals, but they are not commonly true to more than two.—*Note by Dr. Hutton.*

* It is noticed above, by Montucla, that an arithmetical mean between the two axes of an ellipse, has been often taken as the diameter of a circle of equal circumference with the ellipse. It may be added that this rule always gives the perimeter in defect, or less than just.

Another rule, almost as easy, which gives the perimeter always in excess, or more than just, is this: Square each axis, and take the arithmetical mean between these squares; that is, add the squares together, and take half the sum; then extract the square root of this mean, and it will be nearly the diameter of a circle of equal circumference. [As

PROBLEM XLIX.

To determine a straight line nearly equal to the arc of any curve whatever.

We shall suppose that the amplitude of the given arc is not very considerable, as not more than 20 degrees; that is to say, if tangents be drawn at the extremities of the arc, and then perpendiculars to these tangents, the angle included between these perpendiculars shall be at most 20 degrees.

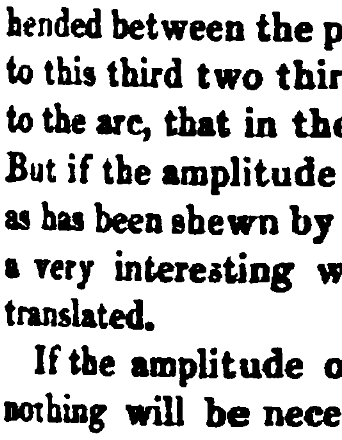
This supposition being made; draw the chord of the arc; and then find, either by calculation, or by means of the compasses, the third of the tangents comprehended between the place where they meet and the points of contact; if we then add to this third two thirds of the chord, we shall have a straight line so nearly equal to the arc, that in the present case the difference will be but a ten thousandth part. But if the amplitude be only about 5 degrees, the error will not be a millionth part, as has been shewn by M. Lambert, member of the Academy of Sciences at Berlin, in a very interesting work, published in German, which is highly worthy of being translated.

If the amplitude of the given arc be greater, as about 50 degrees for example, nothing will be necessary but to divide it into three parts nearly equal, and to draw tangents to the extremities of the arc and to the points of section, which will give a portion of the polygon circumscribed about the curve; if the three chords of the three parts of the arc be then drawn, and if two thirds of these three chords be added to the third of the tangents, forming the circumscribed polygon, the result will be a line equal, within a hundredth thousandth part, to the length of the given arc.

PROBLEM L.

A circle, having a square inscribed in it, being given; to find the diameter of a circle in which an octagon of a perimeter equal to the square can be inscribed.

Fig. 81.



Let A B (Fig. 81.) be the diameter of the given circle, and A D the side of the inscribed square. Divide A D into two equal parts in E, and raise E F perpendicular to A D, meeting the given circle in F; if A F be then drawn, it will be the diameter of a circle, in which if an octagon be inscribed it will be equal in perimeter to the given square.

For it is evident that the circle described on the diameter A F, will pass through the point E, since the angle A E F is a right angle. It is also evident that the line drawn from I, the centre of the second circle, to the point E, will be parallel to D F, because the sides A D and A F of the triangle D A F are bisected in the points E and I. But the angle A F D is half a right angle, being half of D C A, which is a right angle, since the chord of the inscribed square subtends an arc of 90°: consequently the angle A I E is equal to 45°; whence it follows that A E is the side of the octagon inscribed in the circle having A F for its diameter. And it is evident that eight times A E is equal to four times A D.

Remark.—If A E be, in like manner, divided into two equal parts in G, and if G H be drawn perpendicular from the point G, till it meet the second circle; by drawing A H, that line will be the diameter of a third circle, in which, if a polygon of 16 sides be inscribed, it will be isoperimetrous to the above square or octagon.

Hence it follows, that if this operation were infinitely continued, we should obtain

As this latter rule is nearly as much in excess as the former is in defect, if an arithmetical mean between them be taken, that is half their sum, it will be the diameter of a circle of equal circumference, within the fifty thousandth part of the whole, and is the nearest approximating rule yet given. These rules are taken from my treatise on Mensuration, where several others may also be seen, both for the whole circumference and also for any part of it.—*Note by Dr. Hutton.*

a circle or polygon of an infinite number of sides, isoperimetrous to a given square. The circumference of this circle therefore would be equal to the perimeter of the square, and we should thus have the quadrature of the circle.

We have seen a very ingenious attempt to discover the quadrature of the circle on this principle. The author, M. Janot, professor of mathematics in the Royal Military School, reduced the problem to a very exact equation, but complex, by the solution of which he expected to obtain this last diameter; but when he seriously tried to reduce it, he found the two members of his equation to be composed of the same terms, which of course gave him no solution.

PROBLEM LI.

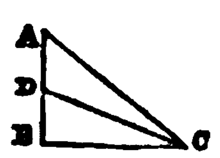
The three sides of a rightangled triangle being given; to find the value of its angles without trigonometrical tables.

We shall first suppose that the ratio of the hypothenuse to the least side is greater or not less than 2 to 1, in order that the angle opposite to that side may be at most about 30°; for the error will be less the more that angle is below 30°.

This being premised; let us suppose, for example, that the hypothenuse of the triangle is equal to 13, the greater side comprehending the right angle 12, and the less 5. We must then make this proportion: as twice the hypothenuse, plus the greater side, or 38, is to the less side or 5, so is three times unity or 3, to a fourth proportional, which will be $\frac{15}{38}$. But $\frac{15}{38}$ reduced to a decimal fraction is 0·39473: if this number be divided by 0·1745, the quotient will be the number of the degrees and parts of a degree contained in the angle opposite to the less side. This quotient is $22\frac{621}{1000}$, which makes 22° 37′ 15″. By the tables it will be found to be 22° 37′ 28″.

If the sides of the triangle are nearly equal, such for example as 3, 4, 5, we must suppose in the triangle a line C D (Fig. 82.) dividing the angle opposite to the side A B, or that represented by 3, into two equal parts. But it is known that in this case the opposite side A B will be divided in the same ratio as the adjacent sides; consequently the segment B D may be found by the following analogy:

Fig. 82.



As the sum of the two other sides or 9, is to 3, the third side, so is C B or 4, to B D, which will be $\frac{12}{9}$ or $\frac{4}{3}$; if the squares of $\frac{4}{3}$ and 4, or of C B and B D, be then added together, by extracting the square root of the sum, which in decimals is 17·777, we shall have for the square root 4·21637, which will be the value of C D. In the last place, by applying the above rule to the triangle B C D, we shall find the angle B C D to be 18° 26′ 7″, and consequently its double, or the angle A C B, 36° 52′ 14″. By trigonometrical tables the latter will be found to be 36° 52′ 15″; so that the difference is only one second.

PROBLEM LII.

An arc of a circle being given, in degrees, minutes, and seconds; to find the corresponding sine, without the help of trigonometrical tables.

The solution we are going to give of this problem, is not so simple and short as the preceding; but it appears to be the best hitherto proposed, especially as it is easy, and may be readily remembered by means of an observation we shall make at the end, and which will shew its source as well as the demonstration of it.

In this problem there are three cases, which require three different methods of operation. The given arc may exceed 60°, or it may be less, or at most not more than 30°; and in the last place it may be greater than 30°, but less than 60°.

1st. We shall suppose that the arc exceeds 60°, and that its sine is required. Take its complement to 90°, and reduce that arc into parts of the radius, which we shall suppose to be 100000; for this purpose, nothing is necessary but to multiply the degrees it contains by $1745\frac{1}{3}$, and the minutes by 29·09, and then to add the

products. Square this arc thus reduced, and raise it also to the fourth power; divide the square of it by 2, and from the quotient subtract unity or the radius; divide the fourth power of it by 24, and add the quotient to the above remainder; the number thence resulting will be nearly the sine of the given arc.

Let the given arc, for example, be 70° 30′; its complement to 90° is 19° 30′, which reduced to parts of the radius, as beforesaid, will give 34025. The square of this number, suppressing the five last figures, which are useless, because we have no occasion for more than 100000 parts of the radius, is 11583, and its half 5792, which taken from 100000 leaves 94208. Square 11583, which will give the fourth power of 34035; and if five figures be suppressed, as useless for the reason before mentioned, we shall have 1341, which must be divided by 24. The quotient, which is somewhat less than 56, being added to 94208, will make 94264, which will be the sine of 70° 30′. And this is exactly what it will be found to be in the tables of sines.

2d. Let us now suppose that the given arc is at most 30°. Find the cube and fifth power of that arc reduced to parts of the radius; then divide the cube by 6, and the fifth power by 120; if the first quotient be subtracted from the arc, and the second be added to the remainder, we shall have the value of the sine, a very small error excepted.

Let the given arc, for example, be 30°. When reduced to 100000th part of the radius it will give 52362, the cube of which, suppressing the last ten figures, will be 14354. The sixth part of this number is 2392, which taken from the arc 52362, leaves 49970. The fifth power of the same number 52362, suppressing the last twenty figures, is 3935, which divided by 120 gives 32; if 32 be added to the above remainder, we shall have 50002 for the sine of 30°; which it is well known is exactly 50000; consequently the error is only two units in the last figure.

3d. If the arc be between 30° and 60°, for example 45°; take the difference between that arc and 60°, which is 15°, and add to it 60°; the sum will be 75°, the sine of which must be found by the first rule.

Then find that of 15° by the second; and subtract it from that of 75°; the remainder will be the sine of 45°; for according to a theorem in trigonometry, that the sines of two arcs, equally distant from 60°, have for difference the sine of that arc by which each of these two arcs differs from that of 60°.

If, instead of the sine of an arc, that of its complement be required, the same rules may be employed: the sine of the complement of 20°, for example, is the right sine of 70°; and on the other hand,' the sine complement of 70°, is the right sine of 20°; by which it may be readily seen, that to find the sine complement of an arc, nothing is necessary but to find the right sine of the complement of the arc.

When the right sine and the sine complement of an arc are known, it will be easy to find the tangent by the following proportion: As the sine complement, or cosine, is to the sine, so is radius to the tangent: nothing therefore is necessary but to divide the sine, increased with any number of ciphers at pleasure, by the cosine.

Remark.—We have here given a method for supplying the place of tables, so necessary in practical trigonometry, or of forming them very expeditiously, in cases when they are not at hand, or cannot be procured. I was once myself in such a situation, having lost my baggage, which was taken from me by a party of the Iroquois Indians, when posted at Oswego in Canada. In that dreary abode, I endeavoured to amuse myself by the study of geometry. An opportunity of performing some trigonometrical operations occurred: but, being destitute of books, I fortunately remembered the theorem of Snellius, which serves as a basis for the solution of the preceding problem: in short, I recollected two expressions, in infinite series, which give the value of the sine and cosine, the arc being given. The first, as is well known, a being made to

represent the arc, is $a - \frac{a^3}{6} + \frac{a^5}{120} - \frac{a^7}{5040}$ &c., and the second $1 - \frac{a^2}{2} + \frac{a^4}{24} - \frac{a^6}{720}$, &c. But when the arc a is very much below the value of the radius or unity, it is evident that the three first terms of each will be sufficient, because all the following terms become excessively small. After what has been said, the demonstration of these rules may be easily discovered.

PROBLEM LIII.

A circle and two points being given; to describe another circle, which shall pass through these points, and touch the former circle.

It is here evident, that these two points must be both within, or both without, the given circle.

Fig. 83.

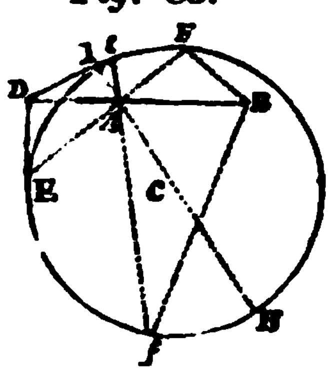
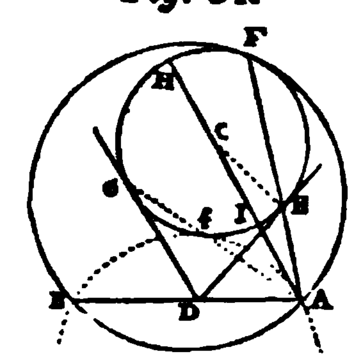


Fig. 84.

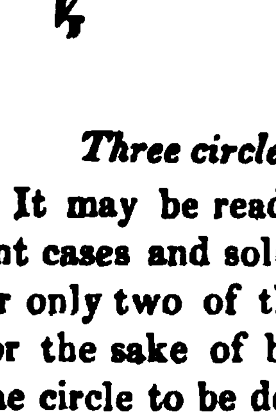


Let the two given points then be A and B, as in the two Figures 83. and 84. Join these points by a straight line A B; and through one of them, for example A, and the centre of the given circle, draw the straight line A I H, intersecting it in the two points H and I; then take A D a fourth proportional to A B, A H, A I, and from the point D draw the two tangents D E, D e; lastly, from the point A, through the two points of contact draw the two lines E A F, e A f, intersecting the circle in F and f: the circle described through the two points A and B, and through F, will touch the given circle in F; and if one be described through the points A, B, and f, it will touch the given circle in f.

PROBLEM LIV.

Two circles and a point being given: to describe a third circle, which shall pass through the given point, and touch the other two circles.

Fig. 85.



Let the centres of the two given circles be the points A and C, (Fig. 85.) and their radii A B, C D. In the line which joins their centres continued, find the point F, the tangent from which to one of the circles shall be the tangent to the other, (by Prob. xii.), and join the point F to the given point E: then make F G a fourth proportional to F E, F B, F D; and by the preceding problem, through the points G and E describe a circle which shall touch one of the two circles A B or C D: this third circle will touch also the other circle.

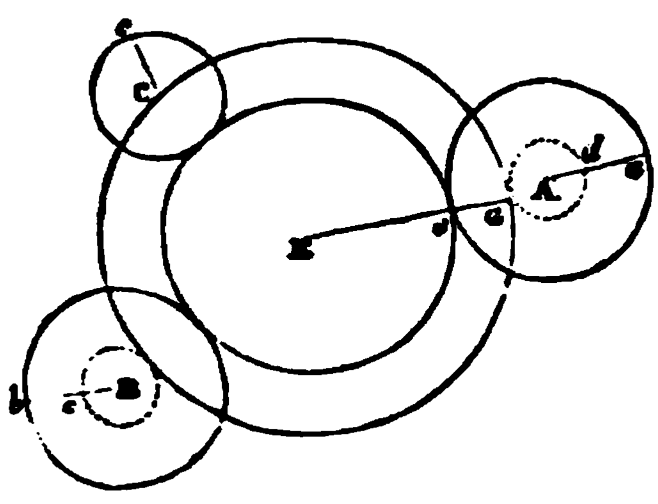
PROBLEM LV.

Three circles being given; to describe a fourth, which shall touch them all.

It may be readily seen that this problem is susceptible of a great number of different cases and solutions; for the required circle may contain the three given circles, or only two of them, or even one; or the given circles may all be without it. But, for the sake of brevity, we shall confine ourselves to one of these cases, that where the circle to be described must leave the other three without it.

Let the three given circles then be denoted by A, B, C, (Fig. 86.) and let their radii be A a, B b, C c; let A also be the greatest, B the mean one, and C the least. In the radius A a make $a\,d$ equal to C c, the radius of the least circle, and from A as a centre, with the radius A d describe a new circle. In the radius B b,

Fig. 86.



make $b\,e$ equal to C c also, and from B as a centre, with the radius B e, describe another circle: then, by the preceding proposition, through the centre C describe a circle, which shall touch the two new circles; let its centre be E, and its radius E G; diminish this radius by the radius C c, and from the same centre E describe another circle, which will evidently touch the three first circles given

For since the circle described from the centre A with the radius A d, is within the proposed circle A, by the quantity $a\,d$ or C c, it is evident that if the radius E G be diminished by that quantity, the circle described with this new radius, instead of touching the interior circle, having A d for its radius, will touch the proposed circle, the radius of which is A a.

It may be seen also that the same circle described with the radius E G, less C c, will touch externally the circle which has for its radius B b. Lastly, it will touch externally the circle having C c for its radius; consequently it will touch them all three externally.

Remark.—This problem had some celebrity among the ancients; and indeed it is attended with a certain degree of difficulty. It terminated a treatise of Apollonius, entitled "De Contactibus," which has been lost, but which Vieta, a celebrated geometrician who lived about the end of the sixteenth century, restored, and which may be found in his works, printed in Latin, at Leyden, in 1646, in folio, with the title of "Apollonius Gallus, seu exsuscitata Apollonii Pergæi de Tactionibus Geometria."

Newton has given a beautiful and ingenious solution of this problem; but that of Vieta appeared to us preferable for the present work, being founded on easier principles. We cannot omit this opportunity of observing, that the above work of Vieta is a most elegant piece of geometry, treated in the manner of the ancients.

PROBLEM LVI.

What bodies are those, the surfaces of which have the same ratio to each other as their solidities?

This problem was proposed, in the form of an enigma, in one of the French Journals, entitled the Mercury, of the year 1773.

"Reponds-moi, d'Alembert, qui découvre les traces
Des plus sublimes vérités;
Quels sont les corps dont les surfaces
Sont en même rapport que leurs solidités?"

We do not find that d'Alembert condescended to answer this problem, for it may be readily seen, by those in the least acquainted with geometry, that two bodies well known, the sphere and the circumscribed cylinder, will solve it. Archimedes demonstrated long ago, that the sphere is equal to two thirds of that cylinder, both in surface and solidity, provided the two bases of the cylinder are comprehended in the former; and this is the answer which was given to the enigma in the following Mercury.

But we may go a little further, and say, that there are a great number of bodies which, when compared with each other, and with the sphere, will answer the problem also: such are all solids formed by the circumvolution of a plane figure circumscribed about the same sphere, and even all plane-faced solids, regular or irregular, that can

be circumscribed about the same sphere; for the solidity of all these bodies is the product of their surfaces by the third of the radius of the inscribed sphere, while the solidity of the sphere is the product of its surface by the third of its radius.

Thus, the equilateral cone is to the inscribed sphere, both in surface and solidity, as 9 to 4.

The case is the same in regard to the sphere and the circumscribed isosceles cone; except that the ratio, instead of 4 to 9, will be different according to the elongation or oblate form of the cone.

If the sphere and the circumscribed cylinder possess this property, it is because the latter is a body produced by the circumvolution of the square circumscribed about the great circle of the sphere, on an axis perpendicular to two of the parallel sides.

If the square and inscribed circle revolved around the diagonal of the square, the surface and solidity of the body, thus produced, would be to each other as $\sqrt{2}$ is to 1.

We shall here propose a similar problem:

What are those figures, the surfaces and perimeters of which are to each other in the same ratio?

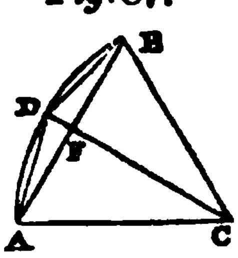
The answer is easy: the circle and all polygons, regular or irregular, circumscriptible of it.

THEOREM VIII.

The dodecagon inscribed in the circle, is $\frac{3}{4}$ of the square of the diameter, or equal to the square of the side of the inscribed triangle.

This theorem, which is exceedingly curious, was first remarked by Snellius, a Dutch geometrician.

Fig. 87.



Let A C (Fig. 87.) be the radius of a circle, in which is inscribed the side A B of the hexagon; and if A D and D B, be the sides of the regular dodecagon, it thence follows that, by drawing the radius D C, it will cut the side A B perpendicularly, and divide it into two equal parts. But it is evident, that the area of the dodecagon is equal to 12 times one of the triangles A D C, or D C B; and as the triangle A D C is equal to the product of the radius by the half of A F, or by the fourth part of the radius, that is to say, is equal to a fourth of the square of the radius, the twelve will be equal to three times the square of the radius, or to three fourths of the square of the diameter.

On the other hand, the side of an equilateral triangle inscribed in a circle, the diameter being unity, is equal to $\sqrt{\frac{3}{4}}$; consequently its square is also equal to $\frac{3}{4}$ of the square of the diameter, or to the dodecagon.

Remark.—Two of the inscribed polygons, viz. the square and the dodecagon, possess the property of having a numerical ratio to the square of the diameter; for the inscribed square is exactly the half; but of the regular polygons circumscribed, this property belongs only to the square.

Irregular polygons however, and even a great variety of them, commensurable to the square of the radius, might be inscribed in a given circle.

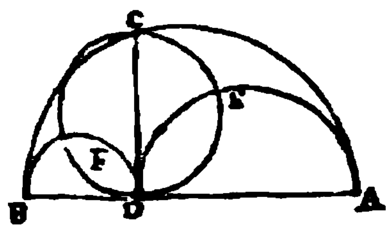
Let the diameter of the circle, for example, be 1; and let the four sides of the inscribed quadrilateral be $\frac{6}{10}$, $\frac{8}{10}$, $\frac{12}{13}$, $\frac{5}{13}$: its surface will be rational, and equal to [illegible] of the square of the diameter.

PROBLEM LVII.

If the diameter A B (*Fig.* 88.) *of a semicircle* A C B, *be divided into any two parts*

whatever, AD *and* DB; *and if on these parts as diameters there be described two semicircles* AED *and* DFB, *a circle is required equal to the remainder of the first semicircle.*

Fig. 88.



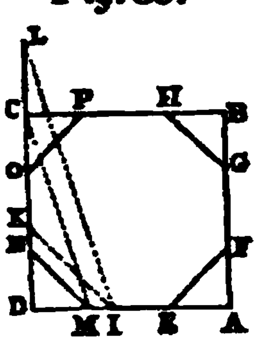
From the point D raise DC perpendicular to AB, till it meet the semicircle ACB: if a circle be then described having DC for its diameter, it will be that required.

The demonstration of this problem, so well known, is deduced from a theorem in the second book of the Elements of Euclid, viz., that the square of AB is equal to the squares of AD and DB, and twice the rectangle of AD and DB; a rectangle to which the square of DC is equal by the property of the circle. Instead of these squares if we substitute semicircles, which are in the same ratio, the problem will be demonstrated.

PROBLEM LVIII.

A square being given; to cut off its angles in such a manner, that it shall be transformed into a regular octagon.

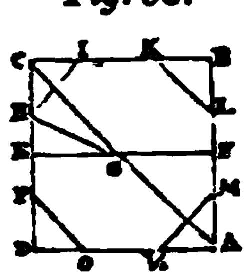
Fig. 89.



Let the given square be ABCD (Fig. 89.) In the two sides DC and DA, which meet in D, take any two equal segments whatever, DI and DK, and draw the diagonal IK; make DL equal to twice DK, plus the diagonal IK, and draw LI; if CM be then drawn parallel to LI, through the point C, it will cut off from the side of the square the quantity DM, to which if DN be made equal, by drawing the line NM, we shall have the side of the octagon required. If AE, AF, BG, BH, CP, and CO be made equal to the line DM, by drawing EF, GH, and OP, the required octagon will be completed.

Remark.—The solution above given, is an example of what often happens in employing the algebraic calculus in the solution of geometrical problems; for there is a solution much more simple, and that of a nature to be self-evident even to a beginner. It is as follows:

Fig. 90.

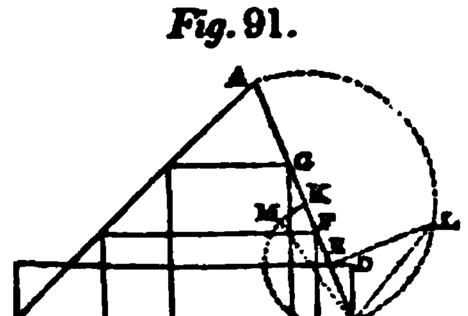


Draw the diagonal AC (Fig. 90.) of the square, and also EF bisecting the opposite sides in E and F, and the diagonal in G. Draw GH so as to bisect the angle CGE; so shall EH be half the side of the octagon. Therefore make CI, BK, BL, AM, AN, DO, DP, each equal to CH, and the angles of the octagon will be found.

PROBLEM LIX.

A triangle ABC *being given; to inscribe in it a rectangle, in such a manner, that* FH *or* GI *shall be equal to a given square.*

Fig. 91.



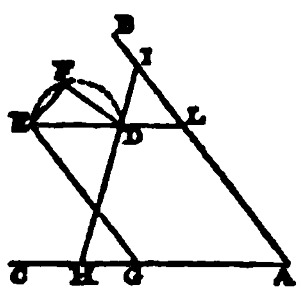
On the base BC (Fig. 91.) describe the rectangle BD, equal to the given square, and let E be the point where AC is intersected by the side of this rectangle parallel to BC. On AC describe a semicircle, and having raised the perpendicular EL, till it meet the circumference, draw CL; on CK, the half of AC, describe also a semicircle, in which make CM equal to CL: if KF and KG be then made equal to KM, we shall have the points F and G, through which if two lines be drawn parallel to the base till they meet AB, and also two other lines perpendicular to the base, they will

form the rectangles F H and G I, equal to each other, as well as to the rectangle D B, which was equal to the given square: therefore, &c.

PROBLEM LX.

Through a given point D (*Fig.* 92.) *within an angle* B A C, *to draw a line* H I, *in such a manner, that the triangle* I H A *shall be equal to a given square.*

Fig. 92.



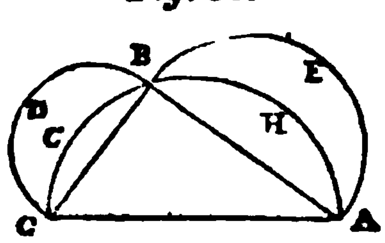
Through the point D draw L E parallel to one of the sides of the given angle, and make the rhombus L E G A, equal to the given square. On the line D E describe a semicircle, in which apply D F equal to D L, and draw E F; lastly, if G H be made equal to E F, and H D I be drawn through the point H, the line H D I will be the one required.

PROBLEM LXI.

Of the Lunule of Hippocrates of Chios.

Though the quadrature of the circle be in all probability impossible, means have been devised to find certain portions of the circle which are demonstrated to be equal to rectilineal spaces. The oldest instance of a circular portion, which may be thus squared, is that of the lunules of Hippocrates of Chios; the construction of which is as follows:

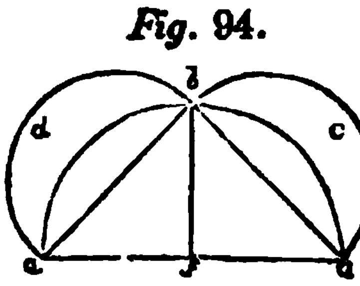
Fig. 93.



Let A B C (Fig. 93.) be a right-angled triangle, on the hypothenuse of which describe the semicircle A B C, touching the right angle B: if semicircles be then described on the sides A B and B C, the spaces in the form of a crescent, A E B H A and B D C G B, will together be equal to the triangle A B C.

For it is well known that the semicircle on the base A C, is equal to the two semicircles A E B and B D C, because circles are to each other as the squares of their diameters: if the segments A H B and B G C, which are common to both, be taken away, there will remain, on the one hand, the triangle A B C, and on the other the two spaces in the form of a crescent, A E B H and B D C G, and these remainders will be equal: therefore, &c.

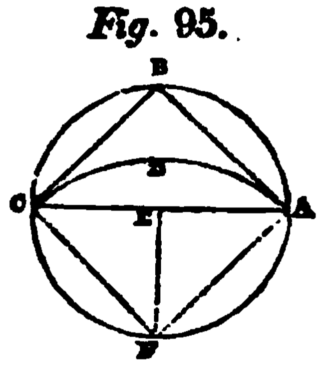
Fig. 94.



If the sides *a b*, *b c*, are equal, as in Fig. 94, the two lunules will evidently be equal, and each will be half of the triangle *a b c*, that is to say, equal to the triangle *b f a* or *b f c*.

Hence we obtain a simpler construction of the lunule of Hippocrates. Let A B C (Fig. 95.) be a semicircle on the diameter A C, and A F C an isosceles right-angled triangle. If from the point F as a centre, there be described through A and C, the arc of a circle A D C on the base A C, the lunule A B C D will be equal to the triangle C A F.

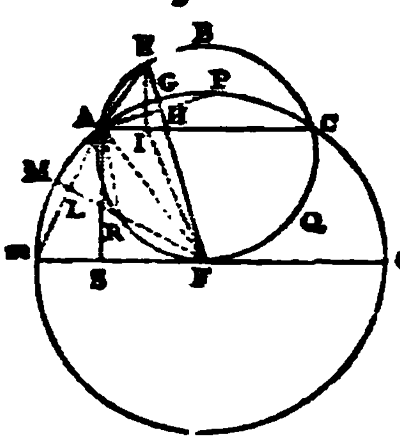
Fig. 95.



Since the square of F C is double the square of E C, or of E F, the circle described with the radius F C will be double that described with the radius E C: consequently a fourth part of the former, or the quadrant F A D C, will be equal to the half of the second, or to the semicircle A B C. If the common segment A D C E A therefore be taken away, the remainders, that is to say, the triangle A F C, on the one hand, and the lunule A B C D A, on the other, will be equal.

Remarks.—We shall take this opportunity of making the reader acquainted with several curious observations, added by modern geometricians to the discovery of Hippocrates.

Fig. 96.



1st. From the centre F (Fig. 96.) if there be drawn any straight line whatever F E, cutting off a portion of the lunule A E G A; that portion will still be squarable, and equal to the rectilineal triangle A H E right-angled at H.

For it may be easily demonstrated, that the segment A E will be equal to the semi-segment A G H.

2d. From the point E, if E I be let fall perpendicular on A C, and if F I and F E be drawn, the same portion of the lunule A E G A, will be equal to the triangle A F I.

For it may be easily demonstrated that the triangle A F I is equal to the triangle A H E.

3d. The lunule therefore may be divided in a given ratio, by a line drawn from the centre F: nothing more is necessary than to divide the diameter A C in such a manner, that A I shall be to C I in that ratio; to raise E I perpendicular to A C, and to draw the line F E; the two segments of the lunule A G E and G E C will be in the ratio of A I to I C.

All these remarks were first made by M. Artus de Lionne, bishop of Gap, who published them in a work entitled "Curvilineorum Amœnior Contemplatio," 1654, 4to, and afterwards by other geometricians.

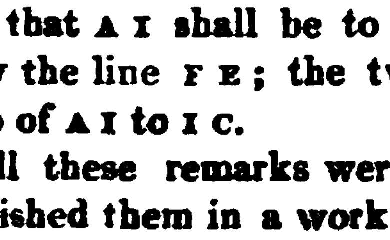
4th. If the two circles, forming the lunule of Hippocrates, be completed, the result will be another lunule, which may be called Conjugate, and in which mixtilineal spaces, absolutely squarable, may be found.

From the point F, if there be drawn any radius F M, intersecting the two circles in R and M; we shall have the mixtilineal space R A M R, equal to the rectilineal triangle L A R: which can be easily demonstrated; for it may be readily seen that the segment A R of the small circle, is equal to the semi-segment L A M of the greater.

Hence it follows, that if the diameter *m* O touch the small circle in F, the mixt triangular space A R F *m* A, will be equal to the triangle A S F, right-angled in S, or to half the lunule A G C B A.

5th. There are also some other portions of the lunule of Hippocrates that are absolutely squarable; which, as far as we know, were never before remarked.

Fig. 97.



Let A B C F A (Fig. 97.) be a lunule, and let A B be a tangent to the interior arc. Draw the lines E A and *e* A making with A B equal angles; if from the point B there be then drawn the chords B E, B *e*, which will be equal, we shall have the mixtilineal space, terminated by the two circular arcs, E B *e*, A G F, and the straight lines A *e* and F E, equal to the rectilineal figure *e* A E B *e*.

This would be true, even if the figure A B C F A were not absolutely squarable; that is to say, though A B C should not be a semicircle, provided the two circles were always in the ratio of 2 to 1.

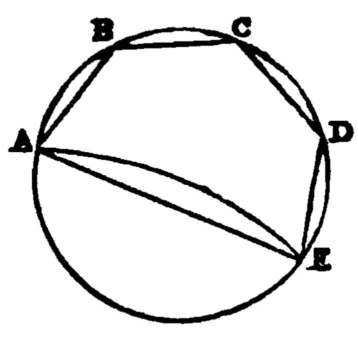
PROBLEM LXII.

To construct other lunules, besides that of Hippocrates, which are absolutely squarable.

The lunule of Hippocrates is absolutely squarable, because the chords A B, B C (Fig. 95.) and A C are such, that the square of the last is equal to the squares of the other two; so that by describing on the last an arc of a circle, similar to those subtended by A B and B C, the two segments A B and B C are equal to A D C.

This method of considering the lunule of Hippocrates conducts us to more general views; for we may conceive in a circle any equal number of chords at pleasure; for example four, as A B, B C, C D, and D E, (Fig. 98.) of such a nature, that by drawing the chord A E, the square of it shall be quadruple of one of them; or more generally, the number of these chords being *n*, the square of A E may be to that of A B, as *n*

Fig. 98.



to 1. Thus, if we describe on A E an arc similar to those subtended by the chords A B, B C, &c., the segment A E will be equal to the segments A B, B C, &c., together: if from the rectilineal figure A B C D E, therefore, we take away the segment A E, and add it to the segments A B, B C, &c., the result will be a lunule formed of the arcs A C E, and A E, which will be equal to the rectilineal polygon A B C D E.

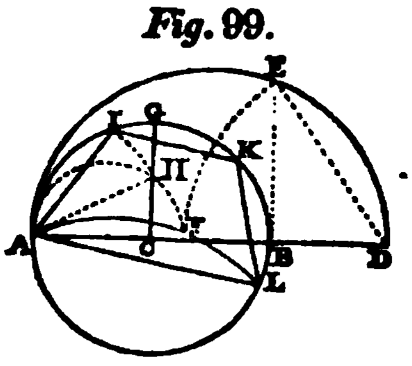
The question then is to resolve the following geometrical problem:

In a given circle to inscribe a series of equal chords, A B, B C, C D, &c., in such a manner, that the square of the chord A E, by which they are all subtended, shall be to the square of one of them, as the whole number of them is to unity; triple, if there are three; quadruple, if there are four, &c.

But we shall confine ourselves to cases that can be constructed by the elements of geometry, which will still give us two lunules similar to those of Hippocrates, the one formed by circles in the ratio of 1 to 3, and the other by two circles in that of 1 to 5, besides two other lunules, formed by circles in the ratio of 2 to 3 and of 3 to 5.

Construction of the first Lunule.

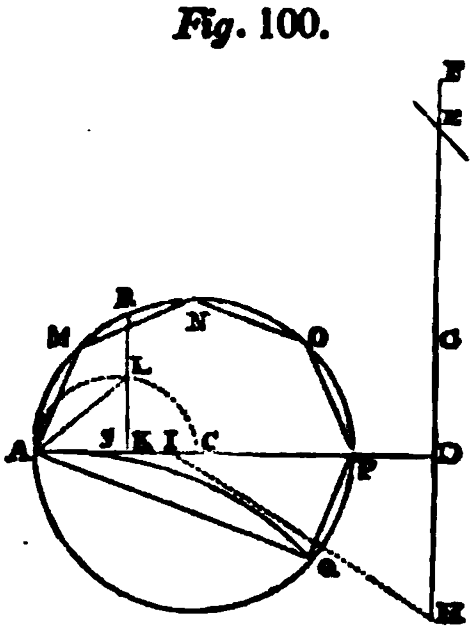
Fig. 99.



Let A B (Fig. 99.) be the diameter of the lesser circle, with which the lunule is to be constructed. Continue A B to D, so that B D shall be equal to the radius, and on A D as a diameter describe the semicircle A E D, cutting B E, drawn perpendicular to A D, in E; draw D E, and make D F equal to it: on A F describe also a semicircle A H F, intersecting the radius C G, perpendicular to A B, in H; draw A H, and in the given circle make the chords A I, I K, and K L, equal to A H; then draw A L, and on that chord, with a radius equal to D E, describe an arc of a circle A L; by these means we shall have the lunule A G B L A, equal to the rectilineal figure A I K L A.

Construction of the second Lunule, where the circles are as 1 to 5.

Fig. 100.

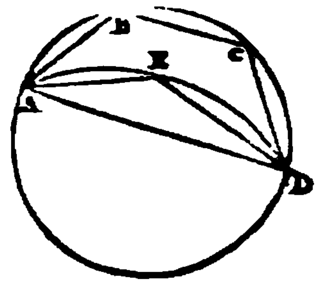


Continue the diameter of the given or less circle, till the part P D (Fig. 100.) be equal to half the radius; and draw the indefinite line D E perpendicular to A D; then from the point S, which divides the radius A C into two equal parts, with a radius equal to 3 times A C, describe an arc of a circle, cutting the before-mentioned perpendicular in E: make E F equal to $\frac{1}{4}$ of A C, and D H equal to the radius; divide H F into two equal parts in G; and from G as a centre, with a radius equal to G H, describe an arc of a circle cutting the straight line A D in I: then make D K equal to H I, and draw K R perpendicular to the diameter, intersecting the semicircle described on A C in L; lastly draw A L, and let the chords A M, M N, N O, O P, P Q, be made equal to it: if an arc of a circle be then described on A Q, with a radius equal to D E, the lunule A N P Q A will be equal to the rectilineal figure A M N O P Q A.

Lunules absolutely squarable may therefore be formed with circles, which are to each other in the ratios of 1 to 2, 1 to 3, and of 1 to 5. But there are no others formed by circles in simple multiple or sub-multiple ratio, which can be constructed

merely by the rule and compasses. Those which might be formed with circles in the ratio of 1 to 4, 1 to 6, or 1 to 7, &c., would require the assistance of the higher geometry. The trisection of an angle, or the finding of two mean proportionals, is a problem of the same nature, and of the same degree, and to be solved only by the same means. But there are still two which may be constructed by the help of simple geometry, and which are formed by circles in the ratio of 2 to 3 and of 3 to 5. For the sake of brevity, we shall confine ourselves to shewing the method of construction.

Fig. 101.



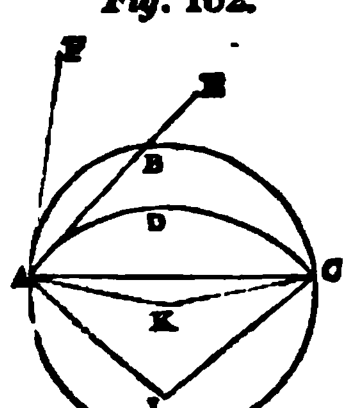
For the first.—Let there be any circle, the radius of which is supposed to be 1; inscribe in it a chord A B (Fig. 101.), equal to $\sqrt{\frac{3}{4}} - \sqrt{\frac{1}{12}}$, and let it be twice repeated, from B to C, and from C to D: draw the chord A D, and on A D describe an arc similar to the arc A B C; if the two equal chords A E and E D be then drawn, the lunule A B C D E A will be equal to the rectilineal polygon A B C D E A.

For the second.—In a circle, the radius of which is 1, inscribe a chord equal to $\sqrt{\frac{5}{8} - \frac{1}{8}\sqrt{5} - \sqrt{\frac{5}{8} - \frac{1}{8}\sqrt{5}}}$, and carry it round five times: draw the chord of the quintuple arc, and describe on it an arc with a radius $= \sqrt{\frac{5}{3}}$; in this arc inscribe the three chords of its three equal parts, which may be done by common geometry, because each of these thirds is similar to a fifth of the first arc already given. We shall then have a lunule equal to a retilineal figure, formed by the five chords of the small circle and the three chords of the greater.

PROBLEM LXIII.

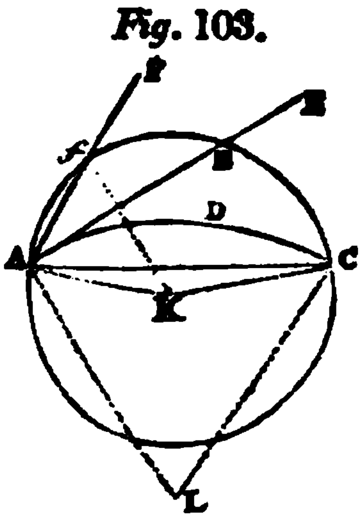
A lunule being given; to find in it portions absolutely squarable, provided the circles by which it is formed are to each other in a certain numerical ratio.

Fig. 102.



Let A B C D A (Fig. 102, 103, 104.) be a lunule, formed by two circles in any of the above ratios, A B C being a portion of the lesser circle, and A D C of the greater. Draw A E the tangent of the arc A D C, and then draw the line A F in such a manner, that the angle E A C shall be to the angle F A C in the same ratio as the less circle is to the greater; then one of the three following things will take place; A F will be a tangent to the circle A B C, (Fig. 102.); or it will cut it, as in f, (Fig. 103.); or as in φ, (Fig. 104.)

Fig. 103.



In the first case, the lunule will be absolutely squarable, and equal to the rectilineal figure K A L C (Fig. 102.)

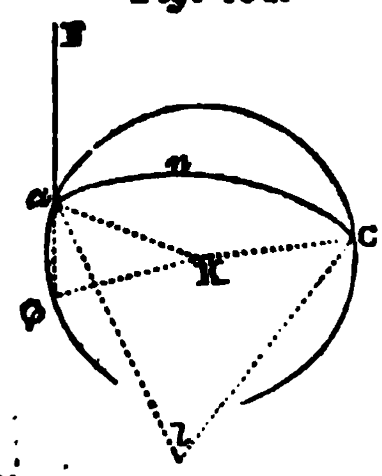
In the second, this lunule, minus the circular segment A f, will be equal to the rectilineal figure A f K C L A, or the space A K C L, plus the triangle A K f, (Fig. 103.)

In the third, the same lunule, plus the circular segment $a\,\varphi$, will be eqyal to the rectilineal space $a\,\varphi$ K $c\,l\,a$, or the space a K $c\,l$ minus the triangle a K φ, (Fig. 104.)

We omit the demonstration, both for the sake of brevity, and because it may be easily conceived from what has been already said.

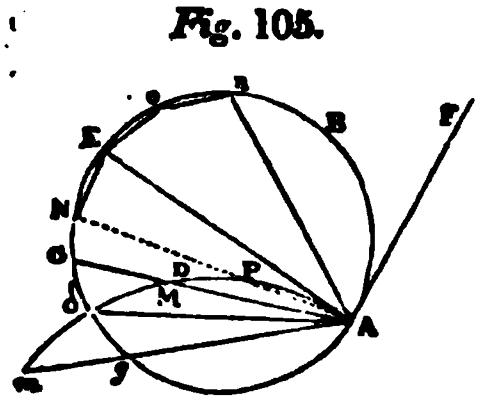
Hence it may be readily seen, that if the given circles have to each other a certain ratio, which will admit of the angle F A C (Fig. 102, 103.) being constructed with the rule and compasses, in such a manner as to be to the angle E A C, in the reciprocal ratio of these circles, we may draw the line F A, which will cut off from

Fig. 104.



the lunule the portion A D C B f A, equal to an assignable rectilineal space. Now this will always be the case when the less circle is to the greater in the ratio of 1 to 2, or 1 to 3, or 1 to 4, or 1 to 5, &c.; for the angle F A C must then be double, triple, quadruple, or quintuple of E A C: in this there is no difficulty. The case will be the same if the less circle is to the greater in the ratio of 2 to 3, or 2 to 5, or 2 to 7, &c.; or the arc A D C, being susceptible of geometrical trisection, as is often the case, if the greater circle be to the less as 3 to 4, or 3 to 5, or 3 to 7, &c.

Fig. 105.



Another method.—Let A F (Fig. 105.) be a tangent to the circle A B C in A, and A E a tangent to the arc A D C in the same point. Draw the line A G in such a manner, that the angle F A G may be to the angle E A G, as the greater circle is to the less; that is, that the angle F A E shall be to E A G, as the greater circle minus the less is to the latter: the line A G will then fall either on A C, or above it, as in A G, or below it as in A g.

Now, in the first case, it may be easily demonstrated, that the lunule is absolutely squarable.

In the second, it may be shewn that the same lunule, minus the mixtilineal triangle M G C M, is equal to an assignable rectilineal space.

In the third, it may be proved that the same lunule, if the mixtilineal triangle C m g be added, will be equal to the same rectilineal space.

Lastly, let there be drawn, in any of the preceding figures, between A C and A E, as Fig. 105, any line whatever A N, forming with the tangent A E any angle N A E; then, in the angle F A E, draw another line A n, in such a manner, that the angle n A E shall be to E A N as F A E to C A E. It may still be demonstrated, that the mixtilineal figure formed of the two arcs N n, A P, and the two lines A n, P N, will be equal to a rectilineal space; which may be found, by dividing the arc N n into as many parts, similar to the arc A P, as the number of times that the less circle is contained in the greater: this may be performed geometrically, if the ratio of the one circle to the other be as 1 to 2, or 1 to 3, or 1 to 4, &c. If we here suppose it, for example, to be as 1 to 3, we shall have three equal chords, n O, O E, and E N, and the portion of the lunule in question will be equal to the rectilineal figure A n O E N A, since the three segments, n O, O E, &c., are together equal to the segment A P.

Remark 1.—We have also proposed and solved the following problem.

A lunule, not squarable, but formed by two circles in the ratio of 1 *to* 2, *being given; to intersect it by a line parallel to its base, which shall cut off from it a portion absolutely squarable.*

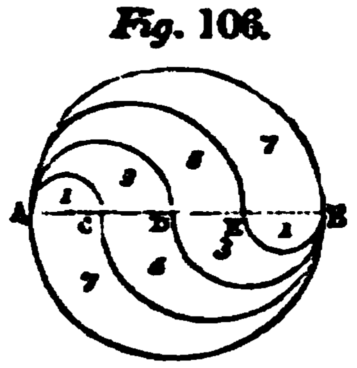
Remark 2.—The following method for dividing circles, &c. is so curious, that it is well deserving of a place here, in addition to the foregoing ways of dividing them into certain portions.

To divide geometrically Circles and Ellipses into any number of parts at pleasure, and in any proposed ratios.

Although the learned labours* of all ages have failed in their attempts at the geometrical quadrature of the circle, and even of the division of the circumference into any number of equal parts at pleasure; yet our own time has furnished the solution of a problem but little less curious, and heretofore esteemed almost, if not altogether, as difficult as it; namely the division of the circle into any proposed number of parts

whatever, of equal perimeter, and the areas either equal or in any proportion to each other. The solution of this seeming paradox was first published by Dr. Hutton, in his quarto volume of Tracts, in 1786. That curious solution was, in substance, as follows:

Fig. 106.



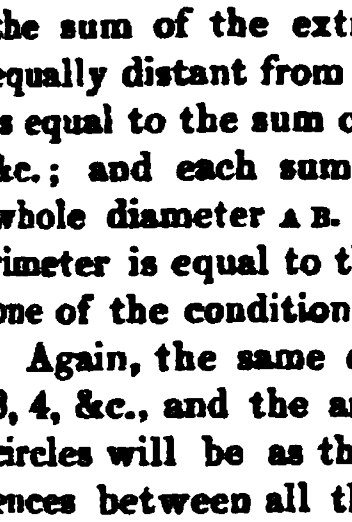
Divide the diameter A B (Fig. 106.) of the given circle into as many equal parts as the circle itself is to be divided into, at the points C, D, E, &c. Then on the lines A C, A D, A E, &c., as diameters, describe semicircles on one side of the diameter A B, and also on the lines B E, B D, B C, &c., on the other side of that diameter; then will these semicircles divide the whole given circle in the manner proposed, viz. into parts which are all equal to each other, both in area and in perimeter.

For, the several diameters of the dividing semicircle being in arithmetical progression, and the diameters of circles being in the same proportion as their circumferences, these also will be in arithmetical progression. But, in such a progression, the sum of the extremes being equal to the sum of each pair of terms that are equally distant from them; therefore the sum of the circumferences on A C and C B, is equal to the sum of those on A D and D B, and to the sum of those on A E and E B, &c.; and each sum equal to the semicircumference of the given circle on the whole diameter A B. Therefore all the parts have equal perimeters; and each perimeter is equal to the whole circumference of the first given circle: which satisfies one of the conditions in the problem.

Again, the same diameters being in proportion to each other as the numbers 1, 2, 3, 4, &c., and the areas of circles being as the squares of their diameters, the semicircles will be as the square numbers 1, 4, 9, 16, &c., and consequently the differences between all the adjacent semicircles are as the terms of the arithmetical progression, 1, 3, 5, 7, &c.: and here again the sums of the extremes and of every two equidistant means, make up the several equal parts of the circle: which is the other condition of the problem.

But this subject admits of a still more geometrical form, and is capable of being rendered very general and extensive, and is moreover very fruitful in curious consequences. For first, in whatever ratio the whole diameter is divided, whether into equal or unequal parts, and whatever be the number of the parts, the perimeters of the parts will always be equal. For since the circumferences of circles are in the same proportion as their diameters, and because A B (Fig. 107.) and A D + D B, and A C + C B are all equal, therefore the semicircumferences c, and $b + d$, and $a + e$, are all equal; and constantly the same, whatever be the ratio of the parts A D, D C, C B, of the diameter. We shall presently find too that the spaces T V, R S, and P Q, will be universally as the same parts, A D, D C, C B, of the diameter.

Fig. 107.



The semicircles having been described as before mentioned, erect C E perpendicular to A B, and join B E. Then will the circle on the diameter B E, be equal to the space P Q. For, join A E. Now the space P is = semicircle on A B − semicircle on A C: but the semicir. on A B = semicir. on A E + semicir. on B E, and the semicir. on A C = semicir. on A E − semicir. on C E; therefore semicir. A B − semicir. A C = semicir. B E + semicir. C E, that is, the space P is = semicir. B E + semicir. C E; to each of these add the space Q, or the semicir. on B C; then P + Q = semicir. B E + semicir. C E + semicir. B C, that is, P + Q = double the semicir. B E, or = the whole circle on B E.

In like manner, the two spaces P Q and R S together, or the whole space P Q R S, is

equal to the circle on the diameter B F. And therefore the space R S alone, is equal to the difference, or the circle on B F minus the circle on B E.

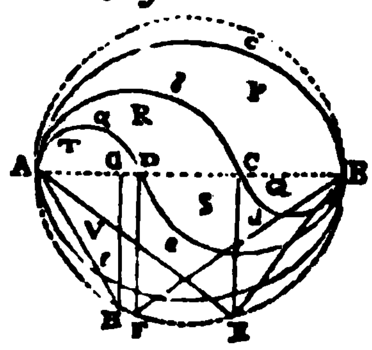
But, circles being as the squares of their diameters, BE^2, BF^2, and these again being as the parts or lines B C, B D, therefore the spaces P Q, P Q R S, R S, T V, are respectively as the lines B C, B D, C D, A D. And if B C be equal to C D, then will P Q be equal to R S, as in the first or simplest case.

Hence to find a circle equal to the space R S, where the points D and C are taken at random: From either end of the diameter, as A, take A G equal to D C, erect G H perpendicular to A B, and join A H; then the circle on A H will be equal to the space R S. For, the space P Q is to the space R S, as B C is to C D or A G, that is as BE^2 to AH^2, the squares of the diameters, or as the circle on B E to the circle on A H. But the circle on B E is equal to the space P Q; therefore the circle on A H is equal to the space R S.

Hence, to divide a circle in this manner, into any proposed number of parts, that shall be in any ratio to one another: Divide the diameter into as many parts, at the points D, C, &c., and in the same ratios as those proposed; then on the several distances of these points from the two ends A and B as diameters, describe the alternate semicircles on the different sides of the whole diameter A B; and they will divide the whole circle in the manner proposed. That is, the spaces T V, R S, P Q, will be as the lines A D, D C, C B.

But these properties are not confined to the circle alone. They are to be found also in the ellipse, as the genus of which the circle is only a species. For if the annexed figure be an ellipse described on the axis A B (Fig. 108.) the area of which is, in like manner, divided by similar semi-ellipses, described on A D, A C, B C, B D, as axes, all the semi-perimeters *f*, *a e*, *b d*, *c*, will be equal to one another, for the same reason as before in the circle, namely, because the peripheries of similar ellipses are in the same proportion as their diameters. And the same property would still hold good, if A B were any other diameter of the ellipse, instead of the axis; describing on the parts of its semi-ellipses which shall be similar to those into which the diameter A B divides the given ellipse.

Fig. 108.



And farther, if a circle be described about the ellipse, on the diameter A B, and lines be drawn similar to those in the second figure; then by a process the very same as before in the circle, substituting only semi-ellipse for semicircle, it is found that the space

P Q = the similar ellipse on the diameter B E,
P Q R S = the similar ellipse on the diameter B F,
R S = the similar ellipse on the diameter A H,

or to the difference of the ellipses on B F and B E; also the elliptic spaces P Q, P Q R S, R S, T V, are respectively as the lines B C, B D, D C, A D; being the same ratios as the the circular spaces. And hence an ellipse is divided into any number of parts, in any assigned ratios, after the same manner as the circle is divided, namely, dividing the axis, or any diameter, in the same manner, and on the parts of it describing similar semi-ellipses.

With respect to the above method of dividing a circle, Dr. Hutton gives the following anecdote, which we think will be interesting to our readers:—

"About the year 1770, Mr. James Ferguson, the ingenious lecturer on astronomy and mechanics, in his peregrinations, came to Newcastle, where I then resided, to give his usual course of public lectures; on which occasion, with the assistance of my friends, I not only procured him a numerous and respectable audience, but also accommodated him with the free use of the new school rooms, which I had lately built, to deliver his lectures in. As Mr. F. commonly amused my family and friends at

evenings, with shewing his ingenious mechanical contrivances and drawings, on one of these occasions he produced a very neat and correct drawing on a large scale, being a construction of this problem in the prolix way in which it had been given by Mr. Hawney; but which he exhibited as a great curiosity. I ventured to state to him, that I thought a much simpler construction might be found out for this problem, which was then new to me. As Mr. F. expressed a wish to see such a thing as a simpler construction, which however he seemed to have his doubts of procuring, I was induced to consider it that evening, before going to rest, and discovered the construction above given.

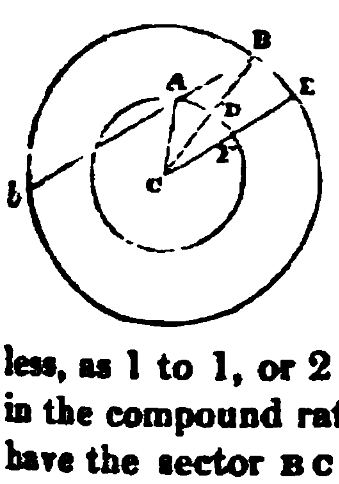
"The next morning I shewed it to him, and he seemed much pleased with its apparent simplicity, but doubted whether it might be exactly true. On referring him to the accompanying demonstration, I was much surprized by his reply that he could not understand that, but he would make the drawing on a large scale, which was always his way to try if such things were true. In my surprise, I asked where he had learned geometry,—by what Euclid or other book,—when he frankly stated that he had never learned any geometry, nor could ever understand the demonstration of any one of Euclid's propositions. Accordingly, next morning, with a joyful countenance, he brought me the construction neatly drawn out on a large sheet of pasteboard, saying he esteemed it a treasure, having found it quite right, as every point and line agreed to a hair's breath, by measurement on the scale."

PROBLEM LXIV.

Of various other Circular Spaces absolutely Squarable.

1st. Let there be two concentric circles, through which is drawn the line *b* B (Fig. 109.) a tangent or secant to the interior circle. Draw C A and C B, forming an angle A C D, and make the arc D F in proportion to the arc D A, as the square of C D is to the difference of the squares of C B and C D: if C E be then drawn, we shall have the mixtilineal space A B E F equal to the rectilineal triangle A C B.

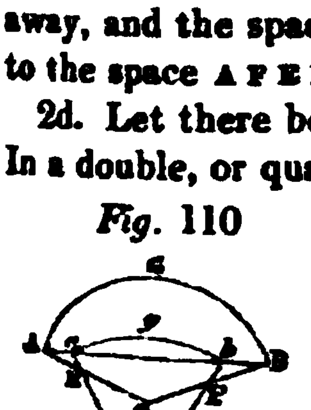
Fig. 109.



It is evident that, to render the position of C E determinable by common geometry, the ratio between the arcs A D and D F must be that of certain numbers as 1 to 1, 1 to 2, 1 to 3, &c., or 2 to 1, 2 to 3, &c. Consequently, the difference of the squares of the radii of the two circles, must be to the square of the less, as 1 to 1, or 2 to 1, or 3 to 1, &c. The sectors of the different circles being then in the compound ratio of the squares of their radii and of their amplitudes, we shall have the sector B C E equal to A C F; if the common sector D C F therefore be taken away, and the space A D B be added to both, the rectilineal triangle A C B will be equal to the space A F E B.

2d. Let there be any sector, as A C B G A (Fig. 110.) of which A B is the chord. In a double, or quadruple, or octuple circle, take a sector *a c b g a*, the angle of which shall be the half or the fourth, or the eighth part of the angle A C B, which it is possible to do with the rule and compasses; let this second sector be disposed as seen in the figure, that is to say, in such a manner, that the arc *a g b* shall stand on the chord A B. We shall then have the space A *a g b* B G A, equal to the rectilineal figure E C F *c*, minus the two triangles A *a* E and *b* B F.

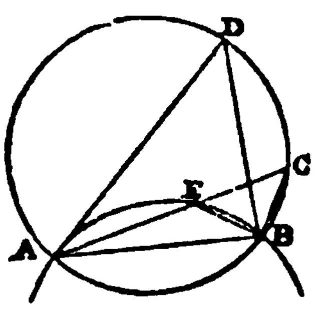
Fig. 110



This is almost evident; for, by the above construction, the sector A C B G is equal to *a c b g*; if the part therefore which is common be taken away, there will be an equality between what remains on the one hand, viz. the kind of lunule A G B *b g a*, plus the two triangles A *a* E, and B *b* F, and what remains on the other, or the recti-

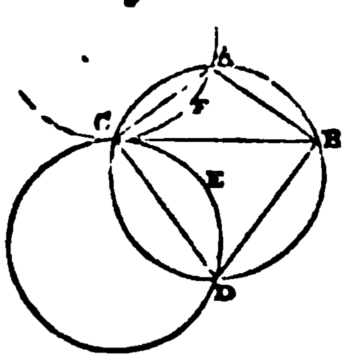
lineal figure E C F C: this kind of lunule therefore is equal to the above rectilineal figure, diminished by these two triangles.

Fig. 111.



3d. If two equal circles cut each other in A and B (Fig. 111.) and if any line A C be drawn intersecting the interior arc in E, and the exterior in C, it is evident that the arc E B will be equal to the arc B C; and consequently the segment E B will be equal to the segment B C. Hence it follows, that the triangle formed by the two arcs E B and B C, and the straight line E C, will be equal to the rectilineal triangle E B C. Lastly, that if A D be a tangent in A, to the arc A E B; the mixtilineal figure A E B C D, will be equal to the rectilineal triangle A D B.

Fig. 112.

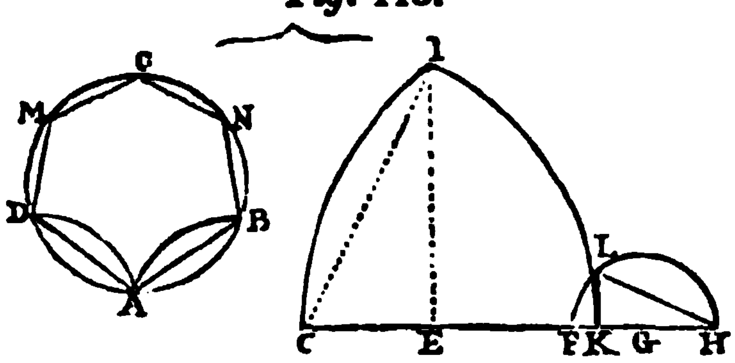


4th. If two equal circles touch each other in C (Fig. 112.) and if a third equal circle be described through the point of contact; the curvilineal space A F C E D B A will be equal to the rectilineal quadrilateral A B D C.

For if C B be drawn a tangent to the first two circles, the space comprehended by the arcs C F A and A B and the straight line C B, is equal to the rectilineal triangle C A B, as has been shewn already. The case is the same with the mixtilineal space C E D B in regard to the triangle C D B: therefore, &c.

5th. The above remark was made by M. Lambert, in the "Acta Helvetica," vol. ii. But other spaces of the same form may be found equal to rectilineal figures, though bounded by circular arcs, two of which only are equal.

Fig. 113.



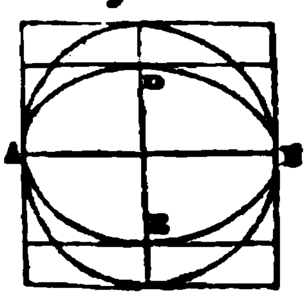
Let A B C D (Fig. 113.) be a circle, from which it is required to cut off, by two other circular arcs, a space of the above kind absolutely squarable. On an indefinite right line make the parts C E, E F, F H, each equal to the side of the square inscribed in the given circle: and let the third part F H be divided into two equal parts in G: on the extremity of C E raise the perpendicular E I, and let it be intersected in I, by a circle described from G as a centre with the radius G C. Draw C I, and make C K equal to it; lastly, on F G describe a semicircle, cutting, in L, the line K L perpendicular to F G; draw H L, and in the given circle make the chords A B and A D equal to it. If with a radius equal to C E, there be then described arcs, passing through the points A and B, A and D, with their convexity turned towards C; we shall have the space bounded by the arcs A B, A D, and B C D, equal to the rectilineal space formed by the chords A B, A D, and the four chords D M, M C, C N, and N B, of the four equal portions of the arc B C D.

But, as enough has been said on this subject, we shall only add one reflection, which is, that these quadratures ought not to be considered as real quadratures of a curvilineal space. All the marvellous in these operations, as M. de Fontenelle has very properly remarked, consists in a kind of geometric legerdemain, by means of which as much is dexterously added on the one hand, to a rectilineal space, as is taken from it on the other. It was not in this manner that Archimedes first squared the parabola, and in which modern geometricians have given the quadrature of so many other curves. All these things however appeared to us sufficiently curious to entitle them to a place in a work of this nature.

PROBLEM LXV.

Of the measure of the Ellipse or Geometrical Oval, and of its parts.

Fig. 114.



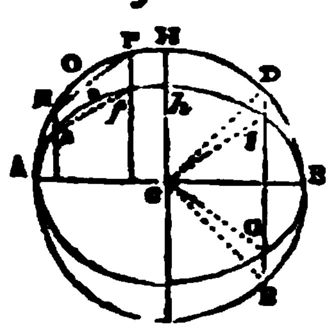
It may be easily demonstrated, that the ellipse (Fig. 114.) is to the rectangle of its axes A B and D E, as the circle is to the rectangle of its axes, or to the square of its diameter A B, since each axis is equal to the diameter.

Thus, as the circle is $\frac{11}{14}$ nearly of the square of its diameter, the ellipse is also $\frac{11}{14}$ of the rectangle of its axes.

Nothing then is necessary, but to multiply the rectangle of the axes of the given ellipse by 11, and to divide the product by 14; the quotient will give the area.

We shall here add, that each segment or sector of the ellipsis is always in a given ratio to the sector or segment of a circle, as is easy to be determined. Let the elliptical sector F C G, (Fig. 115.) for example, or the segment F B G, be given: on the axis A B describe a circle from the centre C; and if G F be continued to D and E, we shall have the elliptical sector F C G B to the circular sector D C E B, as F G to D E, or as the less axis of the ellipsis is to the greater: the elliptical segment B F G will also be to the circular segment D B E, as F G to D E, or as the less axis of the ellipsis to the greater.

Fig. 115.



Let there be likewise, in an ellipsis, any segment whatever, as *n o p*. On the axes let fall two perpendiculars from *n* and *p*, and continue them till they meet the circle in N and P; if N P be then drawn, we shall have the segment *n o p* to the circular segment N O P, in the same ratio as the less axis is to the greater. From this is deduced the solution of the following problem.

PROBLEM LXVI.

To divide the sector of an ellipsis into two equal parts.

Fig. 116.

Let it be required, for example, to divide the elliptical sector D C B (Fig. 116.) into two equal parts, by a line as C G.

On the diameter A B describe a circle; and having drawn D I perpendicular to A B, continue it to E, and draw E C, which will give the circular sector E C B; divide the arc E B into two equal parts in F, and draw F H perpendicular to the axis A B; then from the centre C, to the point G, where that perpendicular cuts the ellipsis, draw the line G C, the elliptical sector B C G will be equal to G C D, as the circular B C F is to F C E.

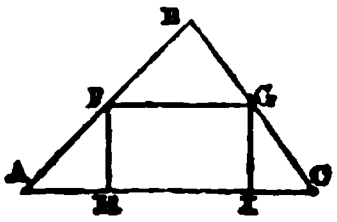
The case would be the same if the sector were equal to the 4th part of an ellipsis, or any higher part; and also if the sector were comprehended between any two semi-diameters of the ellipsis, as D C and *d* C.

In this case, from the points D and *d*, let fall on the axis the perpendiculars D I and *d i*, which when continued will cut the semicircle A E B in E and *e*; divide the arc E *e* into two equal parts in *f*, and draw *f h* perpendicular to A B, cutting the ellipsis in *g*: the line C*g* will divide the sector D C *d* into two equal parts.

PROBLEM LXVII.

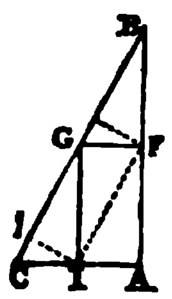
A carpenter has a triangular piece of timber; and, wishing to make the most of it, is desirous to know by what means he can cut from it the greatest right-angled quadrangular table possible. In what manner must he proceed?

Fig. 117.



Let the given triangular piece of timber be A B C (Fig. 117.) Divide the two sides A B, B C, into two equal parts, in F and G, and draw F G: then from the points F and G, draw F H and G I perpendicular to the base: the rectangle F I, will be the greatest possible that can be inscribed in the triangle, and will be exactly the half of it.

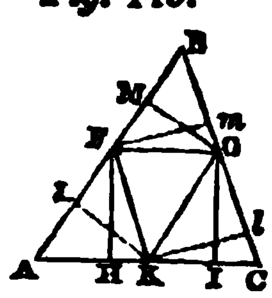
Fig. 118.



If the triangle be right-angled at A (Fig. 118.) the question may be solved in two different ways, by which there may be obtained the two rectangular tables I *i* and F I, which will each be the greatest inscriptible in the given triangle, and both equal.

When the triangle has all its angles acute, the solution will be different according to the side assumed as base. There will consequently be three, and each will give a table more or less elongated, and always of the same area, otherwise the greatest would exclusively solve the problem: such are the rectangles F I, G L, and K *m* (Fig. 119.)

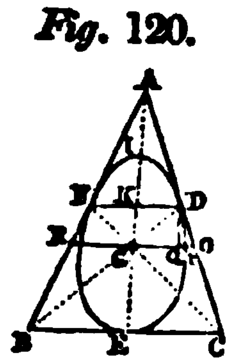
Fig. 119.



But the carpenter having consulted a geometrician, the latter observed that it would be most advantageous to convert this piece of timber into an oval table: and in what manner then must he proceed to trace out on it the greatest oval possible.

Let the given triangular piece of wood, as before, be A B C (Fig. 120.) First divide each side into two equal parts in F, D, and E; these three points will be the points of contact where the ellipsis touches the sides of the triangle; if the lines A E, C F, and B D be then drawn, intersecting each other in G, the point G will be the centre of the ellipsis.

Fig. 120.



Then make G L equal to G E, and through G draw Q O, parallel to B C, and through the point D draw D Q parallel to A E; then take G P, a mean proportional between G Q and G O: if the triangle B A C be isosceles, the lines G L and G P will be the semi-axis of the ellipsis; and we have already shewn in what manner an ellipsis may be described when the two axes are given.

But if the angle L G P be acute or obtuse, the ellipsis may be traced out at once by means of an instrument, described in Prob. xxii.; for it is of little importance whether the angle of the two given diameters be a right angle or not. This method will always be equally successful; with this only difference, that when the above angle is not a right angle, the portions of the ellipsis, described in the two adjacent angles, L G P and L G R, will not be equal and similar.

The two axes may be determined also directly: the method may be found in books on conic sections, and to these we must refer, as the nature of this work will not admit of entering deeply into the subject.

PROBLEM LXVIII.

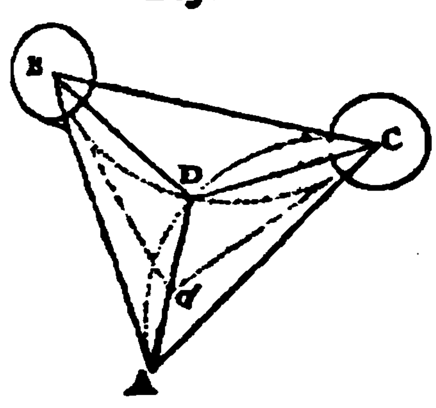
The points B *and* C (*Fig.* 121) *are the adjutors of two basons in a garden, and* A *is the point where a conduit is introduced, and to be divided into two parts, in order to supply* B *and* C *with water. Where must the point of separation be, that the sum of the three conduits,* A D, D B, *and* D C, *and consequently the expense in pipes, shall be the least possible?*

This problem, which belongs to that branch of civil engineering that relates to the conveyance of water, when reduced to geometrical language, may be enounced as follows: In a triangle A B C, to find a point, from which if three lines be drawn to the three angles, the sum of these lines shall be the least possible. Now it is

evident that there must be such a point, and that its position being found, the expense in pipes will be less than if the point of separation were assumed in any other place.

It would be tedious to explain the reasoning by means of which this problem is solved; and it would be impossible to employ calculation without great prolixity. We shall therefore only observe, that it may be demonstrated, that the required point D must be so situated, that the angles A D C, A D B, and B D C shall be equal to each other, and consequently each equal to 120°.

Fig. 121.



To construct this problem, on the side A C as a chord describe an arc of a circle A D C, capable of containing an angle of 120°, or equal to one third of the circle of which it forms a part; if the same thing be done on another of the sides, as B C, the intersection of these two circular arcs will determine the required point D; and it is from this point that the conduit must be divided, in order to be conveyed thence to B and to C.

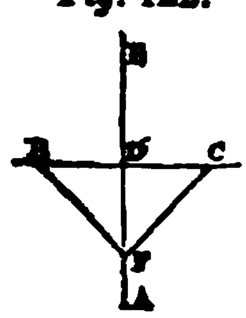
Such, at least, would be the solution of the problem if the three pipes A D, D C, and D B, were all to be of the same bore. But an intelligent engineer will not make the pipes equal in size; he will be sensible that to give greater height to the jet, it will be proper that the pipes D B and D C should not together admit a greater quantity of water than the pipe A D, otherwise the water in these pipes, after coming from the pipe D, would be in a state of stagnation, and would not receive the impulse necessary to make it rise to its greatest height.

The solution of the problem, in this new case, is as follows: We shall suppose that the bore of the pipe A D, or its capacity, is exactly double that of the other two; that is to say, that the diameters are in the ratio of 10 to 7 nearly; for by these means the water will always sustain an equal pressure in the former and in the two latter. We shall suppose also, that the price of the foot of each kind of these pipes is in the same ratio, because, in economical problems of this sort, it is the ratio of the prices that ought chiefly to be considered.

These things being premised, we shall find that the point of separation of the pipes ought to be in *d*, so situated, that the angles C *d* A and B *d* A shall be equal, and of such a nature, that the sine of each shall be to radius as 10 is to 14; or more generally, as the price of the foot of the larger pipe is to double that of the smaller. Hence it will be easy, according to this hypothesis, to determine the angle, which will be found to be 132° 56′ or near 133°.

If on the sides A B and A C then, of the triangle A B C, there be described two circular arcs, each containing an angle of 133°, their point of section will be in *d*, where the main pipe ought to be divided, to convey water to B and C, so as to incur the least possible expense in pipes.

Fig. 122.



Remark.—By extending this problem, we may suppose that the main pipe is to convey water to three given points, B, C, E, (Fig. 122.) In that case it may be demonstrated, that if the four pipes were equal, the point of separation could not be placed more advantageously, at least for diminishing the quantity of the pipes, than in the place where the lines A E and B C intersect each other; but this perhaps would not be the most advantageous disposition for making the water to be thrown up with the greatest force.

The same observation, made in regard to the first solution of the problem, may be made here also. To give greater force to the jet, the main pipe ought to be nearly triple in size to each of the rest. Let us suppose then that the price of a

foot of the former, is to that of a foot of the others, as m is to n; and in the last place, to simplify the problem, the solution of which would be otherwise exceedingly complex, we shall suppose that the lines A E and B C cut each other at right angles: this being the case, it will be found, that the angle E F C ought to be such, that, radius being unity, the cosine of it shall be $\frac{1}{2} n \sqrt{4nn - (m-1)}$, or, what amounts to the same thing, the sine of the angle D C F must be equal to the above expression.

If we suppose then, for example, that m is to n as 5 to 3, we shall have the above expression equal to 0·71496, which is the sine of an angle of 45° 38′. If the angle D C F therefore be made equal to from 45° to 46°, the point F will be that where the principal pipe ought to be divided.

If m were to n as 2 to 1, the above expression would become equal to 0·86600, which is the sine of an angle of 60°; in this case therefore the angle D C F ought to be made equal to 60°, or each of the angles D F C and D F B equal to 30°.

It is here evident that, to render the problem susceptible of a solution, m and n must be such, that the above expression shall not be imaginary, nor greater than unity. In either of these cases there could be no solution; and this would indicate, at most, that the division ought to be made at the point A, or at as great a distance as possible from the line B C. This expression also must not be $= 0$: in that case we ought to conclude that the division should be made at the point D.

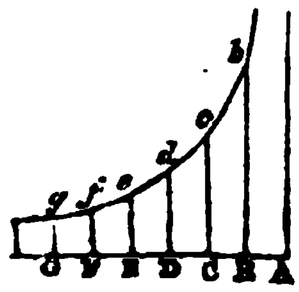
PROBLEM LXIX.

Geometrical paradox of lines which always approach each other, without ever being able to meet or to coincide.

Every person, in the least acquainted with geometry, knows, that if two straight lines, in the same plane, approach each other, they will necessarily meet in a common point of intersection. We say *in the same plane*, for if they were in different planes, it is evident that they might approach till a certain term, without cutting each other, and that they would then diverge from each other more and more. If we suppose, for example, two parallel and vertical planes, on one of which is drawn a horizontal line, and on the other one inclined to the horizon, it may be readily conceived that they will not be parallel, and yet they can never intersect each other, their least distance being necessarily that of the two planes. Here then we have two lines not parallel, which never meet; but this is not the sense in which the problem is understood.

It may be demonstrated that there are many lines, and in the same plane, which continually approach each other, and which however can never meet. They are indeed not straight lines, but a curve combined with a straight line, or two curved lines together. We shall here give a few examples of these lines, which are very familiar to those who are versed in the higher geometry.

Fig. 123.

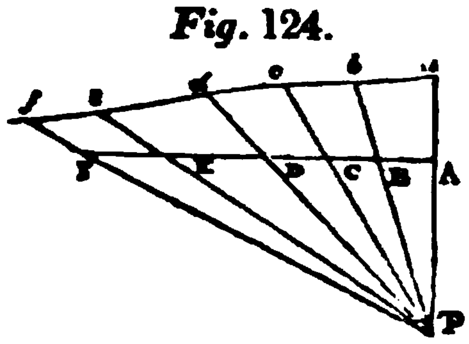


In the indefinite straight line A G, (Fig. 123.) take equal parts A B, B C, C D, &c.; and from the points B, C, D, &c., raise the perpendiculars B b, C c, D d, E e, &c., which decrease, according to a progression, no term of which can become 0, though it may become indefinitely small; let these terms decrease, for example, according to the progression 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c.: it is evident that the curve passing through the summits of the lines, decreasing according to this progression, can never meet the line G, however far continued, since its distance from that line can never become 0; it will however approach it more and more, and in such a manner, as to be nearer it than any quantity, however small. This curve, in the present case, is that so well known to geometricians under the name of the hyperbola; which has the property of being contained between the branches of two rectilineal angles, having their

vertices opposed to each other, towards which it approaches more and more, without ever touching them.

If the progression, according to which these lines B *b*, C *c*, D *d*, &c. decrease, were $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, &c., the line passing through the points *b*, *c*, *d*, *e*, &c., would still approach more and more to the straight line A G, without ever meeting it; for whatever might be the distance of any term of this progression, it could never become $= 0$.

Fig. 124.



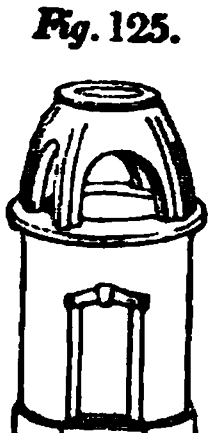
Another example.—Without the indefinite line A F, (Fig. 124.) assume any point P, from which draw P A perpendicular to A F, and any other lines at pleasure, P B, P C, P D, &c., more and more inclined; in the continuation of which make the lines A *a*, B *b*, C *c*, &c., always equal: it is evident that the line passing through the points *a*, *b*, *c*, *d*, &c., never can meet the line A F, though it may approach it more and more, and nearer than any determinate quantity; because F *f* becomes more and more inclined. This curve is that known to geometricians by the name of the *conchoid*, and was invented by Nicomedes, a Greek geometrician, to serve for the solution of the problem respecting two mean proportionals.

A great many other examples might be found in the higher geometry; but these will be sufficient for our purpose.

PROBLEM LXX.

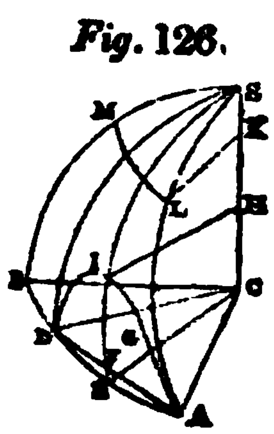
In the island of Delos, a temple consecrated to Geometry was erected, on a circular basis (*Fig.* 125.), *and covered by a hemispherical dome, having four windows in its circumference, with a circular aperture at the top, so combined, that the remainder of the hemispherical surface of the dome was equal to a rectilineal figure; and in the cylindric part of the temple was a door, absolutely squarable, or equal to a rectilineal space. What geometrical means did the architect employ in the construction of this monument?*

Fig. 125.



Every person, acquainted with the principles of geometry, knows that the measure of a hemispherical surface depends on that of the circle, which is equal to the surface of a cylinder having the same base and the same altitude. The ingenuity of this construction then was, 1st. To have cut from the dome, by the apertures above mentioned, spherical portions of such a nature, that the remainder should be equal to a figure purely rectilineal. 2d. To have described in the cylindric part, or circular wall of the temple, another figure which was squarable. The method that might have been employed is as follows:

Fig. 126.

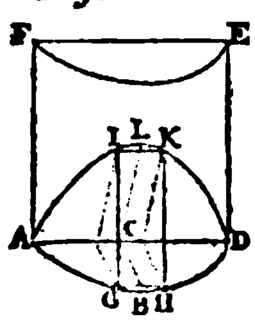


Let us first suppose a fourth part of the hemispherical dome, having for its base the quadrant A C B (Fig. 126.) Take the arc B D, equal to one fourth of the arc A B, as the breadth of the arc that ought to separate the windows; and draw A D the chord of the remainder. Now let S C E be any section whatever, through the axis of the dome S C, and let its intersection with A D be F; make C E, C F, C G, continually proportional; in the axis C S make the line C H equal to E G, and draw H I parallel to C E, which will intersect the quadrant S E in I: then will I be one of the points of the window required; and the series of points I, determined in this manner, will give the contour of that window, the surface of which will be equal to double the segment A E D, while the spherical portion S A I D S will be equal to double the rectilineal triangle C A D.

The whole surface of this fourth part of the dome will be equal then to double this triangle, plus the spherical sector S D B, which is equal to double the circular sector C D B, or to the fourth of the spherical sector S A E B; if from this sector, therefore, there be cut off the fourth part S L M, by a plane parallel to its base, and distant from the vertex S by the fourth part of the radius S C, the remainder of this hemispherical quadrant, that is to say the surface A I D B M L A, will be equal to double the rectilineal triangle C A D. If the other quadrants of the hemispherical dome be then made similar to the present one, the whole dome, the apertures deducted, will be equal to eight times the triangle A C D.

In regard to the aperture to be made in the circular wall of the temple, and which must be equal to a rectilineal space, nothing is easier, though it be a part of a cylindric surface. Let A B D E F (Fig. 127.) represent one half of this surface; assume, as the breadth of the door to be formed, the chord G H, parallel to the diameter A D; make G I and H K, which are perpendicular to the base, of such a size, that the door may have that proportion which good taste and the character of the work require; if through the points I, K, and the line A D, a plane be then made to pass, which by its intersection with the cylindric surface will determine the curve I L K, we shall have the cylindric aperture G B H K I, a little arched at the top, which will be to the rectangle of C B by G H, as the sine of the angle L C B is to the sine of half the right angle. The problem of the Greek geometrician therefore is solved.

Fig. 127.

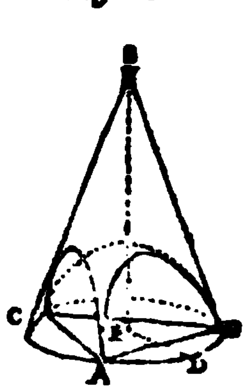


This problem might be varied a great many ways. During my dreary residence, in 1758, at a post in Canada, I amused myself with these variations, and I resolved the problem by making the whole of the surface of the temple absolutely squarable. I left only one aperture in the dome, viz. a hole at the top, like that of the Pantheon at Rome, and I made the four windows in the cylindric part of the temple, &c. All this however will be easy to any one versed in geometry.

Remarks.—1. This problem is nearly the same as that proposed by Viviani, in 1692, under the title of "Enigma Geometricum," which was easily solved by Leibnitz, Bernouilli, and the Marquis de l'Hôpital. An account of it may be seen in my "History of the Mathematics," vol. ii. book i. Viviani's solution is ingenious and elegant; but as the dome, according to this solution, would not be susceptible of construction, because it would bear upon four points, which in architecture is absurd, we have made some changes in the enunciation, by adding the circular aperture at the top. By these means the dome will bear upon parts that have some solidity, each window being separated from the other by an arc which forms a sixth part of the whole circumference.

2. Father Guido-Grandi has remarked, that if a polygon, for example the triangle A B C (Fig. 128.) be inscribed in the circular base of a cone, and if on each side of this polygon a plane be raised perpendicular to the base, the portion of the conical surface, cut off towards the axis, is equal to a rectilineal space. For it may be easily demonstrated that this surface is to that of the rectilineal polygon A B C, which corresponds to it perpendicularly below, as the surface of the cone is to the circle of its base; that is to say, as the inclined side of the cone S D, is to E D the radius of that base.

Fig. 128.



The portions also of the cone cut off by the above planes, towards the base, are evidently in the same ratio with the segments of the circle on which they rest. In fact, whatever figure be inscribed in the base, if we conceive a right cylindric surface

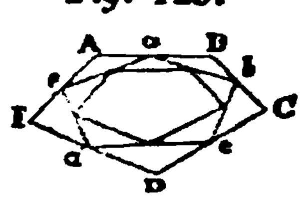
raised from the circumference of the figure, it will cut off from the conical surface a portion which will be to it in the same ratio.

This Italian geometrician, who was of the order of the Camaldules, thought proper to give to this conical portion absolutely squarable, the name of "Velum Camaldulense." In like manner, a Franciscan took it into his head to construct a sun-dial on a body which resembled a sandal, and to print a description of it, under the title of "Sandalion Gnomonicum."

PROBLEM LXXI.

If each of the sides of any irregular polygon whatever, as A B C D E A (Fig. 129.) *be divided into two equal parts, as in* a, b, c, d, e ; *and if the points of division in the contiguous sides be joined; the result will be a new polygon* a b c d e a : *if the same operation be performed on this polygon; then on the one resulting from it; and so on ad infinitum; it is required to find the point where these divisions will terminate.*

Fig. 129.



This problem, impossible to be resolved perhaps by considerations purely geometrical, is susceptible of a very simple solution, deduced from another consideration, and which shall be given in a subsequent page.

In the mean time our readers may exercise their ingenuity upon it, as we shall only add, that it was proposed in 1750, by M. D——, who said he had it from M. Buffon.

A COLLECTION

OF VARIOUS PROBLEMS, BOTH ARITHMETICAL AND GEOMETRICAL; THE SOLUTION OF WHICH IS PROPOSED BY WAY OF EXERCISE TO MATHEMATICAL READERS.

THOSE who study mathematics cannot begin too early to exercise their talents with the solution of the problems presented by that science; for it is by such exercise that the inventive faculty is called forth and strengthened. We have therefore thought it our duty to subjoin to this part of the Mathematical Recreations, a selection of problems proper for exercising and amusing young mathematicians. They are of different degrees of difficulty, that they may be suited to the different capacities of those who read this work. Some curious theorems have been inserted among them; and, as the demonstration of these is required, they may serve also to exercise their ingenuity.

It may be here observed, that, as the most of these problems are far from being difficult if the resources of algebraic calculation be employed, it is therefore proposed that the solutions of them should be found by means of pure geometry; as it is well known that algebraic analysis gives, for the most part, complex solutions, while those which arise from analysis purely geometrical are far more simple and elegant.

ARITHMETICAL AND GEOMETRICAL PROBLEMS AND THEOREMS.

PROBLEM I.—In a right-angled triangle, given the base, the sum or difference of the other two sides, and the area, to determine the triangle?

PROB. II.—Given the base, the ratio of the other two sides, and the area, to determine the triangle.

PROB. III.—The base, the angle comprehended by the two other sides, and the area being given, to determine the triangle.

PROB. IV.—Three lines being given in position, on a plane, to draw another line through them, which shall be cut by them into two parts, in a given ratio.

PROB. V.—Four lines being given in position on a plane, to draw another line through them, which shall be cut into three parts, in a given ratio.

PROB. VI.—What is the probability of throwing an ace, or any one of the faces of a die, in three throws; that is, either at the first, second, or third throw?

PROB. VII.—At the game of Piquet, A is first in hand, and has no ace; what probability is there that he will take in from the pack either one, or two, or three, or four aces.

PROB. VIII.—What is the probability of throwing one ace, and no more, in four successive throws?

PROB. IX.—In a lottery, where the number of blanks is to that of the prizes as 39 to 1, as was the case in the year 1720, how many tickets must be purchased that the buyer may have an equal chance for one or more prizes?

PROB. X.—If a man has in his hand a certain number of pieces of money, as for example 12, how much may be betted to 1 that in tossing them all up at once, or separately, there shall be as many heads as tails.

PROB. XI.—Four lines being given of such a nature, that any three of them are together greater than the fourth, to construct of them a quadrilateral figure inscriptible in a circle, or which can be circumscribed about it?

Theorem 1.—If from the three angles of any right-angled triangle, three lines be drawn perpendicular to the opposite sides, they will all cut each other in the same point.

Theor. 2.—If lines be drawn from these angles, dividing each of them into two equal parts, or cutting the opposite sides into two equal parts, these three lines will all pass through the same point.

PROB. XII.—A trapezium being given, to divide it into two equal parts, or in any given ratio, by a line passing through a given point, either in one of the sides, or within the trapezium, or without it.

PROB. XIII.—In a given circle to inscribe an isosceles triangle of a given magnitude.—It is evident that this triangle must be less than the equilateral triangle inscribed in the given circle; for the latter is the greatest of all those that can be inscribed in it.

PROB. XIV.—To circumscribe about a given circle an isosceles triangle of a given magnitude.—This triangle must be greater than the circumscribed equilateral triangle; since the latter is the least of all those that can be circumscribed.

PROB. XV.—In an isosceles triangle to describe three circles, each of which shall touch two sides of the triangle, and which all three shall touch each other.

PROB. XVI.—To do the same thing in a scalene triangle.

PROB. XVII.—What is the value of this analytical expression, $\sqrt{2\sqrt{2\;\;2}}$, &c., in infinitum?—The answer is 2; but a demonstration is required. In like manner the value of $\sqrt{3\sqrt{3\;\;3}}$, &c., in infinitum, is 3; and so of any other number.

PROB. XVIII.—In a pyramid, of four triangular faces, if the sides of these four triangles be given; required the angles formed by the faces of this pyramid, the perpendicular let fall from any of the angles on the base, and the solidity of the pyramid.

PROB. XIX.—To cut a given trapezium into four equal parts, by lines intersecting each other at right angles.

PROB. XX.—A gentleman has an irregular quadrangular piece of ground, from which he is desirous, for the purpose of making a parterre, to cut the largest oblong possible, with its angles touching the sides of the quadrilateral: how is this to be done?

PROB. XXI.—Given the area of a right-angled triangle, and the sum of the three sides, to determine the triangle.

PROB. XXII.—If from a pack, consisting of 52 cards, 13 of each suit, 5 cards be dealt to one person; what is the chance that two of them shall be trumps, or of any suit that is proposed.

PROB. XXIII.—About a given circle to circumscribe a triangle, of a given perimeter; provided this perimeter be greater than that of the equilateral triangle circumscribed.

PROB. XXIV.—In a triangle, not equilateral, to find a point which, if three perpendiculars be drawn to the three sides, they shall be together equal to a given line. —We have excluded the equilateral triangle, because it may be easily demonstrated, that from whatever point, within such a triangle, perpendiculars are let fall on the sides, their sum will be always the same.

The case is the same in regard to every regular polygon; and even those that are irregular, provided the sides are equal.

PROB. XXV.—In a given circle to inscribe an isosceles triangle, or to circumscribe about it a triangle of a given perimeter.—This problem not being always possible, as may be easily seen, it is required to assign its limitations.

PROB. XXVI.—In a given circle to inscribe, or to circumscribe about it, any triangle whatever, of a determinate perimeter.

PROB. XXVII.—In a given quadrilateral to inscribe an ellipsis; that is to say, to describe in it an ellipsis which shall touch its four sides.

PROB. XXVIII.—A jeweller has a valuable plate of agate, in the form of an irregular trapezium, and is desirous to cut from it the largest oval possible for the lid of a snuff-box: in what manner must he proceed?—It is evident that this problem expressed geometrically is as follows: In a given quadrilateral to inscribe the largest ellipse possible: a problem which is certainly not easy. It is proper to inform those who may be disposed to try it, that it requires a profound knowledge of analysis. The following also might be proposed:

About a given quadrilateral to circumscribe the least ellipsis possible.

PROB. XXIX.—A point and a straight line being given, in what line will be found the centres of all the circles passing through the given point, and touching the given line.

PROB. XXX.—Required the same thing in regard to all the circles that touch a given circle and a given straight line.—This straight line may be without the given circle; or it may touch it, or intersect it.

PROB. XXXI.—Any two circles being given, in what line will be found the centres of all the circles that touch the given circles: whether the touching circle comprehends them both within it, or touches the one without and the other within?

PROB. XXXII.—The base of a triangle, the sum of the two other sides, and the line drawn from the vertex to the middle of the base, being given; to determine the triangle.

PROB. XXXIII.—Given the three lines, drawn from the angles of a triangle to the middle of each of the opposite sides; to determine the triangle.

PROB. XXXIV.—Given the base of a triangle, and the sum and the difference of the squares of the sides, to determine the triangle—This problem is susceptible of a very simple and very elegant construction; for the vertex of this triangle is in the circumference of a certain circle, and is also in a certain straight line.

PROB. XXXV.—Given the three lines drawn from the angles of a triangle to the opposite sides, dividing each of these angles into two equal parts; to determine the triangle.

PROB. XXXVI.—Any number of points being given, to draw a straight line among them in such a manner, that if a perpendicular be let fall on it from each of these

points, the sum of the perpendiculars on the one side, shall be equal to the sum of those on the other side.

PROB. XXXVII.—The same supposition being made, it is required that the sum of the squares of the perpendiculars drawn on the one side, shall be equal to the sum of the squares of those on the other; or that the sum of these perpendiculars, raised to any power whatever n, shall be on both sides equal.

PROB. XXXVIII.—In any trapezium, given the four sides and the area, to determine the trapezium.

PROB. XXXIX.—An angle being given, to find a point from which if two perpendiculars be let fall on its sides, the quadrilateral formed by them and the sides of the angle, shall be equal to a given square.

PROB. XL.—As there are an infinite number of points which will answer the problem, it is proposed to find the line traced out by them, or the curve which they form.

PROB. XLI.—To find four numbers in arithmetical progression, to which if four given numbers, such as 2, 4, 8, 17, be added, their sums shall be in geometrical progression.

PROB. XLII.—Two couriers, A and B, set out at the same time; A from Paris for Orleans, the distance between which is 60 miles, and B from Orleans for Paris; and they travel at such a rate that A reaches Orleans 4 hours after meeting B, and B reaches Paris 6 hours after meeting A: how many miles per hour did each travel?

PROB. XLIII.—A certain sum, placed out at interest, amounted at the end of a year to £1100, and at the end of eighteen months to £1120; what was the sum, and at what rate of interest was it lent?

PROB. XLIV.—Two bills of exchange, one of £1200, payable in 6 months, and the other of £2000, payable in 9 months, were discounted at the same time, and at the same rate of interest, for £120, at what rate of interest were they discounted?

PROB. XLV.—How many ways can £100. be paid by guineas, at 21 shillings, and pistoles at 17 shillings, each?

PROB. XLVI.—An angle and a point within it being given, to draw through that point a straight line intersecting the two sides of the angle, in such a manner, that the rectangle of their segments towards the vertex, shall be equal to a given square. This given square must not be less than a certain square, which gives rise to the following problem.

PROB. XLVII.—The same supposition being made as in the preceding case, required the position of the line passing through the given point, when the rectangle of the sides of the angle cut off towards the vertex is the least possible.

PROB. XLVIII.—Three lines being given in position, to find a point from which the three perpendiculars drawn to these lines shall be in a given ratio.——We shall here observe that this problem is susceptible of a very simple and very elegant solution, without calculation.

PROB. XLIX.—Given two circles in a given ratio, as of 1 to 2, for example, and which cut each other, but in such a manner as not to form a squarable lunule; it is proposed to draw through these circles a line parallel to that which joins the points of intersection, so that the part of the lunule cut off above may be equal to a rectilineal space.

PROB. L.—The same supposition being made, it is proposed to cut the two circular arcs by a third, which shall be of such a nature that the concavo-convex triangle, formed by these three arcs, shall be equal to a rectilineal space, if possible.

PROB. LI.—Three persons have together £100; and it is known that 9 times the money of the first, plus 15 times that of the second, plus 20 times that of the third, is equal to £1500. How much money has each?—It may be here proper to observe, that this problem, as well as the 45th, 52d, 57th, and 58th, is susceptible of several solutions; and to solve them completely it will be necessary to find all the different answers, and to shew that there can be no more; for by repeated trials it would not be difficult to find some of them.

PROB. LII.—A farmer bought 100 calves, sheep, and pigs, for the sum of £100., at the rate of £3. 10*s*. for the calves, £1. 6*s*. 8*d*. for the sheep, and 10*s*. for the pigs: How many of each kind did he purchase?

PROB. LIII.—Three merchants enter into partnership, and agree to advance each £10000. towards a certain adventure; two of them paid down the money, but the third advanced only the half of his share, that is £5000; the adventure having failed, they lost not only their capital but 50 per cent. more: What must each contribute to make good the loss?

PROB. LIV.—In a rectilineal triangle, given the base, the rectangle of the other two sides, and the included angle, to determine and construct the triangle.

PROB. LV.—An arc of a circle being given, to divide it into two parts, the sines of which shall be in a given ratio.

PROB. LVI.—If a person draws 4 cards from a pack, containing 32, what probability is there, or how much may be betted to 1, that among these four cards there will be one of each colour?

PROB. LVII.—It is required to divide 24 into three such parts, that if the first be multiplied by 36, the second by 24, and the third by 8, the sum of those products may be 516?

PROB. LVIII.—How many ways may four sorts of wine, the prices of which are 16d. 10d., 8d. and 6d. per quart, be mixed, so as to make 100 quarts in all, worth 12d. per quart?

PROB. LIX.—To find a number of such a nature, that if 12 and 25 be successively added to it, the sums shall be square numbers.

PROB. LX.—To find three numbers, the squares of which shall be in arithmetical progression.

PROB. LXI.—Any number of points being given, to find another, from which if straight lines be drawn to all the rest, the sum of these lines shall be equal to a given line.

PROB. LXII.—The same supposition being made as before, the sum of the squares of the lines drawn from the required point must be equal to a given square.

It is very singular that the last problem is susceptible of a construction much easier than the preceding. We shall here observe, merely for the purpose of exciting the curiosity of the geometrical reader, that in the latter the required point, and all those that solve the problem, for there are a great many which do so, are situated in the circumference of a certain circle; and it is very remarkable that the centre of this circle is the centre of gravity of the given points, supposing each of them to be charged with the same weight.

It may be observed also, that if it were required that the square of one of the lines drawn, plus the double of the second, plus the triple of the third, &c., should make the same sum, it would be necessary to suppose the first point loaded with a single weight, the second with a double weight, the third with a triple one, &c., and their centre of gravity would still be the centre of the required circle.

The solution of this problem was not unknown to the ancient geometricians. It was one of those of the *Loca plana* of Apollonius; and this may serve to give us a more favourable idea of their analysis than is generally entertained.

COLLECTION OF USEFUL TABLES.

TABLE

OF THE LENGTH OF THE FOOT, OR OTHER LONGITUDINAL MEASURE USED IN ITS STEAD, AMONG THE DIFFERENT NATIONS AND IN THE PRINCIPAL CITIES OF EUROPE.

Having frequently experienced great embarrassment, while engaged in certain researches, from not being able to obtain accurate information respecting the measures of different countries, whenever an opportunity occurred we collected with great care the proportions of these foreign measures, both ancient and modern, as compared with our own, and it is hoped our readers will consider themselves indebted to us for the following table on this subject, which there is reason to think is the fullest and most complete ever given. All the different measures are compared with the English foot, which is here supposed to be divided into 12 inches, each inch into 12 lines, and each line into 10 parts: which makes the foot to consist of 1440 of these parts. The first column in the table shews the number of these parts which each measure contains; and the second the value of it in English feet, inches, lines, and tenths of a line.

ANCIENT FEET.

	parts.	ft.	in.	lin.	pts.
Ancient Roman foot	1392	0	11	7	2
Greek and Ptolemaic	1453	1	0	1	3
Greek Phyleterian	1681	1	2	0	1
Foot of Archimedes, or probably of Sicily and Syracuse	1051	0	8	9	1
Drusian	1570	1	1	1	0
Macedonian	1670	1	1	11	0
Egyptian	2046	1	5	0	6
Hebrew	1745	1	2	6	5
The natural (*hominis vestigium*)	1172	0	9	9	2
Arabian	1577	1	1	1	7
Babylonian	1648	1	1	8	8
———	1635	1	1	7	5

MODERN FEET.

	parts.	ft.	in.	lin.	pts.
English	1440	1	0	0	0
Altorf	1116	0	9	3	6
Amsterdam	1335	0	11	1	5
Ancona and the Ecclesiastical States	1846	1	3	4	6
Antwerp	1353	0	11	3	3
Aquileia	1624	1	1	6	4
Arles	1279	0	10	7	9
Augsburg	1399	0	11	7	9
Avignon	1279	0	10	7	9
Barcelona	1428	0	11	10	8

	parts.	ft.	in.	lin.	pts.
Basle	1360	0	11	4	0
Bergamo	2060	1	5	2	0
Berlin	1428	0	11	10	8
Besançon	1462	1	0	2	2
Bologna	1792	1	2	11	2
Bourg en Bresse and Bugey	1483	1	0	4	3
Bremen	1375	0	11	5	5
Brescia	2247	1	6	8	7
Breslaw	1620	1	1	6	0
Bruges	1079	0	8	11	9
Brussels	1299	0	10	9	9
Chambery and Savoy	1594	1	1	3	4
China—Tribunal of mathematics	1623	1	1	6	3
—— Imperial foot	1513	0	1	7	3
Cologne	1300	0	10	10	0
Constantinople	3161	2	2	4	1
————	1678	1	1	11	8
Copenhagen	1511	1	0	7	1
Cracow	1684	1	2	0	4
Dantzic	1329	0	11	0	9
Delft	787	0	6	6	7
Denmark	1508	1	0	6	8
Dijon	1483	1	0	4	3
Dordrecht	1110	0	9	3	0
Ferrara	1896	1	3	9	6
Florence	1433	0	11	11	3
Franche-Comté	1687	1	2	0	7
Frankfort on the Main	1343	0	11	2	3
Genoa (the palm)	1170	0	9	9	0
Geneva	2763	1	11	0	3
Grenoble and Dauphigny	1611	1	1	5	1
Haerlem	1350	0	11	3	0
Halle in Saxony	1407	0	11	8	7
Hamburgh	1343	0	11	2	3
Heidelberg (Palatinate)	1300	0	10	10	0
Inspruck	1566	1	1	2	6
Leghorn	1428	0	11	10	8
Leipsic	1489	1	0	4	9
Leyden	1473	1	0	3	3
Liege	1360	0	11	4	0
Lisbon	1371	0	11	5	1
Lombardy, foot of Luitprand or Aliprand	2053	1	5	1	3
Lorraine	1377	0	11	5	7
Lubec	1343	0	11	2	3
Lucca	2787	1	11	2	7
Lyons and the Lyonnese, Fores and Baujalois	1611	1	1	5	1
Madrid	1318	0	10	11	8
Maestricht	1319	0	10	11	9
Malta (the palm)	1318	0	10	11	8
Mantua (the brasso)	2190	1	6	3	0
Marseilles	1172	0	9	9	2

	parts.	ft.	lin.	in.	pts.
Mechlin	1084	0	9	0	4
Mentz	1423	0	11	10	3
Milan—Decimal foot	1231	0	10	3	1
—— —Aliprand ditto	2053	1	5	1	3
Modena	2997	2	0	1	7
Monaco	1110	0	9	3	0
Montpellier (the pan)	1119	0	9	3	9
Moscow	1337	0	11	1	7
Munich	1364	0	11	4	4
Naples (the palm)	1240	0	10	4	0
Netherlands, see Maestricht.					
Nuremberg—Town foot	1434	0	11	11	4
————Country foot	1306	0	10	10	6
Padua	2024	1	5	8	4
Palermo	1076	0	8	11	6
Paris—foot	1535	1	0	9	5
—— metre	4731	3	3	5	1
Parma	2692	1	10	5	2
Pavia	2217	1	6	5	7
Prague	1424	0	11	10	4
Provence, see Marseilles.					
Rhinlandish foot	1473	1	0	3	3
Riga	1343	0	11	2	3
Rome (the palm)	1055	0	8	9	5
Rouen, as at Paris	1535	1	0	9	5
Savoy, see Chambery.					
Seville in Andalusia	1428	0	11	10	8
Sienna, common foot	1784	1	2	10	4
Stettin in Pomerania	1763	1	2	8	3
Stockholm	1545	1	0	10	5
Strasburgh—Town foot	1377	0	11	5	7
————Country ditto	1395	0	11	7	5
Toledo	1318	0	10	11	8
Turin (Piedmont)	2414	1	8	1	4
Trent	1729	1	2	4	9
Valladolid	1307	0	10	10	7
Venice	1638	1	1	7	8
Verona	1609	1	1	4	9
Vicenza	1636	1	1	7	6
Vienna	1492	1	0	5	2
Vienne in Dauphigny	1524	1	0	8	4
Ulm	1190	0	9	11	0
Urbino	1673	1	1	11	3
Utrecht	1067	0	8	10	7
Warsaw	1684	1	2	0	4
Wesel	1110	0	9	3	0
Zurich	1410	0	11	9	0

TABLE

OF SOME OTHER MEASURES, BOTH ANCIENT AND MODERN, COMPARED WITH THE ENGLISH STANDARD.

The ancient cubit in general was a foot and a half. The Hebrews however had three cubits.

1st. The common cubit, which was a foot and a half Hebrew measure, or 2617 of those parts of which the English foot contains 1440.

2d. The sacred and modern cubit, which was one Babylonic foot and three quarters, or 2883 or 2861 parts of the English foot.

3d. The great geometric cubit, which was 9 Hebrew feet, or 6 lesser cubits.

	Grecian feet.
The orgya of the Greeks was	6
The arura	50
The plethron	100
The diplethron	200

	Roman feet.
The hexapeda of the Romans was	6
The decempeda	10

MEASURES OF PARIS.

	French feet.	English feet.
Toise of Paris	6	6·3959
Metre, or new measure	$3\frac{12}{153}$	3·2854
The royal perch	22	23·4515
The mean perch	20	21·3195
The lesser perch used at Paris	18	19·1876

The acre is 100 square perches.

The are is 100 square metres.

MEASURES OF CAPACITY FOR LIQUIDS.

The Muid for liquids (Paris measure) contains 8 French cubic feet, or 16744·7071 English cubic inches.

Six French cubic inches make a poinçon, or by corruption poisson, = 7·2677 English cubic inches.

		Eng. cub. inches.
2 poissons make	1 demi-setier	14·5353
2 demi-setiers	1 chopine	29·0707
2 chopines	1 pinte	58·1413
2 pintes	1 quarte	116·2827
4 quartes	1 grand setier	465·1308
36 grand setiers	1 muid	16744·7071

Litre ……… a cubical decimeter = $1\frac{1}{20}$ pinte.

A muid therefore is equal to 72·4871 English wine gallons, or about $1\frac{1}{7}$ hogshead.

FRENCH DRY MEASURES.

The litron contains 36 French cubic inches, or 43·606 English cubic inches.

		Eng. cub. inches.
16 litrons make	1 boisseau	697·696
3 boisseaux	1 minot	2093·088
2 minots	1 mine	4186·176
2 mines	1 setier	8372·352
12 setiers	1 Paris muid	100468·224

Hence the French muid for things dry is equal to 46·72 English bushels, or 5 quarters 6 bushels 2·88 pecks.

The following tables of ancient measures, have been added from Arbuthnot.

ROMAN MEASURES OF LENGTH.

Digitus transversus	0·72525	Eng. in.	Cubitus	1·4505	Eng. ft.
Uncia, the ounce	0·967	—	Gradus	2·4175	—
Palmus minor	2·901	—	Passus	0·967	paces.
Pes, the foot	11·604	—	Stadium	120·875	—
Palmipes	1·20875	Eng. ft.	Milliare	967·0	—

SCRIPTURE MEASURES OF LENGTH.

Digit	0·7425	inches.	Arabian pole	4·62	yards.
Palm	2·97	—	Schænus	46·2	—
Span	8·91	—	Stadium	231·0	—
Lesser cubit	1·485	Eng ft.	Sabbath day's journey	1155·0	—
Greater cubit	1·7325	—	Eastern mile	1·886	miles.
Fathom	2·31	yards.	Parasang	4·158	—
Ezekiel's reed	3·465	—	Day's journey	33·264	—

GRECIAN MEASURES OF LENGTH.

Dactylos	0·75546	inches.	Pygme*	1·13203	Eng. ft.
Doron / Dochme	3·02187	—	Pygon	1·25911	—
Dichas	7·55468	—	Pechys	1·51093	—
Orthodoron	8·31015	—	Orgya	1·00729	—
Spithame	9·06562	Eng. in.	Stadios / Dulos	100·72916	paces.
Pous	12·0875	—	Milion	805·8333	—
Pous	1·00729	Eng. ft.			

ROMAN DRY MEASURES.

Hemina	0·5074	Eng. pints.	Modius	1·0141	Eng. peck.
Sextarius	1·0148	—			

ATTIC DRY MEASURES.

Xestes	0·9903	Eng. pints.	Medimnus	1·0906	Eng. bush.
Chenix	1·486	—			

JEWISH DRY MEASURES, ACCORDING TO JOSEPHUS.

Gachal	0·1949	Eng. pints.	Ephah	1·0961	Eng. bush.
Cab	3·874	—	Latech	5·4807	—
Gomer	7·0152	—	Coron / Chomer	1·3702	Eng. qr.
Seah	1·4615	Eng. peck.			

ROMAN MEASURES FOR LIQUIDS.

Hemina	0·59759	Eng. pints.	Urna	3·5857	Eng. gals.
Sextarius	1·19518	—	Amphora	7·1712	—
Congius	7·1712	—	Culeus	2·2766	Eng. hogs.

ATTIC MEASURES FOR LIQUIDS.

Cotyle	0·5742	Eng. pints.	Chous	6·8900	Eng. pints.
Xestes	1·1483	—	Meteotes	10·3350	Eng. gall.

* From this measure is derived the English word pigmy.

JEWISH MEASURES FOR LIQUIDS.

Caph	0·8612 Eng. pints.	Seah	3·4450 Eng gall.
Log	1·1483 —	Bath	10·3350 —
Cab	4·5933 —	Coron..........	1·6405 Eng hogs.
Hin	1·7225 Eng. gall.		

FRENCH MEASURES.

The Paris foot is to the English foot, as 1 to 1·065977
The Paris square foot is to the English, as 1 to 1·136307
The Paris cube foot is to the English, as 1 to 1·211277

The French wine pint contains 58·1413 English cubical inches; and the English wine pint contains 28·875 cubical inches.

NEW FRENCH MEASURES.

The new French measures were established by a decree of the national convention, on the 7th of April, 1795. The elementary measure on which they are founded, is a decimal part of the distance from the pole to the equator; that is to say, a decimal part of a quarter of the terrestrial meridian: for the *metre*, which is the element of all the rest, is the ten millionth part of that distance, and is equal, in the old French measures, to 36 inches and 11·296 lines. A metre in length, is the element of all the lineal measures; a square metre is the element of all the superficial measures; and a cubic metre is the element of all the measures of capacity.

MEASURES OF LENGTH.

	Eng. inches.
Millimetre	·03937
Centimetre	·39378
Decimetre	3·93786
Metre	39·37860
Decametre	393·78605
Hectometre	3937·86059
Chiliometre	39378·60599
Myriometre	393786·05997

	Miles.	fur.	yds.	ft.	inches.
A Metre is				3	3·37
A Decametre			10	2	9·78
A Hectometre			109	1	1·86
A Chiliometre		4	213	2	6·60
A Myriometre	6	1	158	1	6·05

The distance from the pole to the equator, or fourth part of the terrestrial meridian, according to the late French measurement, is 32815504 English feet.

Centesimal degree = 328155·04 English feet.

MEASURES OF CAPACITY.

	Eng. cub. inches.
Millilitre	·06106
Centilitre	·61063
Decilitre	6·10634
Litre	61·06345
Decalitre	610 63450
Hectolitre	6106·34504
Chiliolitre (cubic metre)	61063·45042
Myriolitre	610634·50427

A litre is 2·114, or nearly 2¼ English wine pints.
A Hectolitre is 2·6434 wine gallons, or 2 gallons 2 quarts 1·14 pint.
A Chiliolitre is 4 hogsheads, 12 gallons, 1·36 quart; or 1 tun 12·34 gallons.
A Myriolitre is 10 tuns, 1 hogshead, 60·4 gallons; or nearly 10½ tuns.

SQUARE OR SUPERFICIAL MEASURES.

	Eng. square feet.
Square millimetre	·01076
Square centimetre	·10768
Square decimetre	1·07685
Centiare (square metre)	10·76856
Deciare	107·68564
Are	1076·85645
Decare	10768·56454
Hectare	107685·64540
Chiliare	1076856·45407
Myriare	10768564·54070

A Hectare is 2·472 English statute acres; or 2 acres, 1 rood, 35·5 poles.

MEASURES FOR FIRE-WOOD.

	Eng. cubic. feet.
Decistere	3·533764
Stere (cubic metre)	35·337645

PART THIRD,

CONTAINING VARIOUS PROBLEMS IN MECHANICS.

After Arithmetic and Geometry, Mechanics is the next of the physico-mathematical sciences, having their certainty resting on the simplest foundations. It is a science also, the principles of which, when combined with geometry, are the most fertile and of the most general use in the other parts of the mixed mathematics. All those mathematicians therefore who have traced out the development of mathematical knowledge, place mechanics immediately after the pure mathematics, and this method we shall here adopt also.

We suppose, as in every other part of the mathematics introduced into this work, that the reader is acquainted with the first principles of the science of which we treat. Thus, in regard to mechanics, we suppose him acquainted with the principles of equilibrium and of hydrostatics; with the chief laws of motion, &c. For it is not our intention to teach these principles; but only to present a few of the most curious and remarkable problems which arise from them.

PROBLEM I.

To cause a ball to proceed in a retrograde direction, though it meets with no apparent obstacle.

Place an ivory ball on a billiard table, and give it a stroke on the side or back part, with the edge of the open hand, in a direction perpendicular to the table, or downward. It will then be seen to proceed a few inches forward, or towards the side where the blow ought to carry it; after which it will roll in a retrograde direction, as it were of itself, and without having met with any obstacle.

Remark.—This effect is not contrary to the well-known principle in mechanics, that a body once put in motion, in any direction, will continue to move in that direction until some foreign cause oppose and prevent or turn it. For, in the present case, the blow given to the ball, communicates to it two kinds of motion; one of rotation about its own centre, and the other direct, by which its centre moves parallel to the table, as impelled by the blow. The latter motion, on account of the friction of the ball on the table, is soon annihilated; but the rotary motion about the centre continues, and when the former has ceased, the latter makes the ball roll in the retrograde direction. In this effect, therefore, there is nothing contrary to the well known laws of mechanics.

PROBLEM II.

To make a false ball, for playing at nine pins.

Make a hole in a common ball used for playing at the above game; but in such a manner as not to proceed entirely to the centre; then put some lead into it, and close it with a piece of wood, so that the joining may not be easily perceived. When this ball is rolled towards the pins, it will not fail to turn aside from the proper direction, unless thrown by chance or dexterity in such a manner, that the lead shall turn exactly at the top and bottom while the ball is rolling.

Remark.—The fault of all balls used for billiards depends on this principle. For, as they are all made of ivory, and as, in every mass of that substance, there are always some parts more solid than others, there is not a single ball perhaps which has the centre of gravity exactly in the centre of the figure. On this account every ball deviates more or less from the line in which it is impelled, when a slight motion is communicated to it, in order to make it proceed towards the other side of the billiard table, unless the heaviest part be placed at the top or bottom. We have heard an eminent maker of these balls declare, that he would give two guineas for a ball that should be uniform throughout; but that he had never been able to find one perfectly free from the above-mentioned fault.

Hence it happens, that when a player strikes the ball gently, he often imagines that he has struck it unskilfully, or played badly: while his want of success is entirely the consequence of a fault in the ball. A good billiard player, before he engages to play for a large sum, ought carefully to try the ball, in order to discover the heaviest and lightest parts. This precaution was communicated to us by a first-rate player.

PROBLEM III.

How to construct a balance, which shall appear just when not loaded, as well as when loaded with unequal weights.

We certainly do not here intend to teach people how to commit a fraud, which ought always to be condemned; but merely to shew that they should be on their guard against false balances, which often appear to be exact; and that in purchasing valuable articles, if they are not well acquainted with the vendor, it is necessary to examine the balance, and to subject it to trial. It is possible indeed to make one, which when unloaded shall be in perfect equilibrium, but which shall nevertheless be false. The method is as follows:

Let A and B be the two scales of a balance, and let A be heavier than B: if the arms of the balance be made of unequal lengths, in the same ratio as the weights of the two scales, and if the heavier scale A be suspended from the shorter arm, and the lighter scale B from the longer, these scales when empty will be in equilibrium. They will be in equilibrium also when they contain weights which are to each other in the same ratio as the scales. A person therefore unacquainted with this artifice will imagine the weights to be equal; and by these means may be imposed on.

Thus, for example, if one of the scales weighs 15, and the other 16; and if the arms of the balance from which they are suspended be, the one 16 and the other 15 inches in length; the scales when empty will be in equilibrium, and they will remain so when loaded with weights which are to each other in the ratio of 15 to 16, the heaviest being put into the heaviest scale. It will even be difficult to observe this inequality in the arms of the balance. Every time therefore that goods are weighed with such a balance, by putting the weight into the heavier scale and the merchandise into the other, the purchaser would be cheated of a sixteenth part, or an ounce in every pound.

But, this deception may be easily detected by transposing the weights; for if they are not then in equilibrium, it is a proof that the balance is not just.

And indeed in this way the true weight of any thing may be discovered, even by such a false balance, namely, by first weighing the thing in the one scale, and then in the other scale; for a mean proportional between the two weights will be the true quantity; that is, multiply the numbers of these two weights together, and take the square root of the product. Thus, if the thing weigh 16 ounces in the one scale, and only 14 in the other: then the product of 16 multiplied by 14 is 224, the square root of which gives 14[illegible] for the true weight, or nearly 15 ounces. Or indeed the just weight is found nearly by barely adding the two numbers together, and dividing

the sum by 2. Thus 16 and 14 make 30, the half of which, or 15, is the true weight very nearly.

PROBLEM IV.

To find the centre of gravity of several weights.

As the solution of various problems in mechanics depends on a knowledge of the nature and place of the centre of gravity, we shall here explain the principles of its theory.

The centre of gravity of a body is that point around which all its parts are balanced, in such a manner, that if it were suspended by that point, the body would remain at rest in every position in which it might be placed around that point.

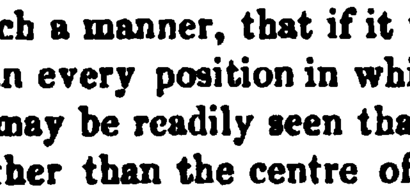
It may be readily seen that, in regular and homogeneous bodies, this point can be no other than the centre of magnitude of the figure. Thus, the centre of gravity in the globe and spheroid, is the centre of these bodies; in the cylinder it is in the middle of the axis.

The centre of gravity between two weights, or bodies of different gravities, is found by dividing the distance between their points of suspension into two parts, which shall be inversely proportional to the weights; so that the shorter part shall be next to the heavier body, and the longer part towards the lighter. This is the principle of balances with unequal arms, by means of which any bodies of different weights may be weighed with the same weight, as in the steel-yard.

When there are several bodies, the centre of gravity of two of them must be found by the above rule: these two are then supposed to be united in that point, and the common centre of gravity between them and the third is to be found in the same manner, and so of the rest.

Let the weights A, B, and C, for example, be suspended from three points of the line or balance D F (Fig. 1.), which we shall suppose to have no weight. Let the body A weigh 108 pounds; B 144, C 180; and let the distance D E be 11 inches, and E F, 9.

Fig. 1.



First find the common centre of gravity of the bodies B and C, by dividing the distance E F, or 9 inches, into two parts, which are to each other as 144 to 180, or as 4 to 5. These two parts will be 4 and 5 inches; the greater of which must be placed towards the smaller weight: the body B being here the smaller we shall have E G equal to 5 inches, and F G to 4; consequently D G will be 16.

If we now suppose the two weights B and C, united into one in the point G, and consequently equal in that point to 324 pounds; the distance D G, or 16 inches, must be divided in the ratio of 108 to 324, or of 1 to 3. One of these parts will be 12 and the other 4; and as A is the less weight, D H must be made equal to 12 inches, and the point H will be the common centre of gravity of all the three bodies, as required.

The result would have been the same, had the bodies A and B been first united.

In short, the rule is the same whatever be the number of the bodies, and whatever be their position in the same straight line or in the same plane.

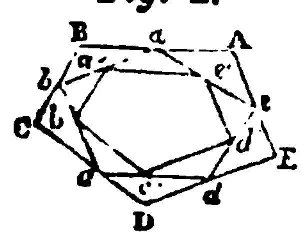
This may suffice here in regard to the centre of gravity. But for many curious truths deduced from this consideration, recourse may be had to books which treat on mechanics. We shall however mention one beautiful principle in this science, deduced from it, which is as follows:

If several bodies or weights be so disposed, that by communicating motion to each other, their common centre of gravity remains at rest, or does not deviate from the horizontal line, that is to say neither rises nor falls, there will then be an equilibrium.

The demonstration of this principle is almost evident from its enunciation; and it may be employed to demonstrate all the properties of machines. But we shall leave the application of it to the reader.

Remark.—As this is the proper place, we shall here discharge a promise made at p. 183, viz. to resolve a geometrical problem, the solution of which, as we said, seems to be only deducible from the property of the centre of gravity.

Fig. 2.

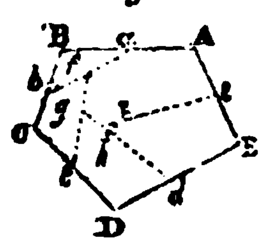


Let the proposed irregular polygon then be ABCDE (Fig. 2.), the sides of which are each divided into two equal parts, in *a*, *b*, *c*, *d*, and *e*, from which results a new polygon *a b c d e a*; let the sides of the latter be each divided also into two equal parts by the points a', b', c', d', e', which when joined will give a third polygon $a'\,b'\,c'\,d'\,e'\,a'$; and so on. In what point will this division terminate ?

To solve this problem, if we suppose equal weights placed at *a*, *b*, *c*, *d*, *e*, their common centre of gravity will be the point required.

But, to find this centre of gravity, we must proceed in the following manner,

Fig. 3.

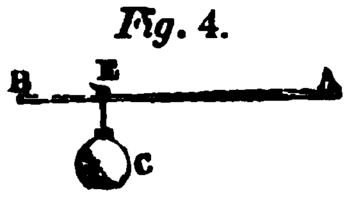


which is exceedingly simple. First draw *a b* (Fig. 3.) and let the middle of it be the point *f*; then draw *f e*, and divide it in *g*, in such a manner that *f g* shall be one third of it; draw also *g d*, and let *g h* be the fourth of it; in the last place draw *h e*, and let *h i* be the fifth of it: the weight *e* being the last, the point *i*, as may be demonstrated from what has been before said, will be the centre of gravity of the five equal weights placed at *a*, *b*, *c*, *d*, and *e*; and will solve the proposed problem.

PROBLEM V.

When two persons carry a burthen, by means of a lever or pole, which they support at the extremities; to find how much of the weight is borne by each person.

Fig. 4.



It may be readily seen that, if the weight C were exactly in the middle of the lever AB (Fig. 4.), the two persons would each bear one half. But if the weight is not in the middle, it can be easily demonstrated, that the parts of the weight borne by the two persons, are in the reciprocal ratio of their distance from the weight. Nothing then is necessary but to divide the weight according to this ratio; and the greater portion will be that supported by the person nearest the weight, and the least that supported by the person farthest distant. The calculation may be made by the following proportion:

As the whole length of the lever AB, is to the length AE, so is the whole weight to the weight supported by the power or person at the other extremity B; or as AB is to BE, so is the whole weight to the part supported by the power or person placed at A.

If AB, for example, be 6 feet, the weight C 150 pounds, AE 4 feet, and BE 2, we shall have this proportion: as 6 is to 4, so is 150 to a fourth term, which will be 100. The person placed at the extremity B, will therefore support 100 pounds, and consequently the one placed at A will have to support only 50.

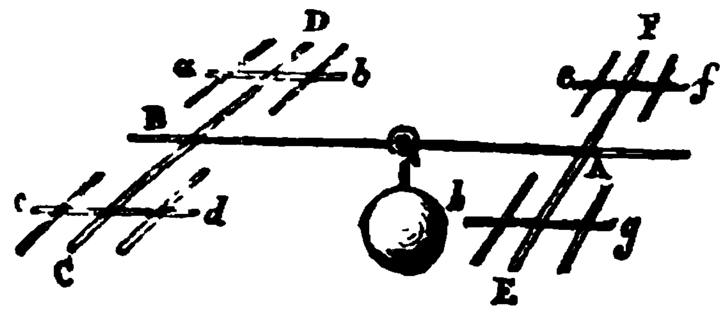
Remark.—The solution of this problem affords the means of dividing a burthen or weight proportionally to the strength of the agents employed to raise it. Thus, for example, if the one has only half the strength of the other, nothing is necessary but to place him at a distance from the weight double to that of the other.

PROBLEM VI.

How 4, 8, 16, *or* 32 *men may be distributed in such a manner as to carry a considerable burthen with ease.*

If the burthen can be carried by four men, after having made it fast to the middle of a large lever AB (Fig 5.), cause the extremities of this lever to rest on two shorter ones CD and EF, and place a man at each of the points C, D, E, and F:

Fig. 5.



it is evident that the weight will then be equally distributed among these four persons.

If eight men are required, pursue the same method with the levers C D and E F, as was employed in regard to the first; that is, let the extremities of C D be supported by the two shorter ones *a b* and *c d*; and those of E F by the levers *e f* and *g h*: if a man be then stationed at each of the points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, they will be all equally loaded.

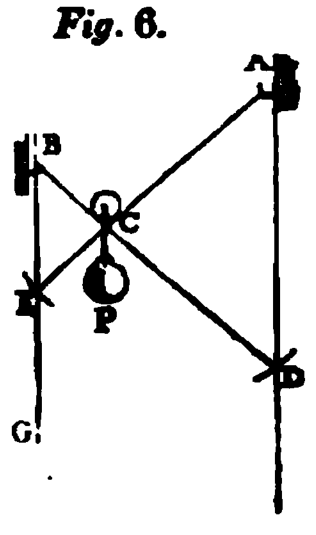
The extremities of the levers or poles *a b*, *c d*, *e f*, and *g h*, might, in like manner, be made to rest on others placed at right angles to them: by means of this artifice the weight would be equally distributed among sixteen men. And so of any other number.

We have heard that this artifice is employed at Constantinople, to raise and carry the heaviest burthens, such as cannons, mortars, enormous stones, &c. The velocity, it is added, with which burthens are transported from one place to another, by this method, is truly astonishing.

PROBLEM VII.

A rope A C B (*Fig.* 6.), *of a determinate length, being made fast by both ends, but not stretched to two points of unequal height,* A *and* B; *what position will be assumed by the weight* P, *suspended from a pulley which rolls freely on that rope.*

Fig. 6.



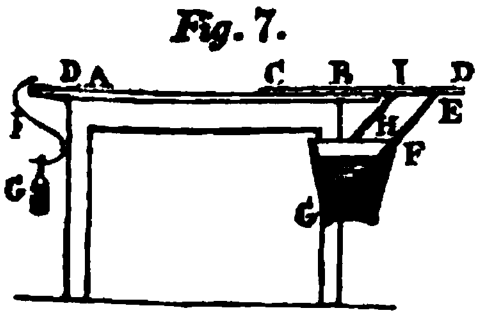
From the points A and B, let fall the indefinite vertical lines A D and B G; then from the point A, with an opening of the compasses equal to the length of the rope, describe an arc of a circle, intersecting the vertical line B G in E: and from the point B describe a similar arc of a circle, intersecting the vertical line A D in D: if the lines A E and B D be then drawn, the point C, where they cut each other, will give the position of the rope A C B, when the weight has assumed that position in which it must rest; and the point C will be that in which the pulley will settle. For it may be easily demonstrated, that in this situation the weight P will be in the lowest position possible, which is an invariable principle of the centre of gravity.

PROBLEM VIII.

To cause a pail full of water to be supported by a stick, one half of which only, or less, rests on the edge of a table.

To make the reader comprehend properly the method of performing this trick, in regard to equilibrium, we have given, in the annexed figure, a section of the table and the bucket.

Fig. 7.



In this figure let A B be the top of the table, on which is placed the stick C D. Convey the handle of the bucket over this stick, in such a manner that it may rest on it in an inclined position; and let the middle of the bucket be within the edge of the table. That the whole apparatus may be fixed in this situation, place another stick G F E, with one of its ends resting against the corner G of the bucket, while the middle part rests against the edge F of the bucket, and its other extremity against the first stick C D, in E, where there ought to be a notch to retain it. By these means the bucket will remain fixed in that situation, without being able to incline to

either side; and if not already full of water, it may be filled with safety; for its centre of gravity being in the vertical line passing through the point H, which itself meets with the table, it is evident that the case is the same as if the pail were suspended from the point of the table where it is met by that vertical. It is also evident that the stick cannot slide along the table, nor move on its edge, without raising the centre of gravity of the bucket, and of the water it contains. The heavier therefore it is, the greater will be the stability.

Remark.—According to this principle, various other tricks of the same kind, which are generally proposed in books on mechanics, may be performed. For example, provide a bent hook D G F, as seen at the opposite end of the same figure, and insert the part F D, in the pipe of a key at D, which must be placed on the edge of a table; from the lower part of the hook suspend a weight G, and dispose the whole in such a manner that the vertical line G D may be a little within the edge of the table. When this arrangement has been made, the weight will not fall, and the case will be the same with the key, which had it been placed alone in that situation would perhaps have fallen; and this resolves the following mechanical problem, proposed in the form of a paradox: *A body having a tendency to fall by its own weight, how to prevent it from falling, by adding to it a weight on the same side on which it tends to fall.*

The weight indeed appears to be added on that side, but in reality it is on the opposite side.

PROBLEM IX.

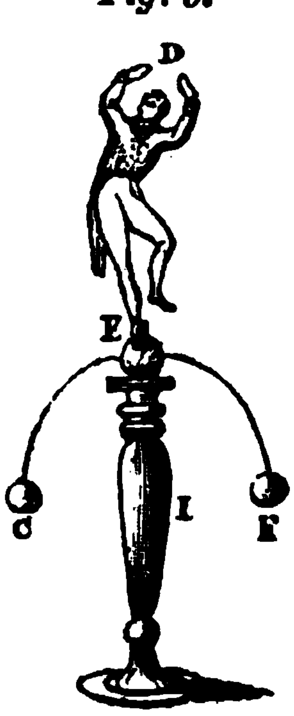
To hold a stick upright on the tip of the finger, without its being able to fall.

Fig. 8.

Affix two knives, or other bodies, to the extremity of the stick, in such a manner that one of them may incline to one side, and the second to the other, as seen in the figure (Fig. 8.): if this extremity be placed on the tip of the finger, the stick will keep itself upright, without falling; and if it be made to incline, it will raise itself again, and recover its former situation.

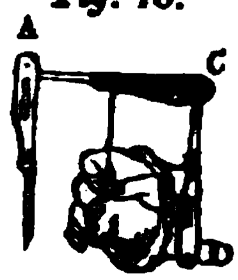
For this purpose, the centre of gravity of the two weights added, and of the stick, must be below the point of suspension, or the extremity of the stick, and not at the extremity, as asserted by Ozanam; for in that case there would be no stability.

Fig. 9.



It is the same principle that keeps in an upright position those small figures furnished with two weights, to counterbalance them; and which are made to turn and balance, while the point of the foot rests on a small ball, loosely placed on a sort of stand. Of this kind is the small figure D (Fig. 9.), supported on the stand I, by a ball E, through which passes a bent wire, having affixed to its extremities two balls of lead, C and F. The centre of gravity of the whole, which is at a considerable distance below the point of support, maintains the figure upright, and makes it resume its perpendicular position, after it has been inclined to either side; for this centre tends to place itself as low as possible, which it cannot do without making the figure stand upright.

Fig. 10.



By the same mechanism, three knives may be disposed in such a manner as to turn on the point of a needle; for being disposed as seen in the figure (Fig. 10.), and placed in equilibrio on the point of a needle held in the hand, they cannot fall, because their common centre of gravity is far below the point of the needle, which is above the point of support.

PROBLEM X.

To construct a figure, which, without any counterpoise, shall always raise itself upright and keep in that position, or regain it, however it may be disturbed.

Make a figure resembling a man, of any substance exceedingly light, such as the pith of the elder tree, which is soft and can be easily cut into any form at pleasure. Then provide for it an hemispherical base of some very heavy substance, such as lead. The half of a leaden bullet, made very smooth on the convex part, will be proper for this purpose. If the figure be cemented to the plane part of this hemisphere; then, in whatever position it may be placed, as soon as it is left to itself, it will rise upright (Fig. 11.); because the centre of gravity of its hemispherical base being in the axis, tends to approach the horizontal plane as much as possible, and this can never be the case till the axis becomes perpendicular to the horizon; for the small figure above scarcely deranges it from its place, on account of the disproportion between its weight and that of its base.

Fig. 11.

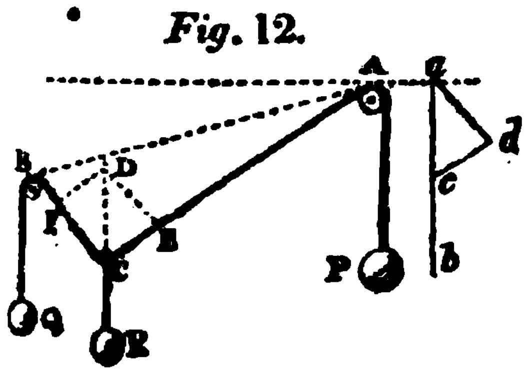
In this manner were constructed those small figures called Prussians, sold at Paris some years ago. They were formed into battalions, and being made to fall down by drawing a rod over them, they immediately started up again as soon as it was removed.

Screens of the same form have been since invented, which always rise up of themselves, when they happen to be pressed down.

PROBLEM XI.

If a rope A C B, *to the extremities of which are affixed the given weights* P *and* Q, *be made to pass over two pulleys* A *and* B; *and if a weight* R *be suspended from the point* C, *by the cord* R C; *what position will be assumed by the three weights and the rope* A C B? (*Fig.* 12.)

Fig. 12.



In the line *a b*, perpendicular to the horizon, assume any part *a c*, and on that part as a base, describe the triangle *a d c*, in such a manner, that *a c* shall be to *c d*, as the weight R to the weight P; and that *a c* shall be to *a d*, as R to Q; then through A, draw the indefinite line A C, parallel to *c d*; and through B, draw B C, parallel to *a d*: the point C, where these two lines intersect each other, will be the point required, and will give the position A C B of the rope.

For, if in R C continued we assume C D, equal to *a c*, and describe the parallelogram E D F C; it is evident that we shall have C F and C E equal to *c d* and *a d*; and therefore the three lines E C, C D, and C F will be as the weights P, R, and Q; consequently the two forces acting from C to F, and from C to E, or in the direction of the lines C A and C B, will be in equilibrio with the force which acts from C towards R.

Remarks.—1st. If the ratio of the weights were such, that the point of intersection C should fall on the line A B, or above it, the problem in this case would be impossible.

The weight Q, or the weight P, would overcome the other two in such a manner, that the point C would fall in B or A; so that the rope would form no angle.

These weights also might be such that it would be impossible to construct the triangle *a c d*, as if one of them were equal to or greater than the other two taken together; for, to make a triangle of three lines, each of them must be less than the other two. In that case the weight equal or superior to the other two would overcome them both, so that no equilibrium could take place.

2d. If instead of a knot at C, we should suppose the weight R suspended from a pulley capable of rolling on the rope A C B, the solution would be still the same; for it is evident that, things being in the same state as in the first case, if a pulley were substituted for the knot C, the equilibrium would not be destroyed. But there would be one limitation more than in the preceding case. It would be necessary that the point of intersection, C, determined as above, should fall below the horizontal line, drawn through the point B; otherwise the pulley would roll to the point B, as if on an inclined plane.

PROBLEM XII.

Calculation of the time which Archimedes would have required to move the earth, with the machine of which he spoke to Hiero.

The expression which Archimedes made use of to Hiero, king of Sicily, is well known, and particularly to mathematicians. "Give me a fixed point," said the philosopher, "and I will move the earth from its place." This affords matter for a very curious calculation, viz. to determine how much time Archimedes would have required to move the earth only one inch, supposing his machine constructed and mathematically perfect; that is to say, without friction, without gravity, and in complete equilibrium.

For this purpose, we shall suppose the matter of which the earth is composed to weigh 300 pounds the cubic foot; being the mean weight nearly of stones mixed with metallic substances, such in all probability as those contained in the bowels of the earth. If the diameter of the earth be 7930 miles, the whole globe will be found to contain 261107411765 cubic miles, which make 1423499120882544640000 cubic yards, or 38434476263828705280000 cubic feet; and allowing 300 pounds to each cubic foot, we shall have 11530342879148611584000000 for the weight of the earth in pounds.

Now, we know by the laws of mechanics that, whatever be the construction of a machine, the space passed over by the weight, is to that passed over by the moving power, in the reciprocal ratio of the latter to the former. It is known also, that a man can act with an effort equal only to about 30 pounds for eight or ten hours, without intermission, and with a velocity of about 10000 feet per hour. If we suppose the machine of Archimedes then to be put in motion by means of a crank, and that the force continually applied to it is equal to 30 pounds, then with the velocity of 10000 feet per hour to raise the earth one inch, the moving power must pass over the space of 38434476263828705280000 inches; and if this space be divided by 10000 feet, or 120000 inches, we shall have for quotient 320287302198572544O, which will be the number of hours required for this motion. But as a year contains 8766 hours, a century will contain 876600; and if we divide the above number of hours by the latter, the quotient, 3653745176803, will be the number of centuries during which it would be necessary to make the crank of the machine continually turn, in order to move the earth only one inch. We have omitted the fraction of a century, as being of little consequence in a calculation of this kind.*

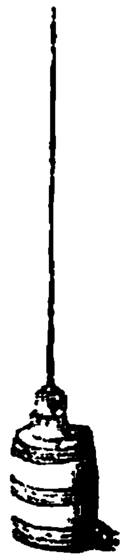
* The machine is here supposed to be constantly in action: but if it should be worked only 8 hours each day, the time required would be three times as long.

PROBLEM XIII.

With a very small quantity of water, such as a few pounds, produce the effect of several thousands. (Fig. 13.)

Place a cask on one of its ends, and make a hole in the other end, capable of admitting a tube, an inch in diameter and from 12 to 15 feet in length; which must be fitted closely into the aperture by means of pitch or tow. Then load the upper end of the cask with several weights, so that it shall be sensibly bent downwards; and having filled the cask with water, continue to pour some in through the tube. The effort of this small cylinder of water will be so great, that not only the weights which pressed the upper end of the cask downwards will be raised up, but very often the end itself will be bent upwards, and form an arch in a contrary direction.

Fig. 13.



Care however must be taken that the lower end of the cask rest on the ground; otherwise the first effort of the water would be directed downwards, and the experiment might seem to fail.

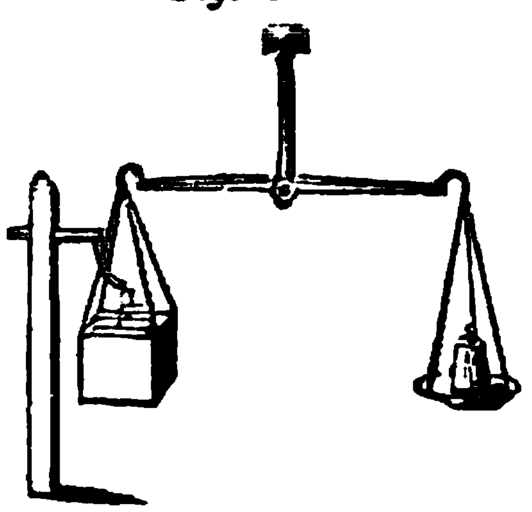
By employing a longer tube, the upper end of the cask might certainly be made to burst.

The reason of this phenomenon may be easily deduced from a property peculiar to fluids, of which it is an ocular demonstration, viz. that when they press upon a base they exercise on it an effort proportioned to the breadth of that base multiplied by the height. Thus, though the tube used in this experiment contains only about 150 or 180 cylindric inches of water, the effort is the same as if the tube were equal in breadth to the cask, and at the same time 12 or 15 feet in height.

Another Method. (Fig. 14.)

Suspend from a hook, well fixed in a wall, or any other firm support, a body weighing 100 pounds or more; then provide a vessel of such dimensions, that between that body and its sides, there shall be room for only one pound of water; and let the vessel be suspended to one of the arms of a balance, the other arm of which has suspended from it a scale, containing a weight of 100 pounds. Pour a pound of water into the vessel suspended from the one arm of the balance, and it will raise the scale containing the 100 pounds.

Fig. 14.



Those who have properly comprehended the preceding explanation, will find no difficulty in conceiving the cause and necessity of this effect; for they are both the same, with this difference only, that the water, instead of being collected in a cylindric tube, is in the narrow interval between the body L and the vessel which surrounds it: but this water exercises on the bottom of the vessel the same pressure that it would experience if entirely full of water.

Another Method.

Provide a cubic foot of very dry oak, weighing about 60 pounds, and a cubical vessel about a line or two larger every way. If the cubic foot of wood be put into the vessel, and water be poured into it, when the latter has risen to nearly two thirds of its height, the cube will be detached from the bottom, and float. Thus we see a weight of about 60 pounds overcome by half-a-pound of water and even less.

Remark.—Hence it appears that the vulgar are in an error, when they imagine a body floats more readily in a large quantity of water than in a small one. It will always float, provided there be a sufficiency to prevent it from touching the bottom. If vessels are lost at the mouths of rivers, it is not because the water is too shallow.

but because the vessels are loaded so much, as to be almost ready to sink, even in salt water. But as the water of the sea is nearly a thirtieth part heavier than fresh water, when a ship passes from the one into the other, it must sink more and go to the bottom. Thus an egg, which sinks in fresh water, will float in water which holds in solution a great deal of salt.

The principle of the foregoing experiments is what has been called the *Hydrostatic paradox*, and it is on the same principle that Brahma's *Hydrostatic press* is constructed.

With this press, it has been justly observed, that "a prodigious force is obtained with the greatest ease and in a very small compass; so that with a machine of the size of a teapot standing on the table before him, a man shall cut through a thick bar of iron as easily as he could clip a piece of pasteboard with a pair of scissars."

The following is a description of Brahma's press, as commonly constructed. A (Fig. 15.) is a strong metal cylinder bored perfectly smooth and cylindrical, and into it is fitted the piston B, made perfectly water-tight, by packing it in the usual way.

Fig. 15.

In the bottom of the cylinder is inserted the end of the tube C, the aperture of which communicates with the inside of the cylinder under the piston B, where it is shut by a small valve, as in the suction pipe of a common pump. The other end of the tube C communicates with the small forcing pump E, worked by the lever H. Now, suppose the diameter of the cylinder A to be 12 inches, and that of the smaller pump E and the tube C only a quarter of an inch, the proportion between the covers of the ends of the pistons will be as 1 to 2304,—and supposing the intermediate span between them to be filled with water, the force of one piston will act on the other in that proportion. Suppose the small piston to be forced down in the act of injecting water with a force of 20 cwt.—which may easily be done by the lever H—the piston B would be forced upwards with a force 2304 times as great. Hence, with this machine, by the aid of a simple lever, a weight of 2304 tons may be raised through any given space better than by any other apparatus constructed on the known principles of mechanics. It may be observed, too, that the effect of all other mechanical combinations is counteracted by an accumulated complication of parts which limits the intent to which they can be usefully carried,—but in machines depending on the principle under consideration, the power may be extended *ad libitum* either by increasing the proportion between the diameters of E and A, or applying greater power to the lever H.

Fig. 16. is a section shewing how, by means of fluids, the power and motion of one machine may be communicated to another, let their distance and relative situations be what they may. A and B are two small smooth cylindrical tubes, inside of each being a portion made air-tight and water-tight. C C is a tube conveyed underground or otherwise, from the one cylinder to that of the other,—and this tube is filled with water till it touch the bottom of both pistons. Then by depressing the piston A, the piston B will be raised, and *vice versa*. Thus bells may be rung, wheels turned, or other machinery put motion, by invisible agency.

Fig. 16.

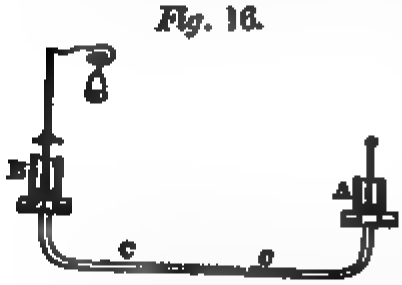
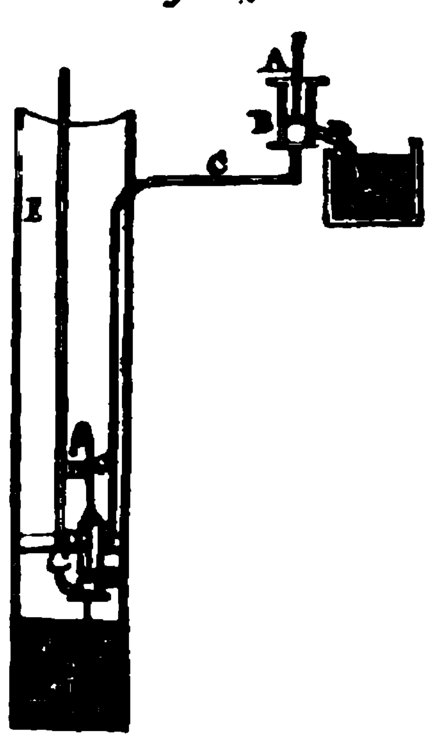


Fig. 17. is a section shewing another instance of action and force communicated from one machine to another,—and how water may be raised from wells of any depth, and at any distance from the place where the power is applied. A is a cylinder of any required dimensions, in which is the working piston B of the Brahma press, described above. Into the bottom of this cylinder is inserted the tube C, which may be of less

Fig. 17



dimensions than the cylinder A. This tube is conveyed in any direction down to the pump cylinder D in the deep well E E, and forms a junction with D above the piston F, which piston has a rod G working through a stuffing box. To this rod G a weight H is connected over a pulley or otherwise, sufficient to overbalance the weight of the water in the tube c, and to raise the piston F when the piston B is lifted. Suppose, now, the piston B is drawn up by its rod, there will be a vacuum made in D below the piston F, which vacuum will be filled with water, through the suction pipe, by the pressure of the atmosphere.

The piston B, being pressed downwards in the cylinder A, will make a stroke in the pump cylinder D, which may be repeated in the usual way by the motion of the piston B and the action of the water in the tube c. The small tube and cock in the cistern I are for the purpose of charging the tube c.

By these means a commodious machine of prodigious power and of the greatest strength may readily be formed: all that is required is accurate execution, which in the present state of the mechanical arts is quite attainable.

PROBLEM XIV.

To find the weight of a cubic foot of water.

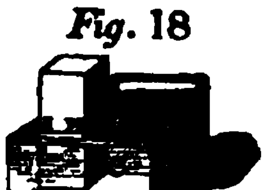
To know the weight of a cubic foot of water is one of the most essential elements of hydrostatics and hydraulics; and for that reason we shall here show how it may be accurately determined.

Provide a vessel, capable of containing exactly a cubic foot, and having first weighed it empty, weigh it again when filled with water. But as liquids always rise considerably above the edges of the vessel that contains them, the result in this case will not be very correct. There are means indeed to remedy this defect; but we are furnished with a very accurate method of doing it by hydrostatics.

Provide a cube of some very homogeneous matter, such as metal, each side of which is exactly four inches; weigh it by a good balance, in order to ascertain its weight within a few grains; then suspend it by a hair, or strong silk thread, from one of the scales of the same balance, and again find its weight when immersed in water. We are taught by hydrostatics that it will lose exactly as much in weight as the weight of an equal volume of water. The difference of these two weights therefore will be the weight of a cube of water, each side of which is four inches, or of the twenty-seventh part of a cubic foot.

If very great precision is not required, provide a cube or rectangular parallelopipedon, of any homogeneous matter, lighter than water—such, for example, as wood; and having weighed it as accurately as possible, immerse it gently in water, in such a manner that the water may not wet it above that point at which it ought to float above the liquid. We shall here suppose that I M D (Fig. 18.) is the line, which exactly marks how much of it is immersed. Find the content of the solid A B C D M I, by multiplying its base by the height; the product will be the volume of water displaced by the body; and this volume, according to the principles of hydrostatics, must weigh as much as the body itself. If this volume of water be 720 cubic inches, for example, and if the weight of the body be 26·0416 pounds, we consequently know that 720 cubic inches of water weigh 26·0416 pounds. Hence it will be easy to determine the weight of a cubic foot, which contains 1728 cubic inches. Nothing is necessary but to make this proportion: as 720

Fig. 18

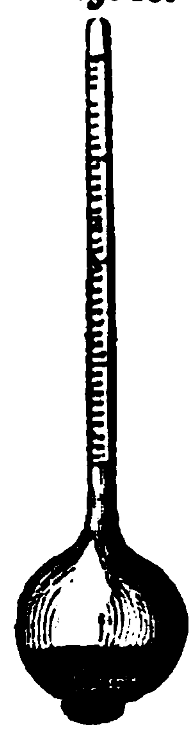


cubic inches are to 1728, so are 26·0416 pounds to a fourth term, which will be 62·5 pounds, or 62 pouuds and a half; which therefore is the weight of a cubic foot of water.

PROBLEM XV.

Two liquors being given; to determine which of them is the lightest.

Fig. 19.



This problem is generally solved by means of a well known instrument called the *areometer* or *hydrometer*. This instrument is nothing else than a small hollow ball, joined to a tube 4 or 5 inches in length (Fig. 19.); a few grains of shot, or a little mercury, being put into the ball, the whole is so combined, that in water of mean gravity, the small ball and part of the tube are immersed.

It may now be easily conceived that when the instrument is put into any fluid, for example river water, care must be taken to observe how far it sinks into it; if it be then placed in another kind of water, such as sea water, for instance, it will sink less; and if immersed in any liquor lighter than the first, such as oil for example, it will sink farther. Thus it can be easily determined, without a balance, which of two liquors is the heavier or lighter. This instrument has commonly on the tube a graduated scale, in order to shew how far it sinks in the fluid.

But this instrument is far inferior to that presented, in 1766, by M. de Parcieux, to the Academy of Sciences, and yet nothing is simpler. This instrument consists of a small glass bottle, two inches, or two inches and a half at most, in diameter, and from six to eight inches in length. The bottom must not be bent inwards, lest air should be lodged in the cavity when it is immersed in any liquid. The mouth is closed with a very tight cork stopper, into which is fixed, without passing through it, a very straight iron wire, 25 or 30 inches in length, and about a line in diameter. The bottle is then loaded in such a manner, by introducing into it grains of small shot, that the instrument, when immersed in the lightest of the liquors to be compared, sinks so as to leave only the end of the iron wire above its surface, and that in the heaviest the wire is immersed some inches. This may be properly regulated by augmenting or diminishing either the weight with which the bottle is loaded, or the diameter of the wire, or both these at the same time. The instrument, when thus constructed, will exhibit, in a very sensible manner, the least difference in the specific gravities of different liquors, or the changes which the same liquor may experience, in this respect, under different circumstances; as by the effect of heat, or by the mixture of various salts, &c.

It may be readily conceived, that to perform experiments of this kind, it will be necessary to have a vessel of a sufficient depth, such as a cylinder of tin-plate, 3 or 4 inches in diameter, and 3 or 4 feet in length.

We have seen an instrument of this kind, the movement of which was so sensible, that when immersed in water, cooled to the usual temperature, it sank several inches, while the rays of the sun fell upon the water; and immediately rose on the rays of that luminary being intercepted. A very small quantity of salt or sugar, thrown into the water, made it also rise some inches.

By means of this instrument, M. de Parcieux examined the gravity of different kinds of the most celebrated waters; among which was that drank at Paris; and he found that the lightest of all was distilled water. The next in succession, according to their lightness, were as in the following order; viz., the water of the Seine, that of the Loire, that of the Yvette, that of Arcueil, that of Sainte-Reine, that of Ville d'Avray, the Bristol water, and well water.

We hence see the error of the vulgar, who imagine that the water of Ville d'Arvay, that of Sainte-Reine, and that of Bristol, particularly the last brought to France, at

so great expense, are better than common river water; for they are, on the contrary, worse, since they are heavier.

If different kinds of water differ in their gravity, the case is the same with wines also. The lightest of all the known wines, at least in France, is the Rhenish. The next in succession are Burgundy, red Champagne, the wines of Bourdeaux, Languedoc, Spain, the Canaries, Cyprus, &c.

The lightest of all the known liquors is ether. The others, which follow in the order of gravity, are: alcohol, oil of turpentine, distilled water, rain water, river water, spring water, well water, mineral waters. Among the tables, annexed to this part of the work, the reader will find one containing the specific gravity of various liquors, compared with that of rain water; which, being the easiest procured, may serve as a common standard, and also the specific gravity of the different solid bodies, whether belonging to the mineral, vegetable, or animal kingdom; which will doubtless be found very useful, as it is often necessary to have recourse to tables of this kind.

As the following rules for calculating the absolute gravity, in English troy weight, of a cubic foot and inch, English measure, of any substance whose specific gravity is known, may be of use to the reader, the translator has thought proper to subjoin them to this article of the original.

In 1696, Mr. Everard, balance-maker to the Exchequer, weighed before the commissioners of the House of Commons, 2145·6 cubical inches, by the Exchequer standard foot, of distilled water, at the temperature of 55° of Fahrenheit, and found that it weighed 1131 oz. 14 drs. troy, of the Exchequer standard. The beam turned with 6 grains, when loaded with 30 pounds in each scale. Hence, supposing the pound averdupois to weigh 7000 grs. troy, a cubic foot of water weighs 62½ pounds averdupois, or 1000 ounces averdupois, wanting 106 grs. troy. If the specific gravity of water therefore be called 1000, the proportion of specific gravities of all other bodies will express nearly the number of averdupois ounces in a cubic foot. Or, more accurately, supposing the specific gravity of water expressed by 1, and that of all other bodies in proportional numbers, as the cubic foot of water weighs, at the above temperature, exactly 437489·4 grains troy, and the cubic inch of water 253·175 grains, the absolute weight of a cubical foot or inch of any body, in troy grains, may be found by multiplying its specific gravity by either of the above numbers respectively.

By Everard's experiment, and the proportions of the English and French foot, as established by the Royal Society and French Academy of Sciences, the following numbers have been ascertained:

Paris grains, in a Paris cube foot of water	645511
English grains, in a Paris cube foot of water	529922
Paris grains, in an English cube foot of water	533247
English grains, in an English cube foot of water	437489·4
English grains in an English cube inch of water	253·175
By an experiment of Picard, with the measure and weight of the Chatelet, the Paris cube foot of water contains of Paris grains	641326
By one of Du Hamel, made with great care	641376
By Homberg	641666

These results shew some uncertainty in measure or in weights; but the above computation from Everard's experiment may be relied on; because the comparison of the English foot with that of France, was made by the joint labour of the Royal Society of London, and the French Academy of Sciences. It agrees likewise, very nearly, with the weight assigned by Lavoisier, which is 70 Paris pounds to the cubical foot of water.

PROBLEM XVI.

To determine whether a mass of gold or silver, suspected to be mixed, is pure or not.

If the mass or piece, the fineness of which is doubtful, be silver for example, provide another mass of good silver equally heavy; so that the two pieces, when put into the scales of a very accurate balance, may remain in equilibrio in the air. Then suspend these two masses of silver from the scales of the balance, by two threads or two horse-hairs, to prevent the scales from being wetted when the two masses are immersed in the water: if the masses are of equal fineness, they will remain in equilibrio in the water, as they did when in the air; but if the proposed mass weighs less in water, it is adulterated; that is to say, is mixed with some other metal, of less specific gravity than that of silver, such as copper for example; and if it weighs more, it is mixed with some metal of greater specific gravity, such as lead.

Remarks.—1. This problem is evidently the same as that whose solution gave so much pleasure to Archimedes. Hiero, king of Syracuse, had delivered to a goldsmith a certain quantity of gold, for the purpose of making a crown. When the crown was finished, the king entertained some suspicion in regard to the fidelity of the goldsmith, and Archimedes was consulted respecting the best means of detecting the fraud, in case one had been committed. The philosopher, having employed the above process, discovered that the gold, of which the crown consisted, was not pure.

If a large mass of metal were to be examined, as in the case of Archimedes, it would be sufficient to immerse the mass of gold or silver, known to be pure, in a vessel of water, and then the suspected mass. If the latter expelled more water from the vessel it would be a proof of the metal being adulterated by another lighter, and of less value.

But notwithstanding what Ozanam says, the difference between the weight in air and that in water will indicate the mixture with more certainty; for every body knows that it is not so easy, as it may at first appear, to measure the quantity of water expelled from any vessel.

2. According to mathematical rigour, the two masses ought first to be weighed in vacuo; for since air is a fluid, it lessens the real gravity of bodies by a quantity equal to the weight of a similar volume of itself. Since the two masses then, the one pure and the other adulterated, are unequal in volume, they ought to lose unequal quantities of their weight in the air. But the great tenuity of air, in regard to that of water, renders this small error insensible.

PROBLEM XVII.

The same supposition made; to determine the quantity of mixture in the gold.

The ingenious artifice employed by Archimedes, is contained in the solution of this problem, and is as follows.

Suspecting that the goldsmith had substituted silver or copper for an equal quantity of gold, he weighed the crown in water, and found that it lost a weight, which we shall call A: he then weighed in the same fluid a mass of pure gold, which in air was in equilibrio with the crown, and found that it lost a weight, which we shall call B; he next took a mass of silver, which in air was equal in weight to the crown, and weighing it in water, found that it lost a quantity C. He then employed this proportion: as the difference of the weights B and C, is to that of the weights A and B, so is the whole weight of the crown to that of the silver mixed in it. The answer, in this case, may be obtained by a very short algebraical calculation, though the reasoning is rather too prolix; we shall however explain it after having illustrated this rule by an example.

Let us suppose that Hiero's crown weighed 20 pounds in the air, and that when

weighed in water it lost a pound and a half. Archimedes, by weighing in air and in water, a mass of gold containing 20 pounds, must have found a difference of $1\frac{1}{19}$ pound; and by weighing in like manner a mass of silver of 20 pounds, he must have found a difference of $1\frac{9}{11}$ pound. As A, in this case, is equal to $\frac{3}{2}$, B to $\frac{20}{19}$, and C to $\frac{20}{11}$; hence the difference of A and B is $\frac{17}{38}$, and that of B and C is $\frac{160}{209}$: we must therefore use the following proportion: as $\frac{160}{209}$ are to $\frac{17}{38}$, so is 20 to a fourth term, which will be $\frac{187}{16}$ = 11 lbs. 8 oz. 5 dwts.

The reasoning which conducted, or might have conducted, the Syracusan philosopher to this solution, is as follows. If the whole mass were of pure gold, it would lose, when weighed in water, $\frac{1}{19}$ of its weight; and if it were of pure silver it would lose, when weighed in water, $\frac{1}{11}$ of its weight: consequently, if it loses less than the latter quantity, and more than the former, it must be a mixture of gold and silver; and the quantity of silver substituted for gold will be greater as the quantity of weight which the crown loses in water approaches nearer to $\frac{1}{11}$, and *vice versa*. This mass of 20 pounds then must be divided into two parts, in the ratio of the following differences: viz. the difference between the loss which the crown experiences and that experienced by the pure gold; and the difference between the loss experienced by the crown and that experienced by the pure silver; these will be the proportions of the gold and silver mixed together in the crown; and from this reasoning is deduced the preceding rule.

We must here observe, that it is not necessary to take two masses, one of gold and another of silver, each equal in weight to the crown. It will be sufficient to ascertain that gold loses a nineteenth of its weight, when weighed in water; and silver one eleventh, and perhaps this was really the method employed by Archimedes.

PROBLEM XVIII.

Suppose there are two boxes exactly of the same size, similar and of equal weight, the one containing gold and the other silver: is it possible, by any mathematical means, to determine which contains the gold, and which the silver? Or, if we suppose two balls the one made of gold and hollow, the other of solid silver gilt, is it possible to distinguish the gold from the silver?

In the first case, if the masses of gold and silver are each placed exactly in the middle of the box which contains it, so that their centres of gravity coincide, whatever may be said in the old books on Mathematical Recreations, we will assert that there are no means of distinguishing them, or at least that the methods proposed are defective.

The case is the same in regard to the two similar globes of equal size and weight.

If we were however under the necessity of making a choice, we would endeavour to distinguish the one from the other in the following manner.

We would suspend both balls by as delicate a thread as possible to the arms of a very accurate balance, such as those which, when loaded with a considerable weight, are sensibly affected by the difference of a grain. We would then immerse the two balls in a large vessel filled with water, heated to the degree of ebullition, and that which should preponderate we would consider as gold For, according to the experiments made on the dilatation of metals, the silver, passing from a mean temperature, to that of boiling water, would probably increase more in volume than the gold; in that case the two masses, which in air and temperate water were in equilibrio, would not be so in boiling water.

Or, we might make a round hole in a plate of copper, of such a size, that both balls should pass exactly through it with ease; we might then bring them to a strong degree of heat, superior even to that of boiling water. Now, if we admit that silver expands more than gold, as above supposed, we might apply each of them to the hole in question, and the one which experienced the greater difficulty in passing, ought to be accounted silver.

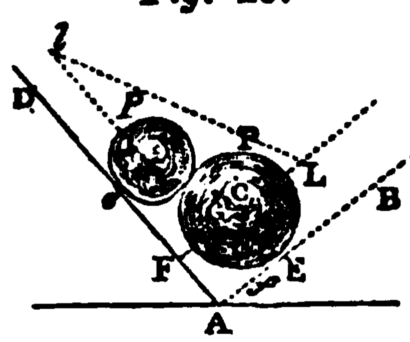
P

PROBLEM XIX.

Two inclined planes A B *and* A D *being given, and two unequal spheres* P *and* p; *to bring them to an equilibrium in the angle, as seen in the figure.* (Fig. 20.)

The globes P and *p*, will be in equilibrio, if the powers with which they repel each other, in the direction of the line C *c*, which joins their centres, are equal.

Fig. 20.



But the force with which the globe P tends to descend along the inclined plane B A, which is known, the inclination of the plane being given, is to the force with which it acts in the direction C *c*, as radius is to the cosine of the angle *c* C F; and in like manner, the force with which the weight *p* descends along D A, is to that with which it tends to move in the direction *c* C, as radius is to the cosine of the angle C *c f*: hence it follows, that as these second forces must be equal, the cosine of the angle *c* must have the same ratio to the cosine of the angle C, as the force with which the globe P tends to roll along B A, has to that with which *p* tends to roll along D A. The ratio of these cosines therefore is known; and as in the triangle C G *c* the angle G is known, since it is equal to the angle D A B, it thence follows that its supplement, or the sum of the two angles C and *c*, is also known; and hence the problem is reduced to this, viz. to dividing a known angle into two such parts, that their cosines shall be in a given ratio; which is a problem purely geometrical.

But, that we may confine ourselves to the simplest case, we shall suppose the angle A to be a right-angle. Nothing then will be necessary but to divide the quadrant into two arcs, the cosines of which shall be in the given ratio, which may be done with great ease.

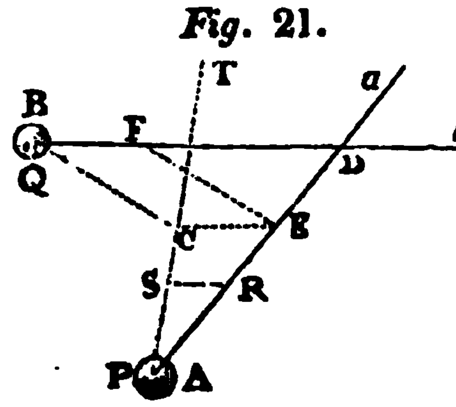
Let the force then with which P tends to move along its inclined plane be equal to M; and that of *p* to roll along its plane equal to *m*. Draw a line parallel to the plane A B, at a distance from it equal to the radius of the globe P, and another parallel to the plane D A, at a distance from it equal to the radius of *p*, which will intersect each other in G; having then made G L to G *l*, as *m* to M, employ the following proportion: as L *l* is to L G, so is the sum of the radii of the two globes to G C; and from the point C, draw C *c* parallel to L *l*: the points C and *c* will be the places of the centres of the two globes, and in this situation alone they will be in equilibrio.

PROBLEM XX.

Two bodies, P *and* Q, *depart at the same time from two points* A *and* B, *of two lines given in position, and move towards* a *and* b, *with given velocities: required their position when they are the nearest to each other possible?* (Fig. 21.)

If their velocities were to each other in the ratio of the lines B D and A D, it is evident that the two bodies would meet in D. But supposing their velocities different from that, there will be a certain point where, without meeting, they will be at the least distance from each other possible; and after that they will continually recede from each other. Here, for example, the lines B D and A D are nearly equal. If we suppose then that the velocity of P is to that of Q, in the ratio of 2 to 1, required the point of the nearest approach.

Fig. 21.



Through any point R, in A D, draw the line R S parallel to B D, and in such a manner, that A R shall be to R S, as the velocity of P is to that of Q; that is to say, in the present case, as 2 to 1; produce indefinitely the line A S T, and from the point B draw B C perpendi-

cular to A T; through the point C draw C E parallel to B D, till it meet A D in E; and having drawn E F parallel to C B, meeting B D in F, the points F and E will be those required.

PROBLEM XXI.

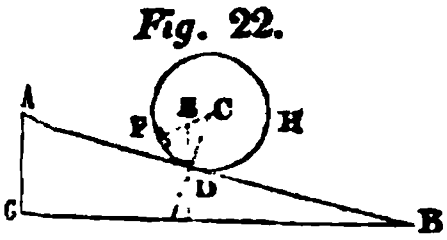
To cause a cylinder to support itself on a plane, inclined to the horizon, without rolling down; and even to ascend a little along that plane. (Fig. 22.)

If a cylinder be homogeneous, and placed on an inclined plane, its axis being in a horizontal situation, it is evident that it will roll down; because its centre of gravity being the same as that of the figure, the vertical line, drawn from this centre, will always fall beyond the point of contact of the lowest side; consequently the body must of necessity roll down towards that side.

But, if the cylinder be heterogeneous, so that its centre of gravity is not that of the figure, it may support itself on an inclined plane, provided the angle which the plane makes with the horizon does not exceed certain limits.

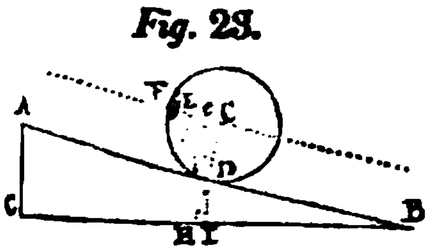
Let there be a cylinder, for example, of which H F D is a section perpendicular to the axis. To remove its centre of gravity from the centre of the figure, make a groove in it parallel to its axis, of a semi-circular form, and fill it with some substance F much heavier, so that the centre of gravity of the cylinder shall be removed from C to E. Let the inclined plane be A B, and let B G be to G A in a less ratio than C F to C E. The cylinder may then support itself on the inclined plane, without rolling down; and if it be moved from that position, in a certain direction, it will even resume it by rolling a little towards the summit of the plane.

Fig. 22.



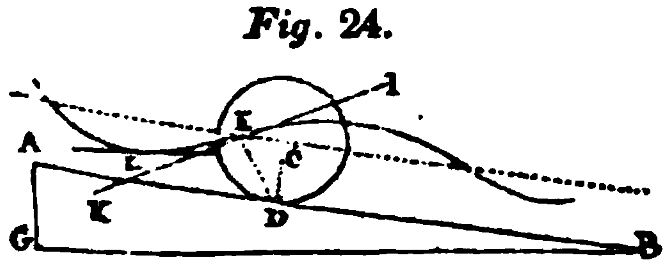
For, let us suppose the cylinder placed on the inclined plane with its axis horizontal, and its centre of gravity in a line parallel to the plane, and passing through the centre, in such a manner that the centre of gravity shall be towards the upper part of the plane (Fig. 23.) Through the point of contact, D, draw C D H perpendicular to the inclined plane, and I D *e* perpendicular to the horizon. We shall then have B G to G A, or B I to I D, as D I to I H, or as D C to C *e*; and since the ratio of B G to G A is less than that of C F or C D to C E, it follows that C *e* is less than C E, consequenly the vertical line drawn from the point E will fall without the point of contact towards A; the body therefore will have a tendency to fall on that side, and will roll towards it, ascending a very little till its centre of gravity E has assumed a position as seen Fig. 22, where it coincides with the vertical line passing through the point of contact. When the cylinder arrives at this situation, it will maintain itself in it, provided neither its surface nor that of the plane be so smooth as to admit of its sliding parallel to itself. In this situation it will even have greater stability, according as the ratio of B G to G A is less than that of C F or C D to C E, or as the angle A B G or C D *e* is less than C D E.

Fig. 23.



This is also a truth which we must demonstrate. For this purpose, it is to be remarked that E, the centre of gravity of the cylinder, in rolling along the inclined plane, describes a curve, such as is seen in Fig. 24.; this is what geometricians call an elongated cycloid, which rises and descends alternately below the line drawn parallel to the inclined plane, through the centre of the cylinder. But the cylinder being in the position represented in Fig. 24, if the line E D be drawn from the centre of gravity to the point of contact, it may be

Fig. 24.



demonstrated that the tangent to the point E of that curve is perpendicular to D E: if the inclination of the plane therefore is less than the angle C D E, that tangent will meet the horizontal line towards the ascending side of the plane; and the centre of gravity of the cylinder will then be as on an inclined plane I K; consequently it must descend to the point L of the hollow of the curve, which it describes, where that curve is touched by the horizontal line.

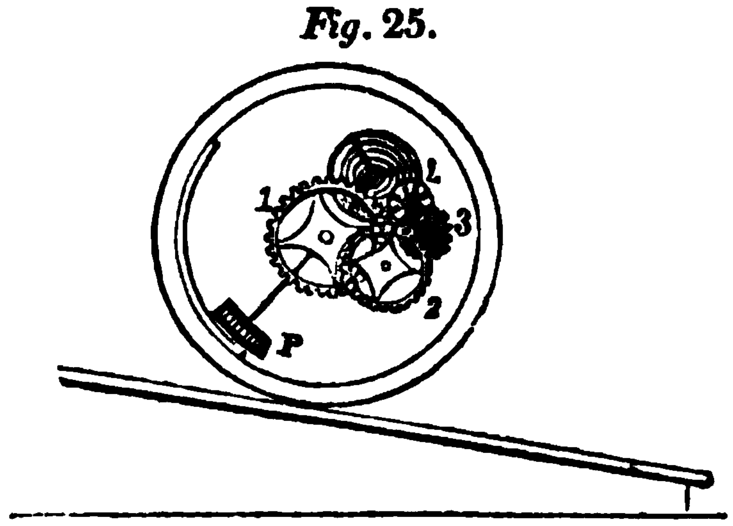
When it reaches this point it cannot deviate from it, without ascending on the one side or the other: if it be then removed a little from this point, it will return to its former position.

PROBLEM XXII.

To construct a clock which shall point out the hours, by rolling down an inclined plane.

This small machine, invented by an Englishman named Wheeler, is exceedingly ingenious, and is founded on the principle contained in the solution of the preceding problem.

Fig. 25.



It consists of a cylindrical box, made of brass, four or five inches in diameter, and having on one side a dial plate, divided into 12 or 24 hours. In the inside, represented by Fig. 25, is a central wheel, which by means of a pinion moves a second wheel, and the latter moves a third, &c., while a scapement, furnished with a balance or spiral spring, acts the part of a moderator, as in common watches. To the central wheel is affixed a weight P, which must be sufficient, with a moderate inclination, as 20 or 30 degrees, to move that wheel, and those which receive motion from it. But, as the machine ought to be perfectly in equilibrio around its central axis, a counteracting weight, of such a nature that the machine shall be absolutely indifferent to every position around this axis, must be placed diametrically opposite to the small system of wheels 2, 3, 4, &c. When this condition has been obtained, the moving weight P must be applied; the effect of which will be to make the central wheel, 1, revolve, and by its means the clock movement 2, 3, 4, &c.; but, at the same time that this motion takes place, the cylinder will roll down the plane a little, which will bring the weight P to its primitive position, so that the effect of this continual pressure will make the cylinder roll while the weight P changes its place relatively, in regard to the cylinder, but not in regard to the vertical line. The weight P, or the inclination of the plane, must be regulated in such a manner, that the machine shall perform a whole revolution in twenty-four or twelve hours. The handle must be affixed to the common axis of the central wheel and weight P; so that it shall always look towards the zenith or the nadir. If more ornaments are required, the axis may support a small globe with a figure placed on it, to point out the hours with its finger raised in a vertical position. It may be readily conceived, that when the machine has got to the lowest part of the inclined plane, to make it continue going, nothing will be necessary but to cause it to ascend to the highest. If it goes rather too slow, its movement may be accelerated by raising up the inclined plane, and *vice versa*.

PROBLEM XXIII.

To construct a dress, by means of which it will be impossible to sink in the water, and which shall leave the person, who wears it, at full freedom to make every kind of movement.

As a man weighs very nearly the same as an equal volume of water, it is evident that a mass of some substance much lighter than that fluid may be added to his body, by which means both together will be lighter than water, and of course must float. It is in consequence of this principle that, in order to learn to swim, some people tie to their breast and back two pieces of cork, or affix full blown bladders below their arms. But these methods are attended with inconveniences, which may be remedied in the following manner.

Between the cloth and lining of a jacket, without arms, place small pieces of cork, an inch and a half square, and about half or three quarters of an inch in thickness. They must be arranged very near to each other, that as little space as possible may be lost; but yet not so close as to affect in any degree the flexibility of the jacket, which must be quilted to prevent their moving from their places. The jacket must be made to button round the body, by means of strong buttons, well-sewed on; and to prevent its slipping off, it ought to be furnished behind with a kind of girdle, so as to pass between the thighs and fasten before.

By means of such a jacket, which will occasion as little embarrassment as a common dress, people may throw themselves into the water with the greatest safety; for if it be properly made the water will not rise over their shoulders. They will sink so little that even a dead body in that situation would infallibly float. The wearers therefore need make no effort to support themselves; and while in the water they may read or write, and even load a pistol and fire it. In the year 1767 an experiment was made of all these things, by the Abbé de la Chapelle, fellow of the Royal Society of London, by whom this jacket was invented.

It is almost needless to observe how useful this invention might be on land as well as at sea. A sufficient number of soldiers, provided with these jackets, might pass a deep and rapid river in the night time, armed with pistols and sabres, and surprise a corps of the enemy. If repulsed, they could throw themselves into the water, and escape without any fear of being pursued.

During sea voyages, the sailors, while employed in dangerous manœuvres, often fall overboard and are lost; others perish in ports and harbours by boats oversetting in consequence of a heavy swell, or some other accident; in short, some vessel or other is daily wrecked on the coasts, and it is not without difficulty that only a part of the crew are saved. If every man, who trusts himself to this perfidious element, were furnished with such a cork jacket, to put on during the moments of danger, it is evident that many of them might escape death.

PROBLEM XXIV.

To construct a boat which cannot be sunk, even if the water should enter it on all sides.

Cause a boat to be made with a false bottom, placed at such a distance from the real one, as may be proportioned to the length of the boat, and to its burthen and the number of persons it is intended to carry. According to the most accurate calculation, this distance, in our opinion, ought to be one foot, for a boat eighteen feet in length, and five or six in breadth. The vacuity between this false bottom and the real one ought to be filled up with pieces of cork, placed as near to each other as possible: and as the false bottom will lessen the sides of the boat, they may be raised proportionally; leaving large apertures, that the water thrown into the vessel may be able to run off. It may be proper also to make the stern higher, and to furnish it with a deck, that the people may take shelter under it, in case the boat should be thrown on its side by the violence of the waves.

Boats constructed in this manner might be of great utility for going on board a vessel lying in a harbour, perhaps several miles from the shore; or for going on shore from a ship anchored at a distance from the land. Unfortunate accidents too

often happen on such occasions, when there is a heavy surf, or in consequence of some sudden gust of wind; and it even appears that sometimes the greatest danger of a voyage is to be apprehended under circumstances of this kind. But boats constructed on the above principle would prevent such accidents.

Much we confess is to be added to this idea, presented here in all its simplicity; for some changes perhaps ought to be made in the form of the vessel; or heavy bodies ought to be added in certain places to increase its stability. This is a subject of research well worth attention, as the result of it might be the preservation of thousands of lives every year.

For this invention we are indebted to M. de Bernieres, one of the four controllers general of bridges and causeways: who in 1769 constructed a boat of this kind for the king. He afterwards constructed another with improvements for the Duke de Chartres; and a third for the Marquis de Marigny. The latter was tried by filling it with water, or endeavouring to make it overset; but it righted as soon as left to itself; and though filled with water, was still able to carry six persons.

By this invention the number of accidents which befall those who lead a seafaring life, may in future be diminished; but the indifference with which the invention of M. de Bernieres was received, shews how regardless men are of the most useful discoveries, when the general interests of humanity only are concerned, and when trouble and expense are required to render them practically useful.*

PROBLEM XXV.

How to raise from the bottom of the sea a vessel which has sunk.

This difficult enterprise has been several times accomplished by means of a very simple hydrostatical principle, viz., that if a boat be loaded as much as possible, and then unloaded, it tends to raise itself with a force equal to that of the weight of the volume of water which it displaced when loaded. And hence we are furnished with the means of employing very powerful forces to raise a vessel that has been sunk.

The number of boats employed for this purpose, must be estimated according to the size of the vessel, and by considering that the vessel weighs in water no more than the excess of its weight over an equal volume of that fluid; unless the vessel is firmly bedded in the mud; for then she must be accounted of her full weight. The boats being arranged in two rows, one on each side of the sunk vessel, the ends of cables, by means of divers, must be made fast to different parts of the vessel, so that there shall be four on each side, for each boat. The ends of these cables, which remain above water, are to be fastened to the head and stern of the boat for which they are intended. Thus, if there are four boats on each side, there must be thirty-two cables, being four for each boat.

When every thing is thus arranged, the boats are to be loaded as much as they will bear without sinking, and the cables must be stretched as much as possible. The boats are then to be unloaded, two and two, and if they raise the vessel, it is a sign that there is a sufficient number of them; but in raising the vessel, the cables affixed to the boats which remain loaded will become slack, and for this reason they must be again stretched as much as possible. The rest of the boats are then to be unloaded, by shifting their lading into the former. The vessel will thus be raised a little more, and the cables of the loaded boats will become slack; these cables being again stretched, the lading of the latter boats must be shifted back into the others, which will raise the vessel still a little higher; and if this operation be repeated as long as

* Vessels constructed on this principle, known under the name of Life Boats, are in very general use; and have been the means of saving the lives of many who would otherwise have perished by shipwreck. They were first constructed at Shields by Mr. Greathead; but a humble mechanic of the name of Wouldhave is said to have been the original inventor.

necessary, she may be brought to the surface of the water, and conveyed into port, or into dock.

An account of the manœuvres employed to raise, in this manner, the Tojo, a Spanish ship belonging to the Indian fleet, sunk in the harbour of Vigo, during the battle on the 10th of October 1702, may be seen in the "Mémoires des Academiciens étrangers," vol. II. But as this vessel had remained more than thirty-six years in that state, it was imbedded in a bank of tenacious clay, so that it required incredible labour to detach it; and when brought to the surface of the water, it contained none of the valuable articles expected. It had been one of those unloaded by the Spaniards themselves, before they were sunk, to prevent them from falling into the hands of the English.

Additions.—On the same principle is constructed the camel, a machine employed by the Dutch for carrying vessels heavily laden over the sand banks in the Zuyder-Zee. In that sea, opposite to the mouth of the river Y, about six miles from the city of Amsterdam, there are two sand banks, between which is a passage, called the Pampus, sufficiently deep for small vessels, but not for those which are large and heavily laden. On this account ships which are outward bound, take in before the city only a small part of their cargo, receiving the rest when they have got through the Pampus. And those that are homeward bound must in a great measure unload before they enter it. For this reason the goods are put into lighters, and in these transported to the warehouses of the merchants in the city; and the large vessels are then made fast to boats, by means of ropes, and in that manner towed through the passage to their stations.

Though measures were adopted, so early as the middle of the sixteenth century, by forbidding ballast to be thrown into the Pampus, to prevent the farther accumulation of sand in this passage, that inconvenience increased so much, from other causes, as to occasion still greater obstruction to trade; and it at length became impossible for ships of war and others heavily laden to get through it. About the year 1672, no other remedy was known, than that of making fast to the bottoms of ships large chests filled with water, which was afterwards pumped out, so that the ships were buoyed up, and rendered sufficiently light to pass the shallow. By this method, which was attended with the utmost difficulty, the Dutch carried out their numerous fleet to sea in the above-mentioned year. This plan however gave rise soon after to the invention of the camel, by which the labour was rendered easier. The camel consists of two half ships, constructed in such a manner that they can be applied, below water, on each side of the hull of a large vessel. On the deck of each part of the camel are a great many horizontal windlasses; from which ropes proceed through apertures in the one half, and, being carried under the keel of the vessel, enter similar apertures in the other, from which they are conveyed to the windlasses on its deck. When they are to be used, as much water as may be necessary is suffered to run into them; all the ropes are cast loose, the vessel is conducted between them, and large beams are placed horizontally through the port holes of the vessel, with their ends resting on the camel, on each side. When the ropes are made fast, so that the ship is secured between the two parts of the camel, the water is pumped from them, by which means they rise, and raise the ship along with them. Each half of the camel is generally 127 feet in length; the breadth at one end is 22, and at the other 13. The hold is divided into several compartments, that the machine may be kept in equilibrio, while the water is flowing into it. An East-India ship that draws 15 feet of water, can by the help of the camel be made to draw only 11; and the heaviest ships of war, of 90 or 100 guns, can be so lightened, as to pass without obstruction all the sand banks of the Zuyder-Zee.

Leupold, in his "Theatrum Machinarum," says that the camel was invented by

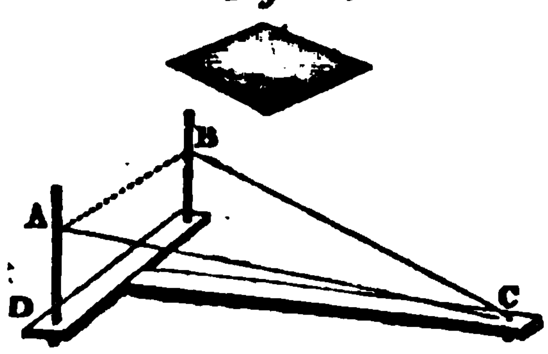
Cornelius Meyer, a Dutch engineer. But the Dutch writers, almost unanimously, ascribe this invention to a citizen of Amsterdam, called Meeuves Meindertszoon Bakker. Some make the year of the invention to have been 1688, and others 1690. However this may be, we are assured, on the testimony of Bakker himself, written in 1692, and still preserved, that in the month of June, when the water was at its usual height, he conveyed, in the course of twenty-four hours, by the help of the camel, a ship of war called *Maagt van Enkhuysen*, which was 156 feet in length, from *Enkhuysen Hooft*, to a place where there was sufficient depth; and that this could have been done much sooner, had not a perfect calm prevailed at the time. In the year 1693, he raised a ship called the *Unie*, six feet, by the help of this machine, and conducted her to a place of safety.

As ships built in the Newa cannot be conveyed into harbour, on account of the sand banks formed by the current of that river, camels are employed also by the Russians, to carry ships over these shoals: and they have them of various sizes. Bernoulli saw one, each half or which was 217 feet in length, and 36 in breadth. Camels are used likewise at Venice.*

PROBLEM XXVI.

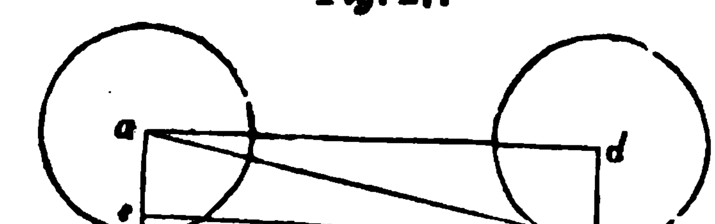
To make a body ascend as if of itself along an inclined plane, in consequence of its own gravity.

Fig. 26.



Provide a double cone (Fig. 26.), that is to say two right cones united at their bases, so as to have a common axis. Then make a supporter, consisting of two branches, forming an angle at the point C, which must be placed in such a manner that the summit C shall be below the horizontal line, and that the two branches or legs shall be equally inclined to the horizon. The line A B must be equal to the distance between the summits of the double cone, and the height A D a little less than the radius of the base. These conditions being supposed, if the double cone be placed between the legs of this angle, it will be seen to roll towards the top; so that the body, instead of descending, will seem to ascend, contrary to the affection of gravity: this however is not the case; for its centre of gravity really descends, as we shall here shew.

Fig. 27.



Let *a c* (Fig. 27.) be the inclined plane, containing the angle A C B; *c e* the horizontal line, passing through the summit *c*, and consequently *e a* will be the elevation of the plane above the horizontal line, which is less than the radius of the circle forming the base of the double cone. It is evident that when this double cone is at the summit of the angle, it will be as seen at *c d*; and when it reaches the highest part of the plane, it will have the position seen at *a f*: its centre then will have passed from *d* to *a*, and since *d c* is equal to *a f*, and *c e* is the horizontal line, *c f* will be a line declining below the horizon; and consequently *d a*, which is parallel to it, will be so also. The centre of gravity of the cone will therefore have descended, while the cone appeared to ascend. But, as has been already seen, it is the descent or ascent of the centre of gravity that determines the real descent or ascent of a body. As long as the centre of gravity can descend, the body therefore really moves in that direction, &c.

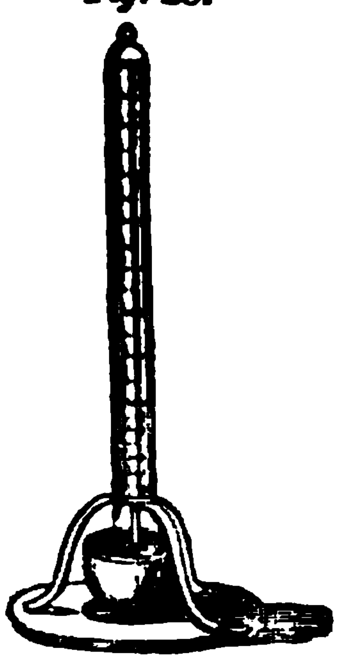
* An engraving of the camel may be seen in "L' Art de batir les Vaisseaux;" Amsterdam, 1719. 4to. vol. ii. p. 98. See also the "Encyclopedie," Paris edition, vol. iii. p. 67.

It will be found, in the present case, that the course of the centre of gravity, in its whole descent, is a straight line. But a parabola or hyperbola might be situated in the same manner, with its summit downwards, and in that case the course of the centre of gravity of the double cone would be a curve. This may furnish a subject of exercise for young geometricians.

PROBLEM XXVII.

To construct a clock with water. (*Fig.* 28.)

Fig. 28.



If the water which issues from a cylindric vessel, through a hole formed in its bottom, flowed in a uniform manner, nothing would be easier than to construct a clock, to indicate the hours by means of water. But it is well known that the greater the height of the water above the orifice through which it issues, the greater is the rapidity with which it flows; so that the vertical divisions ought not to be equal: the solution of the problem therefore consists in determining their ratio.

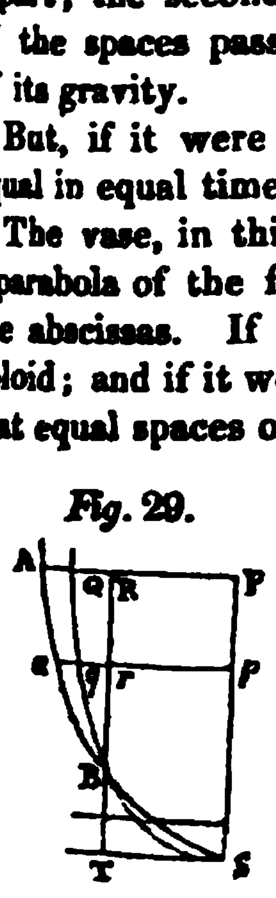
It is demonstrated in hydraulics, that the velocity with which water flows from a vessel, through a very small orifice, is proportional to the square root of the height of the water above the aperture. And hence the following rule, for dividing the height of the vessel, which we suppose to be cylindrical, has been deduced.

If we suppose that the whole water can flow out in twelve hours; divide the whole eight into 144 parts; then 23 of these will be emptied in the first hour; so that there will remain 121 for the other eleven. Of these 121 parts, 21 will be emptied during the second hour; then 19 will be emptied in the third, 17 in the fourth, and so on. As the 144th division therefore corresponds to twelve hours, the 121st will correspond to eleven; the 100th to ten; the 81st to nine, &c.; till the last hour, during which only one division will be emptied. These divisions will comprehend in the retrograde order, beginning at the lowest, the first, 1 part; the second, 3; the third, 5; the fourth, 7, &c.; which is exactly the ratio of the spaces passed over in equal times by a body falling freely in consequence of its gravity.

But, if it were required that the divisions, in the vertical direction, should be equal in equal times, what figure ought to be given to the vessel?

The vase, in this case, ought to be a paraboloid, formed by the circumvolution of a parabola of the fourth degree; or the biquadrates of the ordinates ought to be as the abscissas. If an orifice of a proper size were made in the summit of this paraboloid; and if it were then inverted; the water would flow from it in such a manner, that equal spaces of the vertical height would be emptied in equal times.

Fig. 29.



The method of describing this parabola is as follows. Let A B S (Fig. 29.) be a common parabola, the axis of which is P S, and the summit S. Draw, in any manner, the line R r T, parallel to that axis, and then draw any ordinate of the parabola A P, intersecting R T in R; make P Q a mean proportional between P R and P A; and let $p\,q$ be a mean proportional also between $p\,r$ and $p\,a$; and so on. The curve passing through all the points Q, q, &c., will be the one required; and it may be employed to form a mould for constructing a vessel of the required concavity. To whatever height it shall be filled with water, equal heights will always be emptied in equal times.

In another part of this work, we shall give a method of making equal quantities

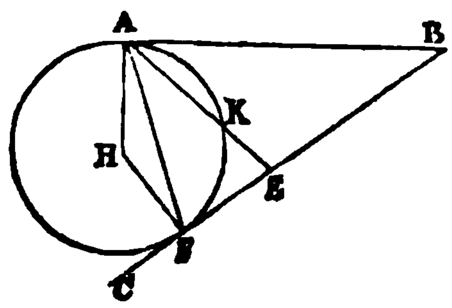
of water flow from a vessel of any form in equal times. As this depends on the property of the siphon, it belongs to a different head.

PROBLEM XXVIII.

A point being given, and a line not horizontal, to find the position of the inclined plane along which, if a body descend, setting out from the given point, it shall reach that line in the least time.

The following solution to this curious problem is by W. Rutherford, Esq., of the Royal Military Academy, Woolwich, and we give it here as much preferable to that by Montucla.

Fig. 30



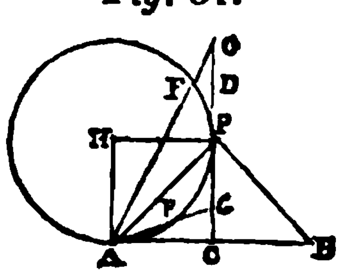
Let A be the given point, and B C the given line. Through A draw A B parallel to the horizon meeting C B in B; make B F = B A, and join A F, and it is the line of quickest descent.

Now draw F H perpendicular to B C meeting the vertical through A in H. Then the angles B A H and B F H being right-angles, are equal,—and because B A and B F are equal, B A F and B F A are equal; therefore the angles H A F and H F A are equal,—and so therefore are H A and H F. Hence a circle described from H on a centre, with radius H A or H F, will touch A B and B F at A and F. And drawing any line A K E from A to B C, and cutting the circle at K, the time of descent down A F is equal to that down A K, and it is therefore less than the time down A E.

Mr. Rutherford solves also the kindred problem, "To determine the slope of a roof of a given width down which the rain may descend in the least time."

Let A B be the breadth of the building, and C its point of bisection; and through A B draw the vertical line C D. Make C P = A C, and draw A P, P B: then A P B is the roof down which the rain or any heavy body will descend in the least time.

Fig. 31.



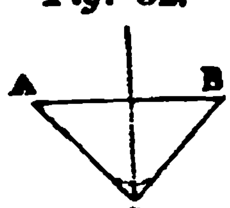
For draw P H parallel to A B meeting the vertical drawn through A at H. Then H P = H A, and a circle described from H, with radius H A or H P, will touch A B and C D at A and P: now the time of descent from P to A is the same as that from F to A, and is consequently less than that from G to A, and A P B is therefore the roof down which rain will descend the quickest.

PROBLEM XXIX.

Two points A *and* B *being given in the same horizontal line; required the position of two planes* A C *and* C B, *of such an inclination, that two bodies descending with accelerated velocity from* A *to* C, *and then ascending along* C B *with the acquired velocity, shall do so in the least time possible.* (Fig. 32.)

It is evident that a body placed at A, on the horizontal line A B, would remain there eternally without moving towards B. To make it proceed therefore by the effect of its own gravity from A to B, it must fall along an inclined plane or a curve; so that, after having descended a certain space, it shall ascend along a second plane, or the remainder of the curve, as far as B. But we shall suppose that this is done by means of two inclined planes. It is here to be observed, that the time employed to descend and ascend, must be longer or shorter according to the inclination and the length of these planes. The question then is, to determine what position of them is most advantageous, in order that the time may be the least. Now it will be found that to obtain the required position, the two planes must be equal and inclined to the horizon at an angle of 45°; that is to say, the triangle A C B ought to be isosceles and right-angled at C.

Fig. 32.

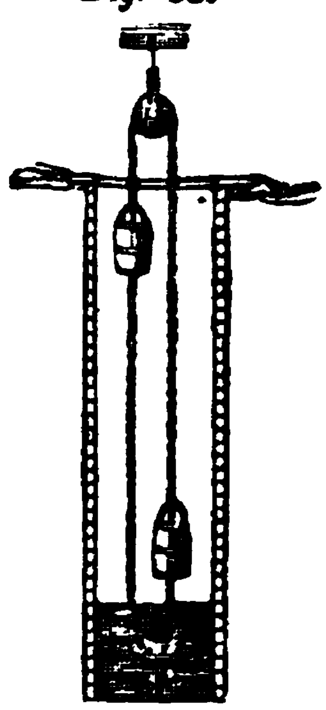


This solution is deduced from that of the preceding problem; for if we conceive a vertical line drawn through the point C, it has been shewn that the plane A C, inclined at an angle of 45 degrees, is the most favourably disposed to make the body, sliding along it, arrive at the vertical line in the least time possible; but the time of the ascent along C B, is equal to that of the descent; whence it follows that their sum, or the double of the former, is also the shortest possible.

PROBLEM XXX.

If a chain and two buckets be employed to draw up water from a well of very great depth; it is required to arrange the apparatus in such a manner, that in every position of the buckets, the weight of the chain shall be destroyed; so that the weight to be raised shall be that only of the water contained in the ascending bucket. (Fig. 33.)

Fig. 33.



If two buckets be suspended from the two ends of a rope or chain, so as to ascend and descend alternately, while the rope rolls round the axis or wheel of the windlass which serves to raise them, it is evident that when one of the buckets is at the bottom, the person who begins to raise it has not only the weight of the bucket to support, but that also of the whole chain or rope from the top to the bottom of the well; and there are some cases, as in mines of three or four hundred feet in depth, where the weight of several quintals must be overcome to raise only two or three hundred pounds to the mouth of the mine. Such were the mines of Pontpeau, until M. Loriot suggested a remedy for this inconvenience.

This remedy is so simple, that it is astonishing no one ever thought of it before. Nothing indeed is necessary but to convert the rope or chain into a complete ring, one of the ends of which descends to the depth where the water or the ore is to be drawn up, and to affix the buckets to two points of the rope in such a manner that when one of them is at the highest part, the other shall be at the lowest. For it is evident that, as equal parts of the chain ascend and descend, these parts will counterbalance each other; and the weight to be raised, were the pit several thousand feet in depth, will be that only of the ore or other substances drawn up.

The case would evidently be the same if there were only one bucket: in every position, the only weight to be raised would be that of the bucket, and the matter it contained; but the machine would be attended with only one half of its advantage; for, by having no more than one bucket, the time which the bucket when emptied would employ in descending would be lost.

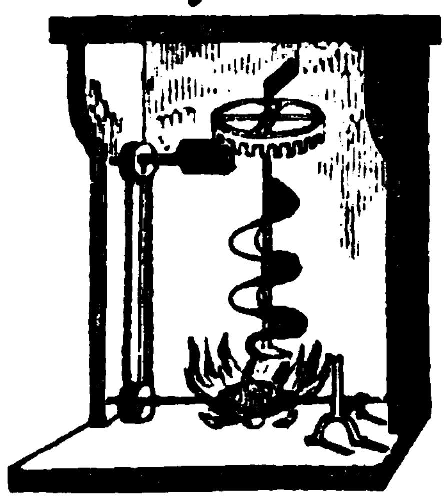
Remark.—In the Memoirs of the Academy of Sciences for 1731, M. Camus gave another method of remedying the above inconvenience. It consists, when there is only one bucket, in employing an axis nearly in the form of a truncated cone; so that when the bucket is at the lowest depth, the rope is rolled round the part which has the least diameter; and when the bucket is at the top, it is rolled round that which has the greatest. By these means, the same force is always required. But it is evident that, in every case, more must be applied than is necessary.

When there are two buckets, M. Camus proposes that one half of the rope should be rolled round one half of the axis, which he divides into two equal parts; so that one half is covered by the rope belonging to the bucket raised up, while the other is uncovered, the bucket which corresponds to it being at the bottom. By these means the two efforts are combined in such a manner, that nearly the same force is always required to overcome them. But these inventions, though ingenious, are inferior to that of M. Loriot.

PROBLEM XXXI.

Method of constructing a jack which moves by means of the smoke of the chimney. (Fig. 34.)

Fig. 34.



The construction of this kind of jack, which is very ingenious, is as follows. An iron bar fixed in the back of the chimney, and projecting from it about a foot, serves to support a perpendicular spindle, the extremity of which turns in a cavity formed in the bar; while the other extremity is fitted into a collar in another bar, placed at some distance above the former. This spindle is surrounded with a helix of tin plate, which makes a couple of revolutions, or turns round the spindle, and which is about a foot in breadth. But instead of this helix, it will be sufficient to cut several pieces of tin plate, or sheet-iron, and to fix them to the spindle in such a manner that their planes shall form with it an angle of about 60 degrees; they must be disposed in several stories, above each other; so that the upper ones may stand over the vacuity left by the lower ones. The spindle, towards it summit, bears a horizontal wheel, the teeth of which turn a pinion having a horizontal axis, and the latter, at its extremity, is furnished with a pulley, around which is rolled the endless chain that turns the spit. Such is the construction of this machine, the action of which may be explained in the following manner. When a fire is kindled in the chimney, the air which by its rarefaction immediately tends to ascend, meeting with the helicoid surface, or kind of inclined vanes, causes the spindle, to which they are affixed, to turn round, and consequently communicates the same motion to the spit. The brisker the fire becomes, the quicker the machine moves, because the air ascends with greater rapidity.

When the machine is not used, it may be taken down, by raising the vertical spindle a little, and removing the point from its cavity; which will allow the summit to be disengaged from the collar in which it is made to turn. When wanted for use, it may be put up with the same ease.

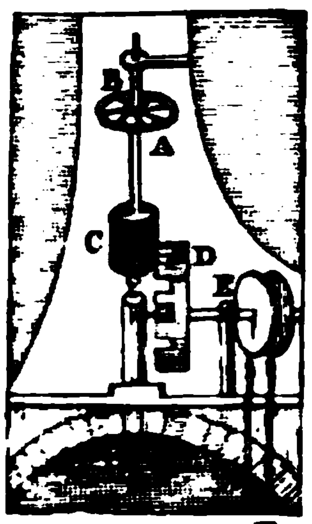
Remarks.—1st. The following mechanical amusement is founded on the same principle. Cut out from a card as large a circle as possible; then cut in this circle a spiral, making three or four revolutions, and ending at a small circle, reserved around the centre, and of about a line or two in diameter; extend this spiral by raising the centre above the first revolution, as if it were cut into a conical surface or paraboloid; then provide a small spit made of iron, terminating in a point, and resting on a supporter. Apply the centre or summit of the helix to this point: and if the whole be placed on the top of a warm stove, the machine will soon put itself in motion, and turn without the assistance of any apparent agent. The agent however in this case is the air, which is rarefied by the contact of a warm body, and which ascending forms a current.

2d. There is no doubt that a similar invention might be applied to works of great utility: it might be employed, for example, in the construction of wheels to be always immersed in water, their axis being placed parallel to the current: to give the water more activity this helicoid wheel might be inclosed in a hollow cylinder, where the water, when it had once entered, being impelled by the current above it, would in our opinion act with a great force.

If the cylinder were placed in an erect position, so as to receive a fall of water through the aperture at the top, the water would turn the wheel and its axis, and

might thus drive the wheel of a mill, or of any other machine. Such is the principle of motion employed in the wheels of Basacle, a famous mill at Toulouse.

*Fig. 34.**



3d. The smoke jacks here in England are made somewhat different from that above described; being mostly after the manner of that exhibited in Fig. 34*, where A B is a circle containing the smoke vanes, of thin sheet iron, all fixed in the centre, but set obliquely at a proper angle of inclination. The other end of the spindle has a pinion C, which turns the toothed wheel D, on the spindle of which is fixed the vertical wheel E, over which passes the chain E F, which turns the spit below. There are other forms of this useful machine also made; but all or most of them having the same kind of vanes in the circle A B, instead of the spiral form in the original.

PROBLEM XXXII.

What is it that supports, in an upright position, a top or tetotum, while it is revolving?

It is the centrifugal force of the parts of the top, or tetotum, put in motion. For a body cannot move circularly without making an effort to fly off from the centre; so that if it be affixed to a string, made fast to that centre, it will stretch it, and in a greater degree according as the circular motion is more rapid.

The top then being in motion, all its parts tend to recede from the axis, and with greater force the more rapidly it revolves; hence it follows that these parts are like so many powers acting in a direction perpendicular to the axis. But as they are all equal, and as they pass all round with rapidity by the rotation, the result must be that the top is in equilibrio on its point of support, or the extremity of the axis on which it turns.

PROBLEM XXXIII.

How comes it that a stick, loaded with a weight at the upper extremity, can be kept in equilibrio, on the point of the finger, much easier than when the weight is near the lower extremity; or that a sword, for example, can be balanced on the finger much better, when the hilt is uppermost?

The reason of this phenomenon, so well known to all those who perform feats of balancing, is as follows. When the weight is at a considerable distance from the point of support, its centre of gravity, in deviating either on the one side or the other from a perpendicular direction, describes a larger circle than when the weight is very near to the centre of rotation, or the point of support. But in a large circle an arc of a determinate magnitude, such as an inch, describes a curve which deviates much less from a horizontal direction than if the radius of the circle were less. The centre of gravity of the weight then may, in the first case, deviate from the perpendicular the quantity of an inch, for example, without having a tendency or force to deviate more, than it would in the second case; for its tendency to deviate altogether from the perpendicular is greater, according as the tangent to that point of the arc where it happens to be approaches more to a vertical direction. The greater therefore the circle described by the centre of gravity of the weight, the less is its tendency to fall, and consequently the greater the facility with which it can be kept in equilibrio.

PROBLEM XXXIV.

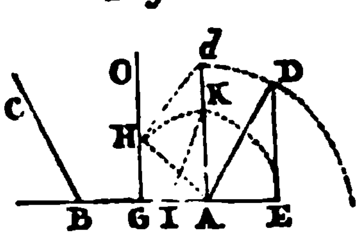
What is the most advantageous position of the feet for standing with firmness, in an erect posture?

It is customary among well-bred people to turn their toes outwards; that is to

say, to place their feet in such a manner, that the line passing through the middle of the sole is more or less oblique to the direction towards which the person is turned. Being induced by this circumstance to inquire whether this custom, to which an idea of gracefulness is attached, be founded on any physical or mechanical reason, we shall here examine it according to the principles of mechanics.

Every body whatever rests with more stability on its base, according as its centre of gravity, on account of its position and the extent of that base, is less exposed to be carried beyond it by the effect of any external shock. The problem then, in consequence of this very simple principle, is reduced to the following: To determine whether the base, within which the line drawn perpendicular to the horizon from the centre of gravity of the human body ought to fall, is susceptible of increase and diminution, according to the position of the feet; and what is the position of the feet which gives to that base the greatest extent. But this becomes a problem of pure geometry, which might be thus expressed: *Two lines* A D *and* B C (Fig. 35.) *of equal length, and moveable around the the points* A *and* B, *as centres, being given; to determine their position when the trapezium or quadrilateral* A B C D *is the greatest possible.* This problem may be solved with the greatest facility, by methods well known to geometricians; and from the solution the following construction is deduced.

Fig. 35.



On the line A d equal to A D, or B C, construct the isosceles triangle A H d, right-angled at H; and make A K equal to A H. Having then assumed A I equal to one half of A G, or one fourth of A B, draw the line K I, and make I E equal to I K: on G E, if an indefinite perpendicular, intersecting in D, the circle described from the point A as a centre, with the radius A D, be then raised, the point D, or the angle D A E, will determine the position of A D, and consequently of B C. If the line A B, and consequently A G or A I, be nothing, or vanish, A E will be found equal to A H; and the angle D A E will be half a right one. Thus, when the heels absolutely touch each other, the angle which the longitudinal lines of the soles of the feet ought to form, is half a right one, or nearly so, on account of the small distance which is then between the two points of rotation, in the middle of the heels.

If the distance A B is equal to A D, the angle D A E ought to be 60 degrees; if A B is equal to twice A D, the angle D A E ought to be nearly 70 degrees; and in the last place, if A B be equal to three times the line A D, it will be found that D A E ought to be nearly 74° 30.

It is thence seen, that in proportion as the feet are at a greater distance from each other, their direction, in order to stand or walk with more stability, ought to approach nearer to parallelism. But, in general, mechanical principles accord with what is taught by custom and gracefulness, as it is called; that is to say, to turn the toes outwards.

PROBLEM XXXV.

Of the game of Billiards.

It is needless to explain here the nature of billiards. It is well known that this game is played on a table covered with green cloth, properly stretched, and surrounded by a stuffed border, the elasticity of which forces back the ivory balls that impinge against it. The winning strokes at this game, are those which, by driving your ball against that of your adversary, force the latter into one of the holes at the corners, and in the middle of the two longer sides, which are called pockets.

The whole art of this game then consists in being able to know in what manner you must strike your adversary's ball with your own, so as to make it fall into

one of the pockets, without driving your own into it also. This problem, and some others belonging to the game of billiards, may be solved by the following principles.

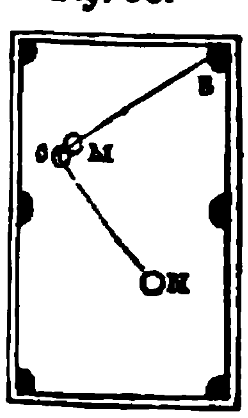
1st. The angle of the incidence of the ball against one of the edges of the table, is equal to the angle of reflection.

2d. When a ball impinges against another, if a straight line be drawn between their centres, which will consequently pass through the point of contact, that line will be the direction of the line described after the stroke.

These things being premised, we shall now give a few of the problems which arise out of this game.

I. *The position of the pocket and that of the two balls* M *and* N *being given* (Fig. 36.) *to strike your adversary's ball* M *in such a manner, that it shall fall into the pocket.*

Fig. 36.

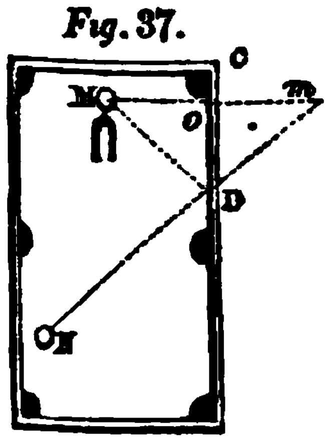


Through the centre of the given pocket and that of the ball, draw, or conceive to be drawn, a straight line; the point where it intersects the surface of the ball, on the side opposite to or farthest from the pocket, will be that where it ought to be touched, in order to make it move in the required direction. If we then suppose the above line continued from one of the radii of the ball, the point *o*, where it terminates, will be that through which the impinging ball ought to pass. It may be readily conceived, that it is in this that the whole dexterity of the game consists; nothing being necessary, but to strike the ball in the proper manner. It is easy to see what ought to be done, but it is not so easy to perform it.

In the last place, it is evident from what has been said, that provided the angle NOB exceeds a right angle ever so little, it is possible to drive the ball M into the pocket.

II.—*To strike the ball by reflection.*

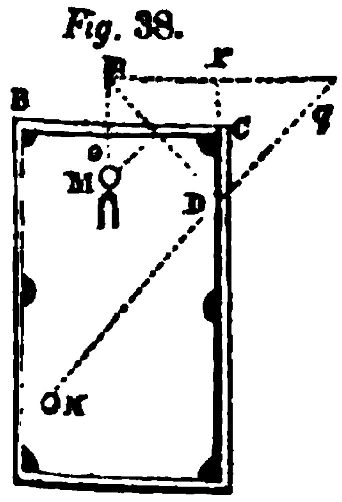
Fig. 37.



The ball M (Fig. 37) being concealed, or almost concealed, behind the iron, in regard to the ball N, so that it would be impossible to touch it directly, without running the risk of striking the iron and failing in the attempt; it is necessary, in that case, to try to touch it by reflection. For this purpose, conceive the line M *o*, drawn perpendicular from M to the edge D C, to be continued to *m*: so that *o m* shall be equal to *o* M. If you aim at the point *m*, the ball N, after touching the edge D C, will strike the ball M.

If it were required to strike the ball M (Fig. 38.) by two reflections, the geometrical solution, in this case, is as follows. Conceive the line M O, drawn perpendicular from the point M, to the edge B C, to be continued till *o m* become equal to *o* M. Conceive also the line *m* P, drawn perpendicular from the point *m* to the edge continued, to be continued to *q*, until P *q* be equal to P *m*; if the ball N be directed to the point *q*, after impinging against the edges D C and C B, it will strike the ball M.

Fig. 38.

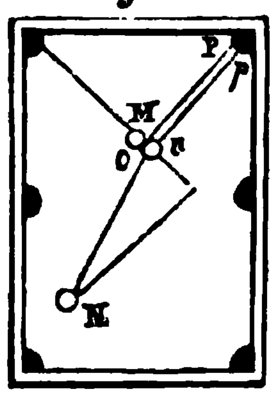


To those in the least acquainted with geometry, the demonstration of this problem will be easy.

III.—*If a ball strikes against another in any direction whatever, what is the direction of the impinging ball after the shock?*

It is of importance, at the game of billiards, to be able to know what will be the direction of your own ball after it strikes that of your adversary obliquely; for every one knows that it is not sufficient to have touched the latter, or to have driven it into the pocket; you must also prevent your own from falling into it.

Fig. 39.



Let M and N (Fig. 39.) be the two balls, the latter of which is to strike the former, touching it in the point *o*. Through this point *o*, let there be drawn the tangent *o* P; and through the centre *n*, of the ball N, when it arrives at the point of contact, draw or conceive to be drawn *n p*, parallel to *o* P: the direction of the impinging ball, after the shock, will be *n p*. A bad player would here be infallibly lost; and indeed this is often the case in this position of the balls. Expert players, when they find that they have to do with novices, often give them this deceitful chance, which makes them lose, by driving their ball into one of the corner pockets. In this case you must not take the ball of your adversary by halfs, according to the technical term of the game, to drive it to one of the corners at the other end of the table; for in doing so, you will not fail to lose yourself in the other corner.

Remark.—In reasoning on this game, we set out from common principles; but we must confess that we have some doubts on this subject, the reason of which we shall here explain.

If the balls had only one progressive movement forwards, without rotation around their centres, the above principles would be evidently and sufficiently demonstrated. But every one knows that, independently of this progessive motion of the centre, a billiard ball rolls upon the table in a plane which is perpendicular to it. When a ball then touches the edge, and is repelled with a force nearly equal to that with which it impinged, it would appear that this motion ought to be compounded of the rotary motion it had at the moment of the shock, and that which it has in a direction parallel to the edge. But since the first of these motions compounded with the latter, gives the angle of reflection equal to the angle of incidence, what then becomes of the second, which ought to alter the first result? In our opinion, this is a dynamical problem, which has never yet been solved, though it deserves to be so.

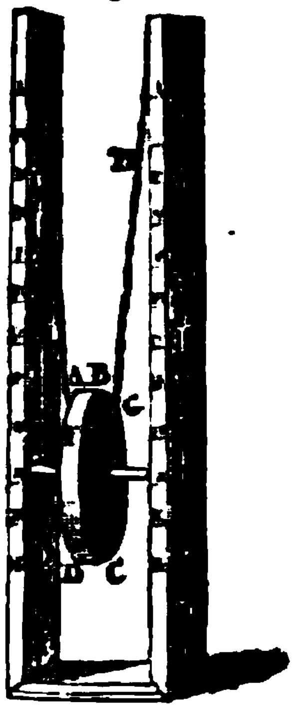
However, this rotary motion, in certain circumstances, gives a result which seems contrary to the laws of the impinging of elastic bodies; for, according to this law, when an elastic body impinges directly and centrally against another which is equal to it, the first ought to stop, in consequence of having communicated, as is supposed, all its velocity to the second. But at the game of billiards, this does not take place; for here the impinging ball continues to move, instead of stopping short. This effect is partly a consequence of the motion of the impinging ball around its centre; a motion which subsists in a great measure after the shock, and it is this motion partly which makes the ball still move forwards. Another cause of the striking ball's moving forward, is the want of perfect elasticity in them both, on which account that ball still retains some portion of its direct forward motion, the other ball, which is struck, receiving the rest of the motion.

PROBLEM XXXVI.

To construct a Water Clock.

This name is given to a clock shaped like a drum or barrel, as A B C D (Fig. 40.), made of metal well soldered, and put in motion by a certain quantity of water

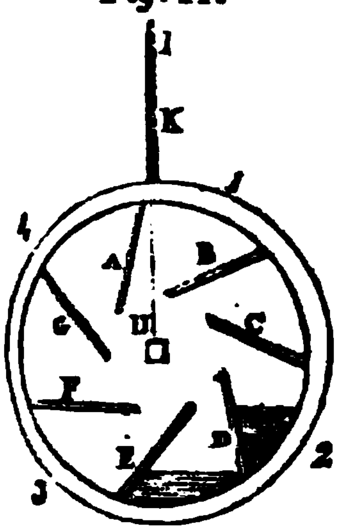
Fig. 40.



contained in the inside of it. The hours are indicated on two vertical pillars, between which it is suspended by small strings or cords, rolled round an axis, every where of the same thickness. The internal mechanism is exceedingly ingenious, and deserves a better explanation than what has been given of it in the preceding editions of the Mathematical Recreations, where Ozanam does not tell us how the machine goes and is supported, as we may say, in the air, without falling, as it seems it ought to do.

Let the circle 1 2 3 4 (Fig. 41.) represent a section of the drum or cylinder, by a plane perpendicular to its axis. We shall here suppose the diameter of it to be six inches; and let A, B, C, D, E, F, G, represent seven cells, the partitions of which are formed of the same metal, and are well soldered to the two circular ends, and to the circular band which forms the circumference. These partitions ought not to proceed from the centre to the circumference, but to be placed in a somewhat transverse direction, so as to be tangents to an interior circle, of about an inch and a half in diameter: the small square H is a section of the axis, which in that part ought to be square, and to fit very exactly into holes, of the same form, made in the centre of each end of the cylinder. Each partition also ought to have in it a small round hole, as near as possible to the circumference of the cylinder, all pierced with the same piercer, that there may be no difference among them.

Fig. 41.



Let us now suppose that a certain quantity of water, about eight or nine ounces, has been put into the cylinder, and that it has already distributed itself as shewn by the horizontal shading lines Fig. 41. If the line I K represent the two strings, G H and E F (Fig. 40.), rolled round the axis of the cylinder, it may be easily seen that the centre of gravity, which, if the machine were empty, would be in the centre of the figure, being thrown out of the line of suspension, and towards the side where the machine has a tendency to fall, it would indeed fall; but the effect of the water behind the partition D, is to throw back the centre of gravity, so that if it were on this side the vertical line K I continued, the cylinder would revolve from D to E, in order to be in that vertical; and in this position the machine would remain in equilibrio, if the water could not proceed from the one cavity to the other; for the cylinder cannot revolve in the direction A G F, without making the centre of gravity ascend towards D: in the like manner it cannot revolve in the direction B C D, without the centre rising on the opposite side. The machine must then remain in equilibrio, until something is changed.

But, if the water flows gradually through the hole in the partition D, which is between the cells D and E, it is evident that the centre of gravity will advance a little beyond K I continued, and the machine will imperceptibly revolve in the direction A G F: and since by descending in this manner, the centre of gravity is thrown towards the vertical line K I produced, the equilibrium will at the same time be restored, and this motion will continue until the whole of the cord be unrolled from the axis. This movement indeed will not be altogether uniform; for it is evident that when the water is almost entirely behind the partition D, the cylinder will revolve faster than when it has nearly flowed off; and the periods of these inequalities, during a whole revolution of the cylinder, will be equal in number to the cells; a circumstance which seems not to have been observed by those who have written on clocks of this kind.

To have an exact division of time by these means, it will therefore be necessary to make a mark on the circumference of the cylinder. If the machine be then wound up as high as possible, and disposed in such a manner that the mark shall be at the top of the cylinder, you must have a good clock, with which to mark, during a whole revolution, the points of the hours elapsed. But care must be taken that the number of hours shall be an integer number, as 2, 4, 6, &c. : and for that purpose the movement of the machine must be retarded or accelerated till the proper precision has been obtained; otherwise it might err some minutes, and perhaps a quarter of an hour. How this movement may be accelerated or retarded, we shall shew hereafter.

In the last place, in winding up the clock, great care must be taken that when the axis is placed opposite to the first division, the mark made in the cylinder shall be in the same position; otherwise there may be an error, as already said, of some minutes. We shall now add some useful observations in regard to this object.

I. It is absolutely necessary that the water employed be distilled water; otherwise it will soon become corrupted, so as to stop up the holes through which it ought to flow; and the machine will consequently stand still.

II. The substance most proper for constructing the cylinder of these machines, is gold or silver; or what is cheaper, copper well tinned on the inside, or even tin itself.

III. This machine is apt to go a little faster in summer than in winter, and therefore ought to be regulated from time to time, and retarded or accelerated. For this purpose it will be necessary to add to it a small weight as a counterpoise, tending to make it revolve outward. This weight ought to have the form of a bucket (Fig. 42.), and to be of some light substance, so that it can be charged more or less by means of small drops of lead. To accelerate the machine, two or three drops of lead may be added; and when it is necessary to retard it, they may be removed; which will be much more convenient than adding or taking away water.

Fig. 42.

IV. The place where the axis passes through the cylinder must be well cemented; otherwise the water would gradually evaporate, by which means the machine would be continually retarded, and at length would stop.

V. Notwithstanding all these precautions, it may be readily seen that a machine of this kind is rather an object of curiosity, than calculated to measure time with accuracy. It may be proper for the cell of a convent, or a cabinet of mechanical curiosities; but it will certainly never be used by the astronomer.

VI. The inventor of this kind of clock is not known. Ozanam, who wrote in 1693, says that the first seen at Paris about that period had been brought from Burgundy; and he adds that Father Timothy, a Barnabite, who excelled in mechanics, had given to this machine all the perfection of which it was susceptible. This monk had constructed one about five feet in height, which required winding up only once a month. Besides the hours, which were marked on a regular dial-plate, at the top of the frame, it indicated the day of the month, the festivals throughout the year, the sun's place, and his rising and setting, as well as the length of the day and night. This was performed by means of a small figure of the sun, which gradually descended, and which, when it reached the bottom of the frame, was raised to the top at the end of every month.

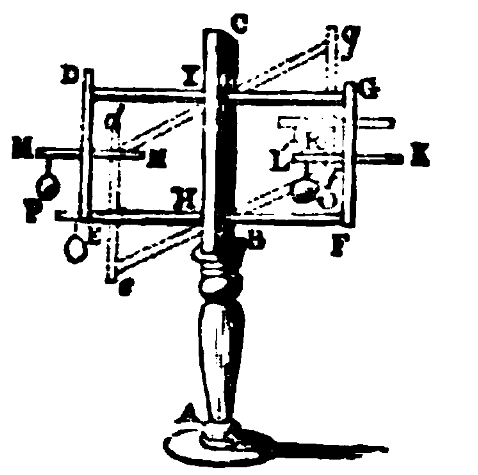
Father Martinelli has treated, at great length, on these clocks, in an Italian work, entitled "Horologi Elementari," in which he delivers methods of making clocks by means of the four elements, water, earth, air, and fire. This work was printed at Venice in 1663, and is very rare. The author shows in it how striking machinery may be adapted to a water-clock; with other curiosities, which are sometimes added to common clocks.

PROBLEM XXXVII.

MECHANICAL PARADOX.—*How equal weights, placed at any distance from the point of support of a balance, shall be in equilibrio.*

Provide a frame in the form of a parallelogram, such as D E F G (Fig. 43.), constructed of four pieces of wood, joined together in such a manner as to move freely at the angles, so that the frame can change its rectangular form into that represented by the letters *e f g d*. The long sides ought to be about twice the length of the others.

Fig. 43.



This frame inserted in a cleft formed in the perpendicular stand B C, so as to be moveable on the two points I and H, where it is fixed in the stand by two small axes: in the last place, two pieces of wood, M N and K L, pass through the shorter sides, in which they are well fixed, and the whole apparatus rests on the stand A B.

Now if the weight P be suspended from the point M, which is almost at the extremity of the arm M N, the most distant from the centre or centres of motion; and if the weight O, equal to the former, be suspended from any point R, of the other arm K L, nearer the centre, and even within the frame, these two weights will always be in equilibrio; though unequally distant from the point of support or of motion in this kind of balance; and they will remain so, whatever situation may be given to the machine, as *e f d g*.

The reason of this effect which at first seems to contradict the principles of statics, is however very simple. For two equal bodies will be in equilibrio, whatever movement may be made by the machine from which they are suspended, if the spaces passed over by these two bodies or weights are equal and similar. But it may be readily seen that this must necessarily be the case here, since the two weights, whatever be their position, are obliged to describe equal and parallel lines.

It may be readily seen also, that, in such a machine, whatever be the position of the weights along the arms M N and K L, the case will always be the same, as if they were suspended from the middle of the short sides E D and F G. But in the latter case, the weights would be in equilibrio, therefore in former also.

PROBLEM XXXVIII.

What velocity must be given to a machine, moved by water, in order that it may produce the greatest effect?

That this is not a matter of indifference, will readily appear from the following observation. If the wheel moved with the same velocity as the fluid, it would experience no pressure; consequently the weight it would be capable of raising would be nothing, or infinitely small. On the other hand, if it were immoveable, it would experience the whole pressure of the current; but in this case there would be an equilibrium, and as no weight would be raised there would consequently be no effect. There is therefore a certain mean velocity, between that of the current and no velocity at all, which will produce the greatest effect—an effect proportional, in a given time, to the product of the weight multiplied by the height to which it is raised.

We shall not here give the analytical reasoning which conducts to the solution of the problem. We shall only observe, that in a machine of the above nature, the velocity of the wheel ought to be equal to one half of that of the current. Consequently the resistance or the weight must be increased, until the velocity be in this ratio. The machine will then produce the greatest effect possible.

PROBLEM XXXIX.

What is the greatest number of float-boards, that ought to be applied to a wheel moved by a current of water, in order to make it produce the greatest effect?

It was long believed that the float-boards of such a wheel ought to be so proportioned, that when one of them was in a vertical position, or at the middle of its immersion, the next one should be just entering the water. A great many reasons were assigned for this mode of construction, which however are contradicted by calculation, as well as by experience.

It is now demonstrated, that the more float-boards such a wheel has, the greater and more uniform will be its effect. This result is proved by the researches of the Abbé de Valernod, of the academy of Lyons, and those of M. du Petit-Vandin, to be found in the first volume of the "Mémoires des Sçavans Etrangers."

The Abbé Bossut, who examined, by the help of experiments, the greater part of the hydraulic theories, has demonstrated also the same thing. According to the experiments which he made, a wheel furnished with 48 float-boards, produced a much greater effect than one furnished with 24; and the latter a greater effect than one with 12; their immersion in the water being equal. M. du Petit Vandin therefore observes, that in Flanders, where running water is so exceedingly scarce, as to render it necessary to turn it to the greatest possible advantage, the wheels of water-mills are furnished with 32 float-boards at least, and even with 48, when the wheel is from 16 to 19 feet in diameter.

PROBLEM XL.

If there be two cylinders, containing exactly the same quantity of matter, the one solid and the other hollow, and both of the same length; which of them will sustain, without breaking, the greatest weight suspended from one of its extremities, the other being fixed?

Some, and perhaps several of our readers, may be inclined to think that, the base of rupture being the same, every thing else ought to be equal. On the first view, one might even be induced to consider the solid cylinder as capable of presenting greater resistance to being broken: this however would be a mistake.

Galileo, who first examined mathematically the resistance of solids to being broken by a weight, has shewn that the hollow cylinder will present the most resistance; and that this resistance will be greater in the transverse direction, according as the hollow part is greater. He even shews, from a theory which approaches very near the truth, that the resistance of the hollow cylinder will be to that of the solid one, as the whole radius of the hollow is to that of the solid. Thus the resistance of a hollow cylinder, having as much vacuity as solid, will be to the resistance of a solid one, as $\sqrt{2}$ to 1, or as 1·141 to 1·000: for the radius of the former will be $\sqrt{2}$, while that of the latter is unity. The resistance of a hollow cylinder, having twice as much vacuity as solid, will be to a solid one, as $\sqrt{3}$ to 1, or as 1·73 to 1·00; for their radii will be in the ratio of $\sqrt{3}$ to 1. The resistance of a hollow cylinder, the solidity of which forms only a twentieth part of the whole volume, will be to that of a solid cylinder of the same mass, as $\sqrt{21}$ to 1, or as 4·31 to 1·00; and so forth.

Remark.—It may be readily observed, and Galileo does not fail to take notice of it, that this mechanism is that which nature, or its Supreme Author, has employed, on various occasions, to combine strength with lightness. Thus the bones of the greater part of animals are hollow: by being solid with the same quantity of matter, they would have lost much of their strength; or to give them the same power of

resistance, it would have been necessary to render them more massy; which would have lessened the facility of motion.

The stems of many plants are hollow also, for the very same reason In the last place, the feathers of birds, in the formation of which it was necessary that great strength should be united with great lightness, are also hollow: and the cavity even occupies the greater part of their whole diameter; so that the sides are exceedingly thin.

PROBLEM XLI.

To construct a lantern, which shall give light at the bottom of the water.

This lantern must be made of leather, which will resist the waves better than any other substance; and must be furnished with two tubes, having a communication with the air above. One of these tubes is destined to admit fresh air for maintaining the combustion of the candle or tape.; and the other to serve as a chimney, by affording a passage to the smoke: both must rise to a sufficient height above the surface of the water, so as not to be covered by the waves when the sea is tempestuous. It may be readily conceived, that the tube which serves to admit fresh air, ought to communicate with the lantern at the bottom; and that the one which serves as a chimney, must be connected with it at the top. Any number of holes at pleasure, into which glasses are fitted, may be made in the leather of which the lantern is constructed; and by these means the light will be diffused on all sides. In the last place, the lantern must be suspended from a piece of cork, that it may rise and fall with the waves.

A lantern of this kind, says Ozanam, might be employed for catching fish by means of light; but this method of fishing has, in some countries, been wisely forbidden under severe penalties.

PROBLEM XLII.

To construct a lamp, which shall preserve its oil in every situation, however moved or inclined.

To construct a lamp of this kind, the body of it, or the vase that contains the oil and the wick, must have the form of a spherical segment, with two pivots at the edge, diametrically opposite to each other, and made to turn in two holes at the extremities of the diameter of a brass or iron circle. This circle must, in like manner, be furnished with two pivots exactly opposite to each other, and at the distance of 90° from the holes in which the former are inserted. These second pivots must be made to turn in two holes diametrically opposite in a second circle; and this second circle must likewise be furnished with two pivots, inserted in some concave body, proper to serve as a covering to the whole lamp.

It may be readily seen that, by this method of suspension, whatever motion be given to the lamp, unless too abruptly, it will always maintain itself in a horizontal position.

This method of suspension is that employed for the mariner's compass, so useful to navigators; and which must always be preserved in a horizonal situation. We have read, in some author, that Charles V. caused a carriage to be suspended in this manner, to guard against the danger of being overturned.

PROBLEM XLIII.

Method of constructing an anemoscope and an anemometer.

These two machines, which in general are confounded, are not however the same. The anemoscope serves for pointing out the direction of the wind, and therefore, properly speaking, is a weather-cock; but in common this term is used to denote a more complex machine, which indicates the direction of the wind by means of a kind of dial plate, placed either on the outside of a house or in an apartment. In regard

to the anemometer, it is a machine which serves to indicate, not only the direction, but the duration and force of the wind.

Fig. 44. The mechanism of the anemoscope is very simple, (Fig. 44.) It consists, in the first place, of a weather-cock, raised above the building, and supported by an axis, one end of which, passing through the roof, is made to turn in a socket fitted to receive it, and with such facility as to obey the least impulse of the wind. On this axis is fixed a crown wheel, the teeth of which being turned downwards, fit into those of a vertical wheel, exactly of the same size, placed on a horizontal axis, which at its extremity is furnished with an index. It is hence evident, that when the vane makes one turn, the index will make one exactly also. If this index then be placed in such a manner as to be vertical, when the wind is north; and if care be taken to observe in what direction it turns, when it changes to the west, it will be easy to divide the dial plate into 32 parts.

Fig. 45. An anemometer, if it be required only to measure the intensity or force of the wind, may be constructed with equal ease. We would propose the following. Let A B (Fig. 45.) be an iron bar, fixed in a horizontal direction to the vertical axis of a vane. The extremities of this bar, which are bent at right angles, serve to support a horizontal axis, around which turns a moveable frame A B C D, of a foot square. To the middle of the lower side of the frame is fastened a very fine but strong silk thread, which passes over a pulley F, fitted into a cleft in the vertical axis of the vane, whence it descends along the axis to an apartment below the roof. The distance G F must be equal to G E. To the end of the silk thread is suspended a small weight, just sufficient to keep it stretched. When the frame which, by the turning of the vane, will be always presented to the wind, is raised up, as will be the case, more or less, according to the force of the wind, the small weight will be raised up also, and will thus indicate, by means of a scale adapted to the axis of the vane, the strength of the wind. It may readily be perceived that the force of the wind will be equal to Zero or nothing, when the small weight is at its lowest point; and that its maximum, or greatest degree, will be when it is at its highest, which will indicate that the wind keeps the frame in a horizontal position, or very nearly so.

The force of the wind, according to the different inclination of the frame, may be determined with still greater precision: for this force will always be equal to the absolute weight of the frame, which is known, multiplied by the sine of the angle which it makes with the vertical line, and divided by the square of the same angle. Nothing then will be necessary, but to ascertain, by the motion of the small weight affixed to the thread E F P, the inclination of the frame. But this is easy; for it may be readily seen that the quantity which it rises above the lowest point, will always be equal to the chord of the angle formed by the frame with the vertical plane, or to double the sine of the half of that angle. The extent therefore of this angle may be marked along the scale, and also the force of the wind, calculated according to the foregoing rule.

In the Memoirs of the Academy of Sciences, for the year 1734, may be found the description of an anemometer invented by M. d'Ons-en-Bray, to indicate at the same time the direction of the wind, its duration in that direction, and its strength. This anemometer merits that we should here give some idea of it.

It consists of three parts, viz., a common clock, and two other machines, one of

which serves to mark the direction of the wind and its duration; the other to indicate its force.

The first of these machines consists, like the common anemoscope, of a vertical axis bearing a vane, which by means of some wheels indicates, on a dial-plate, from what quarter the wind blows; the lower part of this axis passes through a cylinder, in which are implanted thirty-two pins, in a spiral line; and these pins, by the manner in which they present themselves, press against a piece of paper, properly prepared and stretched, between two vertical columns or axes, on one of which it is rolled up, while it is unrolled from the other. This rolling up and unrolling are performed by the simultaneous motion of two axes, which are made to move by the clock above mentioned. It may now be readily conceived that, according to the position of the vane, one of the pins will present itself to the prepared paper, and by pressing gently against it will leave a mark, the length of which will indicate the duration of the wind. If two neighbouring pins make a mark, at the same time, this will indicate that the wind followed a middle direction.

The part of the anemometer which indicates the force of the wind, consists of a mill, after the Polish manner, which revolves faster, according as the wind is stronger. Its vertical axis is furnished with a wheel that drives a small machine, which, after a certain number of turns, forces a pin against a frame of paper, having a motion similar to that of the anemometer above described. The number of these strokes, each of which is marked by a hole, on a determinate length of this moveable paper, denotes the force of the wind, or rather the velocity of the circulation of the mill, which is nearly in the same proportion. But, for a complete explanation of the mechanism, we must refer to the Memoirs of the Academy of Sciences, above quoted; as want of room will not allow us to give a more minute description of it in this place.

Professor Leslie, in his Essay on Heat, has suggested an anemometer, founded on different principles. He found, by experiment, that the cooling power of a stream of air is proportional to its velocity; and putting T for the time in which a body loses an aliquot part of its heat in still air, t the time in which it loses the same quantity when exposed to the wind, and V the velocity of the wind in miles per hour, he gives the following formula:

$$V = 1 - \frac{T}{t} \cdot 4\tfrac{1}{2}.$$

T and t are found by means of a thermometer, whose bulb is a little more than half an inch in diameter, and filled with tinged alcohol.

When the thermometer is held in still air its temperature is marked; it is then heated by the application of the hand, till the alcohol rises a certain number of degrees, and the time which it takes to descend through half that number of degrees is carefully marked. Mr. Leslie calls this time the fundamental measure of cooling. The same observation is made when the ball is exposed to the wind, and the time which the alcohol takes to descend through half the number of degrees that it rose is called the occasional measure of cooling. The former of these is T, and the latter t, in the above formula, which may be thus expressed in words:

Divide the fundamental measure of cooling by the occasional measure of cooling, and multiply the difference between unity and the quotient by 4½, the product is the velocity of the wind, in miles per hour.

As an example, let us suppose that in still air the temperature is 50°, and when warmed by the hand it rises to 70°, and that in 100 seconds it falls to 60°; and further, that when exposed to the wind, and heated by the hand, it takes only 10 seconds to fall through the same number of degrees; then we have the velocity of wind $\overline{\tfrac{100}{10} - 1} \cdot 4\tfrac{1}{2} = 40\tfrac{1}{2}$ miles per hour.

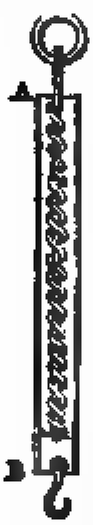
Remark.—Many other forms of anemometers have been invented, in various countries. Of several of these the descriptions may be seen, with their figures and the calculation of their effects, in Dr. Hutton's Dictionary, under the several articles ANEMOMETER, RESISTANCE, WIND, and WIND-GAGE.

PROBLEM XLIV.

Construction of a Steel-yard, by means of which the weight of a body may be ascertained, without weights.

We shall here describe two instruments of this kind; the one portable and adapted for ascertaining moderate weights, such as from 1 to 25 or 30 pounds; the other fixed, and employed for weights much more considerable, and even of several thousand pounds. One of the latter kind was used in the custom house at Paris; and could be employed with great convenience, for weights between 100 and 3000 pounds.

Fig. 46.



The first of these steel-yards is represented Fig. 46. It consists of a metal tube A B, about six inches in length, and eight lines in diameter, a section of which is here given, to shew in the inside of it a spiral steel spring. The upper end A, is pierced with a square hole, to afford a passage to a metal rod, which is also square; and which passes through the spring, so that it is impossible to draw it upwards, without compressing the spring against the upper end within the tube. To the lower part of the tube is affixed a hook, from which the body to be weighed is suspended.

It is here evident, that if bodies of different weights be applied to the hook, while the steel-yard is suspended by its ring, they will draw down the tube more or less, by forcing the upper end of it against the spring. The rod therefore must be divided, by suspending successively from the hook different weights, such as one pound, two pounds, &c., to the greatest which it can weigh; and if the part of the rod drawn out of the tube each time be marked by a line, accompanied with a figure denoting the weight, the instrument will be complete. When you intend to use it, nothing is necessary but to put your finger into the ring, to raise up the article you intend to weigh, suspended from the hook, and to observe, on the divided face of the rod, the division exactly opposite to the edge of the hole: the figure belonging to this division will indicate the number of pounds which the proposed body weighs.

The second steel-yard consists of two bars, placed back to back, or of a single one A B C D E bent in the form seen Fig. 47. The part A B is suspended by a ring from a strong beam, and the part D E terminates in a hook at E, from which the articles to be weighed are suspended. To the part E D is fixed a rack, fitted into a pinion, connected with a wheel, the teeth of which are fitted into another pinion, having on its axis an index; and this index makes just one revolution, when the weight of 3000 pounds is suspended from the hook E. For it may be readily seen, that when any weight is suspended from E, the spring B C D must be more or less stretched; this will give motion to the rack D F, and the latter will turn the pinion into which it is fitted; and consequently will give motion to the wheel and second pinion, having on its axis the index. It is also evident, that in constructing the machine, such a force may be given to the spring, or its wheels may be combined in such a mannner, that a determinate weight, as 3000 pounds, shall cause the index to perform a complete revolution. The centre of motion of this index is in the centre of a

Fig. 47.

circular plate, marked with the divisions, that serve to indicate the weight. These divisions must be formed by suspending, in succession, weights less than the greatest, in the arithmetical progression, as 29 hundred weight, 28, 27, &c. This will give the principal divisions, which without any considerable error may be then subdivided into equal parts.

When the instrument is thus constructed; then to find the weight of any article that weighs less than 3000 pounds, nothing is necessary but to suspend it to the hook E; and the index will point out, on the circular plate, its weight in quintals, or hundreds, quarters, and pounds.

Remark.—It may be proper here to observe, that this method of weighing cannot be perfectly exact, unless we suppose that the temperature of the air always remains the same; for during cold weather springs are stiffer, and during hot weather are less so. On this account, we have no doubt that there is a difference between the same article weighed at the custom-house at Paris in winter and in summer. In winter it must appear to weigh less than it does in summer.

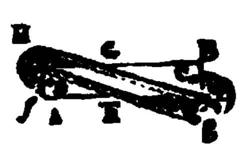
PROBLEM XLV.

Method of constructing a small figure, which when left to itself descends along a small stair on its hands and its feet.

This small machine, the mechanism of which is very ingenious, was a few years ago brought from India. It is called the tumbler, because its motion has a great resemblance to that of those performers at some of the public places of amusement, who throw themselves backwards resting on their hands; then raise their feet, and complete the circle by resuming their former position; but the figure can perform this movement only descending, and along a sort of steps. The artifice of this small machine is as follows:

A B (Fig. 48.) is a small piece of light wood, about two inches in length, 2 lines in thickness, and 6 in breadth. At its two extremities are two holes C and D, which serve to receive two small axes, around which the legs and arms of the figure are made to play. At each extremity of the piece of wood there is also a small receptacle, of the form seen in the figure, that is to say, nearly concentric with the holes C and D; having an oblique prolongation towards the middle of the piece of wood, and from the ends of these two prolongations proceed two grooves G g and F f, formed in the thickness of the wood, and nearly a line in diameter.

Fig. 48.



Quicksilver being put into one of these receptacles till it is nearly full, they are both closed up by means of very light pieces of pasteboard, applied on the sides. To the axis, passing through one of the holes C, are affixed two supporters, cut into the form of legs, with feet somewhat lengthened, to give them more stability. And to the other axis, passing through D, are affixed two supporters shaped like arms, with their hands placed in such a manner as to become a base, when the machine is turned backwards. In the last place, to the part G H is applied a sort of head and visage, made of the pith of the elder tree, and dressed after the manner of tumblers. A belly is constructed of the same substance, and the whole figure is clothed in a silk dress, which descends to the middle of the thighs. Having thus given a general account of the construction of this small machine, we shall now proceed to explain its mode of action.

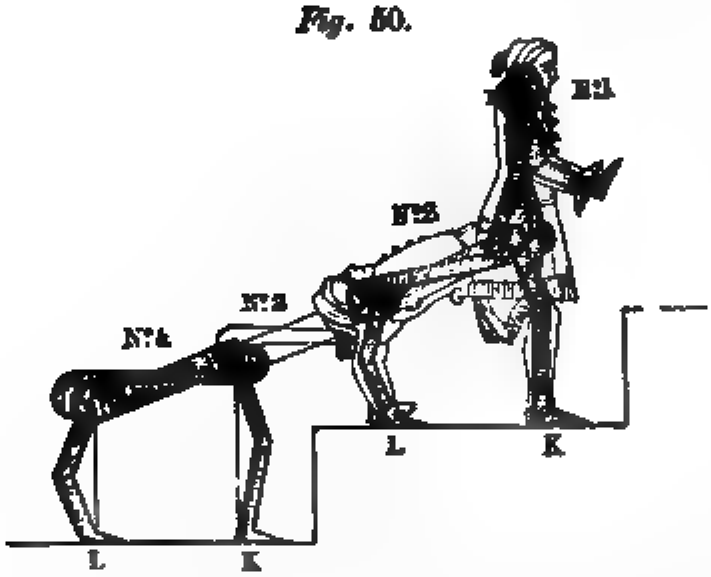
Let us first suppose the machine to be placed upright on its legs, as seen Fig. 49 or 50. No. 1. As all the weight is on one side of the axis of rotation C, because the receptacle of the quicksilver on that side is filled, the machine must incline to that side, and would be thrown entirely backwards, did not the arms or supporters, turning around the axis D, present themselves in a vertical direction; but as they are

Fig. 49.

shorter than the legs, the machine assumes the position represented Fig. 50, No. 2; and the quicksilver finding the small groove $G\,g$, inclined to the horizon, flows with impetuosity into the receptacle placed on the side D.

Let us now suppose that at this moment the machine rests on the supports or arms D L, which turn around the axis D: it is evident that, if the empty part of the machine is very light, the quicksilver being entirely beyond the point of rotation D, will, by its considerable preponderance overcome it, and cause the machine to revolve around the axis D, which will raise it, and make it turn on the other side. But as the supporters C K must necessarily be longer than the others D L that the line C D may have the inclination which is necessary to cause the quicksilver to flow by the small groove $G\,g$, from the one receptacle to the other, the base must make a jump double in height to the difference of these supporters; otherwise the line $G\,g$, instead of assuming a horizontal position, would remain inclined in a direction contrary to that which it ought to have.

Fig. 50.



The machine having then attained to the situation D C, Fig. 50, No. 3, and the quicksilver having passed into the receptacle on the side C; it is evident that the same mechanism which will raise it up, by making it turn round the point C, will overturn it on the other side, where the two supporters, which revolve round the axis C, present it a base. this will make it resume the position of Fig. 50, No. 2: and so on. Hence this motion will be perpetual, as long as the machine meets with steps like the first.

Remarks.—Some particular conditions are required, in order that the supporters of the small figure, that is to say its legs and arms, may present themselves in a proper manner, to keep it in the position in which it ought to be.

1st. It is necessary that the great supporters, or legs, when they have arrived at that point at which the figure, after having thrown itself topsy-turvey, rests upon them, should meet with some obstacle, to prevent them or the figure from turning any more: this may be done by two small pegs, which meet a prolongation of the thighs.

2d. While the figure is raising itself on its legs, it is necessary that the arms should perform, on their axis, a semi-revolution; that they may present themselves perpendicular to the horizon, and in a firm manner, when the figure throws itself backward. This may be accomplished by furnishing the arms of the figure with two small pulleys, concentric to the axis of the motion of these arms, over which are conveyed two silk threads, that unite under the belly of the figure, and are fixed to a small cross bar, joining the thighs towards the middle: this will greatly contribute to their stability. These threads must be lengthened, or shortened, till this semi-revolution of the arms is exactly performed; and until the figure, when placed on its four supporters, with its face turned either up or down, does not waver; which it would do if these sup-

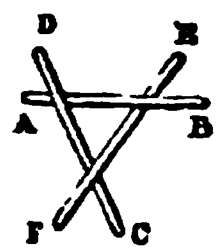
porters were not bound together in this manner; and if the large ones, or legs, did not meet with an obstacle to prevent them from inclining any farther.

PROBLEM XLVI.

To arrange three sticks, on a horizontal plane, in such a manner, that while the lower extremities of each rest on that plane, the other three shall mutually support each other in the air.

This depends merely on a little mechanical address, and may be performed in the following manner.

Fig. 51.



Take the first stick A B (Fig. 51), and rest the end A on the table, holding the other raised up, so that the stick shall be inclined at a very acute angle. Place above it the second stick, with the end C resting on the table. And then dispose the third stick E F, in such a manner, that while the end E rests on the table, it shall pass below the stick A B, towards the upper end B, and rest on the stick C D. These three sticks, by this arrangement, will be so connected with each other, that the ends D, B and F will necessarily remain suspended, each supporting the other.

PROBLEM XLVII.

To make a soft body, such as the end of a candle, pierce a board.

Load a musket with powder, and instead of a ball put over it the end of a candle; if you then fire it against a board, not very thick, the latter will be pierced by the candle-end, as if by a ball.

The cause of this phenomenon, no doubt, is that the rapid motion with which the candle-end is impelled, does not allow it time to be flattened, and therefore it acts as a hard body. It is the effect of the inertia of the parts of matter, as may be easily proved by experiment. Nothing is easier to be divided than water; yet if the palm of the hand be struck with some velocity against the surface of water, a considerable degree of resistance, and even of pain, is experienced from it, as if a hard body had been struck. Nay, a musket ball, when fired against water, is repelled by it, and even flattened. If the musket is fired with a certain obliquity, the ball will be reflected; and, after this reflection, is capable of killing any person who may be in its way. This arises from a certain time being necessary to communicate to any body a sensible motion. When a body then, moving with great velocity, meets with another of a size much more considerable, it experiences almost as much resistance as if the latter were fixed.

PROBLEM XLVIII.

On the principles by which the possible effect of a machine can be determined.

It is customary for quacks, and those who have not a sufficient knowledge of mechanics, to ascribe to machines prodigious effects, far superior to such as are consistent with the principles of sound philosophy. It may therefore be of utility to explain here those principles by which we ought to be guided, in order to form a rational opinion respecting any proposed machine.

Whatever may be the construction of a machine, even supposing it to be mathematically perfect, that is, immaterial and without friction, its effect, that is to say, the weight put in motion, multiplied by the perpendicular height to which it may be raised, in a determinate time, cannot exceed the product of the moving power, multiplied by the space it passes over in the same time. Consequently, since every machine is material, and as it is impossible to get entirely rid of friction, which will necessarily destroy a part of the power, it is evident that the first product will always be less than the latter. Let us apply this to an example.

Should a person propose a machine, which by the strength of one man applied to a crank, or the lever of a capstan, shall raise in an hour 3500 gallons of water, to the height of 24 feet; we might tell him, that he was ignorant of the principles of mechanics.

For the strength of a man applied to a crank, or to draw or push any weight, is only equal to about 26 or 28 pounds, with a velocity at most of 11000 feet per hour; and he could labour no more than 7 or 8 hours in succession. Now, as the product of 11000 by 28 is 308000, if this product be divided by 24, the height to which the water is to be raised, the quotient will be 12833 pounds of water, or 206 cubic feet = 1540 gallons raised to that height; which makes about 60 gallons, per minute, to the height of 10 feet. This is all that could be produced by such a power in the most favourable case. But the more complex the machine, the greater is the resistance to be surmounted; so that the product would never be nearly equal to the above effect.

In a machine, where a man should act by his own weight, and in walking, the advantage would not be much greater: for all that a man could do by walking, without any other weight than that of his body, on a plane inclined at an angle of 30 degrees, would be to pass over 6000 feet per hour, especially if he had to walk in this manner for 7 or 8 hours. But here it is the perpendicular height alone, which in this case is 3000 feet, that is to be considered: the product of 3000 by 150 pounds, which is the average weight of a man, is 450000; the greatest effect therefore of such a machine, would be 450000 pounds, raised to the height of one foot, or 18750 to the height of 24 feet, or about 90 gallons per minute, to the height of 10 feet. By taking an arithmetical mean between this determination and the preceding, it will be found that the mean product possible of the strength of a man, employed to put in motion a hydraulic machine, is at most 75 gallons per minute; especially if continued for 7 or 8 hours in the day.

If the power were to act only for a very short time, as 3, 4, or 5 minutes, the product indeed might appear more considerable, and about double. This is one of the artifices employed by mechanicians, to prove the superiority of their machines. They put them in motion for some minutes, by vigorous people, who make a momentary effort, and thus cause the product to appear much greater than it really is.

The above determination agrees pretty well with that given by Desaguliers in his Treatise on Natural Philosophy: for he assured himself, he says, by calculation, that the effect of the simplest and most perfect machines, put in motion by men, never gives, in the ratio of each man, above 72 gallons of water per minute raised to the height of 10 feet.

A circumstance, very necessary to be known in regard to machines which are to be moved by horses, is as follows: a horse is equal to about seven men*, or can make an

* C. Regnier, in his description of the Dynamometer, an instrument invented by him for the purpose of determining the relative strength of men and horses, published in the "Journal de l'Ecole Polytechnique," vol. ii. p. 160, says, that from the result of all his experiments it appears, that the mean term of the maximum of the strength of ordinary men, to raise a weight, is about 285 pounds averdupois, which agrees with the experiments of Delahire, but which Desaguliers considered as too small. In regard to horses, he says, that by taking the mean results given by four horses, of the middle size, subjected to trial one after the other, the strength of ordinary horses may be estimated at 794 pounds averdupois.

In comparing the relative force of men with that of horses, when the former draw a cart or boat by the help of a rope, after various trials, he found that the maximum of the strength of ordinary men, in dragging a horizontal weight, by the help of a rope, is equal to 110 pounds averdupois, and that of the strongest does not exceed 132 pounds averdupois. These different trials agree pretty well with the general received opinion, that a horse is seven times as strong as a man. This principle, however, cannot be admitted in all cases; for it is known by experiment that a horse would sink under a burden seven times as heavy as that which a man can support when standing upright. It may readily be conceived that what has been here said respecting men and horses, is not applicable to daily and incessant labour; but we may deduce from it this very just consequence, that both can act for a whole day, when employing a fifth of their absolute forces. According to the above results, therefore, the power which an ordinary man can exert for a continuance in dragging or pulling, is equal to no more than about 22 pounds, and that of the strongest to about 26 pounds.

effort in a horizontal direction of 210 pounds, moving with the velocity of 10000 or 11000 feet per hour, supposing he is to work 8 or 10 hours per day. Desaguliers even gives less, and thinks that the force of a man is to be only quintupled to find that of the horse.

Those who are acquainted with these principles, will run no risk of being deceived by ignorant or pretended mechanicians; and it is no small advantage to be able to avoid becoming the dupe of such men, whose aim is often to pick the pockets of those who are so simple as to listen to them.

PROBLEM XLIX.

Of the Perpetual Motion.

The perpetual motion has been the quicksand of mechanicians, as the quadrature of the circle, the trisection of an angle, &c., have been that of geometricians: and as those who pretend to have discovered the solution of the latter problems are, in general, persons scarcely acquainted with the principles of geometry, those who search for, or imagine they have found, the perpetual motion, are always men to whom the most certain and invariable truths in mechanics are unknown.

It may be demonstrated indeed, to all those capable of reasoning in a sound manner on those sciences, that a perpetual motion is impossible: for, to be possible, it is necessary that the effect should become alternately the cause, and the cause the effect. It would be necessary, for example, that a weight, raised to a certain height by another weight, should in its turn raise the second weight to the height from which it descended. But, according to the laws of motion, all that a descending weight could do, in the most perfect machine which the mind can conceive, is to raise another in the same time to a height reciprocally proportional to its mass. But it is impossible to construct a machine in which there shall be neither friction nor the resistance of some medium to be overcome; consequently, at each alternation of ascent and descent, some quantity of motion, however small, will always be lost: each time therefore the weight to be raised will ascend to a less height; and the motion will gradually slacken, and at length cease entirely.

A moving principle has been sought for, but without success, in the magnet, in the gravity of the atmosphere, and in the elasticity of bodies. If a magnet be disposed in such a manner as to facilitate the ascension of a weight, it will afterwards oppose its descent. Springs, after being unbent, require to be bent by a new force equal to that which they exercised; and the gravity of the atmosphere, after forcing one side of the machine to the lowest point, must be itself raised again, like any other weight, in order to continue its action.

We shall however give an account of various attempts to obtain a perpetual motion, because they may serve to shew how much some persons have suffered themselves to be deceived on this subject.

Fig. 52.

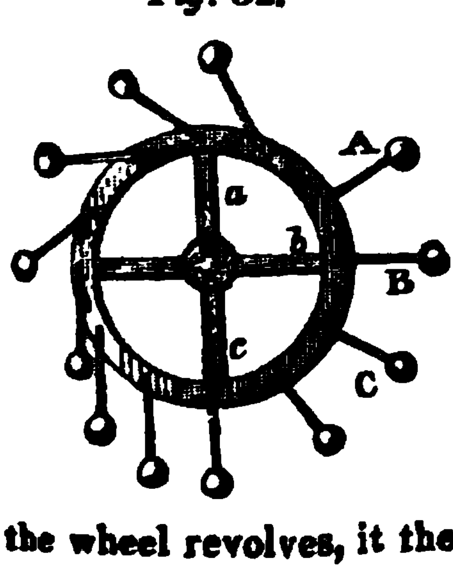


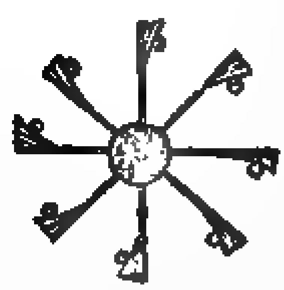
Fig. 52 represents a large wheel, the circumference of which is furnished, at equal distances, with levers, each bearing at its extremity a weight, and moveable on a hinge, so that in one direction they can rest upon the circumference, while on the opposite side, being carried away by the weight at the extremity, they are obliged to arrange themselves in the direction of the radius continued. This being supposed, it is evident that when the wheel turns in the direction *a b c*, the weights A, B and C will recede from the centre; consequently, as they act with more force, they will carry the wheel towards that side; and as a new lever will be thrown out, in proportion as the wheel revolves, it thence follows, say they, that the wheel will continue to move

in the same direction. But notwithstanding the specious appearance of this reasoning, experience has proved that the machine will not go; and it may indeed be demonstrated that there is a certain position, in which the centre of gravity of all these weights is in the vertical plane passing through the point of suspension, and that therefore it must stop.

Fig. 53.

The case is the same with the following machine, which it would appear ought to move also incessantly. In a cylindric drum, in perfect equilibrium on its axis, are formed channels as seen in Fig. 53, which contain balls of lead, or a certain quantity of quicksilver. In consequence of this disposition, the balls or quicksilver must, on the one side, ascend by approaching the centre; and on the other must roll towards the circumference. The machine then ought to turn incessantly towards that side.

Fig. 54.



A third machine of this kind is represented Fig. 54. It consists of a kind of wheel formed of six or eight arms, proceeding from a centre, where the axis of motion is placed. Each of these arms is furnished with a receptacle in the form of a pair of bellows; but those on the opposite arms stand in contrary directions, as seen in the figure. The moveable top of each receptacle has affixed to it a weight, which shuts it in one situation, and opens it in the other In the last place, the bellows of the opposite arms have a communication by means of a canal, and one of them is filled with quicksilver.

These things being supposed, it is visible, that the bellows on the one side must open, and those on the other must shut; consequently the mercury will pass from the latter into the former, while the contrary will be the case on the opposite side.

It might be difficult to point out the deficiency of this reasoning: but those acquainted with the true principles of mechanics will not hesitate to bet a hundred to one that the machine, when constructed, will not answer the intended purpose.

The description of a pretended perpetual motion, in which bellows, to be alternately filled with and emptied of quicksilver, were employed, may be seen in the "Journal des Sçavans," for 1685. It was refuted by Bernoulli, and some others, and it gave rise to a long dispute. The best method which the inventor could have employed to defend his invention, would have been to construct it, and shew it in motion; but this was never done.

We shall here add another curious anecdote on this subject. One Orfyreus announced at Leipsic, in the year 1717, a perpetual motion, consisting of a wheel, which would continually revolve. This machine was constructed for the landgrave of Hesse-Cassel, who caused it to be shut up in a place of safety, and the door to be sealed with his own seal. At the end of forty days, the door was opened, and the machine was found in motion. This however affords no proof in favour of a perpetual motion: for as clocks can be made to go a year without being wound up, Orfyreus's wheel might easily go forty days, and even more.

The result of this pretended discovery is not known: we are informed, by one of the journals, that an Englishman offered 80000 crowns for this machine; but Orfyreus refused to sell it at that price; in this he certainly acted wrong, as there is reason to think that he obtained by his invention neither money nor even the honour of having discovered the perpetual motion.

The Academy of Painting, at Paris, possessed a clock, which had no need of being wound up, and which might be considered as a perpetual motion, though it was not so. But this requires some explanation. The ingenious author of this clock employed the variations in the state of the atmosphere, for winding up his moving weight:

various artifices might be devised for this purpose; but this is no more a perpetual motion, than if the flux and reflux of the sea were employed to keep the machine continually going; for this principle of motion is exterior to the machine, and forms no part of it.

But enough has been said on this chimera of mechanics We sincerely hope that none of our readers will ever lose themselves in the ridiculous and unfortunate labyrinth of such a research.

To conclude, it is false that any reward has been promised by the European powers to the person who shall discover the perpetual motion; and the case is the same in regard to the quadrature of the circle. It is this idea, no doubt, that excites so many to attempt the solution of these problems; and it is proper they should be undeceived.

PROBLEM L.

To determine the height of the arched ceiling of a church, by the vibrations of the lamps suspended from it.

For this invention we are indebted, it is said, to Galileo, who first ascertained the ratio of the duration of the oscillations made by pendulums of different lengths.* But in order that this method may have a certain degree of exactness, the weight of the lamp ought to be several times greater than that of the cord by which it is suspended.

This being supposed, put the lamp in motion by removing it a very little from its perpendicular direction, or carefully observe that communicated to it by the air, which is very common; and with a stop-watch find how many seconds one vibration continues, or, if a stop-watch is not at hand, count the number of vibrations performed in a certain number of minutes: the greater the number of minutes, the more exact will the duration of each vibration be determined; for nothing will then be necessary, but to divide those minutes by the number of vibrations, and the quotient will be the duration of each in minutes or seconds.

We shall here suppose that it has been found, by either of these methods, that the time of each vibration is $5\frac{1}{2}$ seconds; square $5\frac{1}{2}$, which is $30\frac{1}{4}$, and multiply by it $39\frac{1}{8}$ inches, the length of a pendulum that swings seconds in the latitude of London, the product will be 98 ft. 7 in. 6 lin., which will be nearly the height from the point of suspension to the bottom or rather centre of the lamp.

If the distance from the bottom of the lamp to the pavement be then measured, which may be done by means of a stick, and added to the former result, the sum will give the height of the arch above the pavement.

This solution is founded on a property of pendulums, demonstrated in mechanics; which is, that the squares of the times of the vibrations are as the lengths; so that a pendulum four times the length of another, performs vibrations which last twice as long.

But on account of the irregular form of the lamp, and the weight of the rope which sustains it, we must confess that this method is rather curious than exact. We shall, however, present the reader with another problem of the same kind.

PROBLEM LI.

To measure the depth of a well, by the time elapsed between the commencement of the fall of a heavy body, and that when the sound of its fall is conveyed to the ear.

Have in readiness a small pendulum that swings half seconds, that is to say,

* Indeed, it seems it was by that author accidentally observing the uniformity in the intervals of the swing of the suspended lamps, that he first took the hint of employing the oscillations of pendulous bodies, or pendulums, for the purpose of measuring time. And hence the invention of pendulum clocks.

$9\frac{7}{9}$ inches in length, between the centre of the ball, and the point of suspension. You must also employ a weight of some substance as heavy as possible, such for example as lead; as a common stone or pebble experiences a considerable retardation in falling, and therefore would not answer the purpose so well.

Let go the weight and the ball of the pendulum at the same moment of time, and count the number of vibrations the latter makes, till the moment when you hear the sound. We shall here suppose that there were ten vibrations, which make five seconds.

As a heavy body near the earth's surface falls about $16\frac{1}{12}$ feet in one second of time, or for this purpose 16 feet will be exact enough; and as sound moves at the rate of 1142 feet per second; multiply together 1142, 16, and 5, which will give 91360, and to 4 times this product, or 365440, add the square of 1142, which is 1304164, and the sum will be 1669604; and if from the square root of the last number = 1292 the number 1142 be subtracted, the remainder 150, divided by 32, will give 4·69 for the number of seconds which elapsed during the fall of the body: if this remainder be subtracted from 5, the number of seconds during which the body was falling and the sound returning, we shall have 0·31 for the time which the sound alone employed before it reached the ear; and this number multiplied by 1142, will give for product 354 feet = the depth of the well.

This rule, which is rather complex, is founded on the property of falling bodies, which are accelerated in the ratio of the times, so that the spaces passed over increase as the squares of the times*. But as the resistance of the air, which in considerable heights, such as those of several hundred feet, does not fail to retard the fall in a sensible manner, has been neglected, the case of this problem is nearly the same as with the preceding; that is to say, the solution is rather curious than useful.

* For the sake of our algebraical readers we shall here shew how to find the formula from which the above rule is deduced: Let $a = 5$, $b = 16\frac{1}{12}$, $c = 1142$, and let x be the time which the body employs in falling, consequently $a - x$ will be the time of the sound returning. Then as $1^2 : b :: x^2 : b x^2 =$ depth of the well; and $1 : c :: a - x : c a - c x =$ depth of the well also; therefore $b x^2 = c a - c x$, and by transposition and division, $x^2 + \frac{c}{b} x = \frac{c a}{b}$. Completing the square, $x^2 + \frac{c}{b} x + \frac{c^2}{4b^2} = \frac{ca}{b} + \frac{c^2}{4b^2} = \frac{4bca + c^2}{4b^2}$. Hence, $x + \frac{c}{2b} = \sqrt{\frac{4bca + c^2}{4b^2}}$ and $x = \sqrt{\frac{4bca + c^2}{4b^2}} - \frac{c}{2b} = \frac{\sqrt{c^2 + 4abc} - c}{2b} =$ nearly $\frac{ac}{ab + c}$ the time of descent. Consequently $a - \frac{ac}{ab + c} = \frac{a^2 b}{ab + c}$ is nearly the time of the sound's ascent.

Hence, from the expression $\frac{ac}{ab + c}$ a much simpler rule is obtained for the time of the descent, which is as follows: Multiply 1142 by 5, which gives for product 5710; then multiply also 16 by 5, which gives 80, to which add 1142, this gives 1222, by which sum divide the first product 5710, and the quotient 4·68 will be the time of descent, nearly the same as before. This taken from 5 leaves 0·32 for the time of the ascent; which multiplied by 1142, gives 365 for the depth, differing but little from the former more exact number.

HISTORICAL ACCOUNT OF SOME EXTRAORDINARY AND CELEBRATED MECHANICAL WORKS.

An essential part might seem wanting to this work if we neglected to give some account of the various machines most celebrated both among the ancients and moderns. We shall therefore take a cursory view of the rarest and most singular inventions, produced by mechanical genius, in different ages.

I.—*Of the machines or automatons of Archytas, Archimedes, Hero, and Ctesibius.*

Some machines of this kind are mentioned in ancient history, in terms of the utmost admiration. Such were the tripod automatons of Vulcan; and the dove of Archytas, which, as we are told, could fly like a real animal. We have no doubt, however, that the wonderful properties of these machines, if they ever really existed, have been greatly exaggerated by credulity; and by the accounts of them being handed down through such a long series of ages. We are told also of the moving sphere of Archimedes, in which, as appears, that celebrated philosopher had represented all the celestial motions, as they were then known; and this, no doubt, was a master-piece of mechanism for that remote period. Every one is acquainted with the famous verses of Claudian on this machine.

Several wonderful machines were constructed also by Hero and Ctesibius of Alexandria. An account of some of those invented by Hero may be seen in a book called *Spiritalia.* Some of them are very ingenious, and do honour to the talents of that mechanician.

II.—*Of the machines ascribed to Albert the Great, and to Regiomontanus.*

That ignorance, in the darkness of which all Europe was involved, from the sixth or seventh century to the fifteenth, did not entirely extinguish mechanical genius. We are told that the ambassadors sent by the king of Persia to Charlemagne brought, as a present to the latter, a machine, which, according to the description given of it, would have done honour to our modern mechanicians; for it appears to have been a striking clock, which had figures that performed various movements. It is indeed true that, while Europe was immersed in ignorance, the arts and sciences diffused a gleam of light among the nations of the East. In regard to those of the West, if we can believe what is related of Albert the Great, who lived in the thirteenth century, that mathematician constructed an automaton in the human form, which when any one knocked at the door of its cell, came to open it and sent forth some sounds, as if addressing the person who entered. At a period later by some centuries, Regiomontanus, or John Muller of Konigsberg, a celebrated astronomer, constructed an automaton in the figure of a fly, which walked around a table. But these accounts are probably very much disfigured by ignorance and credulity. The following however are instances of mechanical skill, in which there is much more of reality.

III.—*Of various celebrated Clocks.*

In the fourteenth century, James Dondi constructed for the city of Padua a clock, which was long considered as the wonder of that period. Besides indicating the hours, it represented the motion of the sun, moon, and planets, as well as pointed out the different festivals of the year. On this account, Dondi got the surname of Horologio, which became that of his posterity. A little time after, William Zelandin constructed, for the same city, one still more complex; which was repaired in the sixteenth century by Janellus Turrianus, the mechanician of Charles V.

But the most celebrated works of this kind are the clocks of the cathedrals of Strasburgh and Lyons. That of Strasburgh was the work of Conrad Dasypodius, a mathematician of that city, who lived towards the end of the sixteenth century, and who finished it about the year 1573. It is considered as the first in Europe. At any rate there is none but that of Lyons which can dispute pre-eminence with it, or be compared to it in regard to the variety of its effects.

The face of the basement of the clock of Strasburgh exhibits three dial-plates; one of which is round, and consists of several concentric circles; the two interior ones of which perform their revolutions in a year, and serve to mark the days of the year, the festivals, and other circumstances of the calendar. The two lateral dial-plates are square, and serve to indicate the eclipses, both of the sun and the moon.

Above the middle dial-plate, and in the attic space of the basement, the days of the week are represented by different divinities, supposed to preside over the planets from which their common appellations are derived. The divinity of the current day appears in a car rolling over the clouds, and at midnight retires to give place to the succeeding one.

Before the basement is seen a globe, borne on the wings of a pelican, around which the sun and moon revolved; and which in that manner represented the motion of these planets; but this part of the machine, as well as several others, has been deranged for a long time.

The ornamented turret, above this basement, exhibits chiefly a large dial, in the form of an astrolabe; which shews the annual motion of the sun and moon through the ecliptic, the hours of the day, &c. The phases of the moon are seen also marked out on a particular dial-plate above.

This work is remarkable also for a considerable assemblage of bells and figures, which perform different motions. Above the dial-plate last mentioned; for example, the four ages of man are represented by symbolical figures: one passes every quarter of an hour, and marks the quarter by striking on small bells; these figures are followed by death, who is expelled by Jesus Christ risen from the grave; who however permits it to sound the hour, in order to warn man that time is on the wing. Two small angels perform movements also; one striking a bell with a sceptre, while the other turns an hour-glass, at the expiration of an hour.

In the last place, this work was decorated with various animals, which emitted sounds similar to their natural voices; but none of them now remain except the cock, which crows immediately before the hour strikes, first stretching out its neck and clapping its wings. The voice of this figure however is become so hoarse as to be much less harmonious than the voice of that at Lyons, though the latter is attended, in a considerable degree, with the same defect. It is to be regretted that a great part of this machine is entirely deranged. It would be worthy of the illustrious metropolitan chapter of Strasburgh to cause it to be repaired: we have heard indeed that it has been attempted; but that no artist could be found capable of performing it.

The clock of the cathedral of Lyons is of less size than that of Strasburgh; but is not inferior to it in the variety of its movements; and it has the advantage also of being in a good condition. It is the work of Lippius de Basle, and was exceedingly well repaired in the last century by an ingenious clock-maker of Lyons, named Nourisson. Like that of Strasburgh, it exhibits, on different dial-plates, the annual and diurnal progress of the sun and moon, the days of the year, their length, and the whole calendar, civil as well as ecclesiastical. The days of the week are indicated by symbols more analogous to the place where the clock is erected; the hours are announced by the crowing of the cock, three times repeated, after it has clapped its wings, and made various other movements. When the cock has done crowing, angels appear, who, by striking various bells, perform the air of a hymn; the annunciation of the Virgin is represented also by moving figures, and by the descent of a dove from the clouds; and after this mechanical exhibition, the hour strikes. On one of the

sides of the clock is seen an oval dial-plate, where the hours and minutes are indicated by means of an index, which lengthens or contracts itself, according to the length of the semi-diameter of the ellipsis over which it moves.

A very curious clock, the work of Martinot, a celebrated clock-maker of the seventeenth century, was to be seen in the royal apartments at Versailles. Before it struck the hour, two cocks on the corners of a small edifice crowed alternately, clapping their wings; soon after two lateral doors of the edifice opened, at which appeared two figures bearing cymbals, beat upon by a kind of guards with clubs. When these figures had retired, the centre door was thrown open, and a pedestal, supporting an equestrian statue of Louis XIV., issued from it, while a group of clouds separating gave a passage to a figure of Fame, which came and hovered over the statue. An air was then performed by bells; after which the two figures re-entered; the two guards raised up their clubs, which they lowered as if out of respect for the presence of the king, and the hour was then struck. Though all these things are easy for ingenious clock-makers of the present day, when we come to treat of Astronomy we shall give an account of some machines of this kind, purely astronomical, which do honour to the inventive genius of those by whom they were constructed.

IV.—*Automaton machines of Father Truchet, M. Camus, and M. de Vaucanson.*

Towards the end of the seventeenth century, Father Truchet, of the Royal Academy of Sciences, constructed, for the amusement of Louis XIV., moving pictures, which were considered as very remarkable master-pieces of mechanics. One of these pictures, which that monarch called his little opera, represented an opera of five acts, and changed the decorations at the commencement of each. The actors performed their parts in pantomime. The representation could be stopped at pleasure; this effect was produced by letting go a catch, and by means of another the scene could be made to re-commence at the place where it had been interrupted. This moving picture was sixteen inches and a half in breadth, thirteen inches four lines in height, and one inch three lines in thickness, for the play of the machinery. An account of this piece of mechanism may be found in the eulogy on Father Truchet, published in the Memoirs of the Academy of Sciences, for the year 1729.

Another very ingenious machine, and in our opinion much more difficult to be conceived, is that described by M. Camus, a gentleman of Lorrain, who says he constructed it for the amusement of Louis XIV., when a child It consisted of a small coach, drawn by two horses, in which was the figure of a lady, with a footman and page behind.

If we can give credit to what is stated in the work of M. Camus, this coach being placed at the extremity of a table of a determinate size, the coachman smacked his whip, and the horses immediately set out, moving their legs in the same manner as real horses do. When the carriage reached the edge of the table, it turned at a right angle, and proceeded along that edge. When it arrived opposite to the place where the king was seated, it stopped, and the page getting down opened the door, upon which the lady alighted, having in her hand a petition which she presented with a curtsey. After waiting some time, she again curtseyed. and re-entered the carriage; the page then resumed his place, the coachman whipped his horses, which began to move, and the footman, running after the carriage, jumped up behind it.

It is much to be regretted that M. Camus, instead of confining himself to a general account of the mechanism which he employed to produce these effects, did not enter into a more minute description; for, if they are true, it must have required a very singular artifice to produce them, and the same means might be applied to machines of greater utility.

About thirty or thirty-five years ago, three very curious machines were exhibited by M. de Vaucanson, viz., an automaton flute-player, a player on the flageolet and tam-

bourine, and an artificial duck. The first played several airs on the flute, with a precision greater perhaps than was ever attained to by the best living player, and even executed the tonguing, which serves to distinguish the notes. According to M. de Vaucanson, this part of the machinery cost him the greatest trouble. In a word, the tones were really produced in the flute by the proper motion of the fingers.

The player on the flageolet and tambourine performed also some airs on the first of these instruments, and at the same time kept continually beating on the latter.

But the motion of the artificial duck, in our opinion, was still more astonishing; for it extended its neck, raised up its wings, and dressed its feathers with its bill; it picked up barley from a trough, and swallowed it; drank from another, and, after various other movements, voided some matter resembling excrements. The first time I saw these machines I immediately discovered some of the artifices employed in regard to the two former, but I confess that the latter baffled my penetration.

We have also of late been amused, by M. Droz and M. Maillardet, &c., with the surprising performances of the chess-players, the small but sweet singing-bird, the writing figure, the musical lady, the conjurer, the tumbler, &c. &c.

V.—*Of the Machine at Marly.*

It will doubtless be allowed, that the machines above mentioned are, in general, more curious than useful; but there are other two, the celebrity and utility of which require that we should here give them a place. These are the machine of Marly, and that known under the name of the steam engine. We shall begin with the former, of the construction and effects of which the following brief description will give some idea.

The machine of Marly consists of 14 wheels, each about 36 feet in diameter, moved by a stream of water, confined by an estacade, and received into so many separate channels. Each wheel has at the extremities of its axis two cranks, and this forms 28 powers, distributed in the following manner.

It must however be first observed, that the water is raised, to the place to which it is to be conveyed, by three different stages; first from the river to a reservoir, at the elevation of 160 English feet above the level of the Seine; then to a second reservoir 346 feet higher; and from the latter to the summit of a tower, somewhat more than 533 feet above the river.

Of the 28 cranks above mentioned, eight are employed to give motion to 64 pumps; which is done by means of working beams, having four pistons at each extremity of their arms: this makes eight to each working beam, which are drawn up and pushed down alternately. These 64 pumps force up the water to the first reservoir; and this reservoir furnishes water to the first well, on which is established the second set of pumps.

Eleven more cranks are employed to force the water from the first well to the second reservoir. This is done by means of long arms adapted to these cranks, which move large frames, to one of the arms of which are attached strong iron chains, that extend from the bottom of the mountain to the first well. These chains, called *chevalets*, are formed of parallel bars of iron, the extremities of which are bound together by iron bolts, and are supported at certain intervals by transversal pieces of wood, moveable on an axis, that passes through the middle of each; so that when the upper bar of iron, for example, is drawn down by the lower end, all these pieces of wood incline in one direction, and the lower bar moves backwards and pushes in a direction contrary to the upper one. These bars or chains serve to put in motion the working beams, or squares, and the latter move the pistons of 80 sucking and forcing pumps, which raise the water from the first well to the second reservoir.

In the last place, nine other cranks, by a similar mechanism, put in motion those chains, called the *grands chevalets*, which move the pumps of the second well, and raise the water from it to the summit of the tower. These pumps are in number seventy-two.

Such, in a few words, is the mechanism of the machine of Marly. Its mean product, as said, is from 30000 to 40000 gallons of water per hour. We make use of the term mean product, because at certain times it raises 60000 gallons, but only under very favourable circumstances. During inundations, when the Seine is frozen, when the water is very low, or when any repairs are making, the machinery stops, either entirely or in part. We have read that in the year 1685 it raised 70000 gallons per hour; but this we can scarcely believe, if by that quantity is understood its mean product; as it would be above 1000 gallons per minute.

However this may be, the following calculation is founded on details collected on purpose. The annual expense of the machine, including the salaries of those who superintend it, and the wages of the workmen employed, together with repairs, necessary articles, &c., may amount to about £3300 sterling, or £9 per day; which makes about 1 farthing per 90 gallons. But if we take into this account the interest of the £333000 which, it is said, were expended in the construction of it, 90 gallons will cost 3 halfpence, which is at the rate of a farthing for 15 gallons. This is very far from the price which the king of Denmark thought he might set on this water; for that prince, when he paid a visit to Marly, in the year 1769, being astonished, no doubt, at the immensity of the machine, the multitude of its movements, and the number of the workmen it employed, observed that the water perhaps cost as much as wine. By the above calculation, the reader may see how far his majesty was mistaken.

It is an important question to know, whether the machine of Marly could be simplified. On this subject we shall give a few observations, which from some experiments made, and a minute examination of the different parts of the machine, appear to be founded on probability.

People in general are surprised that the inventor of this machine should cause the water, in some measure, to make two rests before it is conveyed to the summit of the tower. It has been humorously said, that he no doubt thought the water would be too much fatigued to ascend to the perpendicular height of more than 533 feet, all at one breath. It is more probable that he thought his moving force would not be sufficient to raise the water to that height; but this is not agreeable to theory; for it is found by calculation, that the force of one crank is more than sufficient to raise a cylinder of water of that altitude, and above 8 inches in diameter. Able mechanicians however are of opinion, that though this be not impossible, to carry it into execution would be attended with great inconveniences, which it would be too tedious to explain.

But it appears certain at present, that the water might be raised in one jet to the second well. This results from two experiments, one made in 1738, and the other in 1775. In the first, M. Camus, of the Royal Academy of Sciences, endeavoured to make the water rise in one jet to the tower: his attempt was not attended with success, but he made it rise to the foot of the tower, which is considerably higher than the second reservoir; hence it follows, that if he had confined himself to making the water rise in one jet to the second reservoir, he would have succeeded. It is said that, during this experiment, the machine was prodigiously strained; that it was even found necessary to secure some parts of it with chains; that it required twenty-four hours to force it to that height, which is about 480 feet, and that it was not possible to make it go farther. The object of the second trial, made in 1775, was to raise the water only to the second well. It indeed ascended thither at different times, and in abundance; but the pipes were exceedingly strained at the bottom,

so that several of them burst; and it was necessary to suspend and recommence the experiment several times. It is however evident that this arose from the age of the tubes and their want of strength, as they had not the proper thickness; a fault which might have been easily remedied. Here then we have one step towards the improvement of the machine; and it results from this trial, that the chains which proceed from the river to the first well, might be suppressed, and even the first well itself.

It still remains to be determined, whether the water could be made to ascend, in one jet, to the summit of the tower. This would be a very curious experiment; but no doubt difficult and expensive, because it would be necessary to make considerable changes in different parts of the machine; and even in the case of its succeeding, the water raised might perhaps be in such small quantity, that it would be better to retain the present mechanism.

It is probable that various improvements might be made in different parts of the machine. In several positions, the moving forces act only obliquely, which occasions a great loss of power, and must tend to render the machine less effectual. The form of the pistons, valves, and aspiration tubes, might perhaps admit also of some change. But as this is not the place for entering into these details, we shall proceed to the Steam Engine, of which we promised to give a short description.

VI.—*Of the Steam Engine.*

The Steam Engine is that perhaps in which the genius of mechanism has been manifested in the highest degree; for no idea could be more happy than that of employing alternately, as moving powers, the expansive force of the steam of water, and the weight of the atmosphere. Such indeed is the principle of this ingenious machine, which is at present employed with so much success in pumping water from mines, and for a variety of other purposes in the arts and manufactures.

The first part of this machine is a large boiler, to the cover of which is adapted a hollow cylinder, two, three, or four feet in diameter. A communication is formed between the boiler and the cylinder by an aperture, capable of being opened or shut. Into this cylinder is fitted a piston, the rod of which is made fast to the extremity of one of the arms of a working beam, having at the extremity of its other arm the weight to be raised, which is generally the piston of a second pump, adapted to raise water from a great depth. The whole must be combined in such a manner that when the air or steam has free access into the cylinder, which communicates with the boiler, the weight alone of the apparatus affixed to the opposite arm shall be capable of raising that piston.

Let us now suppose the boiler filled with water to a certain height, and that it is brought to a state of complete ebullition by a large fire kindled below the boiler. As a part of this water will continually rise in steam, when the communication between the boiler and the cylinder is opened, this vapour, which is elastic, will introduce itself into it, and raise the piston; as its force is equivalent to that of air. Let us suppose also that the piston, when it attains to a certain height, by means of some mechanism, which may be easily conceived, moves a certain part of the machine, which intercepts the communication between the boiler and the cylinder; and, in the last place, that by the same cause a jet of cold water is thrown beneath the bottom of the piston in the cylinder, so as to fall down through the vapour in the form of rain. At that moment the steam will be condensed into water; a vacuum will be formed in the cylinder; and consequently the piston will be then charged with the weight of the atmosphere above it, or a weight equivalent to a column of water of the same base and 32 feet in height. If the piston, for example, be 52 inches in diameter, as is the case in the steam-engines of Montrelais, near Ingrande, this weight will be equal to 29450 pounds: the piston will consequently be obliged to descend with a force equal to nearly 30000 pounds, and the other arm of the working

beam, if it be of the same length, will act with an equal force to overcome the resistance opposed to it. When the piston has made this first stroke, the communication between the boiler and the cylinder is restored; the steam of the boiling water again enters it, and the equilibrium between the air of the atmosphere and the inside of the cylinder being re-established, the weight of the apparatus affixed to the other end of the working beam descends, and raises the piston; the same play as before is renewed; the piston again falls, and the machine continues to produce its effect.

It may be readily conceived, that we must here confine ourselves to this short sketch; for a long description and a variety of figures would be necessary to give a correct idea of the many different parts requisite to produce this effect; such as that which opens and shuts the communication between the boiler and the cylinder; that which injects cold water into the cylinder; those which serve to evacuate the air and water formed in the inside of the cylinder; the regulator necessary to prevent the steam, when it becomes too strong, from bursting the machine, &c. For farther details therefore we must refer the reader to those authors who have purposely treated of this machine; such as Belidor in his "Architecture Hydraulique," vol. ii.; Desaguliers, in his "Cours de Physique Experimentale," vol. ii ; M. Prony, in his "Nouvelle Architecture Hydraulique ;" and several others.

The machine here described is very different from that mentioned by Muschenbrock, in his "Cours de Physique Experimentale." In the latter, the steam acts by its compression on a cylinder of water, which it causes to ascend. This requires steam highly elastic, and very much heated; but in this case there is great danger of the machine bursting. In the new machine, that above described, it is sufficient if the steam has the elasticity of the air: this it will acquire if the water boils only briskly; and therefore the danger of the machine bursting is not nearly so great: it is not even said that this accident ever happened to any of the large steam-engines, which have been long established.

The largest steam-engine with which I am acquainted, is that of Montrelais, near Ingrande, which is employed in freeing the coal mines from water. The cylinder is $52\frac{1}{4}$ inches in diameter.* It raises per hour, to the height of 652 feet, by eight different stages, 1145 cubic feet of water, or 10800 gallons; and as it is estimated, after deducting the time lost by putting it in motion, during accidental repairs, which are necessary from time to time, &c., that it works 22 hours in the 24, its daily effect is to raise, to the above height, and evacuate, 237600 gallons of water. In the same time it consumes about 266 cubic feet of coals. The other expenses attending it must also be considerable.

In the same place is another machine which, in some respects, appears to be constructed on a better principle. Though the cylinder is only 34 inches in diameter, it raises, in 22 hours, to the same height, and at one jet, 22000 cubic feet, or about 165000 gallons, which is above two thirds of the quantity raised by the former, while the moving power, which is in the ratio of the squares of the diameters of the pistons, is only about $\frac{3}{8}$ of that of the other.

An attempt was made, some years ago, to employ the steam-engine to move carriages, and an experiment on this subject was tried at the arsenal of Paris. The carriage indeed moved, but in our opinion this idea must be considered rather as ingenious, than susceptible of being put in practice. It would not be very agreeable to travellers to hear, behind them, the noise of a machine capable, if it should burst, of blowing them to atoms; and we much doubt whether this invention would meet with encouragement. A boat also which, it is said, could be made to

* In some steam-engines in England the cylinder is 63, and even 72 inches in diameter, and their power is equal to that of 250 horses.

move against the current by means of a steam-engine, was seen for a long time in the middle of the Seine, opposite to Passy. Nothing less was hoped from this invention, than to be able to convey a boat, laden with merchandise, in two or three days, from Rouen to Paris; but scarcely was the machine in motion, when the wheels, the float-boards of which were to serve as oars, were broken in pieces by the effect of the too violent and sudden impression they received. Such was the result of this attempt, the failure of which had been predicted by the greater part of those mechanicians who had seen the preparations.

[We have retained in this edition the above paragraph on the application of steam to navigation and locomotive engines on land, as a curious record of the opinions on those subjects entertained by men the most eminent in science, at a very recent period. What would these men say, could they be permitted to view the achievements of modern science, on the application of steam to travelling by sea and by land? Hundreds of people conveyed by the power of a single steam-engine from London to Liverpool at the rate of from 20 to 30 miles an hour; and steam vessels sailing from various ports in England to New York, with the regularity of mail coaches, and completing their voyages in about a fortnight; would doubtless strike them with amazement.]

Remark.—As Montucla has given but a short and imperfect account of that truly noble English invention, we have subjoined the following brief history of it. The Steam Engine was invented by the Marquis of Worcester, in the year 1655; and an account of it was printed in a little book, entitled "A Century of the Names and Scantlings of such Inventions as at present I can call to mind," &c.; in the year 1663.

In the 68th article of that work, the Marquis describes the invention in the following words:—"An admirable and most forcible way to drive up water by fire. Not by drawing or sucking it upwards, for that must be as the philosopher calleth it, *intra sparam activitatis*, which is but at such a distance; but this way hath no bounds, if the vessel be strong enough; for I have taken a piece of a whole cannon, whereof the end was burst, and filled it three quarters full of water, stopping and securing up the broken end, as also the touch-hole, and making a constant fire under it, within 24 hours it burst and made a great crack; so that having a way to make my vessels, so that they are strengthened by the force within them, and the one to fill after the other. I have seen the water run like a constant fountain stream forty feet high; one vessel of water rarefied by fire driveth up forty of cold water. And a man that tends the work is but to turn two cocks, that one vessel of water being consumed, another begins to force and refill with cold water, and so successively, the fire being tended and kept constant, which the self-same person may likewise abundantly perform in the interim between the necessity of turning the said cocks."

But although the above description is a distinct and intelligible one, of the manner of applying steam for raising of water, yet no person, that I have heard of, attempted to erect a machine on these principles until the year 1699; when Captain Savary produced, the 14th of June in that year, a model which was worked before the Royal Society, at their weekly meeting at Gresham College. He afterwards published an account of this machine in the year 1702, in a work entitled "The Miner's Friend."

In Savary's machine, the steam is used for making a vacuum in a vessel placed near to the water to be raised, and communicating with it by a pipe, which has a cock or valve adapted to it. This valve or cock being opened when there is a vacuum in the vessel, the atmosphere presses the water into the vessel; and when this is filled, the valve or cock is shut; and steam being let into it, this presses on the surface

of the water, and forces it upwards through a pipe adapted to the vessel for this purpose.

The disadvantages attending this method of construction were so great, that Capt. Savary never succeeded further than in making some engines for the supply of gentlemen's seats; but he did not succeed for mines, or the supplying of towns with water. This discouragement stopped the progress and improvement of the Steam Engine, till Mr. Newcomen, an ironmonger, and John Ceudley, a glazier at Dartmouth, about the year 1712, invented what is called the Lever or Newcomen engine. In this machine, the steam is made to act in a cylinder distinct from the pumps, and is used merely for the purpose of making and unmaking a vacuum, in this manner—namely, there is a piston in the cylinder, fitted so nicely to it, that it can slide easily up and down without the admission of any air, or other fluid, to pass between its edge and the cylinder. The steam is admitted below the piston, which, being of a strength equal to the atmosphere, brings it into a state of equilibrium, when the weight of the pump rods and volumes of water, at the other end of the lever or balance, raises it up; when the piston has got to the top of the cylinder, a jet of cold water is thrown amongst the steam, which condenses it, and forms a partial vacuum. The atmosphere then acting on the upper side of the piston, forces it down, and raises the column of water at the other end of the beam.

No improvement on this principle took place for above half a century, except in the construction of a variety of contrivances for the purpose of opening and shutting the different cocks and valves, necessary to admit the steam into the cylinder, the water to condense it, to carry off the condensed steam, to make the piston more air-tight, and in general to improve the various working parts of the engine.

Machines of this kind have been constructed in a variety of places; particularly in Great Britain, for the purpose of raising water from mines or for supplying towns, and for raising water to turn wheels. One of the largest of this kind is that which was constructed by the late ingenious Mr. Smeaton, for raising water to turn the wheels of the Blast Furnaces at Carron—the cylinder of this engine is 72 inches in diameter, and I believe it is reckoned the most perfect engine that has been constructed on Newcomen's principle. But although Mr. Smeaton spent much time in the improvements of these engines, and succeeded to a very considerable extent, yet the manner of employing the steam in a cylinder where cold water is to be admitted, for the purpose of condensing it at each stroke, and the piston and cylinder being exposed to the atmosphere, render it so imperfect that above one half of the power of the steam is lost by this construction. And therefore, even with Mr. Smeaton's ingenious improvements, the Steam Engine at that time was but a very imperfect machine, and by no means applicable to such a variety of purposes as it is now in its improved state.

The ingenious Mr. James Watt of Glasgow, perceiving the great loss of steam which was sustained in its use, in Newcomen's engine, about 1768 made a variety of experiments on this subject, and in 1770 obtained a patent for a new mode of applying it; in which the cylinder was made close both at bottom and top, and the rod which connected the piston with the lever, was made to work through a collar of hemp and tallow, so as to be perfectly air-tight. The atmosphere being thus excluded from the cylinder, both the vacuum is made by the steam, and the piston is moved by it. Also the steam is not condensed by throwing cold water into the cylinder, but it is taken out by an air-pump, and condensed in a separate vessel; and in order to keep the cylinder as hot as possible, it is surrounded with steam, and covered with non-conducting substances. By this construction, the engine has been made to perform at least double the effect with the same quanity of fuel, as the best engines on Newcomen's construction. Mr. Watt obtained an extension of his patent right in the year 1775, by an act of parliament, for 25 years; and was joined by the ingenious Mr.

Boulton of Soho, near Birmingham; since which, the same principle has still been followed; but the working parts have undergone various modifications, by the joint abilities of these able mechanicians. The principle which was applied to the working of the piston, only one way, that is, by pushing it downwards, as the atmosphere did in Newcomen's engine, has also been applied to the forcing it up; by which means, engines, where cylinders are of a given diameter, are now made to perform double the effect. This has not only saved great expence in the original construction of the engines, but has enabled them to be applied in cases where immense power has been wanted, and which could not have been performed at all by them on Newcomen's construction. By the same mode of applying the steam, it can now not only be used of the strength of the atmosphere, but as much stronger as necessity or convenience may require; which is a still further consolidation of the power. The celerity also with which the condensation of the steam, and the discharging of the condensed steam and water, are performed, enables them to work quicker, and so to be applied to all kinds of mill work, which are used in the numerous manufactories of this country. Corn is ground by them, cotton spun, silk twisted, the immense machinery used in the new manufactories are worked, and including every kind of mill work to which water can be applied. They are also used in the various branches of the civil engineer. Thus the water is taken from the foundations of Locks, Bridges, Docks, &c. The piles are driven for the foundations, as the mortar manufactured for the building of the walls; earth taken from their canals; and docks and works have been of late performed by their means, which could not have been executed without them.

They are also made so portable for some purposes, that they are even constructed on boats and carriages, to be moved from one place to another; while in others they are made on a large and magnificent scale. Messrs. Boulton and Watt have made them from the power of one, to that of 250 horses; and by their late contrivances in the execution of their different parts, they are so manageable, that even a lad may attend and direct their operations; and so regular in their motions, that water itself cannot be more so.

The quantity of fuel which they consume is comparatively small, to the effect they produce. One bushel of the best Newcastle coal applied to the working of an engine for pumping, will raise about thirty millions of pounds one foot high.—But in these engines, when the steam acts on the piston, both in its ascent and descent, the same quantity of fuel will not produce quite so great an effect, as there is not so much time for performing the condensation, on which account the vacuum is not so complete.

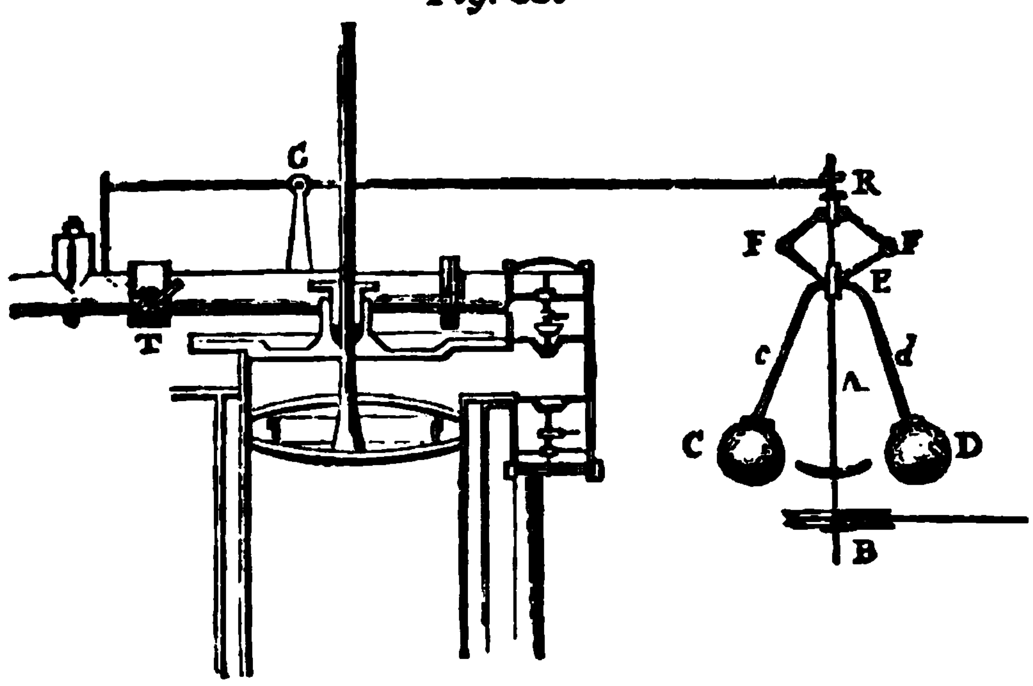
In the application of the steam engine to driving machinery, it is important that a uniform motion should exist. To equalize the variable force communicated by the engine, a large and heavy metal wheel, called a fly wheel, is fixed on the axis turned by the crank which converts the reciprocating motion into a rotatory one, —and this wheel revolves with the axis on which it is fixed. The tendency of this heavy rotating wheel, to retain the velocity which it receives, renders the motion sufficiently uniform for all practical purposes, when the supply of steam from the boiler is nearly uniform, and the resistance to be overcome is also nearly uniform.

To ensure a uniform velocity, however the load or resistance may be varied, it is necessary so to proportion the supply of steam to the resistance, that upon the least change in the velocity the supply of steam may be correspondently raised, so as to keep the engine always going at the same rate.

One of the most remarkable appendages of the steam engine is an apparatus called the *governor*, invented by Mr. Watt, for effecting this object,—viz., for *regulating the supply of steam to the engine.* In the pipe which conducts steam from

the boiler to the cylinder is placed a thin circular plate, which, when its face is presented towards the length of the pipe, nearly stops it,—and when it is inclined more or less, a greater or less quantity of steam is permitted to pass. This plate or valve is called the *throttle* valve; and the following account of the mechanism, which Mr. Watt contrived for making the engine itself turn the plate exactly and always into the precise position in which it is required to be, will, we have no doubt, be interesting to our readers.

Fig. 55.

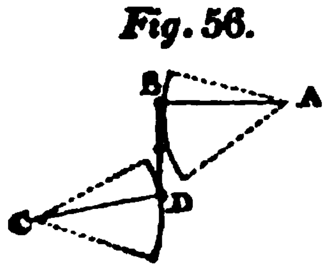


A (Fig. 55.) is a perpendicular axle, to which a grooved wheel B is attached, and which turns with the shaft in the pivots at the top and bottom of it. A strap rolled on the axis of the fly-wheel passes round the groove in the wheel B, as the strap acts in a turning lathe. Thus the rotation of the fly wheel and that of the shaft A will always vary in the same proportion. C and D are two heavy balls at the ends of the rods *c* and *d*, which play on an axis fixed on the revolving shaft at E, and extend beyond the axis to F F. Connected with these rods by joints at F, F, are two other rods F, R, attached to a broad ring of metal which moves freely up and down the revolving shaft; and to this ring a lever is attached, whose centre is at G; and it is connected by a series of levers with the throttle valve T. When the speed of the fly-wheel becomes considerably increased, the spindle A is wheeled more rapidly round; and the balls acquiring greater centrifugal force recede from the axis, depress the metal ring which slides on the spindle, and with it the adjoining end of the lever, raising at the same time the opposite end, and thus partially closing the throttle valve by means of the connecting apparatus, the supply of steam from the boiler to the cylinder is diminished, and a corresponding retardation of motion takes place in consequence. And the contrary effect is produced when the rotation of the fly wheel is diminished.

It will thus be perceived, that when, from any alteration of the load or resistance to be overcome, the velocity of the fly wheel becomes increased or diminished, a corrective is supplied immediately and in accurate perfection, by the action of the *governor*, which has justly been characterised as one of the most elegant and ingenious of mechanical inventions.

Another most important mechanical device, for converting the straight *in and out* motion of the piston rod into a circular motion at the end of the working beam of the engine, merits notice in this place.

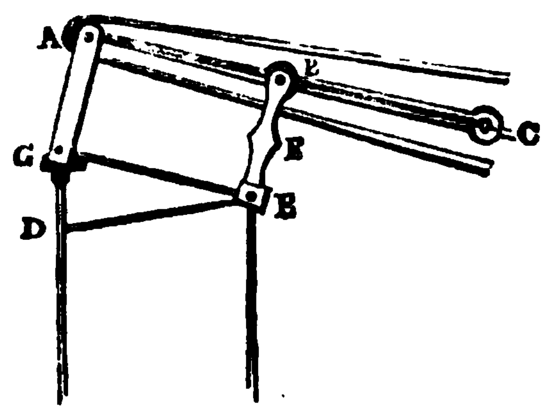
Fig. 56.



Mr. Watt conceived two straight rods, A B, C D (Fig. 56.) moving on pivots at A and C, their extremities B and D being connected by a third rod B D, by pivots at B and D, on which B D can turn freely. Now the pivots B and D will move in circular arcs, whose centres are at A and C; but the middle point of the connecting rod B D will move upwards and downwards in a straight line.

The apparatus which, on this principle, Mr. Watt devised, for accomplishing, in practice, the object in question, may be thus described.

Fig. 57.



The beam moving on its centre C (Fig. 57.), every point in its arm describes the arc of a circle round C as a centre. Let B be the middle of A C, and let D E be a straight rod equal in length to A B or B C, and playing on the pivot D. The end E of the rod is connected by the pivots at B and E, with the straight bar E B. Hence, according to what has just been stated, while the beam and D E revolve round C and D as centres, F, the middle of B E, will move up and down in a straight line.

Again, if a rod A G, equal in length to B F, be attached to A, the end of the beam, by a pivot on which it moves freely: and if G be connected with E by a rod equal in length to A B, the point G will move in a straight line parallel to that in which F moves.

The piston-rod of the steam cylinder is attached to the point G, and that of the air pump to the point F: so that they move in parallel straight lines, the piston-rod having double the stroke of that of the air-pump, being twice the distance from C, the centre of motion.

BALLOONS, TELEGRAPHS, &c.

The latter part of the last century, among many ingenious mechanical inventions, has produced the two remarkable ones relating to air balloons, and to telegraphs, with other means of distant, quick, or secret intelligence; concerning which a brief account may here be added; and first of Aerostation and Air Balloons.

The fundamental principles of aerostation have been long and generally known, as well as speculations on the theory of it; but the successful application of them to practice seems to be altogether a modern discovery. These principles chiefly respect the pressure and elasticity of the air, with its specific gravity, and that of the other bodies to be floated in it. Now any body that is specifically, or bulk for bulk, lighter than the atmosphere, is buoyed up by it, and ascends to such height where the air, by always diminishing in its density upward, becomes of the same specific gravity as the rising body; here this body will float, and move along with the wind or current of air, like clouds at that height. This body then is an aerostatic machine, whatever its form or nature may be; such as an air-balloon, the whole mass of which, including its covering and contents, with the weights annexed to it, is of less weight than the same bulk of air in which it rises.

We know of no solid bodies however that are light enough thus to ascend and float in the atmosphere; and therefore recourse must be had to some fluid or aeriform substance. Among these, that which is called inflammable air is the most proper for that purpose: it is very elastic, and is six, eight, or ten times lighter than common air. So that, if a sufficient quantity of that kind of air be inclosed in any thin bag or covering, the weight of the two together will be less than the weight of the same bulk of common air: consequently this compound mass will rise in the atmosphere, till it attain the height at which the atmosphere is of the same specific gravity as itself; where it will remain or float with the current of air, as long as the inflammable gas does not too much escape through the pores of its covering. And this is an inflammable-air balloon.

Another way is, to make use of common air rendered lighter, by heating it, instead of the inflammable air. Heat rarefies and expands common air, and consequently lessens its specific gravity. So that, if the air, inclosed in any kind of a bag or cover-

ing, be heated, and this dilated, to such a degree that the excess of the weight of an equal volume of common air, above the weight of the heated air, be greater than the weight of the covering and its appendages, the whole compound mass will ascend in the atmosphere, till it arrive at a height where the atmosphere has the same specific gravity with it; where it will remain till, by the cooling and condensation of the included air, the balloon shall gradually contract, and descend again, unless the heat be renewed or kept up. And this is a heated-air balloon, which is also called a Montgolfier, after the name of its inventor.

Various schemes for rising up in the air, and passing through it, have been devised and attempted, both by the ancients and the moderns, on different principles, and with various success. Of these attempts, some have been on mechanical principles, or by the powers of mechanism; and such, it is conceived, were the instances related of the flying pigeon made by Archytas, also the flying eagle, and the fly by Regiomontanus, with many others, both among the ancients and moderns.

Other projects have been vainly formed, by attaching wings to some part of the human body, to be moved either by the hands or the feet, by means of mechanical powers; so that striking the air with them, after the manner of the wings of a bird, the person might raise himself in the air, and transport himself through it, in imitation of that animal. But these attempts belong rather to that species or principle of motion called artificial flying, than to the subject of aerostation, which is properly the sailing or floating in the air by means of a machine rendered specifically lighter than that element, in imitation of aqueous navigation, or the sailing on the water in a ship or vessel, which is specifically lighter than this element.

The first rational account to be found on record, for this sort of sailing, is perhaps that of our countryman Roger Bacon, who died in the year 1292. He not only affirms that the art is feasible, but assures us that he himself knew how to make a machine, in which a man sitting might be able to convey himself through the air like a bird: and he farther affirms that there was another person who had tried it with success. The secret it seems consisted in a couple of large thin shells, or hollow globes, of copper, exhausted of air; so that the whole being thus rendered lighter than air, they would support a chair, in which a person might sit.

Bishop Wilkins too, who died in 1672, in several of his works, makes mention of similar ideas being entertained by divers persons. "It is a pretty notion to this purpose," says he, (in his Discovery of a New World), mentioned by Albertus de Saxonia, and out of him by Francis Mendoza, "that the air is in some part of it navigable. And that upon this static principle, any brass or iron vessel, suppose a kettle, whose substance is much heavier than that of the water; yet being filled with the lighter air, it will swim upon it, and not sink." And again, in his Dedalies, he says, "Scaliger conceives the framing of such volant automata to be very easy. Those ancient motions we thought to be contrived by the force of some included air As if there had been some lamp or other fire within it, which might produce such a forcible rarefaction as should give a motion to the whole frame." From whence it would seem that Bishop Wilkins had some confused notion of such a thing as a heated-air balloon.

Again, father Francisco Lana, in his Prodroma, printed in 1670, proposes the same method with that of Roger Bacon, as his own thought. He considered that a hollow vessel, exhausted of air, would weigh less than when filled with that fluid. He also reasoned that, as the capacity of spherical vessels increases much faster than their surface, the former increasing as the cube of the diameter, but the latter only as the square of the same, it is therefore possible to make a spherical vessel of any given matter and thickness, and of such a size as, when emptied of air, it will be lighter than an equal bulk of that air, and consequently that it will ascend in the atmosphere. After stating these principles, father Lana computes that a round

vessel of plate brass, 14 feet in diameter, weighing 8 ounces the square foot, will only weigh 1848 ounces; whereas a quantity of air of the same bulk will weigh 2156 ounces, allowing only one ounce to the cubic foot; so that the globe will not only ascend in the air, but will also carry up a weight of 308 ounces: and by increasing the bulk of the globe, without increasing the thickness of the metal, he adds, a vessel might be made to carry up a much greater weight.

Such then were the speculations of ingenious men, and the gradual approaches towards this art. But one thing more was yet wanting: although in some degree acquainted with the weight of any quantity of air, considered as a detached substance, it seems they were not aware of its great elasticity, and the universal pressure of the atmosphere; a pressure by which a globe, of the dimensions above described, and exhausted of its air, would immediately be crushed inwards, for want of the equivalent internal counter pressure, to be sought for in some element, much lighter than common air, and yet nearly of equal pressure or elasticity with it; a property and circumstance attending inflammable gas, and also common air when considerably heated.

It is evident then that the schemes of ingenious men hitherto must have gone no farther than mere speculation; otherwise they could never have recorded fancies which, on the first attempt to be put in practice, must have manifested their own insufficiency, by an immediate failure of success. For, instead of exhausting the vessel of air, it must be filled either with common air heated, or with some other equally elastic but lighter air. So that on the whole it appears, that the art of traversing the atmosphere, is an invention of our own time; and the whole history of it is comprehended within a very short period.

The rarefaction and expansion of air by heat, is a property of it that has been long known, not only to philosophers, but even to the vulgar. By this means it is, that the smoke is continually carried up our chimneys: and the effect of heat upon air is made very sensible by bringing a bladder, only partially filled with air, near a fire; when the air presently expands with the heat, and distends the bladder so as almost to burst it. Indeed, so well are the common people acquainted with this effect, that it is the constant practice of those who kick about blown bladders, for foot balls, to bring them from time to time to the fire, to restore the spring of the air, and the distension of the ball, lost by the continual cooling and waste of that fluid.

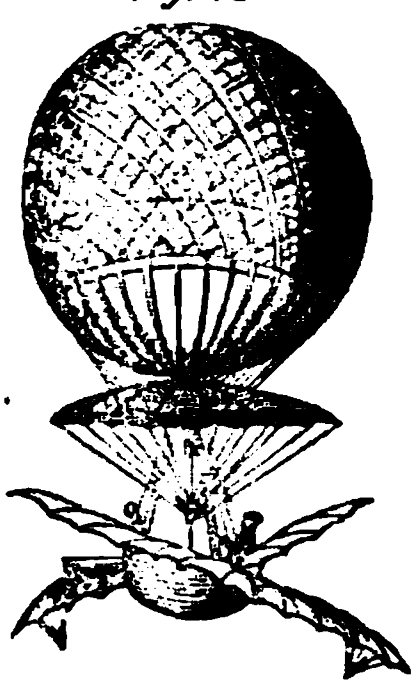
But the great levity, or rather small weight, of inflammable gas, is a modern discovery, namely, within the last 70 or 80 years; a discovery chiefly owing to our own countrymen, Mr. Cavendish and Dr. Black, the latter of whom frequently mentioned also the feasibility of inclosing it in a very thin bag, so as that it might ascend into the atmosphere; an idea which was first put in practice, on a very small scale, by Mr. Cavallo, another ingenious philosopher.

It was however two brothers, of the name of Montgolfier, near the city of Lyons in France, who, in the year 1782, first exhibited to the world what may properly be called air-balloons, of large dimensions, being silken bags of many feet in diameter. These were on the principle of common air heated, by passing through a fire, made near the orifice or bottom of the balloon. This heated air and the smoke thus ascended straight up into the bag, and gradually distended it, till it became quite full, and so much lighter than the atmosphere, that the balloon rapidly ascended, and carried up other weights with it to very great heights. After attaining its utmost height however, partly by the cooling of the included air, and partly by its escape through the pores of the covering, the balloon gradually descends very slowly, and comes at length to the ground, after being sometimes carried to great distances by the wind, or currents of air in the atmosphere.

Other balloons were also soon made by the philosophers in France, and after them in other countries; namely, by filling the balloon case with inflammable gas; a more

troublesome and expensive process, but of much better effect; because, having only to guard against the waste of the fluid through the pores, but not its cooilng, these balloons continue much longer in the air, sometimes for the space of many hours, enabling the passengers to pass over large tracts of country. On one of these occasions, Mr. Blanchard, a noted operator, with a favourable wind, passed over from Dover to Calais, accompanied by another gentleman.

Fig. 58.



Many other persons exhibited balloons, of large dimensions, particularly in France and other parts of the continent, with various success. The people of that country have also successfully applied balloons to the examination of the state of the higher regions of the atmosphere; and also in their armies, to discover the dispositions and operations of an enemy's position and camp. In England they have been less attended to, perhaps owing at first to an unfortunate prejudice, and an idea thrown out, that they could not be turned to any useful purpose in life.

Balloons have become so common within the last few years, that their appearance must be familiar to all our readers, and we therefore give only the annexed cut of Blanchard's balloon with the parachute.

TELEGRAPHS.

A Telegraph is a machine lately brought into use by the French nation, namely in the year 1793; being contrived to communicate words or signals, from one person to another, at a great distance, and in a very short time.

The object proposed is, to obtain an intelligible figurative language, to be distinguished at a distance, to avoid the obvious delay in the dispatch of orders or information by messengers.

On first reflection, we find the practical modes of such distant communication must be confined to sound and vision, but chiefly the latter. Each of these is in a great degree affected by the state of the atmosphere: as, independent of the wind's direction, the air is sometimes so far deprived of its elasticity, or whatever other quality the conveyance of sound depends on, that the heaviest ordnance is scarcely heard farther than the shot flies: and, on the other hand, in thick hazy weather, the largest objects become quite obscured at a short distance. No instrument therefore, designed for the purpose, can be perfect. We can only endeavour to diminish these defects as much as may be.

Some kind of distant signals must have been employed from the earliest antiquity. It seems the Romans had a method in their walled cities, either by a hollow formed in the masonry, or by tubes affixed to it, so to confine and augment sound, as to convey information to any part they wished; and in lofty houses it is now sometimes the custom to have a pipe, by way of speaking trumpet, to give orders from the upper apartments to the lower: by this mode of confining sound, its effect may be carried to a very great distance; but beyond a certain extent, the sound, losing articulation, would only convey alarm, and not give directions.

Every city among the ancients had its watch-towers; and the castra stativa of the Romans had always some spot, elevated either by art or nature, from whence signals were given to the troops cantoned or foraging in the neighbourhood. But they had probably not arrived at greater refinement than that, on seeing a certain signal, they were immediately to repair to their appointed stations.

A beacon, or bonfire made of the first inflammable materials that offered, as the

most obvious, is perhaps the most ancient mode of general alarm, and by being previously concerted, the number or point where the fires appeared might have its particular intelligence affixed. The same observations may be referred to the throwing up of rockets, whose number or the point from whence thrown, may have its affixed signification.

Flags or ensigns, with their various devices, are of earliest invention, especially at sea; where, from the first idea, which was probably that of a vane to shew the direction of the wind, they have been long adopted as the distinguishing mark of nations, and are now so neatly combined by the ingenuity of a great naval commander, that by his system every requisite order and question is received and answered by the most distant ships of a fleet.

To the adopting this, or a similar mode, in land service, the following are objections: that in the latter case, the variety of matter necessary to be conveyed is so exceedingly great, that the combinations would become too complicated. And if the person for whom the information is intended should be in the direction of the wind, the flag would then present a straight line only, and at a little distance be invisible. The Romans were so well aware of this inconvenience of flags, that many of their standards were solid; and the name *manipulus* denotes the rudest of their modes, which was a truss of hay fixed on a pole.

The principle of water always keeping its own level has been suggested, as a possible mode of conveying intelligence, by an ingenious gentleman, and put in practice on a small scale, with a very pleasing effect. As for example, suppose a leaden pipe to reach between too distant places, and to have a perpendicular tube connected to each extremity. Then, if the pipe be constantly filled with water to a certain height, it will always rise to its level on the opposite end; and if but one inch of water be added at one extremity, it will almost instantly produce a similar elevation in the tube at the other end; so that by corresponding letters being adapted to the vertical tubes, at different heights, intelligence may be quickly conveyed. But this method is liable to such objections, that it is not likely it can ever be adopted to facilitate the object of very distant communications.

Full as many, if not greater objections, will perhaps operate against every mode of electricity being used as the vehicle of information. And the requisite magnitude of painted or illuminated letters, offers an insurmountable obstacle; besides in them one object would be lost, that of the language being figurative.

Another idea is perfectly numerical, which is to raise and depress a flag or curtain a certain number of times for each letter, according to a previously concerted system: as, suppose one elevation to mean A, two to mean B, and so on through the alphabet. But in this case, the least inaccuracy in giving or noting the number, changes the letters; and besides the last letters of the alphabet would be a tedious operation.

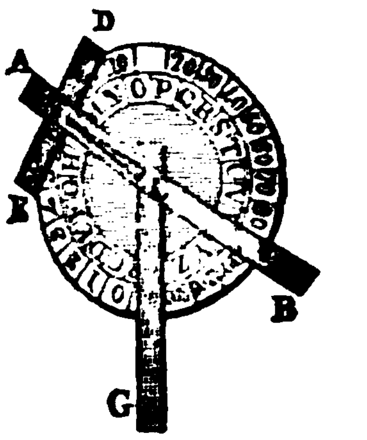
Another method that has been proposed, is an ingenious combination of the magnetical experiment of Comus, and the telescopic micrometer. But as this is only an imperfect idea of Mr. Garnet's very ingenious machine, described below, no farther notice need be taken of it here.

Mr. Garnet's contrivance is merely a bar or plank, turning on a centre like the arm of a windmill; which being moved into any position, an observer or correspondent at a distance turns the tube of a telescope round its axis, into the same position, by bringing a fixed wire within it to coincide with, or become parallel to, the bar, which is a thing extremely easy to do. The centre of motion of the bar has a small circle fixed on it, with letters and figures around the circumference, and a moveable index turning together with the bar, pointing to any letter or mark the operator may wish to set the bar to, or to communicate to the observer. The eye end of the telescope has a like index and circle fixed on the outside of it with the corresponding letters or other marks. The consequence is obvious; the telescope being turned round its axis,

till its wire cover, or become parallel to the bar, the index of the former necessarily points out the same letter or mark on its circle as that of the latter, and the communication of sentiment is immediate and perfect. The use of this machine is so easy, that we have seen it put into the hands of two common labouring men, who had never seen it before, when they have immediately held a quick and distant conversation together.

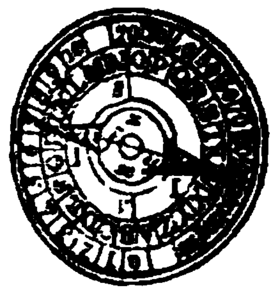
Fig. 59. represents the principal parts of this telescope: A B D E is the telegraph or bar, having on the centre of gravity C, about which it turns, a fixed pin, going through a hole or socket in the firm upright post G, and on the opposite side is fixed an index C F. Concentric to C, on the same post, is fixed a brass circle, of 6 or 8 inches diameter, divided into 48 equal parts, 24 of which represent the letters of the alphabet, and in the other 24, between the letters are numbers. So that the index, by means of the arm A B, may be set or moved to any letter or number. The length of the arm or bar should be 2½ or 3 feet for every mile of distance. Two revolving lamps of different colours, suspended occasionally at A and B, the ends of the arm, would serve equally at night.

Fig. 59.



Let s s (Fig. 60.) represent a transverse section of the outward tube of a telescope, and x x the like section of the sliding or adjusting tube, on which is fixed an index I I. On the part of the outward tube next to the observer, is fixed a circle of letters and numbers, similarly divided and situated as the former circle in Fig. 59; so that the index I I, by means of the sliding or adjusting tube, may be turned to any other letter or number. Now there being a hair, or fine silver wire, *f g*, fixed in the focus of the eye-glass; when the arm A B of the telegraph is viewed at a distance through the telescope, the hair may be turned, by means of the sliding tube, to the same position as the arm A B; then the index I I (Fig. 60.) will point to the same letter or number on its own circle, as the index I (Fig 59.) points to on the telegraphic circle.

Fig. 60.



If, instead of using the letters and numbers to form words at length, they be used as signals, three motions of the arm will give a hundred thousand different signals.

But a telegraph, combined with a telescope, it seems was originally the invention of M. Amontons, an ingenious French philosopher, about the middle of the 17th century; when he pointed out a method to acquaint people at a great distance, and in a very little time, with whatever we please. This method was as follows: Let persons be placed in several stations, at such distances from each other, that, by the help of a telescope, a man in one station may see a signal made by the next before him; this person immediately repeats the same signal to the third man; and this again to a fourth, and so on through all the stations, to the last.

This, with considerable improvements, it seems has lately been brought into use by the French, and called a Telegraph. It is said they have availed themselves of this contrivance to good purpose, in the late war; which has induced the English also to employ a like instrument, in a different form.

The new invented telegraphic language of signals, says a French author, is an artful contrivance to transmit thoughts, in a peculiar way, from one distance to another, by means of machines, which are placed at different distances, of from 12 to 15 miles each, so that the expression reaches a very distant place in the space of a few minutes. The only thing which can interrupt their effects is, if the weather be so bad and turbid, that the objects and signals cannot be distinguished. By this invention, remoteness and distance almost disappear; and all the communications of

correspondence are effected with the rapidity of the twinkling of an eye. The greatest advantage which can be derived from this correspondence, is that, if we choose, its object shall be known to certain individuals only, or to one individual alone, or to the extremities of any distance.

Fig. 61.

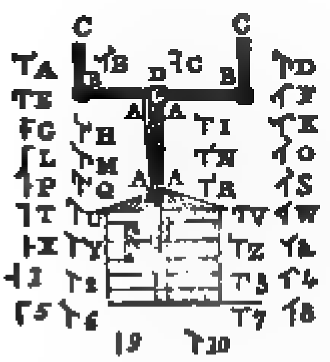


Fig. 61. represents the form of the French Telegraph. A A is a beam or mast of wood, placed upright on a rising ground, and is 15 or 16 feet high. B B is a beam or balance moving on the centre A A. This balance beam may be placed vertically, or horizontally, or any how inclined, by means of strong cords, which are fixed to the wheel D, on the edge of which is a double groove, to receive the two cords. This balance is 11 or 12 feet long, and 9 inches broad, having at the end two bars C C, which likewise turn on the angles by means of four other cords passing through the axis of the main balance. The pieces C are each about three feet long, and may be turned and placed either to the right or left, straight or square with the balance beam. By means of these three, the combination of movements is said to be very extensive, remarkably simple, and easy to perform. Below is a small wooden hut, in which a person is employed to attend the movements of the machine. In the mountain nearest to this, another person is to repeat these movements, and a third to write them down. The signs are sometimes made in words, and sometimes in letters; when in words a small flag is hoisted; and as the alphabet may be changed at pleasure, it is only the corresponding person who knows the meaning of the signs. The alphabet, as well as the numbers to 10, are exhibited in the middle of Fig. 61, annexed to the different forms and positions into which the bars of the machine may be put.

Many improvements and additional contrivances have been since made in England. The following one is by the Rev. J. Gamble. The principle of it is simply that of a Venetian window-blind, or rather what are called the lever boards of a brewhouse, which when horizontal, present so small a surface to the distant observer, as to be lost to his view, but are capable of being in an instant changed into a screen of a magnitude adapted to the required distance of vision. A E B D F C (Fig. 62), is a firm upright frame, supporting nine lever boards working on centres in E B and D F, and opening in three divisions by iron rods. And *a b c d*, *e f g h*, are two lesser frames, fixed to the great one, having also three lever boards in each, and moving by iron rods, in the same manner as the others. If all these rods be brought so near the ground, as to be in the management of the operator, he will then have five keys to play on. Now as each of the handles *i k l m n* commands three lever boards, by raising any one of them, and fixing it in its place by a catch or hook, it will give a different appearance in the machine; and by the proper variation of these five movements, there will be more than 25 of what may be called mutations, in each of which the machine exhibits a different appearance, and to which any letter or figure may be annexed at pleasure.

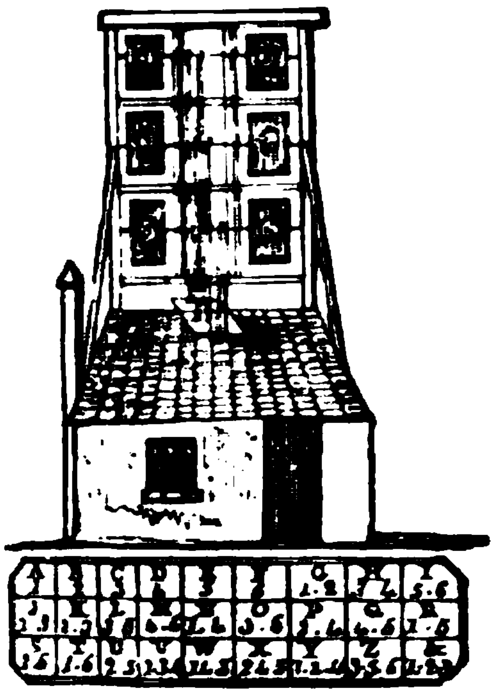
Fig. 62.

Should it be required to give intelligence in more than one direction, the whole machine may easily be made to turn to different points, on a strong centre, after the manner of a single post windmill. To use this machine by night, another frame must be connected with the back part of the Telegraph, for raising five lamps, of different colours, behind the openings of the lever boards; these lamps by night answering for the openings by day.

Fig. 63, represents a front view of the latest form of the Telegraph, now em-

ployed by the English government, by which a signal is conveyed between London and Deal, being 72 miles, by repetition, in three minutes. The corresponding boards forming a scale for the alphabet, and for numbering, is annexed in the engraving.

Fig. 63.



We shall limit ourselves to what has been here said respecting those machines, which have acquired the greatest celebrity; but we shall point out a few books which those who are fond of machines, and who wish to instruct themselves by example, may consult for that purpose. The first of these, which we shall mention, is the *Theatrum Mechanicum* of Leupold, in several volumes folio, the last of which appeared in 1725. This is a curious work, but the author's theory is not always well founded; for he seems not be entirely convinced of the impossibility of the perpetual motion. The next is the *Théâtre des Machines* of James Besson, in Italian and French. And to these we shall add, Bockler's work, in Latin; that of Ramelli in Italian and French, which is rare, and in great request. The *Cabinet des Machines* of de Servieres, 4to, Paris 1733, is one of the most curious works of this kind, on account of the great number of machines described in it, and which were invented by the author. Some of them are very ingenious, and the principles on which they are constructed deserved to have been better explained; but, in general, they are more curious than useful.

The description of the method in which the Chevalier Carlo Fontana raised the famous obelisk, now before St. Peter's at Rome, is likewise a work worthy of a place in the library of every person fond of mechanics. M. Loriot, who has a collection of machines, the invention of which displays great ingenuity, has promised to publish some day a description of them. This, in our opinion, would be a curious and useful work; for the most of his machines bear the stamp of genius. We have seen one, invented by him for driving piles, which acts by a motion always in the same direction, without being obliged to stop or to retrograde, in order to raise up again the weight. Nothing, in our opinion, can be more ingenious than the method in which, after the fall of the weight or rammer, the hook that serves to raise it again, lays hold of it, and by which the cable lengthens itself in order to reach lower and lower, in proportion as the pile sinks deeper. If this mode of construction be compared with those hitherto employed, no one can refuse to give it the preference.

There is also the Collection, in 6 vols. 4to, of Machines and Inventions approved by the Royal Academy of Sciences, containing the engravings and descriptions of a great multitude of machines. In English too we have Desaguliers's Course of Experimental Philosophy, in 2 vols. 4to.; also Emerson's Mechanics, both containing the figures and descriptions of many curious and useful machines. Besides some others of less note.

A TABLE

OF THE SPECIFIC GRAVITIES OF DIFFERENT BODIES, THAT OF RAIN OR DISTILLED WATER BEING SUPPOSED 1000.

METALS.

Gold.	Spec. Grav.
Pure gold of 24 carats, melted but not hammered	19258
The same hammered	19362
Gold, of the Parisian standard, 22 carats fine, not hammered*	17486
The same hammered	17589
Gold of the standard of French coin, 21$\frac{19}{32}$ carats fine, not hammered	17402
The same coined	17647
Gold of the French trinket standard, 20 carats fine, not hammered	15709
The same hammered	15775

Silver.	Spec. Grav.
Pure or virgin silver, 12 deniers fine, not hammered	10474
The same hammered	10511
Silver of the Paris standard, 11 deniers 10 grains fine, not hammered †	10175
The same hammered	10377
Silver, standard of the French coin 10 deniers 21 grains fine, not hammered	10048
The same coined	10408

Platina.	Spec. Grav.
Crude platina, in grains	15602
Purified platina, not hammered	19500
The same hammered	20337
The same drawn into wire	21042
The same rolled	22069

Copper and Brass.	Spec. Grav.
Copper, not hammered	7788
The same wire-drawn	8879
Brass, not hammered	8396
The same wire-drawn	8544
Common cast brass	7824

Iron and Steel.	Spec. Grav.
Cast iron	7207
Bar iron, either hardened or not	7788
Steel, neither tempered nor hardened	7833
Steel hardened under the hammer, but not tempered	7840
Steel tempered and hardened	7818
Steel tempered and not hardened	7816

Other Metals.	Spec. Grav.
Pure tin from Cornwall, melted and not hardened	7291
The same hardened	7299
Malacca tin, not hardened	7296
The same hardened	7307
Molten lead	11352
Molten zinc	7191
Molten bismuth	9823
Molten cobalt	7812
Molten arsenic	5763
Molten nikel	7807
Molten antimony	6702
Crude antimony	4064
Glass of antimony	4946
Molybdena	4739
Tungsten	6067
Mercury	13568
Uranium	6440

PRECIOUS STONES.

White oriental diamond	3521
Rose coloured ditto	3531
Oriental ruby	4283
Spinell ditto	3760

* This is the same as sterling gold.

† This is 10 grs. finer than sterling.

	Spec. Grav.		Spec. Grav.
Ballas ditto	3646	Vermilion	4230
Brasilian ditto	3531	Bohemian garnet	4189
Oriental topaz	4011	Syrian ditto	4000
Saxon ditto	3564	Volcanic ditto with 24 sides	2468
Oriental sapphire	3994	Peruvian emerald	2776
Brasilian ditto	3131	Chrysolite of the jewellers	2782
Girasol	4000	Brasilian ditto	2692
Jargon of Ceylon	4416	Beryl or oriental aqua-marine	3549
Hyacinth	3687	Occidental ditto	2723

SILICEOUS STONES.

Pure rock crystal of Madagascar	2653	Brown Jasper	2691
Ditto of Europe	2655	Yellow ditto	2710
Crystallized quartz	2655	Violet ditto	2711
Oriental agate	2590	Grey ditto	2764
Agate onyx	2638	Black prismatic hexaedral schorl	3385
Transparent calcedony	2664	Black amorphus schorl, called antique basaltes	2923
Cornelian	2614	Paving stone	2416
Sardonyx	2603	Grind-stone	2143
Prasium	2581	Cutler's stone	2111
Onyx pebble	2664	Mill-stone	2484
White jade	2950	White flint	2594
Green ditto	2966	Blackish ditto	2582
Red Jasper	2661		

VARIOUS STONES, &c.

Opake green Italian serpentine	2430	Violet fluor	3176
Coarse Briançon chalk	2727	Red porphyry	2765
Spanish chalk	2790	Red Egyptian granite	2654
Muscovy talc	2792	Pumice stone	915
Common schist or slate	2672	Obsidian stone	2348
New slate	2854	Basaltes from the Giant's Causeway	2864
White razor hone	2876	Touch-stone	2415
Black and white ditto	3131	Bottle glass	2733
Icelandic crystal	2715	Green glass	2642
Pyramidal calcareous spar	2730	White glass	2892
Oriental or white antique alabaster	2714	Leith crystal	3189
Green Campanian marble	2742	Flint glass	3329
Red ditto	2724	Sevres porcelain	2146
White Casara marble	2717	China ditto	2385
White Parian marble	2838	Native sulphur	2033
Ponderous spar	4430	Melted ditto	1991
White fluor	3156	Phosphorus	1714
Red ditto	3191	Hard peat	1329
Green ditto	3182	Ambergris	926
Blue ditto	3169	Yellow transparent amber	1078

LIQUORS.

Distilled water	1000	Rain water	1000

	Spec. Grav.		Spec. Grav.
Sea water *	1026	Distilled acetous acid	1010
Burgundy wine	992	Acetic ditto	1063
Malmsey Madeira	1038	Formic ditto	994
Cyder	1018	Solution of caustic ammonia, or volatile alkali fluor	897
Red beer	1034	Essential oil of turpentine	870
White ditto	1023	Liquid turpentine	991
Highly rectified alcohol	829	Volatile oil of lavender	894
Common spirit of wine	837	Volatile oil of cloves	1036
Sulphuric ether	739	Volatile oil of cinnamon	1044
Nitric ditto	909	Oil of olives	915
Muriatic ditto	730	Oil of sweet almonds	917
Acetic ditto	866	Linseed oil	940
Highly concentrated sulphuric acid	2125	Whale oil	923
Common sulphuric acid	1841	Woman's milk	1020
Highly concentrated nitric acid	1580	Cow's milk	1032
Common nitric acid	1272	Mare's milk	1035
Muriatic acid	1194	Ass milk	1036
Fluoric ditto	1500	Goat's milk	1034
Red acetous ditto	1025	Ewe milk	1041
White acetous ditto	1014		

RESINS AND GUMS.

Common yellow resin	1073	Gum arabic	1452
Mastic	1074	Tragacanth	1316
Storax	1110	Terra Japonica	1398
Opake copal	1140	Socatrine aloes	1380
Madagascar ditto	1060	Opium	1337
Chinese ditto	1063	Indigo	769
Elemi	1018	Yellow wax	965
Labdanum	1186	White ditto	969
Dragon's blood	1205	Spermaceti	943
Gum lac	1139	Beef fat	923
Gum elastic	934	Veal fat	934
Camphor	989	Mutton fat	924
Gum ammoniac	1207	Tallow	942
Gamboge	1222	Hog's fat	937
Myrrh	1360	Lard	948
Galbanum	1212	Butter	942
Assafœtida	1328		

WOODS.

Heart of oak, 60 years old	1170	Willow	585
Cork	240	Male fir	550
Elm plank	671	Female ditto	498
Ash ditto	845	Poplar	383
Beech	852	White Spanish ditto	529
Alder	800	Apple tree	793
Walnut	671	Pear tree	661

* Sea water differs in weight, according to the climate. It is heavier in the torrid zone, and at a distance from the coasts, than in the northern seas and near land.

	Spec. Grav.		Spec. Grav.
Quince tree	705	Spanish ditto	807
Medlar	944	Spanish cyprus	644
Plum tree	785	American cedar	561
Cherry tree	715	Spanish Mulberry tree	897
Filbert tree	600	Pomegranate tree	1354
French box	912	Lignum vitæ	1333
Dutch ditto	1328	Orange tree	705
Dutch yew	788		

Note.—We may here observe, that the numbers in the above table express nearly the absolute weight of an English cubic foot, of each substance, in averdupois ounces.

TABLE OF WEIGHTS,

BOTH ANCIENT AND MODERN, AS COMPARED WITH THE ENGLISH TROY POUND, WHICH CONTAINS 12 OUNCES, OR 5760 GRAINS.

As we gave, at the end of that part which relates to Geometry, a comparative table of the principal longitudinal measures, we think it our duty to give here a similar table of the ancient Hebrew, Greek, and Roman weights; and also of the modern weights of different countries, particularly in Europe, as compared with the English troy pound.

ANCIENT WEIGHTS.

Hebrew Weights.

	Grs. Troy.	lib.	oz.	dwt.	gr.
The obolus called gerah	10·66	0	0	0	10·66
Half shekel or beka	103·37	0	0	4	7·37
Shekel	206·74	0	0	8	14·74
Mina or maneh	12453·67	2	1	18	21·67
Talent or cicar	622683·6	108	1	5	3·6

*Attic Greek Weights.**

	Grs. Troy.	lib.	oz.	dwt.	gr.
Chalcus	·82	0	0	0	0·82
Obolus	8·20	0	0	0	8·20
Drachma	51·89	0	0	2	3·89
Didrachma	103·78	0	0	4	7·78
Tetradrachma	207·56	0	0	8	15·56
Lesser mina of 75 drachms	3891·77	0	8	2	3·77
Greater mina of 100 drachms	5189·03	0	10	16	5·03
Lesser Talent of 60 lesser minæ	233506·20	40	6	9	10·20
Greater talent of 60 greater minæ	311341·8	54	0	12	13·8

Roman Weights.

	Grs. Troy.	lib.	oz.	dwt.	gr.
The denarius	51·89	0	0	2	3·89
Ounce, equal to 8 denarii	415·12	0	0	17	7·12

* It may be proper here to observe, that these weights were at the same time money.

	Grs. Troy.		lb.	oz.	dwt.	grs.
As or pound, equal to 12 ounces	4981·44	..	0	10	7	13·44
Another pound of 10 ounces	4151·2	..	0	8	12	23·2
The lesser talent	233506·20	..	40	6	9	10·20
The greater talent	311341·8	..	54	0	12	13·8

The above tables are taken from a work by M. Christiani, entitled, "Delle Misure d'ogni genere, antiche è moderne," &c.; printed in quarto, at Venice, in the year 1760. As this is an obscure subject, and as some difference prevails among the learned in regard to the value of the ancient weights, the translator has added the following tables from Arbuthnot, in order to render this article more complete.

Jewish weights reduced to English Troy weight.

	lib.	oz.	dwt.	gr.
The shekel	0	0	9	2[illegible]
Maneh	2	3	6	10[illegible]
Talent	113	10	1	10[illegible]

The most ancient Grecian weights, reduced to English Troy weight.

Drachma	0	0	6	2[illegible]
Mina	1	1	0	4[illegible]
Talent	65	0	12	5[illegible]

Less ancient Grecian and Roman weights reduced to English Troy weight.

Lentes	0	0	0	0[illegible]
Siliquæ	0	0	0	3[illegible]
Obolus	0	0	0	9[illegible]
Scriptulum	0	0	0	18[illegible]
Drachma	0	0	2	6[illegible]
Sextula	0	0	3	0[illegible]
Sicilicus	0	0	4	13[illegible]
Duella	0	0	6	1[illegible]
Uncia	0	0	18	5[illegible]
Libra	0	10	18	13[illegible]

The Roman ounce is the English averdupois ounce, which they divided into 7 denarii, as well as 8 drachms: and since they reckoned their denarius equal to the Attic drachm, this will make the Attic weights [illegible] heavier than the correspondent Roman weights.

We shall here observe, that the Greeks divided their obolus into chalci and lepta: thus, Diodorus and Suidas divide the obolus into 6 chalci, and every chalcus into 7 lepta: others divided the obolus into 8 chalci, and every chalcus into 8 lepta, or minuta.

The greater Attic weights, reduced to English Troy weight.

	lb.	oz.	dwt.	grs.
Libra or pound	0	10	18	13[illegible]
Common Attic mina	0	11	7	16[illegible]
Another mina used in medicine	1	2	11	10[illegible]
The common Attic talent	56	11	0	17[illegible]

It is here to be remarked, that there was another Attic talent, said by some to consist of 80, and by others of 100 minæ. Every mina contains 100 drachmæ, and every

talent 60 minæ; but the talents differ in weight, according to the different standard of the drachmæ and minæ of which they are composed. The value of different minæ and talents, in English Troy weight, is exhibited in the following tables:

Table of different Minæ.

	lib.	oz.	dwt.	grs.
Egyptian mina	1	5	6	22[illegible]
Antiochic	1	5	6	22[illegible]
Ptolemaic of Cleopatra	1	6	14	16[illegible]
Alexandrian of Dioscorides	1	8	16	7[illegible]

Table of different Talents.

Egyptian	86	8	16	8
Antiochic	86	8	16	8
Ptolemaic of Cleopatra	93	11	11	0
Alexandrian	104	0	19	14
Of the Islands	130	1	4	12
Antiochian	390	3	13	11

Modern Weights of the principal countries in the world, and particularly in Europe.

	Grs. Troy.		lib.	oz.	dwt.	gr.
Aleppo, the pound, called *rotolo*	30984·86	..	5	4	11	0·86
Alexandria in Egypt	6158·74	..	1	0	16	14·74
Alicant	6908·58	..	1	2	7	20·58
Amsterdam	7460·71	..	1	3	10	20·71
Antwerp, and the Netherlands	7048·15	..	1	2	15	4·15
Avignon	6216·99	..	1	0	19	0·99
Basle	7713·31	..	1	4	1	9·31
Bayonne	7460·71	..	1	3	10	20·71
Bergamo	4663·97	..	0	9	14	7·97
———	11659·52	..	2	0	5	19·52
Berghen	7833·17	..	1	4	6	9·17
Berne	6721·53	..	1	2	0	1·53
Bilboa	7460·71	..	1	3	10	20·71
Bois-le-Duc	7105·48	..	1	2	16	1·48
Bourdeaux, *see* Bayonne.						
Bourg	7073·57	..	1	2	14	17·57
Brescia	4496·61	..	0	9	7	8·61
Cadiz	7038·21	..	1	2	13	6·21
China (*the kin*)	9222·93	..	1	7	4	6·93
Cologne	7220·34	..	1	3	0	20·34
Constantinople	7578·03	..	1	3	15	18·03
Copenhagen	6940·58	..	1	2	9	4·58
Damascus	25612·88	..	4	5	7	4·88
Dantzic	6573·86	..	1	1	13	21·86
Dublin	7774·11	..	1	3	19	18·11
Florence	5286·65	..	0	11	0	6·65
Genoa	4426·05	..	0	9	4	10·05
———	6637·85	..	1	1	16	3·85
Geneva	8407·45	..	1	5	10	7·45
Hamburgh	7314·68	..	1	3	4	18·68
Konigsberg	5968·41	...	1	0	8	16·41
Leghorn	5145·54	...	0	10	14	9·54

	Grs. Troy.		lib.	oz.	dwts.	grs.
Leyden	7038·21	...	1	2	13	6·21
Liege	7089·07	...	1	2	15	9·07
Lille	6544·33	...	1	1	12	16·33
Lisbon	7005·39	...	1	2	11	21·39
Lucca	5272·71	..	0	10	19	16·71
Lyons Silk weight	6946·32	..	1	2	9	10·32
—— Town weight	6431·93	..	1	1	7	23·93
Madrid	6544·33	..	1	1	12	16·33
Malo, St., *see* Bayonne.						
Marseilles	6041·42	..	1	0	11	17·42
Mechlin, *see* Antwerp.						
Melun	4440·82	..	0	9	5	0·82
Messina	4844·46	..	0	10	1	20·46
Montpellier	6217·81	..	1	0	19	1·81
Namur	7174·39	..	1	2	18	22·39
Nancy	7038·21	..	1	2	13	6·21
Nantes, *see* Bayonne.						
Naples	4951·93	..	0	10	6	7·93
Nuremberg	7870·91	..	1	4	7	22·91
Paris	7560·80	..	1	3	15	0·8
Pisa, *see* Florence.						
Revel	6573·86	..	1	1	13	21·86
Riga	6148·89	..	1	0	16	4·89
Rome	5257·12	..	0	10	19	1·12
Rouen	7771·64	..	1	4	3	19·64
Saragossa	4707·45	..	0	9	16	3·45
Seville	7038·21	..	1	2	13	6·21
Smyrna	6544·33	..	1	1	12	16·33
Stettin	6782·24	..	1	2	2	14·24
Stockholm	9211·45	..	1	7	3	19·45
Strasburg	7276·94	..	1	3	3	4·94
Toulouse, and Upper Languedoc	6322·82	..	1	1	3	10·82
Turin and Piedmont, in general	4939·62	..	0	10	5	19·62
Tunis and Tripoli, in Barbary	7139·94	..	1	2	17	11·94
Venice—lesser pound	4215·21	..	0	8	15	15·21
——— greater ditto	6826·54	..	1	2	4	10·54
Verona	5374·44	..	0	11	3	22·44
Vicenza—lesser pound	4676·28	..	0	9	14	20·28
——— greater ditto	6879·05	..	1	2	6	15·05

To reduce any of the weights in the preceding table to English averdupois pounds nothing will be necessary but to divide the grains Troy, in the first column, by 7000.

FRENCH WEIGHTS.

The Paris pound, poids de mark of Charlemagne, contains 9216 Paris grains: it is divided into 16 ounces, each ounce into eight gros, and each gros into 72 grains*. It is equal to 7561 English Troy grains.

The English Troy pound, of 12 ounces, contains 5760 English grains, and is equal to 7021 Paris grains.

* Sometimes the gros is divided into 3 deniers, and each denier into 24 grains.

The English averdupois pound, of 16 ounces, contains 7000 English Troy grains; and is equal to 8538 Paris grains.

NEW FRENCH WEIGHTS.

	Eng. Troy grains.
Milligramme	·01544
Centigramme	·15445
Decigramme	1·54457
Gramme	15·44579
Decagramme	154·45793
Hectogramme	1544·57938
Chiliogramme	15445·79386
Myriagramme	154457·93860

A decagramme is 6 dwts. 10·45 grs. Troy, or 2 drs. 1 scr. 14·45 grs. apoth. weight, or 5·648 drams averdupois.

A hectogramme is 3 oz. 8·48 drams averdupois.

A chiliogramme is 2 lbs. 3 oz. 4·87 drams averdupois.

A myriagramme is 22 lbs. 1 oz. 0·73 drams averdupois.

PART FOURTH,

CONTAINING MANY CURIOUS PROBLEMS IN OPTICS.

The properties of light, and the phenomena of vision, form the object of that part of the mixed mathematics, called *Optics*; which is commonly divided into four branches, viz. Direct Optics, or vision, Catoptrics, Dioptrics, and Perspective.

Light indeed may reach the eye three ways: either directly, or after having been reflected, or after having been refracted. Considered under the first point of view, it gives rise to the first branch of optics, called Direct Optics, or vision: in which is explained every thing that relates to the direct propagation of light, or by a straight line from the object to the eye, with the manner in which objects are perceived, &c.

Catoptrics treat of the effects of reflected light, and the phenomena produced by the reflection of light from surfaces of different forms,—plane, concave, convex, &c.

When light, by passing through transparent bodies, is turned aside from its direct course, which is called refraction, it becomes the object of Dioptrics. It is this branch of optics that explains the effects of refracting telescopes, and of microscopes.

Perspective ought to form a part of direct optics, as it is merely a solution of the different cases of the following problem: On a given surface to trace out the image of an object in such a manner, that it shall make on the eye, when placed in a proper station, the same impression as the object itself—a problem purely geometrical, and in which nothing is required but to determine, on a plane given in position, the points where it is intersected by straight lines drawn to the eye from every point of the object. Consequently, the only thing here borrowed from optics, is the principle of the rectitude of the rays of light, as long as they pass through the same medium; the rest is pure geometry.

Without confining ourselves to any other order than that of method, we shall now take a view of the most curious problems and phenomena in this interesting part of the mathematics.

ON THE NATURE OF LIGHT.

Before we enter into any details respecting optics, we cannot help saying a few words on the nature and properties of light in general.

Philosophers are still divided, and in all probability will be so for a long time to come, in regard to the nature of light. Some are of opinion, that it is produced by an extremely fine and elastic fluid, in consequence of an undulatory motion communicated to it by the vibrations of luminous bodies, and which is propagated circularly to immense distances, and with an inconceivable rapidity. Light, according to this hypothesis, is entirely analogous to sound, which, as is well known, consists in a similar undulation of the air, the vehicle of it. Several very specious reasons give to this opinion a considerable degree of probability, nothwithstanding some physical difficulties which it is not easy to obviate.

According to Newton, light is produced from luminous bodies by the emission of particles highly rarefied, and projected with prodigious velocity. The physical diffi-

culties which militate against the former opinion, seem to serve as proofs of the present one; for the nature and propagation of light can be conceived only in these two ways.

But whatever may be the nature of light, it is proved that it moves with astonishing velocity, since it is well known that it employs only seven or eight minutes in passing from the sun to the earth; and as the distance of the sun from the earth, according to the best observations, is 24000 semi-diameters of the latter, or about 95 millions of miles, light moves at the rate of about two hundred thousand miles per second: at which rate it goes from the earth to the moon, and returns from the moon to the earth, in less than three seconds.

The principal properties of light, or those which form the foundation of Optics, are the following:

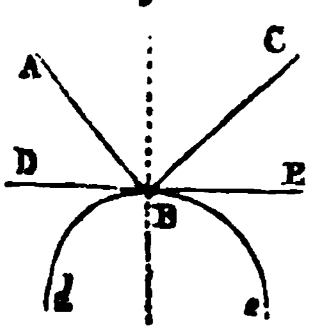
I.—*Light moves in a straight line, as long as it passes through the same transparent medium.*

This property is a necessary consequence of the nature of light; for whatever it may be, it is a body in motion. But a body moves in a straight line if nothing obstructs or tends to turn it aside from its course; and as every thing in the same medium is equal in all directions, the light which passes through it must move in a straight lined course.

This principle of optics, as well as the following, may be proved by experiment.

II.—*Light, when it meets with a polished plane, is reflected, making the angle of reflection equal to the angle of incidence; and the reflection always takes place in a plane perpendicular to the reflecting surface, at the point of reflection.*

Fig. 1.



That is to say, if A B (Fig. 1), be a ray of light, falling on a plane surface D E; and if B be the point of reflection, to find the direction of the reflected ray B C, we must conceive to be drawn through the line A B, a plane perpendicular to the surface D E, and intersecting it in the point B; if the angle C B E in this plane be then made equal to A B D, the line C B will be the reflected ray.

If the reflecting surface be a curve, as *d* B *e*, a plane touching that surface must be conceived passing through B, the point of reflection: the reflection will take place the same as if it were produced by the point B; for it is evident that the curved surface and the plane, a tangent to it in the point B, coincide in that infinitely small part, which may be considered as a plane common to the curved surface and to the tangent plane: the ray of light therefore must be reflected from the curved surface, in the same manner as from the point B of the plane which touches it.

III.—*Light, in passing obliquely from one medium into another of a different density, is turned aside from its rectilineal direction, so as to incline towards the perpendicular when it passes from a rare medium into one that is denser, as from the air into glass or water, and vice versa.*

This proposition may be proved by two experiments, which are a kind of optical illusions.

Experiment 1.

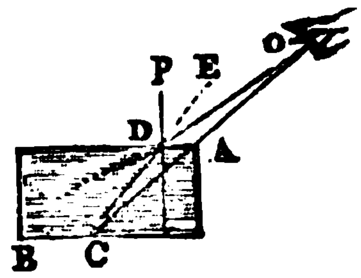
Fig. 2.

Expose to the sun, or to any other light, a vessel A B C D (Fig. 2.), the sides of which are opake, and examine at what point of the bottom the shadow terminates. We shall here suppose that it is at E. Then fill it to the brim with water or oil, and it will be found that the shadow, instead of terminating at the point E, will reach no farther than to F. This difference

can arise only from the inflection of the ray of light S A, which touches the edge of the vessel. When the vessel is empty, this ray, proceeding in the straight line S A E, makes the shadow terminate at the point E; but when the vessel is filled with a fluid denser than air, it falls back to A F. This inflection of a ray of light, in passing obliquely from one medium into another, is called *refraction*.

Experiment 2.

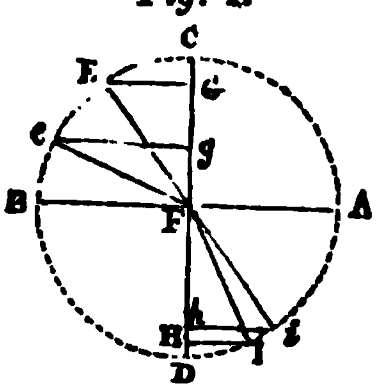
Fig. 3.



Place at the bottom of a vessel, the sides of which are opake, at C for example (Fig. 3.), a piece of money, or any other object, and move backwards from the vessel till the object disappears; if water be then poured into the vessel, the object will immediately become visible, as well as that part of the bottom which was concealed from your sight. The reason of this is as follows:

When the vessel is empty, the eye at O can see the point C only by the direct ray C A O, which is intercepted by the edge A of the vessel; but when the vessel is full of water, the ray C D, instead of continuing its course directly to E, is refracted into D O, by diverging further from the perpendicular D P. This ray conveys to the eye the appearance of the point C, which is seen as at *c*, in the straight line O D continued: the bottom therefore, in this case, appears to be raised. For the same reason, a straight stick or rod, when immersed in water, appears to be bent at the point where it meets with the surface, unless it be immersed in a perpendicular direction.

Fig. 4.



Philosophers have carefully examined the law according to which this inflection takes place, and have found that when a ray, as E F (Fig. 4.) passes from air into glass, it is refracted into F I, in such a manner, that the sine of the angle C F E, and that of D F I, are in a constant ratio. Thus, if the ray E F be refracted into F I, and the ray *e* F into F *i*, the sine of the angle C F E will have the same ratio to the sine of D F I, as the sine of the angle C F *e* has to that of D F *i*. This ratio, when the ray passes from air into common glass, is always as 3 to 2; that is to say, the sine of the angle which the refracted ray forms with the perpendicular to the refracting substance, is always two thirds of the sine of the angle formed by the incident ray with the same perpendicular.

It is to be observed, that when the latter angle, that is the distance of the incident ray from the perpendicular, which is called the angle of inclination, is very small, the angle of refraction may be considered as two thirds of it, because small angles have nearly the same ratio as their sines. We here suppose that the ray passes from air into glass; for it is well known, and may be easily proved by the table of sines, that when two angles are very small, that is if they do not exceed 5 or 6 degrees, they are sensibly in the same ratio as their sines. Thus, in the case above, the angle of refraction I F D, will be two thirds of the angle of inclination G F E; and consequently the angle formed by the refracted ray and the incident, continued in a straight line, will be one third of it.

When the passage takes place from air into water, the ratio of the sine of the angle of inclination, and that of the angle of refraction, is that of 4 to 3; that is to say, the sine of the angle D F I is constantly $\frac{3}{4}$ of the sine of G F E, the angle of inclination of the ray incident in air. Consequently, when these angles are very small, they may be considered as being in the same ratio; and the angle of refraction will be $\frac{3}{4}$ of the angle of inclination.

This proportion is the basis of all the calculations of dioptrics; and on that

account ought to be well imprinted in the memory. For the discovery of it we are indebted to the celebrated Descartes; though it appears certain, by the testimony of Huygens, that a law of refraction equally constant, and which in fact is the same, was discovered before by Willebrod Snell, a Dutch mathematician. But Vossius is wrong when he asserts, as he does in his book "De Natura Lucis," that the expression of Snellius was more convenient. This learned man did not know what he said, when he attempted to speak of natural philosophy.

PROBLEM I.

To exhibit, in a darkened room, external objects, in their natural colours and proportions.

Shut the door and windows of the apartment, in such a manner, that no light can enter it, but through a small hole very neatly cut in one of the window shutters, opposite to some well frequented place or landscape; then hold a white cloth or piece of white paper opposite to the hole, and if the external objects are strongly illuminated, and the room very dark, they will appear as if painted on the cloth or paper, in their natural colours, but inverted.

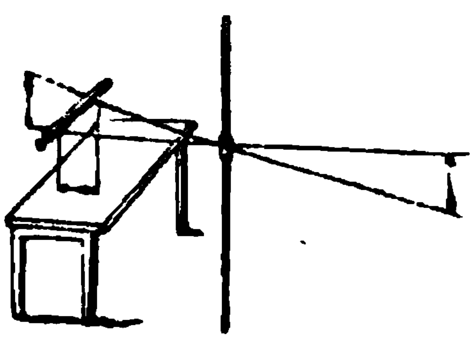
The experiment, performed in this simple manner, will succeed well enough to surprise those who see it for the first time; but it may be rendered much more striking by means of a lens.

Adapt to the hole of the shutter, which in this case must be some inches in diameter, a tube having at its internal extremity a convex lens, of 4, 5, or 6 feet focus; if a piece of white cloth, or a sheet of paper, be then held at that distance from the glass, and in a direction perpendicular to the axis of the tube, the external objects will be painted on the cloth or paper, with much more distinctness and vivacity of colouring, than in the preceding experiment; and in so accurate a manner, that the features of the person seen may be distinguished. This spectacle is highly amusing, especially when a public place, a promenade filled with people, &c. are exhibited.

This painting indeed is inverted, which destroys a little of the effect; but different methods may be employed to make it appear in its natural position: it is however to be regretted that this cannot be done without injuring the distinctness, or lessening the field of the picture. Those who may be desirous of seeing the objects erect, must proceed in the following manner:

At about half the focal distance of the lens place a plane mirror, inclined at an angle of 45°, so that it may reflect downwards the rays proceeding from the lens; if you then place horizontally below it a sheet of paper, the image of the external objects will appear painted on the paper, and in their natural situation to those who have their backs turned towards the window. Fig. 5. represents the mechanism of this inversion, of which a clear idea cannot be formed without some knowledge of catoptrics.

Fig. 5.



The sheet of paper may be extended on a table, and nothing will be necessary but to dispose the glass and mirror at such a height from the paper, that the objects may be distinctly painted on it. By these means a landscape, or edifice, &c., may be exactly delineated with great ease.

PROBLEM II.

To construct a portable Camera Obscura.

Construct a wooden box A B C D (Fig. 6.) about a foot in height, as much in breadth, and two or three feet in length, according to the focus of the lenses em-

ployed. To one of the sides adapt a tube E F, consisting of two, one thrust within the other, that it may be lengthened or shortened at pleasure; and in the anterior aperture of the first tube fix two lenses, convex on both sides, and about seven inches in diameter, so as almost to touch each other; place another of about 5 inches focal distance in the interior aperture; and at about the middle of the box, taken lengthwise, dispose in a perpendicular direction a piece of oiled paper G H, stretched on a frame: in the last place, make a round hole I, in the side opposite to the tube, and sufficiently large to receive both eyes.

Fig. 6.

When you are desirous of viewing any objects, turn the tube, furnished with the lenses, towards them; and adjust it either by drawing it out or pushing it in, till the image of the objects is painted distinctly on the oiled paper.

The following is the description of another camera obscura, invented by Gravesande, and of which he gave an account at the end of his Essay on Perspective.

This machine is shaped almost like a hackney chair; the top of it is rounded off towards the back part, and before it swells out into an arch at about the middle of its height. (See Fig. 7. where this machine is represented with the side opposite to the door taken off, in order that the interior part of it may be exhibited to view.)

Fig. 7.

1st. The board A, in the inside, serves as a table: it turns on two iron hinges fastened to the fore part of the machine, and is supported by two small chains, that it may be raised to facilitate entrance into the machine.

2d. To the back of the machine, on the outside, are affixed four small staples C, C, C, C, in which slide two pieces of wood D E, D E, three inches in breadth; and through these pass two other pieces, serving to keep fast a small board F, which by their means can be moved forwards or backwards.

3d. At the top of the machine is a slit O Q, nine or ten inches in length, and four in breadth, to the edges of which are affixed two rules in the form of a dove tail: between these slides a board of the same length, having a round hole, of about three inches diameter, in the middle, furnished with a nut, that serves to raise or lower a tube about four inches in height, which has a screw corresponding to the nut. This tube is intended for receiving a convex glass.

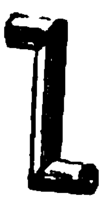
4th. The moveable board, above described, supports a square box X, about seven and a half inches in breadth, and ten in height, the fore part of which can be opened by a small door, and in the back part of the box towards the bottom is a square aperture N, of about four inches in breadth, which may be shut at pleasure by a moveable board.

5th. Above this square aperture is a slit parallel to the horizon, and which occupies the whole breadth of the box. It serves for introducing into the box a plane mirror, which slides between two rules so that the angle it makes with the horizon towards the door B is 112½°, or $\frac{5}{4}$ of a right angle.

6th. The same mirror, when necessary, may be placed in a direction perpendicular to the horizon, as seen at H, by means of a small iron plate adapted to one of its sides, and furnished with a screw which enters a slit formed in the top of the machine, and which may be screwed fast by a nut.

7th. Within the box is another small mirror, L L, which turns on two pivots, fixed a little above the slit of No. 5, and which being drawn up or pushed down by the small rod S, may be inclined to the horizon at any angle whatever.

Fig. 8.



8th. That the machine may be supplied with air, a tube of tin-plate, bent at both ends, as seen Fig. 8, may be fitted into one of the sides, this will give access to the air without admitting light. But if this should not be sufficient, a small pair of bellows, to be moved by the foot, may be placed below the seat, and in this manner the air may be continually renewed.

The different uses of this machine are as follow.

I.—*To represent objects in their natural situation.*

When objects are to be represented in this machine, extend a sheet of paper on the table, or rather stretched on a frame, or you may employ a piece of strong card, and fix it in such a manner as to remain immoveable.

In the tube c, (Fig. 7.) place a convex glass, the focus of which is nearly equal to the height of the machine above the table; open the back part of the box x, and having removed the mirror H, as well as the board F, and the rules D E, incline the moveable mirror L L, till it make with the horizon an angle of nearly 45°, if you intend to represent objects at a considerable distance, and which form a perpendicular landscape. When this is done, all those objects which transmit rays to the mirror L L, so as to be reflected on the convex glass, will appear painted on the paper frame: the point where the images are most distinct may be found, if the tube which contains the lens be lowered or raised, by screwing it up or down.

By these means any landscape, view of a city, &c., may be exhibited with the greatest precision.

II.—*To represent objects in such a manner, as to make that which is on the right appear on the left, and vice versa.*

The box x being in the situation represented in the figure, open the door B, and having placed the mirror H in the slit, and in the situation already mentioned No. 5, raise the mirror L L till it make with the horizon an angle of 22½ degrees; if the fore part of the machine be then turned towards the objects to be represented, which we here suppose to be at a considerable distance, they will be seen painted on the paper, but transposed from right to left.

It may sometimes be useful to make a drawing where the objects are transposed in this manner; for example, in the case when it is intended to be engraved; for as the impression of the plate will transpose the figures from right to left, they will then appear in their natural position.

III.—*To represent in succession all the objects in the neighbourhood, and quite around the machine.*

Place the mirror H in a vertical position, as seen in the figure, and incline the mirror L at an angle of 45 degrees; if the former be then turned round vertically, the lateral objects will be seen painted in succession on the paper, in a very pleasant manner.

It must here be observed, that it will be necessary to cover the mirror H with a kind of box made of pasteboard, open towards the objects, and also towards the aperture N of the box X; for if the mirror H were left entirely exposed, it would reflect on the mirror L a great many lateral rays, which would considerably weaken the effect.

IV.—*To represent the image of Paintings or Prints.*

Affix the painting or print to the side of the board F, which is next to the mirror L, and in such a manner that it may be illuminated by the sun. But as the object

in this case must be at a very small distance, the tube must be furnished with a glass, having its focal distance nearly equal to half the height of the machine above the paper: if the distance of the painting from the glass be then equal to that of the glass from the paper, the figures of the painting will be represented on the paper exacfly of the same size.

The point at which the figures have the greatest distinctness, may be found, by moving backwards or forwards the board F, till the representation be very distinct.

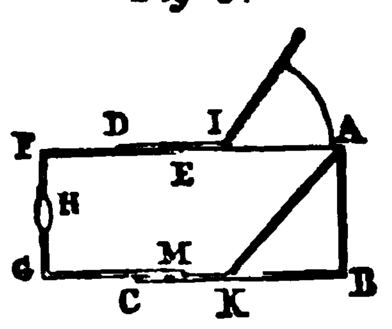
Some attention is necessary in regard to the aperture of the convex glass.

In the first place, the same aperture may in general be given to the glass as to a telescope of the same length.

Secondly, this aperture must be diminished when the objects are very much illuminated; and *vice versa.*

Thirdly, as the traits appear more distinct when the aperture is small, than when it is large, if you intend to delineate the objects, it will be necessary to give to the glass as small an aperture as possible; but taking care not to extenuate the light: it will therefore be proper to have different circles of copper or of blackened pasteboard to be employed for altering the size of the aperture, according to circumstances.

Fig. 9.



A small portable camera obscura is shewn in section (Fig. 9.), where D A B C is a rectangular box, in which slides another E F G M, open at the end E M, and in the centre of the end F G is a double convex lens H. A K is a mirror inclined at an angle of 45°, and I A is a piece of ground glass, on which the image formed in the focus of the lens is depicted, the rays which would have formed an image on A B being reflected to A I by the mirror A K. The box F M is drawn out till the image on the ground glass is quite distinct. Objects seen in this instrument are erect, but inverted; but this defect may be remedied by placing the camera before a mirror, and forming the picture on A I from the object as seen in the mirror, where already *right* has been exchanged for *left*; and a like change being made in the camera, the picture appears as in nature.

Remark.—On the top of the Royal Observatory, at Greenwich, was an excellent camera obscura, capable of containing five or six persons, all viewing the exhibition together. All the motions of the glasses were easily performed by one of the persons within, by means of attached rods; and the images were thrown on a large and smooth concave table, cast of plaster of Paris, and moveable up and down, so as to suit the distances of the objects. But this fine instrument has been removed since Mr. Airy became Astronomer Royal.

THE CAMERA LUCIDA.

The Camera Lucida was invented by Dr. Wollaston in 1807, and has for its purpose the facilitation of the operations of the draughtsman. It consists of a quadrangular glass prism, by which the rays from an object are twice reflected; the second reflection correcting the transposition of right for left, caused by the first reflection. The form of the prism is shewn in the annexed figure. The faces A B and A D are at right angles to each other, and A C and C D make with A B and A D respectively, angles of 67½ degrees, and therefore with each other an angle of 135 degrees.

Fig 10.

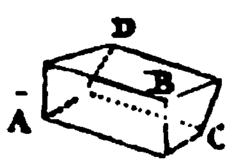
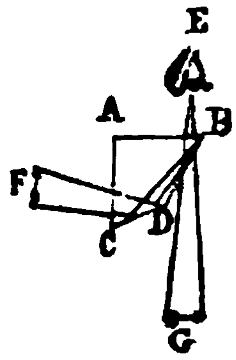
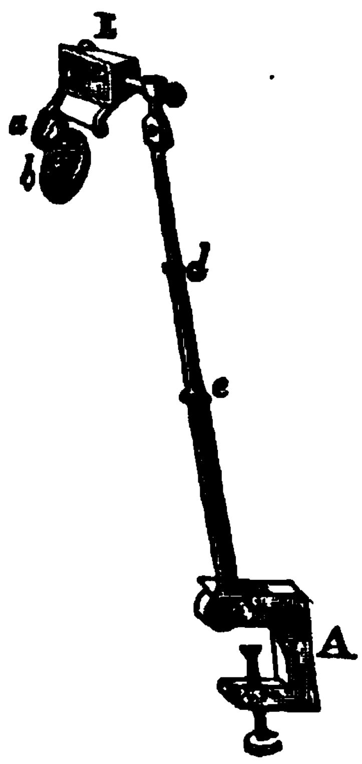


Fig. 11.



The object F (Fig. 11), being opposite the face A C of the prism; the rays proceeding from F pass through it, and are reflected from the surface C D to D B, and thence they are reflected to the eye at E. The rays proceeding upwards to the eye by the last reflection, the observer is led to imagine the image below the instrument at G, and placing his paper there the image may be traced with a pencil.

Fig. 12.

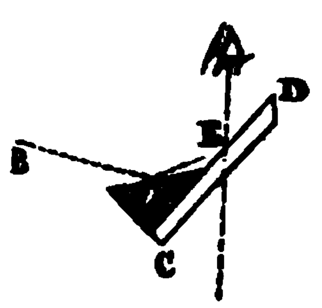


The prism is mounted in a brass frame supported by brass tubes sliding stiffly within one another, the tubes being slit at *d* and *e* (Fig. 12), so as to form a spring for that purpose; and the instrument is furnished with a clamp at A, to fix it to the drawing board or sketching book. The prism is furnished at E with a piece of brass blackened, and perforated with a small hole for directing the position of the eye.

The instrument should be leaned forward until the prism is directly over the middle of the space intended to be occupied by the drawing; the upper surface A B being parallel to the paper. If the eye be altogether over this surface, the pencil cannot be seen, the rays from it do not pass directly through the prism; but by placing the hole of the eye-piece so as to be divided by the edge of the prism, and then applying the eye to it; with one part of the pupil the image of the object is seen, and with the other part the pencil.

It must be observed, that there is no image actually formed upon the paper, as in the camera obscura, but the image appears as far below the instrument as the object is before it; and therefore the eye cannot, in the same state, see both the pencil and the image distinctly. To remedy this inconvenience; to the brass frame of the prism are attached two lenses *a*, *b*, (Fig. 12.), the one concave and the other convex, the former to be turned up in front of the instrument, for short-sighted persons, and the latter to be turned below for long sights. The size of the picture always bears the same relation to the size of the object, that the distance from the eye to the paper does to the distance of the object from the eye. Then by increasing the distance of the prism from the paper, by lengthening the tubes *d*, *e*, the drawing is increased in size, and *vice versa*.

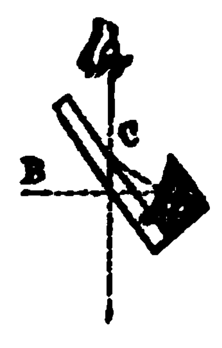
Fig. 13.



The above is the form of the instrument generally employed; but it is not the only one, nor perhaps the most ingenious. There are two forms of it invented by S. Amici, which we shall briefly describe.

The first consists of a parallel piece of plate glass, C D, joined to a reflecting speculum A. The rays from an object at B are thrown on the speculum A, which is inclined to the surface of the glass C D, at an angle of 135°, and are reflected by it to the glass at E, and thence to the eye above.

Fig. 14.



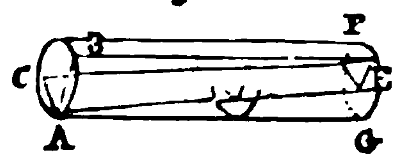
In the other, which Amici esteems the best, the rays are made to pass through the plate glass before striking on the speculum. Thus the ray from B (Fig. 12), passing through the glass is reflected by the speculum to C, and thence by the surface of the glass, to the eye.

THE KALEIDESCOPE.

The Kaleidescope was invented by Sir D. Brewster, in 1814, while experimenting on the polarization of light. In a patent which he obtained for it, it was described as a new optical instrument "for creating and exhibiting beautiful forms." The extreme simplicity of the instrument however destroyed the effect of the patent, as any one possessing the least mechanical skill can make one for himself.

The name of the Kaleidescope is derived from three Greek words, καλος beautiful, ειδος a form, and σκοπεω to see. Two reflecting plates, tapered from one end to the other, to about half the breadth, are inclined to each other at an angle, which must be an even aliquot part of 360°, and placed in a tube so that the point of meeting E at the narrower ends of the plates is near the centre of the circular end F G of the tube, in which there is a hole for the eye; and the edges C A and A B of the mirrors are in the plane of the other end of the tube, to which is fitted a box made of two circular pieces of glass, the outer one being greyed to render the light uniform. This box contains the objects, such as beads, small pieces of coloured glass, &c.

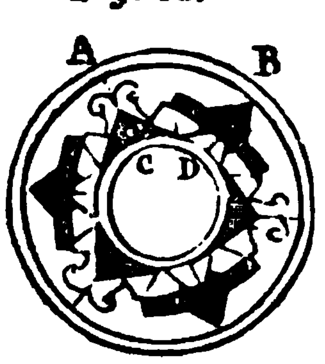
Fig. 15.



On looking in at E, a beautiful coloured star is seen, consisting of a number of identical sectors; a greater number the smaller the angle between the reflecting plates.

There are several modifications of the instrument depending on the number and position of the mirrors.

Fig. 16.



The *Polyangular* Kaleidescope, for instance, is so constructed that by means of screws the angle of the mirrors can be varied, thus increasing or diminishing the number of the sectors.

In the *Annular* Kaleidescope, by which we may obtain patterns for circular borders, the mirrors are not placed edge to edge, but as A C and B D (Fig. 16.), producing a pattern surrounding a black circle.

When the plates A C and B D are parallel, the pattern becomes rectilineal. (Fig. 17.)

Fig. 17.

Kaleidescopes may be constructed also, with three or four reflecting plates, but those of three are limited to these three cases:

The mirrors at angles of 90°, 45° and 45°.
The mirrors at angles of 90°, 60° and 30°.
The mirrors at angles of 60°, 60° and 60°.

Those of four reflecting plates must be of the form of a hollow square or rectangle.

And lastly, by taking off the object box, and placing a lens so that the focus falls in the plane of C A and A B, (Fig. 15.) a star is formed of the distant objects.

PROBLEM III.

To explain the nature of Vision, and its principal phenomena.

Before we explain in what manner objects are perceived, it will be necessary to begin with a description of the wonderful organ destined for that purpose.

The eye is a hollow globe, formed of three membranes, which contain humours of different densities, and which produces, in regard to external objects, the same effect as the Camera Obscura. The first or outermost of these membranes, called the *sclerotica*, is only a prolongation of that which lines the inside of the eye-lids. The second, called the *choroides*, is a prolongation of the membrane which covers

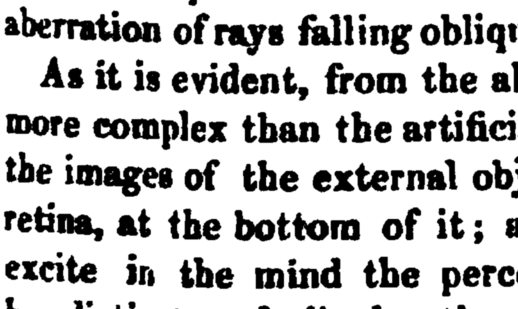
the optic nerve, as well as all the other nerves. And the third, which lines the inside of the eye, is an expansion of the optic nerve itself: it is this membrane, entirely nervous, which is the organ of vision; for, notwithstanding the experiments in consequence of which this function has been ascribed to the choroides, we cannot look for sensation any where else than in the nerves and nervous parts.

In the front of the eye the sclerotica changes its nature, and assumes a more convex form than the ball of the eye, forming here what is called the *transparent cornea*. The choroides, by being prolonged below the cornea, must necessarily leave a small vacuity, which forms the anterior receptacle of the aqueous humour. This prolongation of the choroides terminates at a circular aperture well known under the name of the *pupil*. The coloured part which surrounds this aperture is called the *iris* or *uvea*; it is susceptible of dilatation and contraction, so that when exposed to a strong light, the aperture of the pupil contracts, and in a dark place it dilates.

This aperture of the pupil is similar to that of the camera obscura. Behind it is suspended, by a circular ligament, a transparent body of a certain consistence, and having the form of a lens: it is called the *crystalline* humour, and in this natural camera obscura performs the same office as the glass in the artificial one.

By this description it may be seen, that between the cornea and the crystalline humour there is a sort of chamber, divided into nearly two equal parts, and another between the crystalline humour and the retina. The first is filled with a transparent humour similar to water, on which account it has been called the *aqueous humour*. The second chamber is filled with a humour of the same consistence almost as the white of an egg: it is known by the name of the *vitreous humour*. All these parts may be seen represented Fig. 18; where *a* is the sclerotica, *b* the cornea, *c* the choroides, *d* the retina, *e* the aperture of the pupil, *f f* the uvea, *h* the crystalline humour, *i i* the aqueous, *k k* the vitreous, and *l* the optic nerve.

Fig. 18.



The form of the cornea is a prolate spheroid, whose axis is that of the eye; the surfaces of the crystalline are also spheroidal, the curvature of the inner surface being much greater than that of the outer,—and their axes neither coincide with each other, nor with that of the cornea. The density of the crystalline lens increases from the surface to the centre. This variable density aids in correcting aberration: and the elliptic form of the surface is supposed to have the effect of correcting the aberration of rays falling obliquely on the eye.

As it is evident, from the above description, that the eye is a camera obscura, but more complex than the artificial one before described, it may readily be conceived that the images of the external objects will be painted in an inverted situation, on the retina, at the bottom of it; and these images, by affecting the nervous membrane, excite in the mind the perception of light, colours, and figures. If the image be distinct and lively, the impression received by the mind is the same; but if it be confused and obscure, the perception is confused and obscure also: this is sufficiently proved by experiment. That such images really exist, may be easily shewn by employing the eye of any animal, such as that of a sheep or bullock; for if the back part of it be cut off, so as to leave only the retina; and if the cornea of it be placed before the hole of a camera obscura, the image of the external objects will be seen painted on the retina at the bottom of it.

But it may here be asked, since the images of the objects are inverted, how comes it that they are seen in their proper position? This question can have no difficulty but to those who are ignorant of metaphysics. The ideas indeed which we have of the upright or inverted situation of objects, in regard to ourselves, as well as of their distance, are merely the result of the two senses, seeing and touching combined.

The moment we begin to make use of our sight, we experience, by means of touching, that the objects which affect the upper part of the retina, are towards our feet in regard to those that affect the lower part, which touching tells us are at a greater distance. Hence is established the invariable connection that subsists between the sensation of an object which affects the upper part of the eye, and the idea of the lowness of that object.

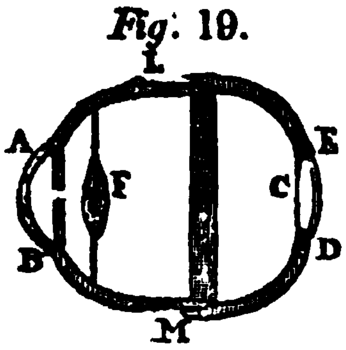
But what is meant by lowness? It is being nearer the lower part of our body. In the representation of any object, the image of the lower part is painted nearer that of our feet than the image of the upper part: in whatever place the image of our feet may be painted on the retina, this image is necessarily connected with the idea of inferiority; consequently, whatever is nearest to it necessarily produces in the mind the same idea. The two sticks of the blind man of Descartes prove nothing here and Descartes would certainly have expressed himself in the same manner, had he not adopted the doctrine of innate ideas, proscribed by modern metaphysics.

PROBLEM IV.

To construct an artificial eye, for exhibiting and explaining all the phenomena of vision.

This machine may be easily constructed from the following description. A B D E is a hollow ball of wood (Fig. 19.) five or six inches in diameter, formed of two hemispheres joined together at L M, and in such a manner, that they can be brought nearer to or separated from each other about half an inch. The segment A B of the anterior hemisphere is a glass of uniform thickness, like that of a watch; below which is a diaphragm, with a round hole about six lines in diameter in the middle of it; F is a lens, convex on both sides, supported by a diaphragm, and having its focus equal to F C when the two hemispheres are at their mean distance. In the last place, the part D C E is formed of a glass of uniform thickness, and concentric to the sphere, the interior surface of which, instead of being polished, is only rendered smooth, so as to be semi-transparent. Such is the artificial eye to which scarcely any thing is wanting but the aqueous and vitreous humours; and these might be represented also, by putting into the first cavity common water, and into the other water charged with a strong solution of salt. But for the experiments we have in view, this is entirely useless.

Fig. 19.



This small machine however may be greatly simplified, by reducing it to two tubes of an inch and a half or two inches in diameter, one thrust into the other. The first, or anterior one, ought to be furnished with a lens of about three inches focus; but care must be taken to cover the whole of it except the part nearest the axis, which may be done by means of a circular piece of card, having in the middle of it a hole about half an inch in diameter. The extremity of the other tube may be covered with oiled paper, which will perform the part of the retina. The whole must then be arranged in such a manner that the distance of the glass from the oiled paper, may be varied from about two to four inches, by pushing in or drawing out one of the tubes. A machine of this kind may easily be procured by any one, and at a very small expence.

Experiment 1.

The glass or the oiled paper being exactly in the focus of the lens, if the machine be turned towards very distant objects, they will be seen painted with great distinctness on the retina. If the machine be lengthened or shortened, till the bottom part be no longer in the focus of the lens, the objects will be seen painted, not in a distinct, but in a confused manner.

Experiment 2.

Present a taper, or any other enlightened object, to the machine at a moderate distance, such as three or four feet, and cause it to be painted in a distinct manner on the retina, by moving the bottom of the machine nearer to or farther from the glass. If you then bring the object nearer, it will cease to be painted distinctly; but the image will become distinct if the machine be lengthened. On the other hand, if the object be removed to a considerable distance, it will cease to be painted distinctly, and the image will not become distinct till the machine is shortened.

Experiment 3.

A distinct image, however, may be obtained in another manner, without touching the machine. In the first case, if a concave glass be presented to the eye, at a distance which must be found by trial, the painting of the object will be seen to become distinct. In the second case, if a convex glass be presented to it, the same effect will be produced.

These experiments serve to explain, in the most sensible manner, all the phenomena of vision, as well as the origin of those defects to which the sight is subject, and the means by which they may be remedied.

Objects are never seen distinctly unless when they are painted in a distinct manner on the retina; but when the conformation of the eye is such, that objects at a moderate distance are painted in a distinct manner, those which are much nearer, or at a much greater distance, cannot be painted with distinctness. In the first case, the point of distinct vision is beyond the retina; and if it were possible to change the form of the eye, so as to move the retina farther from that point, or the crystalline humour farther from the retina, the objects would be painted in a distinct manner. In the second case, the effect is contrary: the point of distinct vision is on this side of the retina; and, to produce distinct vision, the retina ought to be brought nearer to the crystalline humour, or the latter nearer to the retina. We are taught by experience that in either case a change is produced, which is not made without some effort. But in what does this change consist? Is it in a prolongation or flattening of the eye; or is it in a displacement of the crystalline humour? This has never yet been properly ascertained.

In regard to sight, there are two defects, of a contrary nature, one of which consists in not seeing distinctly any objects but such as are at a distance; and as this is generally a failing in old persons, those subject to it are called *presbytæ:* the other consists in not seeing distinctly but very near objects; and those who have this failing are called *myopes*.

The cause of the first of these defects, is a certain conformation of the eye, in consequence of which the image of near objects is not painted in a distinct manner but beyond the retina. But the image of distant objects is nearer than that of neighbouring objects, or objects at a moderate distance; the image of the former may therefore fall on the retina, and distant objects will then be distinctly seen, while neighbouring objects will be seen only in a confused manner.

But to render the view of neighbouring objects distinct, nothing else is necessary than to employ a convex glass, as has been seen in the third experiment: for a convex glass, by making the rays converge sooner, brings a distinct image of the objects nearer; consequently it will produce on the retina a distinct picture, which otherwise would have fallen beyond it.

In regard to the myopes, the case will be exactly the reverse. As the defect of their sight is occasioned by a conformation of the eye which unites the rays too soon, and causes the point, where the image of objects moderately distant are painted with distinctness, to fall on this side the retina, they will receive relief from concave

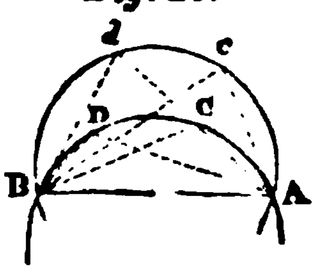
glasses interposed between the eye and the object; for these glasses, by causing the rays to diverge, remove to a greater distance the distinct image, according to the third experiment; the distinct image of objects which was before painted on this side the retina, will be painted distinctly on that membrane when a concave glass is employed.

Besides, myopes will discern small objects within the reach of their sight much better than the presbytæ, or persons endowed with common sight; for an object placed at a smaller distance from the eye, forms in the bottom of it a larger image, nearly in the reciprocal ratio of the distance. Thus a myope, who sees distinctly an object placed at the distance of six inches, receives in the bottom of the eye an image three times as large as that painted in the eye of the person who does not see distinctly but at the distance of eighteen inches: consequently all the small parts of this object will be magnified in the same proportion, and will become sensible to the myope, while they will escape the observation of the presbytæ. If a myope were in such a state as not to see distinctly but at the distance of half an inch, objects would appear to him sixteen times as large as to persons of ordinary sight, whose boundary of distinct vision is about eight inches: his eye would be an excellent microscope, and he would observe in objects what persons of ordinary sight cannot discover without the assistance of that instrument.

PROBLEM V.

To cause an object, whether seen near hand or at a great distance, to appear always of the same size.

Fig. 20.



The apparent magnitude of objects, every thing else being alike, is greater according as the image of the object painted on the retina occupies a greater space. But the space occupied by an image on the retina, is nearly proportioned to the angle formed by the extremities of the object, as may be readily seen by inspecting Fig. 20; consequently it is on the size of the angle formed by the extreme rays, proceeding from the object, which cross each other in the eye, that the apparent magnitude of the object depends.

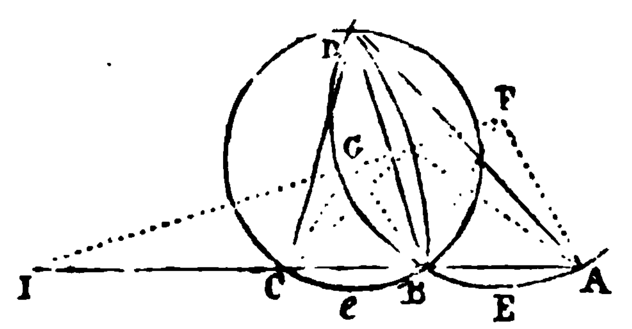
This being premised, let A B be the object, which is to be viewed at different distances, and always under the same angle. On A B, as a chord, describe any arc of a circle, as A C D B: from every point of this arc, as A, C, D, B, the object A B will be seen under the same angle, and consequently of the same size; for every one knows that all the angles having A B for their base, and their summits in the segment A C D B, are equal.

The case will be the same with any other arc, as A *c d* B.

PROBLEM VI.

Two unequal parts of the same straight line being given, whether adjacent or not; to find the point where they will appear equal.

Fig. 21.



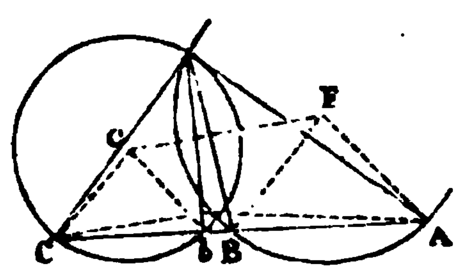
On A B and B C (Fig. 21.), and on the same side, construct the two similar isosceles triangles A F B and B G C; then from the centre F, with the radius F B, describe a circle, and from the point G, with the radius G B, describe another circle; intersecting the former in D: the point D will be the place required, where the two lines appear equal.

For the circular arcs A E B D and B *e* C D are, by construction, similar; and hence it follows, that the angle A D B is equal to B D C, as the point D is common to both the arcs.

Remarks.—1st. There are a great many points, such as D, which will answer the problem; and it may be demonstrated, that all these points are in the circumference of a semicircle, described from the point I as a centre. This centre may be found by drawing, through the summits F and G of the similar triangles A F B and B G C, the line F G, till it meet A C produced, in I.

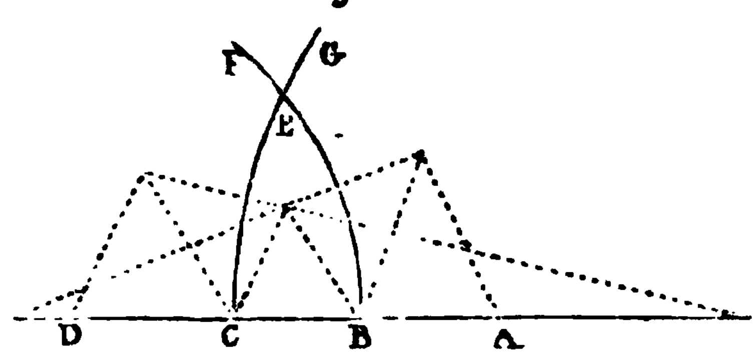
2d. If the lines A B and B C form an angle, the solution of the problem will be still the same; the two similar arcs described on A B and B C will necessarily intersect each other in some point D, unless they touch in B; and this point D will, in like manner, give the solution of the problem.

Fig. 22.



3d. The solution of the problem will be still the same, even if the unequal lines proposed, A B and b C (Fig. 22.), are not contiguous; only care must be taken that the radii F B and G b, of the two circles, be such, that the circles shall at least touch each other. If A B $= a$, B $b = c$, b C $= b$, and A C $= d = a + b + c$, that the two circles touch each other, F B must be at least $= \frac{1}{2} a \sqrt{\frac{ab + ac + bc + c^2}{ab}}$ or $\frac{1}{2} a \sqrt{\frac{ab + cd}{ab}}$, and G $b = \frac{1}{2} b \sqrt{\frac{ab + ac + bc + c^2}{ab}}$ or $\frac{1}{2} b \sqrt{\frac{ab + cd}{ab}}$. If these lines be less, the two circles will neither touch nor cut each other. If they be greater, the circles will intersect each other in two points, which will each give a solution of the problem. Let a, for example, be $= 3$, $b = 2$, and $c = 1$: in this case G b will be found $= \sqrt{2} =$ 1·4142, and B F $= \frac{3}{2} \sqrt{2} = 2·1213$, when the circles just touch each other.

Fig. 23.



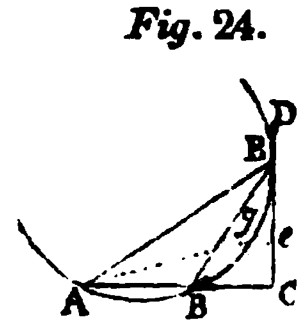
4th. In the last place, if we suppose three unequal and contiguous lines, as A B, B C, C D (Fig. 23.), and if the point from which they shall all appear under the same angle, be required, find, by the first article, the circumference B E F, &c., from every point of which the lines A B and B C appear under the same angle; find also the arc C E G from which B C and C D appear under the same angle; then the point where these two arcs intersect each other will be the point required. But to make these two circles touch each other, the least of the given lines must be between the other two, or they must follow each other in this order, the greatest, the mean, and the least.

If the lines A B, B C, and C D be not contiguous, or in one straight line, the problem becomes too difficult to be admitted into this work. We shall therefore leave it to the ingenuity of such of our readers as have made a more considerable progress in the mathematics.

PROBLEM VII.

If A B *be the length of a parterre, situated before an edifice, the front of which is* C D, *required the point in that front from which the apparent magnitude of the parterre* A B *will be the greatest.* (Fig. 24.)

Fig. 24.



Take the height C E a mean proportional between C B and C A: this height will give the point required. For if a circle be described through the points A, B, E, it will touch the line C E, in consequence of a well-known property of tangents and secants. But it may be readily seen that the angle A E B is

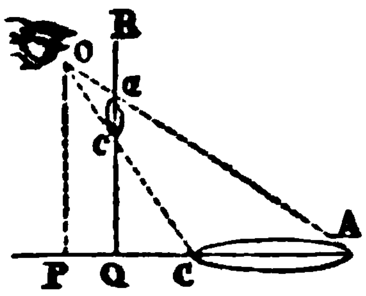
greater than any other A e B, the summit of which is in the line C D; for the angle A e B is less than A g B, which is equal to A E B.

PROBLEM VIII.

A circle on a horizontal plane being given; it is required to find the position of the eye where its image on the perspective plane will still a circle.

We here suppose that the reader is acquainted with the fundamental principle of perspective representation, which consists in supporting a vertical transparent plane between the eye and the object, called the perspective plane. As rays are supposed to proceed from every point of the object to the eye, if these rays leave traces on the vertical plane, it is evident that they will there produce the same effect on the eye as the object itself, since they will paint the same image on the retina. The traces made by these rays are called the *perspective image*.

Fig. 25.



Let A C (Fig. 25.) then be the diameter of the circle on the horizontal plane A C P, perpendicular to the perspective plane, Q B, a section of the perspective plane, and P O a plane perpendicular to the horizon and to the line A P, in which it is required to find the point O, where, if the eye be placed, the representation *a c*, of the circle A C, shall be also a circle.

If P O be made a mean proportional between A P and C P, the point O will be the one required.

For if P A be to P O, as P O to P C, the triangles P A O and P C O will be similar, and the angles P A O and C O P will be equal: the angles P A O and C *c* Q, or P A O and B *c* O, will also be equal; hence it follows that in the small triangle *a c* O, the angle at *c* will be equal to the angle O A C, and the angle at O being common to the triangles A O C and *a* O *c*, the other two, A C O and *c a* O, will be also equal: A O then will be to C O, as *c* O to *a* O; hence the oblique cone A C O will be cut in a sub-contrary manner, or sub-contrary position, by the vertical plane Q B, and consequently the new section will be a circle, as is demonstrated in conic sections.

PROBLEM IX.

Why is the image of the sun, which passes into a darkened apartment through a square or triangular hole, always circular?

This problem was formerly proposed by Aristotle, who gave a very bad solution of it; for he said it arose from the rays of the sun affecting a certain roundness, which they resumed when they had surmounted the restraint imposed on them by the hole being of a different figure. This reason is entirely void of foundation.

To account for this phenomenon, it must be observed that the rays proceeding from any object, whether luminous or not, which pass through a very small hole into a darkened chamber, form there an image exactly similar to the object itself; for these rays, passing through the same point, form beyond it a kind of pyramid similar to the first, and having its summit joined to that of the first, and which, being cut by a plane parallel to that of the object, must give the same figure, but inverted.

This being understood, it may be readily conceived that each point of the triangular hole, for example, paints on paper, or on the floor, its solar image round; for every one of these points is the summit of a cone of which the solar disk is the base.

Describe then on paper a figure similar and equal to that of the hole, whether square or triangular, and from every point of its periphery, as a centre, describe equal circles; while these circles are small, you will have at first a triangular figure with rounded angles; but if the magnitude of the circles be increased more and more, till the radius be much greater than any of the dimensions of the figure, it will be observed to become rounder and rounder, and at length to be sensibly converted into a circle.

But this is exactly what takes place in the darkened apartment; for when the paper is held very near to the triangular hole, you have a mixed image of the triangle and the circle; but if it be removed to a considerable distance, as each circular image of the sun becomes then very large, in regard to the diameter of the hole, the image is sensibly round. If the disk of the sun were square, and the hole round, the image at a certain distance would, for the same reason, be a square, or in general of the same figure as the disk. The image of the moon therefore, when increasing, is always, at a sufficient distance, a similar crescent, as is proved by experience.

PROBLEM X.

To make an object which is too near the eye to be distinctly perceived, to be seen in a distinct manner, without the interposition of any glass.

Make a hole in a card with a needle, and without changing the place of the eye or of the object, look at the latter through the hole; the object will then be seen distinctly, and even considerably magnified.

The reason of this phenomenon may be deduced from the following observations: When an object is not distinctly seen, on account of its nearness to the eye, it is because the rays proceeding from each of its points, and falling on the aperture of the pupil, do not converge to a point, as when the object is at a proper distance: the image of each point is a small circle, and as all the small circles, produced by the different points of the object, encroach on each other, all distinction is destroyed. But when the object is viewed through a very small hole, each pencil of rays, proceeding from each point of the object, has no other diameter than that of the hole; and consequently the image of that point is considerably confined, in an extent which scarcely surpasses the size it would have, if the object were at the necessary distance; it must therefore be seen distinctly.

PROBLEM XI.

When the eyes are directed in such a manner as to see a very distant object; why do near objects appear double, and vice versa?

The reason of this appearance is as follows. When we look at an object, we are accustomed, from habit, to direct the optical axis of our eyes towards that point which we principally consider. As the images of objects are, in other respects, entirely similar, it thence results that, being painted around that principal point of the retina at which the optical axis terminates, the lateral parts of an object, those on the right for example, are painted in each eye to the left of that axis; and the parts on the left are painted on the right of it. Hence there has been established between these parts of the eye such a correspondence, that when an object is painted at the same time in the left part of each eye, and at the same distance from the optical axis, we think there is only one, and on the right; but if by a forced movement of the eyes we cause the image of an object to be painted in one eye, on the right of the optical axis, and in the other on the left, we see double. But this is what takes place when, in directing our sight to a distant object, we pay attention to a neighbouring object situated between the optical axes; it may be easily seen that the two images which are formed in the two eyes are placed, one to the right and the other to the left, of the optical axis; that is to say, on the right of it in the right eye, and on the left of it in the left. If the optical axis be directed to a near object, and if attention be at the same time paid to a distant object, in a direct line, the contrary will be the case. By the effect then of the habit above-mentioned, we must by one eye judge the object to be on the right and by the other to be on the left; the two eyes are thus in contradiction to each other, and the object appears double.

This explanation, founded on the manner in which we acquire ideas by sight,

is confirmed by the following fact. Cheselden relates that a man having sustained a hurt in one of his eyes by a blow, so that he could not direct the optical axes of both eyes to the same point, saw all objects double: but this inconvenience was not lasting; the most familiar objects gradually began to appear single, and his sight was at length restored to its natural state.

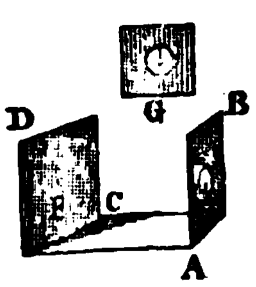
What takes place here in regard to the sight, takes place also in regard to the touch; for when two parts of the body which do not habitually correspond, in feeling one and the same object, are employed to touch the same body, we imagine it to be double. This is a common experiment. If one of the fingers be placed over the other, and if any small body, such as a pea for example, be put between them, so as to touch the one on the right side, and the other on the left, you would almost swear that you felt two peas. The explanation of this illusion depends on the same principles.

PROBLEM XII.

To cause an object, seen distinctly, and without the interposition of any opake or diaphonous body, to appear to the naked eye inverted.

Construct a small machine, such as that represented Fig. 26. It consists of two parallel ends, A B and C D, joined together by a third piece A C, half an inch in breadth, and an inch and a half in length. This may be easily done by means of a slip of card. In the middle of the end A B make a round hole E, about a line and a half in diameter; and in the centre of it fix the head of a pin, or the point of a needle, as seen in the figure: opposite to it in the other end make a hole F with a large pin; if you then apply your eye to E, turning the hole F towards the light or the flame of a candle, you will see the head of the pin greatly magnified, and in an inverted position, as represented at G.

Fig. 26.



The reason of this inversion is, that the head of the pin being exceedingly near the pupil of the eye, the rays which proceed from the point F are greatly divergent, on account of the hole F; and instead of a distinct and inverted image, there is painted at the bottom of the eye a kind of shadow in an upright position. But inverted images on the retina convey to the mind the idea of upright objects: consequently, as this kind of image is upright, it must convey to the mind the idea of an inverted object.

PROBLEM XIII.

To cause an object, without the interposition of any body, to disappear from the naked eye, when turned towards it.

For this experiment we are indebted to Mariotte: and though the consequences he deduced from it have not been adopted, it is no less singular, and seems to prove a particular fact in the animal economy.

Fix, at the height of the eye, on a dark ground, a small round piece of white paper, and a little lower, at the distance of two feet to the right, fix up another of about three inches in diameter; then place yourself opposite to the first piece of paper, and having shut the left eye, retire backwards, keeping your eye still fixed on the first object: when you have got to the distance of nine or ten feet, the second will entirely disappear from your sight.

This phenomenon is accounted for by observing, that when the eye has got to the above distance, the image of the second paper falls on the place where the optic nerve is inserted into the eye, and that according to every appearance this place of the retina does not possess the property of transmitting the impression of objects; for while the nervous fibres in the rest of the retina are struck directly on the side by the rays proceeding from the objects, they are struck here altogether obliquely, which destroys the shock of the particle of light.

The above experiment may be performed more neatly in the following manner. Having made three dots, thus—

● ● ●

close the left eye, and bring the right directly over the left-hand dot, and about three or four inches from the paper. Though the eye be directed steadily to that dot, the other two will be distinctly recognised as existing. But drawing the head gradually back, when the eye is about five or six inches from the paper the middle dot will disappear. Withdrawing a little further, it will re-appear; and all three will be again visible: but moving a little further, the dot in the right will disappear: and on withdrawing a little further, still it will again appear, and all three will be visible.

If the right eye be closed, and the left one brought over the dot on the right, similar phenomena will be observed.

PROBLEM XIV.

To cause an object to disappear to both eyes at once, though it may be seen by each of them separately.

Affix to a dark wall a round piece of paper, an inch or two in diameter, and a little lower, at the distance of two feet on each side, make two marks: then place yourself directly opposite to the paper, and hold the end of your finger before your face, in such a manner that when the right eye is open it shall conceal the mark on the left, and when the left eye is open the mark on the right; if you then look with both eyes to the end of your finger, the paper, which is not at all concealed by it from either of your eyes, will nevertheless disappear.

This experiment is explained in the same manner as the former; for by the means here employed, the image of the paper is made to fall on the insertion of the optic nerve of each eye, and hence the disappearance of the object from both.

PROBLEM XV.

An optical game, which proves that with one eye a person cannot judge well of the distance of an object.

Present to any one a ring, or place it at some distance, and in such a manner that the plane of it shall be turned towards the person's face: then bid him shut one of his eyes, and try to push through it a crooked stick, of sufficient length to reach it: he will very seldom succeed.

The reason of this difficulty may be easily given: it depends on the habit we have acquired of judging of the distances of objects by means of both our eyes; but when we use only one, we judge of them very imperfectly.

A person with one eye would not experience the same difficulty: being accustomed to make use of only one eye, he acquires the habit of judging of distances with great correctness.

PROBLEM XVI.

A person born blind, having recovered the use of his sight; if a globe and a cube which he has learnt to distinguish by the touch are presented to him, will he be able, on the first view, without the aid of touching, to tell which is the cube, and which is the globe?

This is the famous problem of Mr. Molyneux, which he proposed to Locke, and which has much exercised the ingenuity of metaphysicians.

Both these celebrated men thought—not without reason, and it is the general opinion—that the blind man, on acquiring the use of his sight, would not be able to distinguish the cube from the globe, or at least without the aid of reasoning; and indeed, as Mr. Molyneux said, though this blind man has learned by experience in what manner the

cube and the globe affect his sense of touching, he does not yet know how those objects which affect the touch will affect the sight; nor that the salient angle, which presses on his hand unequally when he feels the cube, ought to make the same impression on his eyes that it does on his sense of touching. He has no means therefore of discerning the globe from the cube.

The most he could do, would be to reason in the following manner, after carefully examining the two bodies on all sides: "On whatever side I feel the globe," he would say, "I find it absolutely uniform; all its faces in regard to my touch are the same; one of these bodies, on whatever side I examine it, presents the same figure and the same face; consequently it must be the globe." But is not this reasoning, which supposes a sort of analogy between the sense of touching and that of seeing, rather too learned for a man born blind? It could only be expected from a Saunderson. But it would be improper here to enter into farther details respecting this question, which has been discussed by Molyneux, Locke, and the greater part of the modern metaphysicians.

What was observed in regard to the blind man restored to sight by the celebrated Cheselden, has since confirmed the justness of the solution given by Locke and Molyneux.

When this man, who had been born blind, recovered his sight, the impressions he experienced, immediately after the operation, were carefully observed; and the following is a short account of them.

When he began to see, he at first imagined that all objects touched his eyes, as those with which he was acquainted by feeling touched his skin. He knew no figure, and was incapable of distinguishing one body from another. He had an idea that soft and polished bodies, which affected his sense of touching in an agreeable manner, ought to affect his eyes in the same way; and he was much surprised to find that these two things had no sort of connection. In a word, some months elapsed before he was able to distinguish any form in a painting; for a long time it appeared to him a surface daubed over with colours; and he was greatly astonished when he at length saw his father in a miniature picture: he could not comprehend how so large a visage could be put into so small a space; it appeared to him as impossible, says the author from whom this account is extracted, as to put a cask of liquor into a pint bottle.

PROBLEM XVII.

To construct a machine by means of which any objects whatever may be delineated in perspective, by any person, though unacquainted with the rules of that science.

The principle of this machine consists in making the point of a pencil, which continually presses against a piece of paper, to describe a line parallel to that described by a point made to pass over the outlines of the objects, the eye being in a fixed position, and looking through an immoveable sight.

Fig. 27.

The frame T, T, T, T (Fig. 27.), supported in a perpendicular direction by the two pieces of wood S G, S G, passing through the two lower corners of it, is adapted for receiving a sheet of paper, on which the objects are to be traced out in perspective. The paper is extended on it, and kept in that position by being cemented at the four corners. E F is a cross bar perpendicular to the two pieces S G, S G, and having at its extremity another piece E D, moveable on an axis at E. The latter serves to support the perpendicular rod D C, bearing a moveable sight B A, to which the eye is applied.

The piece of wood N P is moveable, and its extremity P is furnished with a slender point, terminating in a small button. Near its two extremities are fixed two pulleys, under which pass two small cords M M: these two cords are conveyed over the pulleys L, L, fixed at the corners T, T, of the frame, and then around two horizontal ones R, R: by these means they fall on the other side of the frame, where they are fastened to the weight Q, which moves in a groove, so that when the weight Q rises or falls, the moveable piece of wood N P remains always in a situation parallel to itself. This piece of wood ought to be nearly in equilibrium with the weight, that it may be easily moved, when it is necessary to raise or to lower it a little: in the middle of it is fixed the pencil or crayon I.

It may now be readily conceived that, if the eye be applied to the hole A, and if the moveable piece of wood N P be moved with the hand, in such a manner as to make the end P pass over the outlines of a distant object, the point of the pencil I will necessarily describe a line parallel and equal to that described by the point P; and consequently will trace out on the paper O, O, against which it presses, the image of the object in exact perspective.

This machine was invented by Sir Cristopher Wren, a celebrated mathematician, and the architect who built St. Paul's. But if it be required to trace out any object whatever, according to the rules of perspective, the very simple means described in the following problem may be employed.

PROBLEM XVIII.

Another method, by which a person may represent an object in perspective, without any knowledge of the principles of the art.

This method of representing an object in perspective requires, in the same manner as the preceding, no acquaintance with the rules of the art; and the kind of machine employed is much simpler; but it supposes a considerable degree of expertness in the art of drawing, or at least enough to be able to delineate in one small space what is seen in another.

To put this method in practice, construct a frame of such a size, that when looking at the object from a determinate point, it may be contained within that frame. Then fix the place of the eye before the frame, and, in regard to its plane, in whatever manner you think proper. The best position for the eye, unless you intend to make a drawing somewhat fantastical by the position of the objects, will be in a line perpendicular to the plane of the frame, at a distance nearly equal to the breadth of the frame, and at the height of about two-thirds of that of the frame. This place must be marked by means of a sight or hole, about two lines in diameter, made in the middle of a square or circular vertical plane, of about an inch or two in breadth. Then divide the field of the frame into squares of an inch or two in size, by means of threads extended from the sides, and crossing each other at right angles.

Then provide a piece of paper, and divide it, by lines drawn with a black lead pencil, into the same number of squares as the frame. When these preparations have been made, nothing is necessary but to apply your eye to the sight above-mentioned, and to draw in each square of the paper that part of the object observed in the corresponding square of the frame. By these means you will obtain an exact representation of the object in perspective; for it is evident that it will be delineated such as it appears to the eye, and perfectly similar to the figure which would remain on any transparent substance extended on the frame, if the rays, proceeding from each point of the object to the eye, or the place of sight, should leave traces on that substance. The object, or assemblage of objects, will therefore be represented in perspective with great accuracy.

Remark.—The same means may be employed to demonstrate, in a sensible manner, without the least knowledge of geometry, the truth of the greater part of the rules of perspective; for if a straight line be placed behind the frame, in a direction perpendicular to its plane, you will see its image pass through the point of sight, or through that point of the plane of the frame which corresponds to the perpendicular let fall from the eye on that plane. If the line be placed horizontally, and if you cause it to make an angle of 45 degrees with the plane of the picture, you will see the image of it pass through one of those points called the points of distance. If this line be placed in any direction whatever, you will see its image concur with one of the accidental points. It is in these three rules that the whole of perspective almost consists.

PROBLEM XIX.

Of the apparent magnitude of the heavenly bodies on the horizon.

It is a well known phenomenon that the moon and sun, when near the horizon, appear much larger than when they are at a mean altitude, or near the zenith. This phenomenon has been the subject of much research to philosophers; and some of them have given very bad explanations of it.

Those indeed who reason superficially, ascribe it to a very simple cause, viz. refraction; for if we look obliquely, say they, at a crown piece immersed in water, it appears to be sensibly magnified. But every body knows that the rays which proceed from the celestial bodies, experience a refraction when they enter the atmosphere of the earth. The sun and moon are then like the crown immersed in water.

But those who reason in this manner do not reflect that, if a crown piece immersed in a denser medium appears magnified to the eye situated in a rarer medium, the contrary ought to be the case when the eye is situated in a dense medium, while the crown piece is immersed in a rarer. A fish would see the crown piece out of the water much smaller than if it were in the water. But we are placed in the dense medium of the atmosphere, while the moon and sun are in a rarer. Instead therefore of appearing larger, they ought to appear smaller; and this indeed is the case, as is proved by the instruments employed to measure the apparent magnitude of the celestial bodies: these instruments shew that the perpendicular diameter of the sun and moon, when on the horizon, is shortened about two minutes, which gives them that oval form, pretty apparent, under which they often appear.

The cause of this phenomenon must therefore be sought for in a mere optical illusion; and in our opinion the following explanation is the most probable.

When an object paints on the retina an image of a determinate size, the object appears to us larger, according as we judge it to be at a greater distance: and this is the consequence of a tacit reasoning pretty just; for an object which, at the distance of six hundred feet, is painted in the eye under the diameter of the line, must be much larger than that which is painted under the same diameter, though only at the distance of sixty feet. But when the sun and moon are on the horizon, a multitude of intervening objects give us an idea of great distance; whereas when they are near the zenith, as no object intervenes, they appear to be nearer us. In the former situation then they must excite an idea of magnitude, quite different from what they do in the latter.

We must however confess that this explanation is attended with some difficulties.

1st. When we look at the moon on the horizon through a tube, or through the fingers bent into the form of one, the size of it appears to be much diminished, though the fingers conceal the intervening objects in a very imperfect manner. 2d.

We often see the moon rising behind a hill at a small distance, and on such occasions she appears to be exceedingly large.

These facts, which seem to overturn the explanation before given, have induced other philosophers to endeavour to find out a different one. The following is that of Dr. Smith, a celebrated writer on optics.

The celestial arch does not exhibit to us the appearance of a hemisphere, but that of a very oblate surface, the elevation of which towards the zenith is much less than its extension towards the horizon. The sun and moon also appear under the same angle, whether at the horizon or near the zenith. But the intersection of a determinate angle, at a mean distance from the summit, is less than at a greater. The projection therefore of the sun and moon, or their perspective image on the celestial arch, is less at a great distance from the horizon than in the neighbourhood of it. Consequently, when at a distance from the horizon they must appear less than when they are near it.

This explanation of the phenomenon is very specious. But may it not here be asked, why these two images, though seen under the same angle, appear one greater than the other? Are we not still obliged to have recourse to the former explanation? But for the sake of brevity we shall leave the discussion of these two questions to the reader.

It is sufficient that it is fully demonstrated that this apparent magnification is not produced by a larger image painted on the retina. In regard to the moon, it is even somewhat less; since that luminary, when on the horizon, is farther from us, by about a semi-diameter of the earth, or a sixtieth part, than when she is very much elevated above the horizon. In a word, this phenomenon is merely an optical illusion, whatever may be the cause, which is still very obscure, but in our opinion it seems to depend chiefly on the idea of great distance exited by the intervening objects.

PROBLEM XX.

On the converging appearance of parallel rows of trees.

The phenomenon which is the subject of this problem, is well known. Every person must have observed, that when at the extremity of a very long walk, planted on each side with trees, the sides instead of appearing parallel, as they really are, seem to converge towards the other end. The case is the same with the ceiling of a long gallery; and indeed when it is necessary to represent these objects in perspective, the sides of the walk or ceiling must be represented by converging lines; for they are really so in the small image or picture painted at the bottom of the eye.

Other considerations however are necessary, in order to give a complete explanation of the phenomenon; for as the apparent magnitude of objects is not measured by the real magnitude of the images painted in the eye, but is always the result of the judgment formed of their distance by the mind, combined with the magnitude of the image present in the eye, the sides of a walk are far from appearing to converge with so much rapidity, as the lines which form the image of them in the perspective plane, or in the eye. M. Bouguer first gave a complete explanation of what takes place on this occasion; it is as follows:

As the ceiling of a long gallery appears to an eye, placed at one extremity of it, to become lower, the case is the same with a long level walk, the sides of which are parallel; the plane of that walk, instead of appearing horizontal, seems still to rise. For the same reason, as when on the sea shore, the water appears like an inclined plane which threatens the earth with an inundation. Some superstitious persons, little acquainted with the principles of philosophy, have considered this inclination as real, and the apparent suspension of the waters as a visible and continued miracle. In like manner, in the middle of an immense plain we see it rise around us, as if we were at the bottom of a very broad and shallow funnel. M. Boguer has taught us a very

ingenious method of determining this apparent inclination; but it will be sufficient here to say that, to most men, it is about 2 or 3 degrees.

Let us then suppose two horizontal and parallel lines, and an inclined plane of 2 or 3 degrees passing below our feet: it is evident that these two horizontal lines will appear to our eye as if projected on that inclined plane. But their projection on that plane will be two lines concurring in one point, viz., that where the horizontal drawn from the eye would meet it. We must therefore see these lines as convergent.

It thence follows, that if, by any illusion peculiar to the sight, the plane where the parallel lines are situated, instead of appearing inclined upwards, should appear declined downwards, the sides of the walk would appear divergent. This, Dr. Smith, in his Treatise of Optics, says is the case with the avenue at the seat of Mr. North, in the county of Norfolk. But it is to be wished that Dr. Smith had described, in a more minute manner, the position of the places. However we shall solve, according to these principles, another curious problem, which has been much celebrated among opticians.

PROBLEM XXI.

In what manner must we proceed to trace out an avenue, the sides of which, when seen from one of its extremities, shall appear parallel?

Suppose an inclined plane of two degrees and a half, and that two parallel lines are traced out on it. From the eye suppose two planes passing through these lines, and which being continued, cut the horizontal plane in two other lines; these two lines will be convergent, and if continued backwards will meet behind the spectator.

Nothing then is necessary but to find this point of concurrence, which is very easy; for any one in the least acquainted with geometry, must perceive that it is the point where a line drawn through the eye, parallel to the above inclined plane, and in the direction of the middle of the avenue, meets with the horizontal plane. Let a line then inclined to the horizon two or three degrees, be drawn through the eye of the spectator, and in the vertical plane passing through the middle of the avenue; the point where it meets the horizontal plane will be that where the two sides of the avenue must unite. If from this point, therefore, two straight lines be drawn through the two extremities of the initial breadth of the avenue, they will trace out where all the trees ought to be planted, so as to appear to form parallel sides.

If the height of the eye be supposed equal to 5 feet, and the breadth of the commencement of the avenue to be 36, the point of concurrence will be found by calculation to be 102 backwards, and the angle formed by the sides of the avenue ought to be about 18 degrees. It is difficult however to believe, that lines which form so sensible an angle will ever appear parallel to an eye within them, in whatever point it may be placed.

PROBLEM XXII.

To form a picture which, according to the side on which it is viewed, shall exhibit two different subjects.

Provide a sufficient number of small equilateral prisms, a few lines only in breadth, and in length equal to the height of the painting which you intend to make, and place them all close to each other on the ground to be occupied by the painting.

Then cut the painting into bands equal to each of the faces of the prisms, and cement them, in order, to the faces of the same side.

When this is done, take a painting quite different from the former, and having divided it into bands in the same manner, cement them to the faces of the opposite side.

It is hence evident, that when on one side you can see only the faces of the prism turned towards that side, one of the paintings will be seen; and if the picture be looked at on the opposite side, the first will disappear, and the second only will be seen.

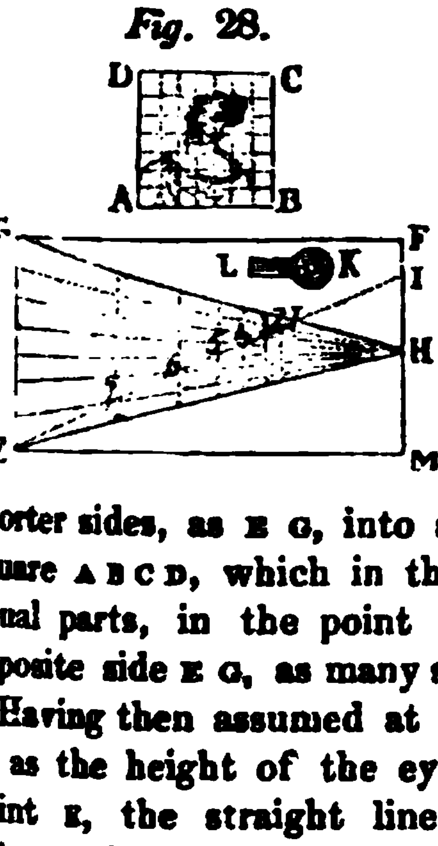
A painting may even be made, which when seen in front, and on the two sides, shall exhibit three different subjects. For this purpose, the picture of the ground must be cut into bands, and be cemented to that ground in such a manner, that a space shall be left between them, equal to the thickness of a very fine card. On these intervals raise, in a direction perpendicular to the ground, bands of the same card, nearly equal in height to the interval between them; and on the right faces of these pieces of card cement the parts of a second painting, cut also into bands. In the last place, cement the parts of a third picture, cut in the same manner, on the left faces of the pieces of card. It is evident that when this picture is viewed in front, at a certain distance, the bottom painting only will be seen; but if you stand on one side, in such a manner that the height of the slips of card conceals from you the bottom, you will see only the picture cemented in detached portions to the faces turned towards that side: if you move to the other side, a third painting will be seen.

PROBLEM XXIII.

To describe on a plane a distorted figure, which when seen from a determinate point shall appear in its just proportions.

A figure, such for example as a head, may be disguised, that is to say distorted, in such a manner, as to exhibit no proportion, when the plane on which it has been drawn is viewed in front; but when viewed from a certain point it shall appear beautiful, that is to say, in its just proportions. This may be done in the following manner:

Fig. 28.



Having drawn on a piece of paper, in its just proportions, the figure you intend to disguise, describe a square around it, as A B C D (Fig. 28), and divide it into several other small squares, which may be done by dividing the sides into equal parts, for example seven, and then drawing straight lines through the corresponding points of division, as the engravers do when they intend to make a reduced drawing from a picture.

Then describe, at pleasure, on the proposed plane, a parallelogram E M F G, and divide one of the two shorter sides, as E G, into as many equal parts as D C, one of the sides of the square A B C D, which in this case are seven. Divide the other side M F, into two equal parts, in the point H, and draw from it to the points of division of the opposite side E G, as many straight lines, the two last of which will be H E and H G. Having then assumed at pleasure, in the side M F, the point I, above the point H, as the height of the eye above the plane of the picture, draw from I to the point E, the straight line E I, which will cut those lines proceeding from the point H, in the points 1, 2, 3, 4, 5, 6, 7. Through these points of intersection draw straight lines parallel to each other, and to the base E G of the triangle E G H, which will thus be divided into as many trapeziums as there are little squares in the square A B C D. Hence, if the figure in the square A B C D be transferred to the triangle E G H, by making those parts of the outline contained in the different natural squares of A B C D, to pass through the corresponding trapeziums or perspective squares, the figure will be found to be distorted. But it may be seen exactly like its prototype, that is to say as in the square A B C D, if it be viewed through a hole K, which ought to be small towards the eye and wide towards the object, made in a small board L, placed perpendicularly in H, so that the height L K shall be equal to H I, which must never be very great, in order that the figure may be more distorted in the picture.

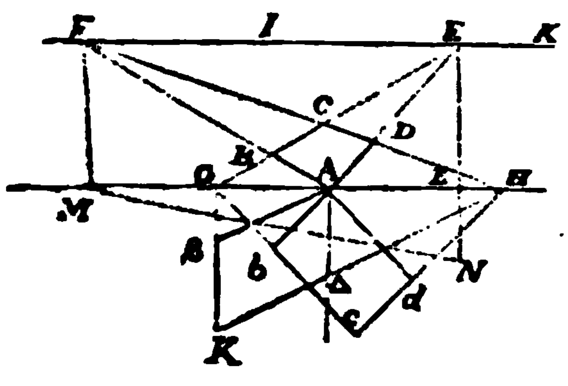
In the convent of the Minimes *de la Place Royale* there is a Magdalen at prayers distorted in the same manner, which has some celebrity. It is the work of Father Niceron of that order, who frequently employed himself on this kind of optical amusement.

Several other anamorphoses may be made in the same manner, by painting, for example, on a curved surface, either cylindric, or conical, or spherical, a certain figure, which when seen from a determinate point shall appear regular; but as this does not succeed so well in practice as in theory, we think it needless to say any thing further on the subject, while there are so many others much more curious. Those persons who are fond of such optical curiosities may consult *La Perspective Curieuse* of Father Niceron, where they will find the subject treated of at full length.

PROBLEM XXIV.

Any quadrilateral figure being given; to find the different parallelograms or rectangles of which it may be the perspective representation. Or any parallelogram, whether right-angled or not, being given, to find its position, and that of the eye, which shall cause its perspective representation to be a given quadrilateral.

Fig. 29.



Let the given quadrilateral be the trapezium A B C D (Fig. 29), which we shall suppose the most irregular possible, having none of its sides parallel. Continue the sides A B and C D, till they meet in F, and the sides A D and B C, till they meet in E; then draw E F, and through the point A, draw G H, parallel to it. Whatever be the position of the eye, provided what is called the point of sight be in the line E F, or not only in E F, but in the continuation of it on both sides; the object, of which the quadrilateral A B C D is the perspective representation, will be a parallelogram.

For all persons acquainted with the rules of perspective, know that lines parallel to each other on a horizontal plane, when represented in perspective meet in one point of the line parallel to the horizon, drawn through the point of sight. Thus all the lines perpendicular to the ground line, meet in the point of sight itself: all those which form with that line an angle of 45°, concur in what is called the points of distance; and those which form a greater or less angle, concur in other points which are always determined by drawing from the eye to the picture a line parallel to those of which the perspective representation is required. All the lines then, which in the picture concur in points situated in the line of the point of sight, are images of horizontal and parallel lines. Thus, the lines on the horizontal plane, which have as representatives in the picture the lines B C and A D, are parallel; and the case is the same with those which give the lineal images A B and D C. But two pairs of parallel lines necessarily form, by their intersection, a parallelogram. The object then of which the quadrilateral A B C D is the image, to an eye situated in the line F E, wherever the point of sight may be, is a parallelogram.

This being demonstrated, we shall first suppose that the required object is a rectangle. To find in this case the place of the eye, divide the distance F E into two equal parts in I, and suppose the eye situated in such a manner that the perpendicular, drawn from its place to the painting, shall fall on the point I; and that the distance is equal to I E or I F: the points F and I will then be what, in the language of perspective, is called the points of distance. Continue the lines C B and C D to G and H in the ground line: the lines H C F and A B F will be the images of the lines which form with the ground line angles of 45 degrees. The case will be the same with those of which G C E and A D E are the images. If the indefinite lines H *d c* and

A b, inclined to the ground line at an angle of 45 degrees, be then drawn on the one side and on the other, and in a contrary direction, the lines G b c and A d, inclined also at half a right angle, these lines will necessarily meet at right angles, and form the rectangle A b c d.

If the point of sight be supposed in another point, for example E, that is to say, if we suppose the eye to be directly opposite to the point E, and at a distance equal to E K, after drawing E L and F M perpendicular to the ground line in the plane of the picture, we must draw to the same ground line, in the horizontal plane, the perpendicular L N, equal to E K, and then the line N M, making with the ground line the angle L M N. If we then draw to the points G and A the indefinite perpendiculars A Δ and G K, and through the points A and H the indefinite lines H K and A β, making with the ground line angles equal to L M N, and in a contrary direction; these two pairs of lines will meet in β, K, Δ, and evidently form an oblique parallelogram, which will be the object of which B C D A is the representation, to an eye situated opposite to E, and at a distance from the picture equal to E K.

If the sides A b and c d, in the rectangle A b c d, were divided into equal parts by lines parallel to the other sides, it is evident that these parallels, being continued, would cut the line A G into as many equal parts. The case would be the same with lines parallel to A b and c d dividing into equal portions the sides A d and b c: the line A H would likewise be divided by them into equal parts. Thus we have the means of dividing the trapezium A B C D, if necessary, into lozenges, which would be the representation of the squares into which A b c d might be divided.

We shall give hereafter the solution of a very curious problem, in regard to ornamental gardening, which is a consequence of the one here solved.

OF PLANE MIRRORS.

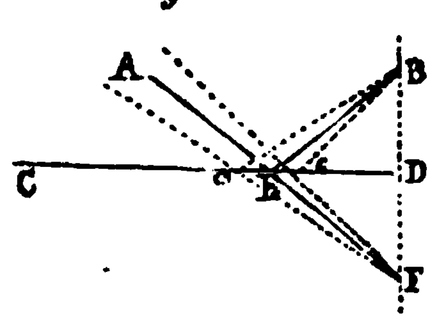
Plane mirrors are those the reflecting surface of which is plane; as is the case with the common glass mirrors used for decorating apartments. Plane mirrors may be made also of metal. Of this kind were those of the ancients; but since the invention of glass, metallic mirrors are never used, except small ones for certain optical instruments, where it is necessary to avoid the double reflection produced by glass, one from the anterior and the other from the posterior surface. It is the latter which gives the liveliest image; for if the silvering be scraped from the back of a mirror, you will see the bright image immediately disappear almost entirely, and that which remains in its place will scarcely be equal to that produced by the nearer surface.

But in catoptrics, in general, the two surfaces of a mirror are supposed to be at such a small distance from each other, as to produce only one image; otherwise the determinations given by this science would require to be greatly modified.

PROBLEM XXV.

A point of the object B, *and the place of the eye* A, *being given; to find the point of reflection on the surface of a plane mirror.* (Fig. 30.)

Fig. 30.



Through B, the given point of the object, and A, the place of the eye, conceive a plane perpendicular to the mirror, and cutting it in the line C D: from the point B, draw B D perpendicular to C D, and continue it to F, so that D F and D B shall be equal: if through the points F and A, the line A F be drawn, intersecting C D in E, the point E will be the point of reflection; B E will be the incident ray; E A the reflected ray; and B E D, the angle of incidence, and A E C, the angle of reflection, will be equal.

For it is evident, by the construction, that the angles B E D and D E F are equal; but the angles D E F and A E C are also equal, being vertical angles; therefore, &c.

PROBLEM XXVI.

The same supposition being made as before; to find the place of the image of the point B.

The place of the image of the point B is exactly in the point F. But we shall not assign as the reason what is commonly given in books on catoptrics, viz., that in mirrors of every kind the place of the image is in the continuation of the reflected ray, where it is intersected by the perpendicular drawn from the point of the object to the reflecting surface: for what effect can this perpendicular, which is merely imaginary, have to fix the image in this manner, in the point where it meets with the reflected ray continued, rather than in any other point? This principle then is ridiculous, and void of foundation.

It is however true that, in plane mirrors, the place where the object is perceived, is in the point where the above perpendicular meets with the reflected ray produced; but this is accidental, and the reason is as follows.

All the rays which emanate from the object B, and are reflected by the mirror, meet, if produced, in the point F: their arrangement then, in regard to the eye, is the same as if they proceeded from the point F. Consequently they must make the same impression on the eye, as if the object were in F; for the eye would not be otherwise affected, if they really proceeded from that point.

Hence it may be concluded that, in a plane mirror, the object appears to be as far behind, as it is distant from the mirror.

It therefore follows, that A F, the distance of the image F from the eye, is equal to the sum of B E, the ray of incidence, and A E the ray of reflection, since B E and E F are equal.

It thence follows also, that when the plane mirror is parallel to the horizon, as C D, a perpendicular object, such as B D, must appear inverted.

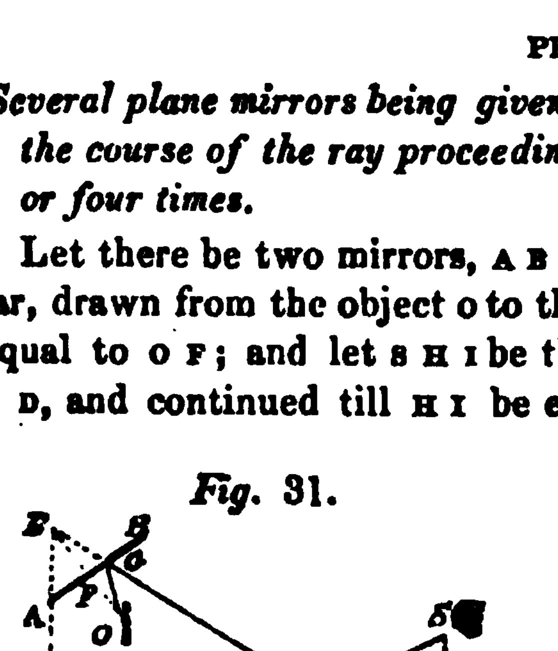
In the last place, when we look at ourselves in a mirror, the left seems to be on the right, and the right on the left.

PROBLEM XXVII.

Several plane mirrors being given, and the place of the eye, and of the object; to find the course of the ray proceeding from the object to the eye, when reflected two, three, or four times.

Let there be two mirrors, A B and C D, (Fig. 31.), and let O F E be the perpendicular, drawn from the object O to the mirror A B, and continued beyond it, so that F E be equal to O F; and let S H I be the perpendicular drawn from the eye to the mirror C D, and continued till H I be equal to H S; join the points I and E by the line E I, which will intersect the mirrors in G and K; and if the lines O G, G K, and K S be then drawn, they will represent the course of the ray, proceeding from the point O to the eye by two reflections.

Fig. 31.

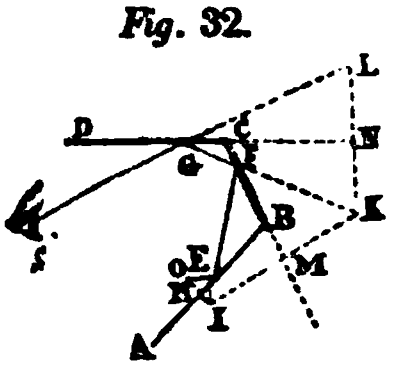


Or, from the point E, the first part of the construction remaining the same, let fall, on the mirror C D, the perpendicular E L M, and continue it beyond it, till L M be equal to L E; draw the line S M, intersecting C D in K; and from the point K, the line K E, intersecting A B in G; if G O be also drawn, the lines O G, G K, and K S will represent the course of the ray proceeding from the point O, and conveyed to the eye by two reflections.

In this case, the point M will be the image of the point O, and the distance S M will be equal to the sum of the rays S K, K G, and G O.

If we suppose three mirrors, and three reflections, the course which the incident ray must pursue, in order to reach the eye, may be found in the same manner. For this purpose, let O I (Fig. 32.) be the perpendicular drawn from the object to the mirror A B, and let H I be equal to H O. From the point I draw I K perpendicular to C B, produced if necessary, and make K M equal to M I: from the point K let fall on D C produced the perpendicular K N, and continue it to L, so that L N shall be equal to K N: draw S L, which will intersect C D in G; and from the point G the line G K, which will intersect C B in F; if the line F I, intersecting A B in E, be then drawn from the point F, and also the line E O, then the line E O will be that according to which the incident ray must fall on the first mirror, to reach to eye S, after three reflections at E, F, and G.

Fig. 32.



In this case the point L will be the apparent place of the image of the object, to an eye situated in S; and the distance S L will be equal to S G, G F, F E, and E O, taken all together.

The application of this problem is generally shewn at the game of billiards; but as we have already treated that subject, under the head mechanics, the reader is referred to that article.

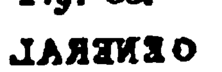
PROBLEM XXVIII.

Various properties of plane mirrors.

I. In plane mirrors the image is always equal and similar to the object. For it may be easily demonstrated, that as each point of the image seems to be as far within the mirror as the object is distant from it, each point of the image is similarly situated, and at an equal distance in regard to all the rest, as in the object: the result must therefore necessarily be the equality and similarity of the image and object.

II. In a plane mirror, what is on the right appears in the object to be on the left, and *vice versa*. This may be easily proved in the following manner. If a piece of common writing be held before a mirror, it cannot be read: as the word GENERAL, for example, will appear under the annexed form; but on the other hand, if the latter word be presented to the mirror, GENERAL will appear. This affords the means of forming a sort of secret writing; for if we write from right to left, it cannot be read by those ignorant of the artifice; but those acquainted with it, by holding the writing before a mirror, will see it appear like common writing. This method however must not be employed for concealing secrets of great importance, as there are few persons to whom it is not known.

Fig. 33.



III. In a plane mirror, when you can see yourself at full length, at whatever distance you remove from it, you will always see your whole body; and the height of the mirror occupied by your image will always be equal to the half of your height.

IV. If one of the sun's rays be made to fall on a plane mirror, and if an angular motion be given to the mirror, the ray will be seen to move with a double angular motion; so that when the mirror has passed over 90°, the ray will have passed over 180°.

V. If a plane mirror be inclined to a horizontal surface, at an angle of 45°, its image will be vertical.

VI. If two plane mirrors be disposed parallel to each other; and if any object, such as a lighted taper, be placed between them; you will see in each a long series of tapers which would be extended *in infinitum*, did not each image become fainter in proportion as the reflections by which they are produced become more numerous.

VII. When two mirrors are disposed in such a manner as to form an angle of at

least 120°, several images will be seen, according to the position of the eye. If the angle of the mirrors be diminished, without changing the place of the eye, these images will be seen to increase in number, as if they emerged from behind an opake body.

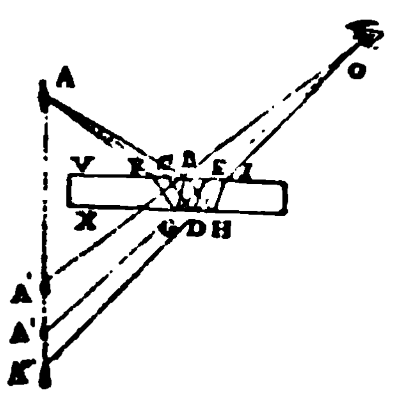
It must be observed, that all these images are in the circumference of a circle, described from the point where the mirrors meet, and passing through the place of the object.

Father Zacharias Traber, a Jesuit, in his *Nervus Opticus*, and Father Tacquet, in his Optics, have carefully examined all the cases resulting from the different angles of these mirrors, as well as from the different positions of the eye and the object. To these we refer the reader.

VIII. When a luminous object, such as the flame of a taper, is viewed in a plane glass mirror of some thickness, several images of that object are perceived; the first of which, or that nearest the surface of the glass, is less brilliant than the second; the latter is the most brilliant of the whole; and after it, a series of images less and less brilliant are observed, to the number sometimes of five or six.

The first of these images is produced by the anterior surface of the glass, and the second by the posterior, which being covered with tin-foil, must produce a more lively reflection: it is therefore the most brilliant of the whole. The rest are produced by the rays of the object, which reach the eye after being several times reflected from the anterior, as well as posterior, side of the mirror. This phenomenon may be explained as follows.

Fig. 34.



Let V X (Fig. 34.) be the thickness of the glass, A the object, and O the place of the eye, which we shall suppose to be both equally distant from the mirror. Of all the small bundles of incident rays, there is one A B, which being reflected by the anterior surface in B, is conveyed to the eye by the line B O, and forms at A′ the first image of the object. Another, as A C, penetrates the glass, and being refracted into the line C D, is wholly reflected into D E, on account of the opacity of the posterior side of the mirror, and being again refracted at E proceeds to O, and forms at A″ the liveliest image of the point A.

Another small bundle A F penetrates also the glass, is refracted along the line F G, and reflected in the direction of G B, from which a part of it issues, but cannot reach the eye; the other part is reflected in the direction B H, and then into H I, from which a small part is still reflected, but the remainder issues from the glass and is refracted in the direction of the line I O, by which it reaches the eye: consequently it produces the third image, at A‴, weaker than the other two.

The fourth image is formed by a bundle of incident rays, which experience two refractions like the rest, and five reflections, viz., three from the posterior surface of the glass, and two from the anterior. In regard to the fifth, it requires two refractions and seven reflections, viz., three from the anterior surface, and four from the posterior; and so of the rest. It may hence be easily conceived how much the brightness of the images must be dimished, and therefore it is very uncommon to see more than four or five.

PROBLEM XXIX.

To dispose several mirrors in such a manner, that you can see yourself in each of them, at the same time.

To produce this effect, nothing is necessary but to dispose the mirrors on the circumference of a circle, in such a manner, that they shall correspond with the chords

of that circle; if you then place yourself in the centre, you will see your image in all the mirrors at the same time.

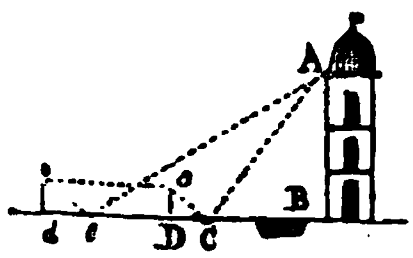
Remark.—If these mirrors are disposed according to the sides of a regular polygon, of an equal number of sides, such as a hexagon or octagon, which seem to be fittest for the purpose, and if all the mirrors are perfectly vertical and plane, they will form a sort of cabinet, which will appear of an immense extent, and in whatever part of it you place yourself, you will see your image, and immensely multiplied.

If this cabinet be illuminated in the inside, by a lustre placed in its centre, it will exhibit a very agreeable spectacle, as you will see long rows of lights towards whatever side your sight is directed.

PROBLEM XXX.

To measure, by means of reflection, a vertical height, the bottom of which is inaccessible.

Fig. 35.



We shall here suppose that A B (Fig. 35.), the vertical height to be measured, is that of a tower, steeple, or such like. Place a mirror at C, in a direction perfectly horizontal; or, because this is very difficult, and as the least aberration might produce a great error in the measurement, place in C a vessel containing water, which will reflect the light in the same manner as a mirror. The eye which receives the reflected ray being at O, measure with care the height O D above the horizontal plane of the mirror at C; measure also D C as well as C B, if the latter is accessible, and then say: As C D, is to D O, so is C B to a fourth proportional B A, which will be the height required.

But if the bottom of the tower be not accessible, to measure the height A B, we must proceed as follows:

Having performed every part of the preceding operation, except measuring C B, which by the supposition is impossible, take another station, as *c*, and place there a mirror or vessel of water: then taking your station in *d*, from which you can see the point A, by means of the reflected ray *c o*, measure *c d* and *d o*. When this is done, you must employ the following proportion: As the difference between C D and *c d* is to C D, so is C *c*, the distance between the two points of reflection, to a fourth proportional, which will be the distance B C, before unknown.

When B C is known, nothing is necessary but to make use of the proportion indicated in the first case, which will give the height A B.

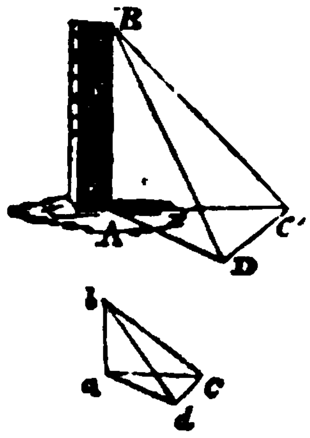
We do not consider this operation as susceptible of much accuracy in practice. Methods purely geometrical, if good instruments are employed, ought always to be preferred; but we should perhaps have been considered as guilty of an omission, had we taken no notice of this geometrico-catoptric speculation, though it has never perhaps been put in practice.

PROBLEM XXXI.

To measure an inaccessible height by means of its shadow.

Fix a stick in a perpendicular direction, on a plane perfectly horizontal, and measure the height of it above that plane, which we shall suppose to be exactly 6 feet. When the sun begins to sink towards the horizon in the afternoon, mark on the ground which is accessible the point C (Fig. 36.) where the shadow of the summit of the tower falls, and also the point *c* the extremity of the shadow of the stick erected perpendicularly on the same plane: at the end of two hours, more or less, mark, as speedily as possible, the two points D and *d*, which will be the summits of the shadows at that period; then join the two points of the shadow

Fig. 36.



of the summit of the tower by means of a straight line, and measure their distance; measure also, in like manner, the line which joins the two points *c* and *d* of the shadow of the stick; after which you will have nothing to do but to employ the following proportion: As the length of the line *c d* which joins the two points of the shadow of the stick, is to the height of the stick *a b*, so is the length of the line C D which joins the two points of the shadow of the tower, to the height of the tower A B.

It requires only an acquaintance with the first principles of geometry to be able to perceive, merely by inspecting Fig. 36, that the pyramids B A D C and *b a d c* are similar; consequently, that *c d* is to *a b* as C D to A B, which is the height required.

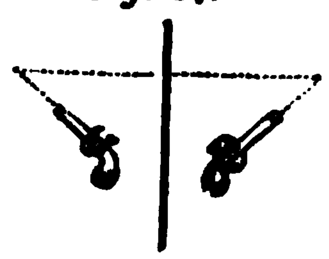
PROBLEM XXXII.

Of some tricks or kinds of illusion, which may be performed by means of plane mirrors.

Many curious tricks, capable of astonishing those who have no idea of catoptrics, may be performed by the combination of several plane mirrors. Some of these we shall here describe.

1st. ***To fire a pistol over your shoulder and hit a mark, with as much certainty as if you took aim at it in the usual manner.*** (Fig. 37.)

Fig. 37.

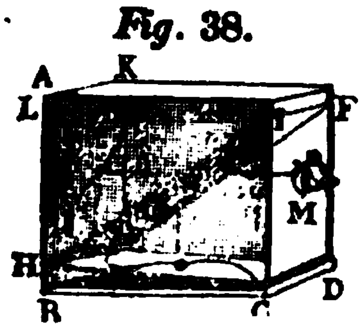


To perform this trick, place before you a plane mirror, so disposed, that you can see in it the object you propose to hit; then rest the barrel of the pistol on your shoulder and take aim, looking at the image of the pistol in the glass as if it were the pistol itself; that is, in such a manner, that the image of the object may be concealed by the barrel of the pistol: it is evident that if the pistol be then fired, you will hit the mark.

2d. ***To construct a box in which heavy bodies, such as a ball of lead, will appear to ascend contrary to their natural inclination.***

Construct a square box, as A B C D (Fig. 38.), where one of the sides is supposed to be taken off, in order to shew the inside; and fix in it a board H G D C, so as to form a plane, somewhat inclined, with a serpentine groove in it of such a size, that a ball of lead can freely roll in it and descend.

Fig. 38.



Then place the mirror H G F I in an inclined position, as seen in the figure, and make an aperture opposite to it at M, in the side of the box, but so disposed that the eye, when applied to it, can see only the mirror, and not the inclined plane H D. It may be easily perceived that the image of this plane, viz. H L K G, will seem to be a plane almost vertical, and that a body which rolls from G to C, along the serpentine groove, will appear to ascend in a similar direction from G to L. Hence, if the mirror is very clean, so as not to be observed, or if only a faint light be admitted into the box, which will tend to conceal the artifice, the illusion will be greater, and those not acquainted with the deception will have a good deal of difficulty to discover it.

3d. ***To construct a box in which objects shall be seen through one hole, different from what were seen through another, though in both cases they seem to occupy the whole box.***

Provide a square box, which, on account of its right angles, is the fittest for this purpose, and divide it into four parts, by partitions perpendicular to the bottom,

crossing each other in the centre. To these partitions apply plane mirrors, and make a hole in each face of the box, to look through; but disposed in such a manner, that the eye can see only the mirrors applied to the partitions, and not the bottom of the box. In each right angle of division formed by the partitions, place some object, which, being repeated in the lateral mirrors, may form a regular representation, such as a parterre, a fortification or citadel, a pavement divided into compartments, &c. That the inside of the box may be sufficiently lighted, it ought to be covered with a piece of transparent parchment.

It is evident that, if the eye be applied to each of the small apertures formed in the sides of the box, it will perceive as many different objects, which however will seem to occupy the whole inside of it. The first will be a regular parterre, the second a fortification, the third a pavement in compartments, and the fourth some other object.

If several persons look at the same time through these holes, and then ask each other what they have seen, a scene highly comic to those acquainted with the secret may ensue, as each will assert that he saw a different object.

Remark.—To render the parchment employed for covering optical machines, such as the above, more transparent, it ought to be repeatedly washed in a clear ley, which must be changed each time: it is then to be carefully extended, and exposed to the air to dry.

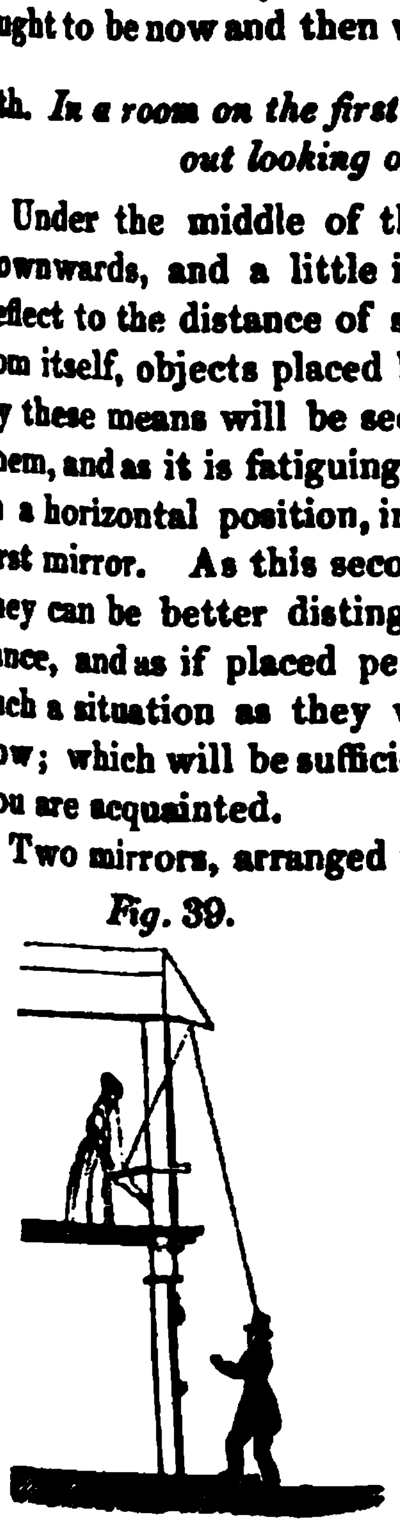
If you are desirous of giving it some colour, you may employ, for green, verdigrise diluted in vinegar, with the addition of a little dark green; for red, an infusion of Brazil wood; for yellow, an infusion of yellow berries; the parchment afterwards ought to be now and then varnished.

4th. *In a room on the first floor, to see those who approach the door of the house, without looking out at the window, and without being observed.*

Under the middle of the architrave of the window place a mirror, with its face downwards, and a little inclined towards the side of the apartment, so that it shall reflect to the distance of some feet from the bottom of the window, or on the bottom itself, objects placed before and near the door of the house. But as the objects by these means will be seen inverted, in which case it will be difficult to distinguish them, and as it is fatiguing and inconvenient to look upwards, fix another plane mirror in a horizontal position, in the place to which the image of objects is reflected by the first mirror. As this second mirror will exhibit the objects in their proper position, they can be better distinguished. They will appear however at a much greater distance, and as if placed perpendicularly on a plane, somewhat inclined, and almost in such a situation as they would be seen in if you looked downwards from the window; which will be sufficient in general to enable you to distinguish those with whom you are acquainted.

Two mirrors, arranged in this manner, are represented figure 39.

Fig. 39.



Ozanam, and others before him, who published Mathematical Recreations, propose by way of problem, to shew a jealous husband what his wife is doing in another apartment. To bore a hole near the ceiling in the partition wall which separates two apartments, and fix a horizontal mirror, half in the one room and half in the other, to reflect, by means of another mirror placed opposite to it, the image of what might take place in one of these rooms, is certainly an ingenious idea; but there is reason to think that neither Ozanam nor his predecessor were jealous husbands, or that they had a singular dependance on the folly and stupidity of the two lovers.

PROBLEM XXXIII.

To inflame objects, at a considerable distance, by means of plane mirrors.

Arrange a great number of plane mirrors, each about six or eight inches square, in such a manner that the solar rays reflected from them may be united in one focus. It is evident, and has been proved by experience, that if there are a sufficient number of these mirrors, as 100 or 150 for example, they will produce in their common focus a heat capable of inflaming combustible bodies, and even at a very great distance.

This was, no doubt, the invention employed by Archimedes, if he really burnt the fleet of Marcellus by means of burning mirrors, as we are told in history; for Kircher, when at Syracuse, observed that the Roman ships could not have been at a less distance from the walls of the city than twenty-three paces. But it is well known that the focus of a concave spherical mirror is at the distance of half its radius; consequently the mirror employed by Archimedes must have been a portion of a sphere of at least 46 paces radius, the construction of which would be attended with insurmountable difficulties. Besides, can it be believed that the Romans, at so short a distance, would have suffered him to make use of his machine without interruption? On the contrary, would they not have destroyed it by a shower of missile weapons?

Anthemius of Tralles, the architect and engineer who lived under Justinian, is the first who, according to the account of Vitellio, conceived the idea of employing plane mirrors for burning;* but we are not told whether he ever carried this method into execution. It is to Buffon that we are indebted for a proof of its being practicable. In the year 1747, this eminent naturalist caused to be constructed a machine consisting of 360 plane mirrors, each 8 inches square, and all moveable on hinges, in such a manner that they could be made to assume any position at pleasure. By means of this machine he was able to burn wood at the distance of 200 feet. Buffon's curious paper on this subject may be seen in the Memoirs of the Academy of Sciences for the year 1748.

That the ancients made use of burning glasses is evident from a passage in a play of Aristophanes, called The Clouds, where Strepsiades tells Socrates, that he had found out an excellent method to defeat his creditors, if they should bring an action against him. His contrivance was, that he would get from the jewellers a certain transparent stone, which was used for kindling fire, and then, standing at a distance, he would hold it to the sun, and melt down the wax on which the action was written.

The astonishing philosophico-military exploit of Archimedes may deserve some farther notice. That exploit has been recorded by Diodorus Siculus. Lucian, Dion, Zonaras, Galen, Anthemius, Tzetzes, and other ancient writers. The account of Tzetzes is so particular, that it suggested to father Kircher the specific method by which Archimedes probably effected his purpose. "Arthimedes," says that author, "set fire to the fleet of Marcellus by a burning glass, composed of small square mirrors, moving every way upon hinges; and which, when placed in the sun's rays, reflected them on the Roman fleet, so as to reduce it to ashes at the distance of a bow-shot." This account gained additional probability by the effect which Zonaras ascribes to the burning mirror of Proclus, by which he affirms, that the fleet of Vitellius, when besieging Byzantium, now Constantinople, was utterly consumed. But perhaps no historical testimony could have gained belief to such extraordinary facts, if similar ones had not been seen in modern times. In the Memoirs of the French Academy of Sciences for 1726, p. 172, we read of a plane mirror, of twelve inches square, reflecting the sun's rays to a concave mirror sixteen inches in diameter,

* Histoire des Mathematiques, par Montucla, vol. i. p. 338.

in the focus of which bodies were burnt at the distance of 600 paces. Speaking of this mirror, father Regnault asks, (in his Physics, vol. 3. disc. 10.), "What would be the effect of a number of plane mirrors, placed in a hollow truncated pyramid, and directing the sun's rays to the same point? Throw the focus, said he, a little farther, and you re-discover or verify the secret of Archimedes." This was actually effected by M. Buffon: in the year 1747 he read to the Academy an account of a mirror, which he had composed of an assemblage of plane mirrors, which made the sun's rays converge to a point at a great distance.

OF SPHERICAL MIRRORS, BOTH CONCAVE AND CONVEX.

A spherical mirror is nothing else than a portion of a sphere, the surface of which is polished so as to reflect the light in a regular manner. If it be the convex surface that is polished, it will form a convex spherical mirror; if it be the concave surface, it will be a concave mirror.

We must here first observe, that when a ray of light falls on any curved surface whatever, it will be reflected in the same manner as from a plane touching the point of that surface where it falls. Thus, if a tangent be drawn at the point of reflection to the surface of a spherical mirror, in the plane of the incident ray and of the centre, the ray will be reflected, making with that tangent an angle of reflection equal to the angle of incidence.

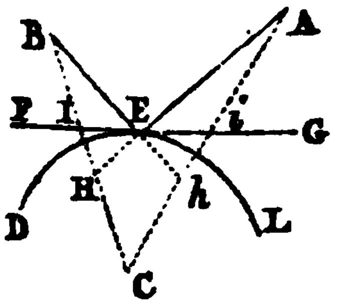
PROBLEM XXXIV.

The place of an object, and that of the eye, being given; to determine, in a spherical mirror, the point of reflection, and the place of the image.

The solution of these two problems is not so easy in regard to spherical as to plane mirrors; for when the eye and the object are at unequal distances from the mirror, the determination of the point of reflection necessarily depends on principles which require the assistance of the higher geometry; and this point cannot be assigned in the circumference of the circle without employing one of the conic sections. For this reason, we shall omit the construction, and only observe that there is one extremely simple, in which two hyperbolas between their asymptotes are employed: one of these determines the point of reflection on the convex surface, and the second the point of reflection on the concave surface.

It will be sufficient for us here to take notice of one property belonging to this point. Let B be the object (Fig. 40.), A the place of the eye, E the point of reflection from the convex surface of the spherical mirror D E L, the centre of which is C; also let F G be a tangent to the point E, in the plane of the lines B C and A C, which it meets in I and i; and let the reflected ray A E, when produced, intersect the line B C in H: the points H and I will be so situated, that we shall have the following proportion: as B C is to C H, so is B I to I H.

Fig. 40.



In like manner, if B E be produced till it meet A C in h, we shall have A C : C h : A i : i h; proportions which will be equally true in the case of reflection from a concave surface.

In regard to the place of the image, opticians have long admitted it as a principle that it is in the point H, where the reflected ray meets the perpendicular drawn from the object to the mirror. But this supposition (though it serves pretty well to shew how the images of objects are less in convex, and larger in concave, than they are in plane mirrors,) has no foundation.

A more philosophical principle advanced by Dr. Barrow is, that the eye perceives the image of the object in that point where the rays forming the small divergent

bundle, which enters the pupil of the eye, meet together. It is indeed natural to think that this divergency, as it is greater when the object is near and less when it is distant, ought to enable the eye to judge of distance.

By this principle also we are enabled to assign a pretty plausible reason for the diminution of objects in convex, and their enlargement in concave mirrors; for the convexity of the former renders the rays, which compose each bundle that enters the eye, more divergent than if they fell on a plane mirror; consequently the point where they meet in the central ray produced, is much nearer. It may even be demonstrated, that in convex mirrors it is much nearer, and in concave much farther distant, than the point H, considered by the ancients, and the greater part of the moderns, as the place of the image. In a word, it is concluded that in our convex mirrors this image will be still more contracted, and in concave ones more extended, than the ancients supposed; which will account for the apparent enlargement of objects in the latter, and their diminution in the former.

This principle however is attended with difficulties, which Dr. Barrow, the author of it, does not conceal, and to which he confesses he never saw a satisfactory answer. This induced Dr. Smith, in his Treatise on Optics, to propose another; but we shall not here enter into a discussion on this subject, as it would be too dry and abstruse for the generality of readers.

PROBLEM XXXV.

The principal properties of Spherical Mirrors, both convex and concave.

1. The first and principal property of convex mirrors is, that they represent objects less than they would be if seen in a plane mirror at the same distance. This may be demonstrated independently of the place of the image; for it can be shewn that the extreme rays of an object, however placed, which enter the eye after being reflected by a convex mirror, form a less angle, and consequently paint a less image on the retina than if they had been reflected by a plane mirror, which never changes that angle. But, the judgment which the eye in general forms respecting the magnitude of objects, depends on the magnitude of that angle, and that image, unless modified by some particular cause.

On the other hand, in concave mirrors it may be easily demonstrated, that the extreme rays of an object, in whatever manner situated, make a greater angle on arriving at the eye, than they would do if reflected from a plane mirror; consequently the appearance of the object, for the above reason, must be much greater.

2. In a convex mirror, however great be the distance of the object, its image is never farther from the surface than half the radius; so that a straight line perpendicular to the mirror, were it even infinite, would not appear to extend farther within the mirror, than the fourth part of the diameter of the circle of which it is a segment.

But in a concave mirror, the image of a line perpendicular to the mirror is always longer than the line itself; and if this line be equal to half the radius, its image will appear to be infinitely produced.

3. In convex mirrors, the appearance of a curved line concentric to the mirror, is a circular line also concentric to the mirror; but the appearance of a straight line, or plane surface, presented to the mirror, is always convex on the outside, or towards the eye.

In a concave mirror, the contrary is the case: the image of a rectilineal or plane object appears concave towards the eye.

4. A convex mirror disperses the rays; that is to say, if they fall on its surface parallel, it reflects them divergent; if they fall divergent, it reflects them still more divergent, according to circumstances.

On this property of concave spherical mirrors, is founded the use made of them for collecting the sun's rays into a small space, where their heat, multiplied in the ratio of their condensation, produces astonishing effects. But this subject deserves to be treated of separately.

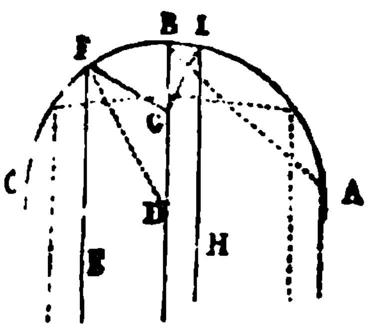
OF BURNING MIRRORS.

The properties of burning mirrors may be deduced from the following proposition:

PROBLEM XXXVI.

If a ray of light fall very near the axis of a concave spherical surface, and parallel to that axis, it will be reflected in such a manner, as to meet it at a distance from the mirror nearly equal to half the radius.

Fig. 41.



For let ABC (Fig. 41.) be the concave surface of a well polished spherical mirror, of which D is the centre, and DB the semi-diameter in the direction of the axis; if EF be a ray of light parallel to BD, it will be reflected in the direction of FG, which will intersect the diameter BD in a certain point G. But the point G will always be nearer to the surface of the mirror than to the centre. For if the radius DF be drawn, we shall have the angles DFE and DFG equal; consequently the angles DFE and GDF will also be equal; since the latter, on account of the parallel lines FE and BD, is equal to DFE: the triangle DGF then is isosceles, and GD is equal to GF; but GF is always greater than GB; whence it follows that DG also is greater than GB; the point G therefore is nearer the surface of the mirror than the centre.

But when the arc BF is exceedingly small, it is well known that the difference between GF and GB will be insensible; consequently, in this case, the point G will be nearly in the middle of the radius.

This is confirmed by trigonometry; for if the arc BF be only 5 degrees, and if we suppose the semi-diameter DB to be 100000 parts, the line BG will be 49809, which differs from half the radius but $\frac{191}{100000}$ part only, or less then $\frac{1}{523}$.* It is even found, that as long as the arc BF does not exceed 15 degrees, the distance of the point G from half the semi-diameter is scarcely a 56th part. Hence it appears, that all the rays which fall on a concave mirror, in a direction parallel to its axis, and at a distance from its summit not exceeding 15 degrees, unite at a distance from the mirror nearly equal to half the semi-diameter. Thus, the solar rays, which are sensibly parallel when they fall on this concave surface, will be there condensed, if not into one point, at least into a very small space, where they will produce a powerful heat, so much the stronger as the breadth of the mirror is greater. For this reason, the place where the rays meet is called the focus, or burning point.

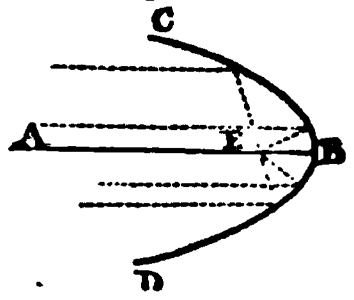
The focus of a concave mirror then is not a point: it has even a pretty sensible magnitude. Thus, for example, if a mirror be the portion of a sphere of six feet radius, and form an arc of 30 degrees, which gives a breadth of somewhat more than three feet, its focus ought to be about the 56th part of that size, or between seven and eight lines. The rays, therefore, which fall on a circle of three feet diameter, will for the most part be collected in a circle of a diameter 56 times less, and which consequently is only the 3136th part of the space or surface. It may hence be easily

* The calculation in this case is easy. For the arc BF being given, we have given also the angle BDF, as well as GFD, which is equal to it, and consequently the angle DGF, which is the supplement of their sum to two right angles. In the triangle DGF then, we have given the three angles and a side, viz. DF, which is the radius; and therefore, by a very simple trigonometrical analogy, we can find the side DG or GF, which is equal to it.

conceived what degree of heat such mirrors must produce, since the heat of boiling water is only triple that of the direct rays of the sun on a fine summer's day.

Attempts however have been made to construct mirrors to collect all the rays of the sun into one point. For this purpose it would be necessary to give to the polished surface a parabolic curve. For let C B D be a parabola (Fig. 42.), the axis of which is A B: we here suppose that the reader has some knowledge of conic sections. It is well known that in this axis there is a certain point F, so situated, that every ray, parallel to the axis of this parabola, will be reflected exactly to that point, which on this account has been called the *focus*. If the concave surface therefore of a parabolic spheroid be well polished, all the solar rays, parallel to each other, and to the axis, will be united in one point, and will produce there a heat much stronger than if the surface had been spherical.

Fig. 42.



Remarks.—I. As the focus of a spherical mirror is at the distance of a fourth part of the diameter, the impossibility of Archimedes being able, with such a mirror, to burn the Roman ships, supposing their distance to have been only 30 paces, as Kircher says he remarked when at Syracuse, may be easily conceived; for it would have been necessary that the sphere, of which his mirror was a portion, should have had a radius of 60 paces; and to construct such a sphere would be impossible. A parabolic mirror would be attended with the same inconvenience. Besides, the Romans must have been wonderfully condescending, to suffer themselves to be burnt so near, without deranging the machine. If the mathematician of Syracuse therefore burnt the Roman ships by means of the solar rays, and if Proclus, as we are told, treated in the same manner the ships of Vitellius, which were besieging Byzantium, they must have employed mirrors of another kind, and could succeed only by an invention similar to that revived by Buffon, and of which he shewed the possibility. (See Prob. 33.)

The ancients made use of concave mirrors to rekindle the vestal fires. Plutarch, in his life of Numa, says that the instruments used for this purpose, were dishes which were placed opposite to the sun, and the combustible matter placed in the centre; by which it is probable he meant the focus, conceiving that to be at the centre of the mirror's concavity.

II. We cannot here omit to mention some mirrors celebrated on account of their size, and the effects they produced; one of them was the work of Settala, a canon of Milan: it was parabolic, and, according to the account of father Schott, inflamed wood at the distance of 15 or 16 paces.

Villette, an artist and optician of Lyons, constructed three, about the year 1670, one of which was purchased by Tavernier, and presented to the king of Persia; the second was purchased by the king of Denmark, and the third by the king of France. The one last mentioned was 30 inches in diameter, and of about 3 feet focus. The rays of the sun were collected by it into the space of about half-a-guinea. It immediately set fire to the greenest wood; it fused silver and copper in a few seconds; and in one minute, more or less, vitrified brick, flint, and other vitrifiable substances.

These mirrors however were inferior to that constructed by Baron von Tchirnhausen, about 1687, and of which a description may be found in the Transactions of Leipsic for that year. This mirror consisted of a metal plate, twice as thick as the blade of a common knife; it was about 3 Leipsic ells, or 5 feet 3 inches, in breadth, and its focal distance was two of these ells, or 3 feet 6 inches: it produced the following effects:

Wood, exposed to its focus, immediately took fire; and the most violent wind was not able to extinguish it.

Water, contained in an earthen vessel, was instantly thrown into a state of ebullition; so that eggs were boiled in it in a moment, and soon after the whole water was evaporated.

Copper and silver passed into fusion in a few minutes, and slate was transformed into a kind of black glass, which, when laid hold of with a pair of pincers, could be drawn out into filaments.

Brick was fused into a kind of yellow glass; pumice stone and fragments of crucibles, which had withstood the most violent furnaces, were also vitrified, &c.

Such were the effects of the celebrated mirror of Baron von Tchirnhausen; which afterwards came into the possession of the king of France, and which was kept in the *Jardin du Roi*, exposed to the injuries of the air, which in a great measure destroyed its polish.

But metal is not the only substance of which burning mirrors have been made. We are told by Wolf, that an artist of Dresden, named Gærtner, constructed one in imitation of Tchirnhausen's mirror, composed only of wood, and which produced effects equally astonishing. But this author does not inform us in what manner Gærtner was able to give to the wood the necessary polish.

Father Zacharias Truber however seems to have supplied this deficiency, by informing us in what manner a burning mirror may be constructed with wood and leaf-gold; for nothing is necessary but to give to a piece of exceedingly dry and very hard wood the form of the segment of a concave sphere, by means of a turning machine; to cover it in a uniform manner with a mixture of pitch and wax, and then to apply bits of gold-leaf, about three or four inches in breadth. Instead of gold-leaf, small plane mirrors, he says, might be adapted to the concavity; and it will be seen with astonishment, that the effect of such a mirror is little inferior to that of a mirror made entirely of metal.

Father Zahn mentions something more singular than what is related by Wolf of the artist of Dresden, for he says that an engineer of Vienna, in the year 1699, made a mirror of paste-board, covered on the inside with straw cemented to it, which was so powerful as to fuse all metals.

Concave mirrors of a considerable diameter, and which produce the same effect as the preceding, may be procured at present at much less expense. For this advantage we are indebted to M. de Bernieres, one of the controllers general of bridges and causeways, who discovered a method of giving the figure of any curve to glass mirrors; an invention which, besides its utility in Optics, may be applied to various purposes in the arts. The concave mirrors which he constructed, were round pieces of glass bent into a spherical form; concave on one side and convex on the other, and silvered on the convex side. M. de Bernieres constructed one for the king of France, of 3 feet 6 inches in diameter, which was presented to his majesty in 1757. Forged iron exposed to its focus was fused in two seconds: silver ran in such a manner that when dropped into water it extended itself in the form of a spider's web; flint became vitrified, &c.

These mirrors have considerable advantage over those of metal. Their reflection from the posterior surface, notwithstanding the loss of rays, occasioned by their passage through the first surface, is still more lively than that from the best polished metallic surface; besides, they are not subject, like metallic mirrors, to lose their polish by the contact of the air, always charged with vapours which corrode metal, but which make no impression on glass: in a word, nothing is necessary but to preserve them from moisture, which destroys the silvering.

PROBLEM XXXVII.

Some properties of concave mirrors, in regard to vision, or the formation of images.

I. If an object be placed between a concave mirror and its focus, its image is

seen within the mirror, and more magnified the nearer the object is to the focus; so that when the object is in the focus itself, it seems to occupy the whole capacity of the mirror, and nothing is seen distinct.

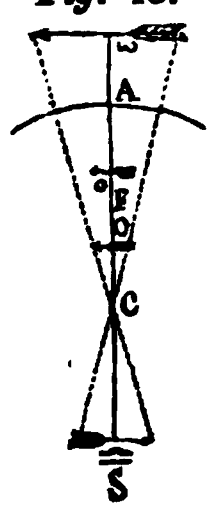
If the object, placed in the focus, be a luminous body, the rays which proceed from it, after being reflected by the mirror, proceed parallel to each other, so that they form a cylinder of light, extended to a very great distance, and almost without diminution. This column of light, if the observer stands on one side, will be easily perceived when it is dark; and at the distance of more than a hundred paces from the mirror, if a book be held before this light, it may be read.

II. If the object be placed between the focus and the centre, and if the eye be either beyond the centre, or between the centre and the focus, it cannot be distinctly perceived, as the rays reflected by the mirror are convergent. But if the object be strongly illuminated, or if it be a luminous body itself, such as a candle, by the union of its rays there will be formed, beyond the centre, an image in an inverted situation, which will be painted on a piece of paper or cloth at the proper distance, or which, to an eye placed beyond it, will appear suspended in the air.

III. The case will be nearly the same when the object is beyond the centre, in regard to the mirror; an inverted image of the object will be painted then between the focus and the centre; and this image will approach the centre in proportion as the object itself approaches it; or will approach the focus as the object removes from it.

In regard to the place where the image will be painted in both these cases, it may be found by the following rule.

Fig. 43.



Let A C S (Fig. 43.) be the axis of the mirror, indefinitely produced; F the focus, C the centre, and O the place of the object, between the centre and the focus. If F ω be taken a third proportional to F O and F C, it will represent the distance at which the image of the point placed in O will be painted.

If the object be in ω, by employing the same proportion, with the proper changes, that is by making F O a third proportional to F ω and F C, as in *o*, the image of it will be found in O.

In the last place, if the object be between the focus and the glass, the place where it will be observed within the mirror, may be found by making F ω to F A, as F A to F *o*.

Remarks.—1. This property which concave mirrors have, of forming between the centre and the focus, or beyond the centre, an image of the objects presented to them, is one of those which excite the greatest surprise in persons not acquainted with this theory. For if a man advance towards a large concave mirror, presenting a sword to it, then he comes to the proper distance, he will see a sword blade, with the point turned towards him, dart itself from the mirror; if he retires, the image of the blade will retire; if he advances in such a manner that the point shall be between the centre and the focus, the image of the sword will cross the real sword as if two people were engaged in fighting.

2. If, instead of a sword blade, the hand be presented at a certain distance, you will see a hand formed in the air in an inverted situation; which will approach the real hand, when the latter approaches the centre, so that they will seem to meet each other.

3. If you place yourself a little beyond the centre of the mirror, and then look directly into it, you will see beyond the centre the image of your face inverted. If you then continue to approach, this phantastic image will approach also, so that you can kiss it.

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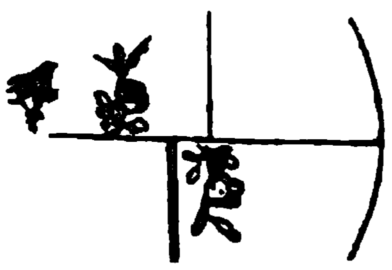


4. If a nosegay be suspended in an inverted situation (Fig. 44.), between the centre and the focus, a little below the axis, and if it be concealed from the view of the spectator, by means of a piece of black pasteboard, an upright image of the nosegay will be formed above the pasteboard, and will excite the greater astonishment, as the object which produces it is not seen; for this reason those not acquainted with the deception will take it for a real object, and attempt to touch it.*

5. If a concave mirror be placed at the end of a hall, at an inclination nearly equal to 45°, and if a print or drawing be laid on a table before the mirror, with the bottom part turned towards it, the figures in the print or drawing will be seen greatly magnified; and if a proper arrangement be made so as to favour the illusion, that is if the mirror be concealed, and only a small hole left for looking through, you will imagine that you see the objects themselves.

On this principle are constructed what are called *Optical boxes*, which are now very common: the method of constructing them will be found in the following problem.

PROBLEM XXXVIII.

To construct an optical box or chamber, in which objects are seen much larger than the box itself.

Provide a square box, of a size proper to contain the concave mirror you intend to employ; that is to say, let each side be a little less than the focal distance of the mirror; and cover the top of it with transparent parchment, or white silk, or glass made smooth, but not polished.

Apply the mirror to one of the vertical sides of the box, and on the opposite side place a coloured print or drawing, representing a landscape, or seaport, or buildings, &c. The print ought to be introduced into the box by means of a slit, so that it can be drawn out, and another substituted in its place at pleasure.

At the top of the side opposite to the mirror, a round hole or aperture must be made, for the purpose of looking through; and if the eye be applied to this hole, the objects represented in the print will be seen very much magnified: those who look at them will think they really behold buildings, trees, &c.

We have seen some of these machines, which by their construction, the size of the mirror, and the correctness of the colouring, exhibited a spectacle highly agreeable and amusing.

OF LENTICULAR GLASSES, OR LENSES.

A lens is a bit of glass having a spherical form on both sides, or at least on one side. Some of them are convex on the one side and plane on the other; and others are convex on both sides: some are concave on one side, or on both; and others are convex on one side and concave on the other. Those convex on both sides, as they resemble a lentil, are in general distinguished by the name of lenticular glasses, or lenses.

The uses to which these glasses are applied, are well known. Those which are convex magnify the appearance of objects, and aid the sight of old people; on the other hand, the concave glasses diminish objects, and assist those who are short-sighted. The former collect the rays of the sun around one point, called the *focus;* and when of a considerable size, produce heat and combustion The concave glasses, on the contrary, disperse the rays of the sun. Both kinds are employed in the construction of telescopes and microscopes.

* Curious spectres and appearances, formed in this manner, have of late years been exhibited as shows to spectators in London.

PROBLEM XXXIX.

To find the focus of a Glass Globe.

As glass globes supply, on many occasions, the place of lenses, it is proper that we should here say a few words respecting their focal distance. The method of determining it is as follows.

Let B C D (Fig. 45.) be a glass sphere, the centre of which is F, and C D a diameter to which the incident ray A B is parallel. This ray, when it meets with the surface of the sphere in B, will not continue its course in a straight line, as would be the case if it did not enter a new medium, but will approach the perpendicular drawn from the centre F to B the point of incidence. Consequently, when it issues from the sphere at the point I, it would meet the diameter in a point E, if it did not deviate from the perpendicular F I, which makes it take the direction I O, and proceed to the point O, the focus required.

Fig. 45.

I

To determine the focus O, first find the point of meeting E, which may be easily done by observing that in the triangle F B E, the side F B is to F E as the sine of the angle F E B is to the sine of the angle F B E; or, on account of the smallness of these angles, as the angle F E B, or its equal G B E, is to the angle F B E; for we here suppose the incident ray to be very near the diameter C D; consequently the angle A B H is very small, as well as its equal F B G; and angles extremely small have the same ratio as their sines. But, by the laws of refraction, when a ray passes from air into glass, the ratio of the angle of incidence A B H, or G B F, to the angle of refraction F B I, if the angles be very small, is as 3 to 2, and therefore the angle F B E is nearly the double of E B G: it thence follows that the side F E, of the triangle F B E, is nearly the double of F B, or equal to twice the radius; consequently D E is equal to the radius.

To find the point O, where the ray, when it issues from the sphere, and deviates from the perpendicular, ought to meet the line D E, the like reasoning may be employed. In the triangle I O E, the side I O is to O E nearly as the angle I E O, or its equal I F E, is to the angle O I E. Now these two angles are equal; for the angle I F D is the one third of the angle of incidence F B G or A B H; but, by the law of refraction, the angle O I E is nearly the half of the angle of incidence E I K, or of its equal F I B, which is $\frac{2}{3}$ of the angle F B G: like the preceding it is therefore the third of F B G or H B A, and consequently the angles O I E and O E I are equal; whence it follows that O E is equal to O I, which is itself equal to D O, on account of their very great proximity. Therefore D O, or the distance of the focus of a glass globe from the surface, is equal to half the radius, or the fourth part of the diameter. Q. E. D.

PROBLEM XL.

To find the focus of any lens.

The same reasoning, as that employed to determine the course of a ray passing through a glass sphere, might be employed in the present case. But for the sake of brevity, we shall only give a general rule, demonstrated by opticians, which includes all the cases possible in regard to lenses, whatever combinations may be formed of convexities and concavities. We shall then shew the application of it to a few of the principal cases. It is as follows:

As the sum of the semi-diameters of the two convexities, is to one of them, so is the diameter of the other, to the focal distance.

In the use of this rule, one thing in particular is to be observed. When one of the

faces of the glass is plane, the radius of its sphericity must be considered as infinite; and when concave, the radius of the sphere, of which this concavity forms a part, must be considered as negative. This will be easily understood by those who are in the least familiar with algebra.

Case i.—*When the lens is equally convex on both sides.*

Let the radius of the convexity of each of the faces be, for example, equal to 12 inches. By the general rule we shall have this proportion: As the sum of the radii, or 24 inches, is to one of them, or 12 inches, so is the diameter of the other, or 24 inches, to a fourth term, which will be 12 inches, the focal distance. Hence it appears that a lens equally convex on both sides unites the solar rays, or in general rays parallel to its axis, at the distance of the radius of one of the two sphericities.

Case ii.—*When the lens is unequally convex on both sides.*

If the radii of the convexities be 12 and 24, for instance, the following proportion must be employed: As $12+24$, or 36, is to 12, the radius of one of the convexities, so is 48, the diameter of the other, to 16; or as $12+24$, or 36, is to 24, the radius of one of the convexities, so is 24, the diameter of the other, to 16: the distance of the focus therefore will be 16 inches.

Case iii.—*When the lens has one side plane.*

If the sphericity on the one side be as in the preceding case, we must say, by applying the general rule: As the sum of the radii of the two sphericities, viz. 12, an infinite quantity, is to one of them, or the infinite quantity, so is 24, the diameter of the other convexity, to a fourth term, which will be 24; for the two first terms are equal, for an infinite quantity increased or diminished by a finite quantity, is always the same: the two last terms therefore are equal; and it hence follows, that a plano-convex glass has its focus at the distance of the diameter from its convexity.

Case iv.—*When the lens is convex on the one side, and concave on the other.*

Let the radius of the convexity be still 12, and that of the concavity 27. As a concavity is a negative convexity, this number 27 must be taken with the sign $-$ prefixed. We shall therefore have this proportion:

As 12 inches -27, or -15 inches, is to the radius of the concavity -27 (or as 15 is to 27), so is 24 inches, the diameter of the convexity, to $43\frac{1}{5}$. This is the focal distance of the lens, and is positive or real; that is to say, the rays falling parallel to the axis, will really be united beyond the glass. The concavity indeed having a greater diameter than the convexity, this must cause the rays to diverge less than the convexity causes them to converge. But if the concavity be of a less diameter than the convexity, the rays, instead of converging when they issue from the glass, will be divergent, and the focus will be before the glass: in this case it is called *virtual*. Thus, if the radius of the concavity be 12, and that of the convexity 27, we shall have, by the general rule: As $27-12$, or 15, is to 27, so is -24 to $-43\frac{1}{5}$. The last term being negative, it indicates that the focus is before the glass, and that the rays will issue from it divergent, as if they came from that point.

Case v.—*When the lens is concave on both sides.*

If the radii of the two concavities be 12 and 27 inches, we shall have this proportion: As $-12-27$ is to -27, or as 39 is to 27, so is -24 to $-16\frac{8}{13}$. The last term being negative, it shews that the focus is only virtual, and that the rays, when they issue from the glass, will proceed diverging, as if they came from a point situated at the distance of $16\frac{8}{13}$ inches before the glass.

CASE VI.—*When the lens is concave on one side, and plane on the other.*

If the radius of the concavity be still 12, the above rule will give the following proportion: As — 12 + an infinite quantity, is to an infinite quantity, so is — 24 to — 24; for an infinite quantity, when it is diminished by a finite quantity, remains still the same. Thus it is seen that in this case the virtual focus of a plano-concave glass, or the point where the rays after their refraction seem to diverge, is at a distance equal to the diameter of the concavity, as the point to which they converge is in the case of the plano-convex glass.

These are all the cases that can occur in regard to lenses: for that where the two concavities might be supposed equal is comprehended in the fifth.

Remark.—In all these calculations, we have supposed the thickness of the glass to be of no consequence in regard to the diameter of the sphericity, which is the most common case; but if the thickness of glass were taken into consideration, the determinations would be different.

OF BURNING GLASSES.

Lenticular glasses furnish a third method of solving the problem, already solved by means of mirrors, viz. to unite the rays of the sun in such a manner as to produce fire and inflammation: for a glass of a few inches diameter will produce a heat sufficiently strong to set fire to tinder, linen, black or grey paper, &c.

The ancients were acquainted with this property in glass globes, and they even sometimes employed them for the above purpose. It was probably by means of a glass globe that the vestal fire was kindled. Some indeed have endeavoured to prove that they produced this effect by lenses: but De la Hire has shewn that this idea is entirely void of foundation, and that the burning glasses of the ancients were only glass globes, and consequently incapable of producing a very remarkable effect.

Baron von Tchirnhausen, who constructed the celebrated mirror already mentioned, made also a burning glass, the largest that had ever been seen. This mathematician, being near the Saxon glass manufactories, was enabled, about the year 1696, to procure plates of glass sufficiently thick and broad, to be converted into lenses several feet in diameter. One of them, of this size, inflamed combustible substances at the distance of 12 feet. Its focus at this distance was about 1½ inch in diameter. But when it was required to make it produce its greatest effects, the focus was diminished by means of a second lens, placed parallel to the former, and at the distance of four feet. In this manner the diameter of the focus was reduced to 8 lines, and it then fused metals, vitrified flint, tiles and slate, earthen ware, &c.; in a word, it produced the same effects as the burning mirrors of which we have already spoken.

Some years ago a lens, which one might have taken for that of Tchirnhausen, was exhibited at Paris. The glass of which it consisted was radiated and yellowish; and the person to whom it belonged asked no less for it than £500 sterling.

For the means of obtaining, at a less expence, glasses capable of producing the same effects, we are indebted to M. de Bernieres, of whom we have already spoken. By his invention for bending glass, two round plates are bent into a spherical form, and being then applied to each other the interval between them is filled with distilled water, or spirit of wine. These glasses, or rather water lenses, have their focus a little farther distant, and *ceteris paribus* ought to produce a somewhat less effect; but the thinness of the glass and the transparency of the water occasion less loss in the rays than in lens of several inches in thickness. In a word, it is far easier to procure a lens of this construction than solid ones like that of Tchirnhausen. M. de Trudaine,

some years ago, caused to be constructed, by M. de Bernieres, one of these water lenses 4 feet in diameter, with which some philosophical experiments have been already made in regard to the calcination of metals and other substances. The heat produced by this instrument is much superior to that of all the burning glasses and mirrors hitherto known, and even to that of all furnaces. We have reason to expect from it new discoveries in chemistry. We shall here add that with water lenses, of a much smaller size, M. de Bernieres has fused metals, vitrifiable stones, &c.

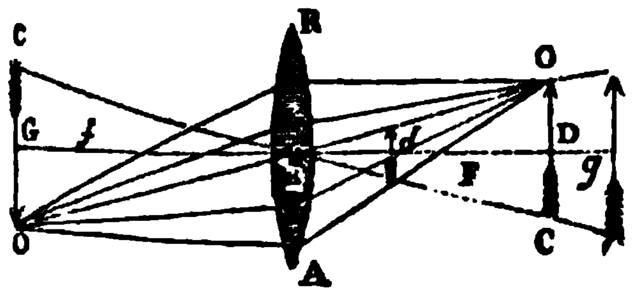
PROBLEM XLI.

Of some other properties of Lenticular Glasses.

1. If an object be exceedingly remote, so that there is no proportion between its distance and the focal distance of the glass, there is painted in the focus of the lens an image of the object in an inverted situation. This experiment serves as the basis of the construction of the camera obscura. In this manner the rays of the sun, or of the moon, unite in the focus of a glass lens, and form a small circle, which is nothing else than the image of the sun or moon, as may be easily perceived.

2. In proportion as the object approaches the glass, the image formed by the rays proceeding from the object, recedes from the glass; so that when the distance of the object is double that of the focus, the image is painted exactly at the double of that distance; if the object continues to approach, the image recedes more and more; and when the object is in the focus, no image is formed; for it is at an infinite distance that it is supposed to form itself. In this case therefore the rays which fall on the glass, diverging from each point of the object, are refracted in such a manner as to proceed parallel to each other.

Fig. 46.



The method of determining, in general, the distance from the lens at which the image of the object is formed, is as follows. Let O C be the object (Fig. 46.), D E its distance from the glass, and E F the focal distance of the glass; if we make use of this proportion: as F D is to F E, so is E F to E G, taking E G on the other side of the glass when E D is greater than E F, the point G will be that of the axis to which the point D of the object, situated in the axis, will correspond.

Hence it may be easily seen, that when the distance of the object from the focus is equal to nothing, the distance E G must be infinite, that is to say there can be no image.

It must also be observed, that when E F is greater than E D, or when the object is between the glass and the focus, the distance E G must be taken in a contrary direction, or on this side of the glass, as E g; which indicates that the rays proceeding from the object, instead of forming an image beyond the glass, diverge as if they proceeded from an object placed at g.

OF TELESCOPES, BOTH REFRACTING AND REFLECTING.

Of all optical inventions, none is equal to that of the telescope: for without mentioning the numerous advantages derived from the common use of this wonderful instrument, it is to it we are indebted for the most interesting discoveries in astronomy. It is by its means that the human mind has been able to soar to those regions otherwise inaccessible to man, and to examine the principal facts which serve as the foundation of our knowledge respecting the heavenly bodies.

The first telescope was constructed in Holland, about the year 1609; but there

is much uncertainty in regard to the name of the inventor, and the means he employed in the formation of his instrument. A dissertation on this subject may be seen in Montucla's History of the Mathematics. We shall confine ourselves at present to a description of the different kinds of telescopes, both refracting and reflecting, and of the manner in which they produce their effect.

Of Refracting Telescopes.

1st. The first kind of telescope, and that most commonly used, is composed of a convex glass, called the *object glass*, because it is that nearest the objects, and a concave one called the *eye-glass*, because it is nearest the eye. These glasses must be disposed in such a manner, that the posterior focus of the object glass shall coincide with the concave glass. By means of this disposition, the object appears magnified in the ratio of the focal distance of the object glass, to that of the eye-glass. Thus, if the focal distance of the object glass be ten inches, and that of the eye-glass one inch, the instrument will be nine inches in length, and will magnify objects ten times.

This kind of telescope is called the *Batavian*, on account of the place where it was invented. It is known also by the name of the Galilean, because Galileo, having heard of it, constructed one of the same kind, and by its means was enabled to make those discoveries in the heavens which have immortalized his name. At present, very short telescopes only are made according to this principle; because they are attended with one defect, which is, that when of a considerable length they have a very confined field.

2d. The second kind of telescope is called the *astronomical*, because employed chiefly by astronomers. It is composed of two convex glasses, disposed in such a manner, that the posterior focus of the object glass and the anterior focus of the eye-glass coincide together, or very nearly so. The eye must be applied to a small aperture, at a distance from the eye-glass, equal to that of its focus. It will then have a field of large extent, and it will shew the objects inverted, and magnified in the ratio of the focal distances of the object glass and eye-glass. If we take, by way of example, the proportions already employed, the astronomical telescope will be 12 inches in length, and will magnify ten times.

Telescopes of very great length may be constructed according to this combination. It is common for astronomers to have them of 12, 15, 20, and 30 feet. Huygens constructed one for himself of 123 feet, and Hevelius employed one of 140. But the inconvenience which attends the use of such long telescopes, in consequence of their weight, and the bending of the tubes, has made them be laid aside, and another instrument more commodious has been substitued in their stead. Hartsoecker made an object glass of 600 feet focus, which would have produced an extraordinary effect had it been possible to use it.

3d. The inconvenience of the Batavian telescopes, which suffer only a small quantity of objects to be seen at once, and that of the astronomical telescope, which represents them inverted, have induced opticians to devise a third arrangement of glasses, all convex, which represents the objects upright, gives the same field as the astronomical telescope, and which is therefore proper for terrestrial objects: on this account it is called the terrestrial telescope. It consists of a convex object glass, and three equal eye-glasses. The posterior focus of the object glass generally coincides with the anterior one of the first eye-glass; the posterior focus of the latter coincides also with the anterior focus of the second, and in like manner the posterior focus of the second with the anterior one of the third, at the posterior focus of which the eye ought to be placed. This instrument always magnifies in the ratio of the focal distances of the object glass and one of the eye-glasses. But

it may be readily seen that the length is increased four times the focal distance of the eye-glass.

4th. The image of objects might be made to appear upright by employing only two eye-glasses: for this purpose it would be necessary that the first should be at a distance from the focus of the object glass equal to twice its own focal distance; and that the anterior focus of the second should be at twice that distance. Such is the terrestrial telescope with three glasses; but experience has shewn that, by this arrangement, the objects are somewhat deformed, for which reason it is no longer used.

5th. Telescopes with five glasses have also been proposed, in order to bend the rays gradually, as we may say, and to obviate the inconveniences of the too strong refraction, which suddenly takes place at the first eye-glass; and also to increase the field of vision. We have even heard of some telescopes of this kind which were attended with great success; but we do not find that this combination of glasses has been adopted.

6th. Some years ago a new kind of telescope was invented, under the name of the *achromatic*, because it is free from those faults occasioned by the different refrangibility of light, which in other telescopes produces colours and indistinctness. The only difference between this and other telescopes is, that the object glass, instead of being formed of one lens, is composed of two or three made of different kinds of glass, which have been found by experience to disperse unequally the different coloured rays of which light is composed. One of these glasses is of crown-glass, and the other of flint-glass. An object glass of this kind, constructed according to certain dimensions determined by geometricians, produces in its focus an image far more distinct than the common ones; on which account much smaller eye-glasses may be employed without affecting the distinctness, as is confirmed by experience. These telescopes are called also *Dollond's telescopes*, after the name of the English artist who invented them. By the above means, the English opticians construct telescopes of a moderate length, which are equal to others of a far greater size; and small ones, not much longer than opera-glasses, with which the satellites of Jupiter may be seen, are sold under Dollond's name at Paris. M. Antheaume, according to the dimensions given by M. Clairault, made, in that capital, an achromatic telescope of 7 feet focal distance, which, when compared with a common one of 30 or 35 feet, was found to produce the same effect.

This invention gives us reason to hope that discoveries will be made in the heavens, which a few years ago would have appeared altogether impossible. It is not improbable even that astronomers will be able to discover in the moon habitations and animals, spots in Saturn and Mercury, and the satellite of Venus, so often seen and so often lost.

[Since the publication of the former editions of this work, achromatic telescopes, greatly exceeding in size any that had before been contemplated, have been constructed in Paris, and at Munich. M. Guinand, of Neufchatel, succeeded in overcoming, to a certain extent, the difficulties which had been experienced in the manufacture of large and homogeneous discs of flint glass; and some of the Parisian glass-makers are understood to be in possession of processes by which the same object may be effected. This has given a decided impulse to the efforts of opticians, and the result has been, that at the observatories of Cambridge, Munich, Dorpat, and other places, astronomical telescopes, of more than 12 inches aperture and 20 feet focal length, have been mounted, and are in action; and equatorial motion being communicated by clock work, they are as manageable as instruments of a very moderate size. With the Dorpat telescope, Professor Struve has already rendered important services

to astronomical science. A full description of this instrument may be seen in the Memoirs of the Royal Astronomical Society, vol. ii. part 1.]

To give an accurate idea of the manner in which telescopes magnify the appearance of objects, we shall take, by way of example, that called the astronomical telescope, as being the simplest. If it be recollected that a convex lens produces in its focus an inverted image of objects which are at a very great distance, it will not be difficult to conceive, that the object glass of this telescope will form behind it, at its focal distance, an inverted image of any object towards which it is directed. But, by the construction of the instrument, this image is in the anterior focus of the eye-glass, to which the eye is applied; consequently the eye will perceive it distinctly; for it is well known, that when an object is placed in the focus of a lens, or a little on this side of it, it will be seen distinctly through the glass, and in the same direction. The image of the object, which here supplies its place, being then inverted, the eye-glass, through which it is viewed, will not make it appear upright, and consequently the object will be seen inverted.

In regard to the size, it is demonstrated, that the angle under which the image is seen is to that under which the object is seen from the same place, as the focal distance of the object glass is to that of the eye-glass: hence the magnified appearance of the object.

In terrestrial telescopes, the two first eye-glasses only invert the image; and this telescope therefore must represent objects upright. But having said enough respecting refracting telescopes, we shall now proceed to reflecting ones.

Of Reflecting Telescopes.

Those who are well acquainted with the manner in which objects are represented by common telescopes, will readily conceive that the same effect may be produced by reflection; for a concave mirror, like a lens, paints in its focus an image of distant objects. If means then are found to reflect the image on one side, or backwards, in such a manner as to be made to fall in the focus of a convex glass, and to view it through this glass, we shall have a reflecting telescope. It need therefore excite no surprise that before Newton, and in the time of Descartes and Mersenne, telescopes on this principle were proposed.

Newton was led to this invention while endeavouring to discover some method of remedying the want of distinctness in the images formed by glasses; a fault which arises from the different refrangibility of the rays of light that are decomposed. Every ray, of whatever colour, being reflected under an angle equal to the angle of incidence, the image is much more distinct, and better terminated in all its parts, as may be easily proved by means of a concave mirror. On this account he was able to apply an eye-glass much smaller, which would produce a greater magnifying power; and this reasoning was confirmed by experience.

Newton never constructed telescopes of more than fifteen inches in length. According to his method, the mirror was placed in the bottom of the tube, and reflected the image of the object towards its aperture: near this aperture was placed a plane mirror, that is to say, the base of a small isosceles rectangular prism, silvered at the back, and inclined at an angle of 45 degrees. This small mirror reflected the image towards the side of the tube, where there was a hole, into which was fitted a lens of a very short focal distance, to serve as the eye-glass. The object then was viewed from the side, a method, in many cases, exceedingly convenient. Mr. Hadley, a fellow of the Royal Society, constructed, in the year 1723, a telescope of this kind, 5 feet in length, which was found to produce the same effect as the telescope of 123 feet, presented to the Royal Society by Huygens.

The reflecting telescopes used at present are constructed in a manner somewhat different. The concave mirror, at the bottom of the tube, has a round hole in the middle, and towards the other end is a mirror, sometimes plane, turned directly towards the other one, which, receiving the image near the middle of the focal distance, reflects it towards the hole in the other mirror. Against this hole is applied a lens of a short focal distance, which serves as an eye-glass, or for viewing terrestrial objects, in order that they may appear upright; and three eye-glasses are used, arranged in the same manner as in terrestrial telescopes.

A telescope however may be made to magnify much more by the following construction. The large mirror, as in all the others, is placed at the bottom, and has a hole in the centre, before which the eye-glass is applied. At the other end of the tube is another concave mirror, of a less focal distance than the former, and so disposed that the image reflected by the former is painted very near its focus, but at a little farther distance than the focus from its surface. This produces another image beyond the centre, which is greater as the first one is nearer the focus: this image is formed very near the hole in the centre of the large mirror, opposite to which the eye-glass is in general placed.

This kind of reflecting telescope is called the Gregorian, because proposed by Mr. James Gregory, even before Newton conceived the idea of his; and it this kind which is at present most in use.

There is also the telescope of Cassegrain, who employs a convex mirror to magnify the image formed by the first concave one. Dr. Smith thought it attended with so many advantages, that he was induced to analyse it in his Treatise on Optics. Cassegrain was a French artist, who proposed this method of construction about the year 1665, and nearly at the same time that Gregory proposed his. It is certain that the length of the telescope is by these means considerably diminished.

The English, for a long time, have enjoyed a superiority in works of this kind. The art of casting and polishing the metallic mirrors, necessary for these instruments, is indeed exceedingly difficult. M. Passement, a celebrated French artist, and the brothers Paris and Gonichon, opticians at Paris, are the first who attempted to vie with them in this branch of manufacture; and both have constructed a great number of reflecting telescopes, some of which are 5 or 6 feet in length. Among the English, no artist distinguished himself more in this respect than Short, though his telescopes were not of great length: besides some of 4, 5, and 6 feet, he made one of 12, which belonged some years ago to the physician of Lord Macclesfield. By applying a lens of the shortest focal distance which it could bear, it magnified about 1200 times. The satellites of Jupiter therefore, seen through this telescope, are said to have had a sensible apparent diameter. But this telescope, as we have heard, is no longer in existence, the large mirror being lost.

The longest of all the reflecting telescopes ever yet constructed, if we except that lately made by Herschel, is one in the king's collection of philosophical and optical instruments at La Meute; it is the work of Dom Noel, a Benedictine, the keeper of the collection, and was begun several years before he was placed at the head of that establishment, where he finished it, and where the curious were allowed to see it, and to contemplate with it the heavens. It is mounted on a kind of moveable pedestal, and, notwithstanding its enormous weight, can be moved in every direction, along with the observer, by a very simple mechanism. But what would be most interesting, is to ascertain the degree of its power, and whether it produces an effect proportioned to its length, or at least considerably greater than the largest and best reflecting telescopes constructed before; for we know that the effects of these instruments, supposing the same excellence in the workmanship, do not increase in proportion to the length.

Huygens' telescope of 123 feet, which he presented to the Royal Society, did not

produce an effect quadruple that of a good telescope of 30 feet; and the case must be the same in regard to reflecting telescopes, where the difficulties of the labour are still greater; so that if a telescope of 24 feet produced one half more effect than another of 12, or only the double of one of 6 feet, it ought, in our opinion, to be considered as a good instrument.

We have heard that Dom Noel was desirous of making this comparison, and the method he proposed was rational. We have long considered it as the only one proper for comparing such instruments. It is to place at the distance of several hundred feet printed characters of every size, composingbarbarous words without any meaning, in order that those who make the experiment may not be assisted by one or two words to guess the rest. The telescope, by means of which the smallest characters are read, will undoubtedly be the best. We have seen stuck up, on the dome of the Hospital of Invalids, pieces of paper of this kind, which Dom Noel had placed there for the purpose of making this comparison; but unfortunately such instruments cannot be brought to one place. Printed characters, such as above described, might therefore be fixed up at a convenient distance from each without removing the instruments, and persons appointed for the purpose ought to go to the different observatories, at times when the weather is exactly similar, and examine what characters can be read by each telescope. By this method a positive answer to the above question would be obtained.

But the largest and the most powerful of all the reflecting telescopes has been lately made by Dr. Herschel, under the auspices of the British monarch; a consequence of which was the discovery of his new primary planet, and of many additional satellites. After a long perseverance in a series of improvements of reflecting telescopes, of the Newtonian form, making them successively larger and more accurate, this gentleman came at length to make one of the amazing size of forty feet in length. This telescope was begun in the year 1785, and completed in 1789. The length of the sheet-iron tube is 40 feet, and diameter 4 feet 10 inches. The great mirror is 49½ inches in diameter, 3½ inches thick, and weighs 2118 lb. The whole is managed by a large apparatus of machinery, of wheels and pulleys, by means of which it is easily moved in any direction, vertically and sideways. The observer looks in at the outer or object end; from whence proceeds a pipe to a small house near the instrument for conveying information by sound, backward and forward to an assistant, who thus under cover sets down the time and observations made by the principal observer. The consequences of this, and the other powerful machines of this gentleman, have been new discoveries in the heavens of the most important nature. *

PROBLEM XLII.

Method of constructing a telescope, by means of which an object may be seen, even when then instrument appears to be directed towards another.

As it is not polite to gaze at any one, a sort of glass has been invented in England, by means of which, when the person who uses it seems to be viewing one object, he is really looking at another. The construction of this instrument is very simple.

Adapt to the end of an opera glass (Fig. 47.), the object glass of which in this case becomes useless, a tube with a lateral aperture as large as the diameter of the

* This celebrated telescope, by which its maker, Sir William Herschel, LL.D., made discoveries of the greatest importance in the heavens, has long been in a state unfit for use: the surface of the great mirror having become covered with a crystallised crust. Sir J. Herschel, the distinguished son of Sir William, uses a twenty-feet reflector, of his own construction; and he has found its powers quite sufficient for the most delicate observations of astronomical phenomena.

The late Mr. Ramage, an Aberdeen tradesman, made a considerable number of excellent mirrors of large dimensions, some of which were fitted up as telescopes, and mounted in a manner similar to the great one of Sir William Herschel, with some simplifications. One was for several years on view at the Greenwich Observatory. Mr. Ramage had one fitted up for himself; and we believe Sir John Ross, the northern navigator, had one set up at his house at Stranraer, with which he occasionally made observations on the eclipses of Jupiter's satellites.

Fig. 47.

tube will admit, and opposite to this aperture place a small mirror inclined to the axis of the tube at an angle of 45 degrees, and having its reflecting surface turned towards the object glass. It is evident that when this telescope is directed straight forwards, you will see only some of the lateral objects, viz. those situated near the line drawn from the eye in the direction of the axis of the telescope, and reflected by the mirror. These objects will appear upright, but transposed from right to left. To conceal the artifice better, the fore part of the telescope may be furnished with a plane glass, which will have the appearance of an object glass placed in the usual manner.

This instrument, which is not very common in France, is exceedingly convenient for gratifying one's curiosity in the playhouse, and other places of public amusement, especially if the mirror be so fixed as to be susceptible of being more or less inclined; for those who use it, while they seem to look at the stage and the performers, may without affectation, and without violating the rules of politeness, examine an interesting figure in the boxes.

We must however observe that the first idea of this instrument is not very new; for the celebrated Hevelius, who it seems was afraid of being shot, proposed many years ago his *polemoscope*, or telescope for viewing under cover, and without danger, warlike operations, and those in particular which take place during the time of a siege. It consisted of a tube bent in such a manner as to form two elbows, in each of which was a plane mirror inclined at an angle of 45 degrees. The first part of the tube was made to rest on the parapet towards the enemy; the image reflected by the first inclined mirror passed through the tube in a perpendicular direction, and meeting with the second mirror was reflected horizontally towards the eye glass, where the eye was applied: by these means a person behind a strong parapet could see what the enemy were doing without the walls. The chief thing to be apprehended in regard to this instrument was, that the object glass might be broken by a ball; but this was certainly a trifling misfortune, and not very likely to happen.

OF MICROSCOPES.

What the telescope has performed in the philosophy of the heavenly bodies, the microscope has done in regard to that of the terrestrial: for by the assistance of the latter we have been able to discover an order of beings which would otherwise have escaped our notice; to examine the texture of the smallest of the productions of nature, and to observe phenomena which take place only among the most minute parts of matter. Nothing can be more curious than the facts which have been ascertained by the assistance of the microscope: but in this part of science much still remains to be done.

There are two sorts of microscopes—simple and compound: we shall speak of both, and begin with the former.

PROBLEM XLIII.

Method of constructing a Simple Microscope.

I. Every convex lens of a short focal distance is a microscope; for it is shewn that a lens magnifies in the ratio of the focal distance to the least distance at which the object can be placed to be distinctly seen; which, in regard to most men who are not short-sighted, is about 8 inches. Thus a lens, the focal distance of which is 6 lines, will magnify the dimensions of the object 16 times; if its focal distance be only one line, it will magnify 96 times.

II. It is difficult to construct a lens of so short a focus; as it is necessary that the radius of each of its convexities should be only a line; for this reason small

glass globes, fused at an enameller's lamp, or the flame of a taper, are employed in their stead. The method by which this is done, is as follows.

Break off a piece of very pure transparent glass, either by means of an instrument made for that purpose, or the wards of a key; then take up one of these fragments by applying to it the point of a needle a little moistened with saliva, which will make it adhere, and present it to the blue flame of a taper, which must be kept somewhat inclined, that the fragment of glass may not fall upon the wax. As soon almost as it is held to the flame, it will be fused into a round globule, and drop down: a piece of paper therefore, with a turned up border, must be placed below, in order to receive it.

It is here to be observed that there are some kinds of glass which it is difficult to fuse: in this case it will be necessary to employ another kind.

Of these globules select the brightest and roundest; then take a plate of copper, 5 or 6 inches in length, and about 6 lines in breadth, and having folded it double, make a hole in it somewhat less in diameter than the globule, and raise up the edges. If you then fix one of these globules in this hole, between the two plates, and bind them firmly together, you will have a single microscope.

As it is easy to obtain globules of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of a line in diameter, and as the focus of a globule is at the distance of a quarter of its diameter without it, we are enabled by this process to magnify objects in a very high degree; for if the diameter of the globule be only $\frac{1}{2}$ line, by employing this proportion: as $\frac{5}{4}$ of half a line, or $\frac{5}{8}$, are to 96 lines, so is 1 to a fourth term, we shall have as fourth term the number 153, which will express the increase of the diameter of the object. The object therefore, in regard to surface, will be magnified 23409 times, and in regard to solidity 3581677 times.

The celebrated Lewenhoeck, so well known on account of his microscopical observations, never employed microscopes of any other kind. It is however certain that they are attended with many inconveniences, and can be used only for objects which are transparent, or at least semi-transparent, as it may be readily conceived that it is not possible to illuminate a surface which is viewed in any other way than from behind. By means of these microscopes Lewenhoeck made a great number of curious observations, an account of which will be found hereafter, under the head Microscopical Observations.

III. The water microscope of Gray, which is much simpler, may be constructed in the following manner.

Provide a plate of lead, $\frac{1}{2}$ of a line in thickness at most, and make a round hole in it with a needle or a large pin; pare the edges of this hole, and put into it, with the point of a feather, a small drop of water: the anterior and posterior surfaces of the water will assume a convex spherical form, and thus you will have a microscope.

The focus of such a globule is at a distance somewhat greater than that of a glass globule of equal size; for the focus of a globule of water is at the distance of the radius from its surface. A globule of water, therefore, $\frac{1}{2}$ a line in diameter, will magnify only 128 times; but this deficiency is fully compensated by the ease with which a globule of any diameter, however small, may be obtained.

If water be employed in which leaves, wood, pepper, or flour has been infused, in the open air, the microscope will be both object and instrument; for by this means the small microscopic animals which the water contains will be seen. Mr. Gray was very much astonished, the first time he observed this phenomenon; but it afterwards occurred to him that the posterior surface of the drop produced, in regard to those animals placed between it and its focus, the same effects as a concave mirror, and magnified their image, which was still farther enlarged by the kind of convex lens of the anterior surface.

IV. Another kind of microscope may be also procured at a very small expense, by

making a hole of about the fourth or fifth part of a line in diameter, in a card or very thin plate of metal. If very small objects be viewed through this hole, they will appear magnified in the ratio of their distance from the eye, to that at which an object can be distinctly seen by the naked eye.

This kind of microscope is much extolled in the "Journal de Trevoux;" but we must confess that we never could see small objects distinctly through such holes, unless at the distance of an inch, or half an inch; and even then they did not appear to be much magnified.

PROBLEM XLIV.

Of Compound Microscopes.

The compound microscope consists of an object glass, which is a lens of a very short focus, such for example as 4 or 6 lines, and an eye-glass of 2 inches focus, at the distance from it of about 6 or 8 inches. The object must be placed a little beyond the focus of the object glass, and the distance of the eye from the eye-glass ought to be equal to the focal distance of the latter. Having formed such a combination of glasses, if the object be made to approach gently to the object glass, there will be a certain point at which it will appear to be considerably magnified.

If the focal distance of the object glass be 4 lines, for example, and if the object be $4\frac{1}{2}$ lines from it, the image will be formed at the distance of 64 lines, or five inches 4 lines: it will therefore be 14 times as large as the object, for 64 is to $4\frac{1}{2}$ nearly as 14 to 1. If the focal distance of the eye-glass, in the focus of which this image is formed, be two inches, it will magnify about 4 times more: but $14 \times 4 = 56$, which expresses the number of times that the diameter of the object will appear to be magnified.

If you are desirous that it should not be magnified so much, remove gradually the object from the object glass, and bring the eye-glass nearer; the image will then be seen not so large, but more distinct.

On the other hand, if you wish it to be magnified more, move the object gradually towards the object glass, or move the latter towards the object, and remove the eye-glass: the object will then appear much larger; but there are certain limits beyond which every thing seems confused.

Instead of one eye-glass, two are sometimes used to increase the field of vision; the first of which has a focal distance of 4 or 5 inches, while that of the second is much less; but this is still the same thing. The image of the small object must be placed, in regard to this compound eye-glass, in the same point where an object ought to be, to be seen distinctly when viewed through it.

A concave object glass might be employed by making its posterior focus coincide with the image: this would form a kind of microscope similar to the Batavian telescope; but it would be attended with the same inconvenience, that of having too contracted a field.

There are also reflecting microscopes as well as telescopes: the principle of both is the same, a minute object placed very near the focus of a concave mirror, and on this side of it, in regard to the centre, reflects an image of it beyond the centre; and this image will be larger the nearer it is to the focus. The image is viewed through a convex lens, and in this kind of microscope an object glass of a much shorter focus may be employed, which will contribute to the amplification of the object.

Every thing relating to this subject may be found in a very curious work by Baker, entitled the "Microscope made Easy." The reader may consult also Smith's Optics, part 4. These works, and particularly the first, contain a great variety of curious details respecting the method of employing microscopes, and the observations made by means of them. See also "Essais de Physique de Muschenbroeck."

We intend to give an account of the most curious observations which have been made by the assistance of the microscope; but to avoid confusion we shall reserve that article for the end of this part of our work.

PROBLEM LXV.

A very simple method of ascertaining the real magnitude of objects, seen through a microscope.

It is often useful, and may sometimes gratify curiosity, to be able to determine the real magnitude of certain objects examined by means of the microscope: the following very simple and ingenious method for this purpose was invented by Dr. Jurin, a celebrated philosopher, and a fellow of the Royal Society of London.

Take a piece of the finest silver wire possible to be obtained, and roll it as close as you can around an iron cylinder, a few inches in length. It will be necessary to examine it with a microscope, in order to discover whether there be any vacuity or opening between the folds: by these means you will ascertain, with great precision, the diameter of the silver wire. For if we suppose that there are 520 turns in the space of an inch, it is evident that the diameter of the wire will be the 520th part of an inch; a measure which cannot be obtained in any other manner.

Then cut this silver wire into very small bits, and scatter a certain quantity of them over the small plate on which the objects, submitted to examination, are placed: if you look at these bits of wire along with the objects, you will be enabled, by comparing them together, to judge of the size of the latter.

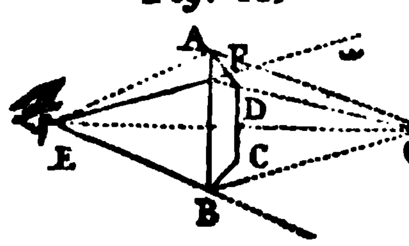
It was by a similar process that Dr. Jurin determined the size of the globules which give to blood its red colour. He first found that the diameter of his silver wire was the 485th part or an inch, and then judged by comparison that the diameter of a red globule of blood was the fourth part of that of the wire; from which he concluded that the diameter of the globule was the 1940th part of an inch.

PROBLEM XLVI.

To construct a Magic Picture, which being seen in a certain point through a glass, shall exhibit an object, different from that seen with the naked eye.

As this opticial problem is solved by means of a glass cut into facets, or what is called a multiplying glass, we shall first explain the nature of such glasses.

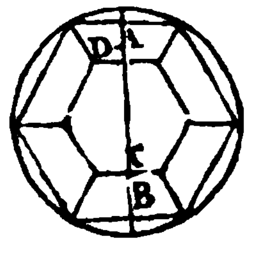
Fig. 48.



Multiplying glasses are generally lenses, plane on one side, and on the other cut into several facets in the form of a polyedron, of this kind is the glass represented Fig. 48. and 49. where it is seen in front, and also edgewise. It consists of a plane hexagonal facet in the centre, and six trapeziums arranged round the circumference.

These glasses have the property of representing the object as many times as there are facets; for if we suppose the object to be O, the rays which proceed from it fall upon all the facets of the glass A D, D C, C B. Those which traverse the facet D C, pass through it as through a plane glass interposed between the eye and the object; but the rays that proceed from O, to the inclined facet A D, experience a double refraction, which makes them converge towards the axis O E, nearly as they would do if they fell upon the spherical surface, in which the glass polyedron might be inscribed. The eye, being placed in the common point of concurrence, sees the point O, at ∞, in the continuation of the radius E F; consequently an image of the point O, different from the former, will be observed. As the same thing takes place in regard to each facet, the object will be seen as many times as there are facets on the glass, and in different places.

Fig. 49.



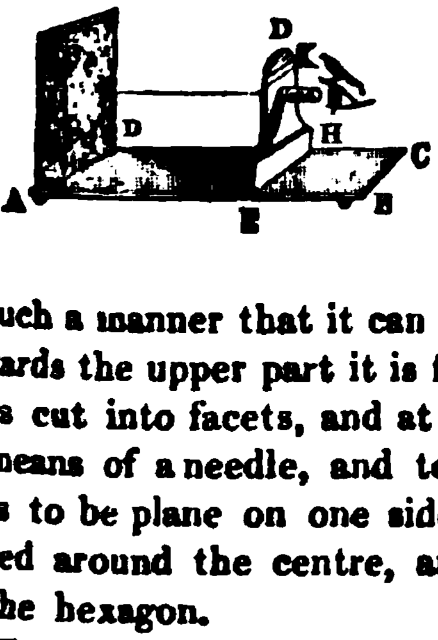
Now, if we suppose a luminous point in the axis of the glass, and at a proper distance, all the rays which fall on one facet will, after a double refraction, proceed

to a piece of white paper placed perpendicular to the axis continued, and paint on it an image of that facet of a greater or less size, and which at a certain distance will be inverted. Consequently, if we suppose the eye to be substituted instead of the luminous point, and that the image itself is luminous or coloured, the rays which proceed from that image, or part of the paper, will terminate at the eye; and they will be the only ones that reach it after experiencing a double refraction on the same facet. If the like reasoning be employed in regard to the rest, it may be easily seen that, when the eye is placed in a fixed point, it will observe through each facet only a certain portion of the paper, and that the whole together will fill the field of vision, though detached on the paper; so that if a certain part of a regular and continued picture be painted on each, they will all together represent that picture.

The artifice then of the proposed magic picture, after having fixed the place of the eye, that of the glass and the field of the picture, is to determine those portions of the picture which shall alone be seen through the glass; to paint upon each the determinate portion, according to a given subject, such as a portrait, so that when united together they may produce the painting itself; and in the last place, to fill up the remainder of the field of the picture with any thing at pleasure; but arranging the whole in such a manner as to form a regular subject.

Having thus explained the principle of this optical amusement, we shall now shew how it is to be put in practice.

Fig. 50.



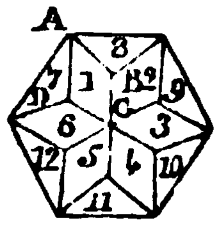
Let A B C D (Fig. 50.) represent a board, at the extremity of which is fixed another in a perpendicular direction, having at its edges two pieces of wood with grooves, to receive a piece of pasteboard, covered with white paper or canvass. This pasteboard, which may be pushed in or drawn out at pleasure, is the field of the intended picture: E D H is a vertical board, the bottom part of which must be contrived in such a manner that it can be brought nearer to or farther from the painting; and towards the upper part it is furnished with a tube, having at its anterior extremity a glass cut into facets, and at the other a piece of card, in which is a small hole made by means of a needle, and to which the eye is applied. We shall here suppose the glass to be plane on one side, and on the other to consist of six rhomboidal facets, placed around the centre, and of six triangular ones which occupy the remainder of the hexagon.

When every thing is thus prepared, fix the support E D H at a certain distance from the field of the picture, according as you are desirous that the parts to be delineated should be nearer to or farther from each other. But this distance ought, at least, to be four times the diameter of the sphere in which the polyedron of the glass could be inscribed; and the distance from the eye to the glass may be equal to twice that diameter. Then place the eye at the hole hole K, the distance of which has been thus determined, and with a stick having a pencil at the end of it, if the hand cannot reach the pasteboard, trace out, in as light a manner as possible, the outline of the space observed through one facet, and do the same thing in regard to the rest. This operation will require a great deal of accuracy and patience; for, to render the work perfect, no perceptible interval must be left between the two spaces seen through two contiguous facets: it will be better on the whole if they rather encroach a little on each other. Care must also be taken to mark each space with the same number as that assigned to each facet, in order that they may be again known. This however will be easy, by observing that the space corresponding to each facet is always transferred parallel to itself from top to bottom, or from right to left, on the other side of the centre.

The next thing is to delineate the regular picture intended to be seen, and to

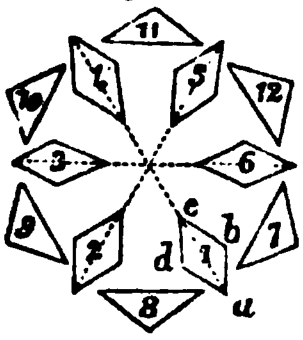
transpose it into the spaces where it appears distorted. According to mathematical accuracy, it would be necessary for this purpose to form a projection of the glass cut into facets, supposing the eye at the distance at which it is really placed; but as we suppose it a little more remote, we may without any sensible error assume, as the field of the regular picture, the vertical projection, as seen Fig. 51, where it is represented such as it would appear to the eye placed perpendicularly above its centre, and at a very considerable distance.

Fig. 51.



Delineate in the field, which in this case will be hexagonal, and composed of six rhomboids and six triangles, any figure whatever, as a portrait for example, and then, considering that the space *a b c d* (Fig. 52.) is that where the portion of the picture marked 1 ought to appear, it must be transferred thither with as much care as possible; do the same thing in regard to the rest; and by these means the principal part of the picture will be completed. But as it is intended to shew something else beside what ought to be seen, it must be disguised by means of some other objects painted in the remaining part of the field, making them to harmonize with what is already painted, in such a manner, that the whole shall appear to form one regular and connected subject. All this however must depend on the taste and genius of the artist.

Fig. 52.



In the "Perspective Curieuse" of father Niceron, a much more minute explanation of the whole process may be found. Those to whom what is here said does not seem sufficient, must consult that work. Niceron tells us that he executed, at Paris, and deposited in the library of the Minimes, of the Place Royale, a picture of this kind, which, when seen with the naked eye, represented fifteen portraits of Turkish Sultans; but, when viewed through the glass, was the portrait of Louis XIII.

A picture by Amadeus Vanloo, much more ingenious, was shewn in the year 1759, in the exhibition room of the Royal Academy of Painting. To the naked eye, it was an allegorical picture, which represented the Virtues, with their attributes, properly grouped; but, when seen through the glass, it exhibited the portrait of Louis XV.

Remarks.—1st. It is necessary to observe, that the place of the glass, when once fixed, must be invariable; for as glasses perfectly regular cannot be obtained, if they are moved, it will be almost impossible to replace them in the proper point; hence it will be necessary to be assured that the glass is of a good quality; for, if it be too alkaline, and happen to lose its polish by the contact of the air, another capable of producing the same effect cannot be substituted in its stead. This is an accident which, according to what we have heard, happened to the glass of Vanloo's picture.

2d. Instead of a glass, like that employed in the above example, or of one more compounded, a plain pyramidal glass might be employed, by which the problem would be greatly simplified.

3d. A glass, the portion of a prism, cut into a great number of planes parallel to its axis, might also be employed; in this case the painting to be viewed through the glass ought to be delineated on parallel bands.

4th. A glass might be formed of several concentric conical surfaces, or of several spherical surfaces of different diameters, likewise concentric: in this case the picture to be viewed through the glass ought to be distributed in different concentric rings.

5th. A magic picture might be formed by reflection. For this purpose, provide a

metal mirror with facets well polished, and having very sharp edges; place before it, in a direction parallel to its axis, a piece of white paper or card, and by means of the principles above explained delineate a picture, which when viewed in front by the naked eye, shall represent a certain subject; if you then make a hole in the middle of the picture, and look through this hole at the image of it formed by the mirror, it will appear to be entirely different.

PROBLEM XLVII.

To construct a lantern, by means of which a book can be read at a great distance, at night.

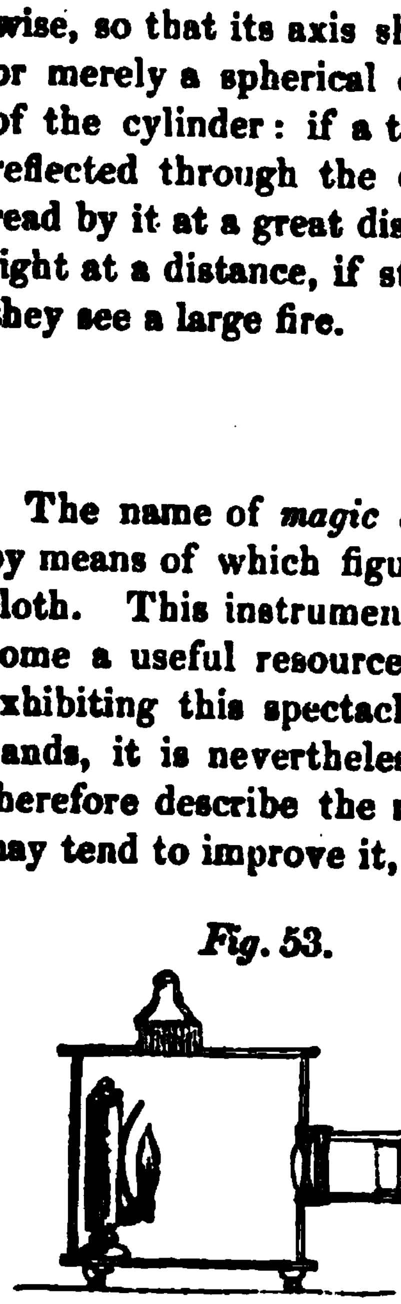
Construct a lantern of a cylindric form, or shaped like a small cask placed lengthwise, so that its axis shall be horizontal; and in one end of it fix a parabolic mirror, or merely a spherical one, the focus of which falls about the middle of the length of the cylinder: if a taper or lamp be then placed in this focus, the light will be reflected through the open end, and will be so strong that very small print may be read by it at a great distance, if looked at through a telescope. Those who see this light at a distance, if standing in the axis of the lantern continued, will imagine that they see a large fire.

PROBLEM XLVIII.

To construct a Magic Lantern.

The name of *magic lantern*, as is well known, is given to an optical instrument, by means of which figures greatly magnified may be represented on a white wall or cloth. This instrument, invented, we believe, by Father Kircher, a jesuit, has become a useful resource to a great number of people, who gain their livelihood by exhibiting this spectacle to the populace. But though it has fallen into vulgar hands, it is nevertheless ingenious, and deserves a place in this work. We shall therefore describe the method of constructing it, and add a few observations, which may tend to improve it, and to render it more interesting.

Fig. 53.



First, provide a box about a foot square (Fig. 53.) of tin-plate, or copper, or wood, and make a hole towards the middle of the fore-part of it, about three inches in diameter: into this hole let there be soldered a tube, the interior aperture of which must be furnished with a very transparent lens, having its focus within the box, and at the distance of two-thirds or three-fourths of the breadth of the box. In this focus place a lamp with a large wick, in order that it may produce a strong light; and that the machine may be more perfect, the lamp ought to be moveable, so that it can be placed exactly in the focus of the lens. To avoid the aberration of sphericity, the lens in question may be formed of two lenses, each of a double focus. This, in our opinion, would greatly contribute to the distinctness of the picture.

At a small distance from the aperture of the box, let there be a slit in the tube, for which purpose this part of it must be square, capable of receiving a slip of glass surrounded by a frame, four inches in breadth, and of any length at pleasure. Various objects, according to fancy, are painted on this slip of glass, with transparent colours; but in general the subjects chosen are of the comic and grotesque kind.

Another tube, furnished with a lens of about three inches focal distance, must be fitted into the former one, and in such a manner, that it can be drawn out or pushed in as may be found necessary.

Having thus given a description of the machine, we shall now explain its effect. The lamp being lighted, and the machine placed on the table opposite to a white

wall, if it be exhibited in the day time, shut the windows of the apartment, and introduce into the slit above mentioned one of the painted slips of glass, but in such a manner that the figures may be inverted: if the moveable tube be then pushed in or drawn out, till the proper focus is obtained, the figures on the glass will be seen painted on the wall, in their proper colours, and greatly magnified.

If the other end of the moveable tube be furnished with a lens of a much greater focal distance, the luminous field will be increased, and the figures will be magnified in proportion. It will be of advantage to place a diaphragm in this moveable tube, at nearly the focal distance of the first lens, as it will exclude the rays of the lateral objects, and thereby contribute to render the painting much more distinct.

We have already said that the small figures on the glass must be painted with transparent colours. The colours for this purpose may be made in the following manner: red, by a strong infusion of Brasil wood, or cochineal, or carmine, according to the tint required; green, by a solution of verdigris; or for dark greens, of martial vitriol (sulphate of iron); yellow, by an infusion of yellow berries; blue, by a solution of vitriol of copper (sulphate of copper); these three or four colours, as is well known, will be sufficient to form all the rest: they may be mixed up and rendered tenacious by means of very pure and transparent gum-water, after which they will be fit for painting on glass. In most machines of this kind, the paintings are so coarsely executed, that they cannot fail to excite disgust; but if they are neatly designed, and well finished, this small optical exhibition must afford a considerable degree of pleasure.

PROBLEM XLIX.

Method of constructing a Solar Microscope.

The Solar Microscope, for the invention of which we are indebted to Mr. Lieberkun, is nothing else, properly speaking, than a kind of magic lantern, where the sun performs the part of the lamp, and the small objects exposed on a glass or the point of a pin, that of the figures painted on the glass slips of the latter. But the following is a more minute description of it.

Make a round hole in the window shutter, about three inches in diameter, and place in it a glass lens of about twelve inches focal distance. To the inside of the hole adapt a tube, having, at a small distance from the lens, a slit or aperture, capable of receiving one or two very thin plates of glass, to which the objects to be viewed must be affixed by means of a little gum-water exceedingly transparent. Into this tube fit another, furnished at its anterior extremity with a lens of a short focal distance, such for example as half an inch. If a mirror be then placed before the hole in the window-shutter on the outside, in such a manner as to throw the light of the sun into the tube, you will have a solar microscope. The method of employing it is as follows.

Having darkened the room, and by means of the mirror reflected the sun's rays on the glasses in a direction parallel to their axes, place some small object between the two moveable plates of glass, or affix it to one of them with very transparent gum-water, and bring it exactly into the axis of the tube: if the moveable tube be then pushed in or drawn out, till the object be a little beyond the focus, it will be seen painted very dictinctly on a card or piece of white paper, held at a proper distance; and will appear to be greatly magnified. A small insect, such as a flea for example, may be made to appear as large as a sheep, or a hair as large as a walking-stick: by means of this instrument the eels in vinegar, or flour paste, will have the appearance of small serpents.

Remark.—As the sun is not stationary, this instrument is attended with one inconvenience, which is, that as this luminary moves with great rapidity, the mirror on

the outside requires to be continually adjusted. This defect however S'Gravesande remedied by means of a very ingenious machine, which moves the mirror in such a manner that it always throws the sun's rays into the tube. This machine, therefore, has been distinguished by the name of the *sol-sta.*

Some curious details respecting the solar microscope may be seen in the French Translation of Smith's Optics, where several useful inventions for improving it, and for which we are indebted to Euler, are explained. A method, invented by Æpinus, of rendering it proper for representing opake objects, will be found there also. It consists in reflecting, by means of a large lens and a mirror, the condensed light of the sun on the surface of the object, presented to the object glass of the microscope. M. Mumenthaler, a Swiss optician, proposed a different expedient. But solar microscopes are still attended with another inconvenience: as the objects are very near the focus of the first lens, they are subjected to a heat which soon destroys or disfigures them. Dr. Hill, who made great use of this microscope, proposed therefore to employ several lamps, the light of which united into one focus is exceedingly bright and free from the above inconvenience; but we do not know whether he ever carried this idea into practice, and with what success.

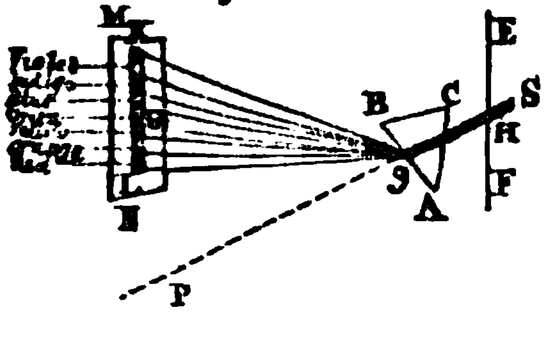
PROBLEM L.

Of Colours, and the different Refrangibility of Light.

One of the noblest discoveries of the 17th century, is that made by the celebrated Newton, in 1666, respecting the composition of light, and the cause of colours. Who could have believed that white, which appears to be a colour so pure, is the result of the seven primitive unalterable colours mixed together in a certain proportion! This however has been proved by his experiments.

The instrument which he employed for decomposing light in this manner, was the prism, now well known, but at that time a mere object of curiosity on account of the colours, with which every thing viewed through it seems to be bordered. But on this subject we shall confine ourselves to two of Newton's experiments, and a deduction of the consequences which result from them.

Fig. 54.



If a ray of a solar light, an inch or half an inch in diameter (Fig. 54.), be admitted into a darkened room, so as to fall on a prism placed horizontally, with a piece of white paper behind it, and if the prism be turned in such a manner, that the image seems to stop; instead of an image of the sun nearly round, you will observe a long perpendicular band, consisting of seven colours, in this invariable order, red, orange, yellow, green, blue, indigo, violet. When the angle of the prism is turned downwards, the red will be at the bottom, and *vice versa;* but the order will be always the same.

From this, and various other experiments of a similar kind, Newton concludes:

1. That the light of the sun contains these seven primitive colours.

2. That these colours are formed by the rays experiencing different refractions; and the red, in particular, is that which is the least broken or refracted; the next is the orange, &c.: in the last place, that the violet is that which, under the same inclination, suffers the greatest refraction. The truth of these consequences cannot be denied by those who are in the least acquainted with geometry.

But the nicest experiment is that by which Newton proved that these differently coloured rays are afterwards unalterable. To make this experiment in a proper manner, it will be necessary to proceed as follows:

In the first place, the hole in the window shutter of the darkened room must be reduced to the diameter of a line at most; and the light every where else must be

carefully excluded. When this is done, receive the solar rays on a large lens, of 7 or 8 feet focus, placed at the distance of 15 feet from the hole, and a little beyond the lens place a prism in such a manner that the stream of light may fall upon it. Then hold a piece of white card at such a distance that the image of the sun would be painted upon it without the interposition of the prism, and you will see painted on the card, instead of a round image, a very narrow coloured band, containing the seven primitive colours.

Then pierce a hole in the card, about a line in diameter, and suffer any one of the colours to pass through it, taking care that it shall do so in the middle of the space which it occupies, and receive it on a second card placed behind the former. If intercepted by another prism, it will be found that it no longer produces a lengthened, but a round image, and all of the same colour. Besides, if you hold in that colour any object whatever, it will be tinged by it; and if you look at the object with a third prism, it will be seen of no other colour but that in which it is immersed, and without any elongation, as when it is immersed in light susceptible of decomposition.

This experiment, which is now easy to those tolerably well versed in philosophy, proves the third of the principal facts advanced by Newton.

3. That when a colour is freed from the mixture of others, it is unalterable; that a red ray, whatever refraction it may be made to experience, will always remain red, and so of the rest.

It does no great honour to the French philosophers of the 17th century to have disputed, and even declared false, this assertion of the English philosopher, especially on no better foundation than an experiment so badly performed, and so incomplete as that of Mariotte. We even cannot help accusing that philosopher, who in other respects deserves great praise, of too much precipitation; for his experiment was not the same as that described by Newton in the "Philosophical Transactions" for 1666; and it may be readily seen that, if performed according to Mariotte's manner, it is impossible it should succeed.

However, it is at present certain, notwithstanding the remonstrances of Father Castel and the Sieur Gautier*, that there are in nature seven primitive, homogeneous colours, unequally refrangible, unalterable, and which are the cause of the different colours of bodies; that white contains them all, and that all of them together compose white; that what makes a body be of one colour rather than another, is the configuration of its minute parts, which causes it to reflect in greater number the rays of that particular colour; and in the last place, that black is the privation of all reflection; but this is understood of perfect black, for the material and common black is only an exceedingly dark blue.

Some people, such as Father Castel, have admitted only three primitive colours, viz., red, yellow, and blue; because red and yellow form orange, yellow and blue green, and blue and red violet or indigo, according as the former or the latter predominates. But this is another error. It is very true that with two rays, one yellow and the other blue, green can be formed; and this holds good also in regard to material colours; but the green of the coloured image of the prism is totally different: it is primitive, and stands the same proof as red, yellow, or blue, without being decomposed. The case is the same with orange, indigo, and violet.

We extract the following remarks on this subject from Dr. Brewster's treatise on Optics, in Dr. Lardner's Encyclopædia.

* The Sieur Gautier, who pretended to be the inventor of the method of engraving in colours, opposed with great violence, in the year 1750, the theory of Newton, both in regard to colours and to the system of the universe. His reasoning and experiments, however, are as conclusive as experiments made with a faulty air-pump would be against the gravity of the atmosphere. For this reason, he never had any partisans but a few of his own countrymen, one of whom was a poet, who had found out that objects are not painted on the retina in an inverted position.

Decomposition of Light by Absorption.

"If we measure the quantity of light, which is reflected from the surfaces, and transmitted through the substance of transparent bodies, we shall find that the sum of these quantities is always less than the quantity of light that falls upon the body. Hence we may conclude that a certain portion of light is lost in passing through the most transparent bodies. This loss arises from two causes. A part of the light is scattered in all directions by irregular reflection from the imperfectly polished surface of particular media, or from imperfect union of its parts; while another, and generally a greater portion, is *absorbed*, or stopped by the particles of the body. Coloured fluids, such as black and red ink, though equally homogeneous, stop or absorb different kinds of rays, and when exposed to the sun they become heated in different degrees, while pure water seems to transmit all the rays equally, and scarcely receives any heat from the passing light of the sun.

"When we examine more minutely the action of coloured fluids in absorbing light, many remarkable phenomena present themselves, which throw much light upon this curious subject.

"If we take a piece of blue glass, like that generally used for finger glasses, and transmit through it a beam of white light, the light will be a fine deep blue. This blue is not a simple homogeneous colour, like the blue or indigo of the spectrum, but is a mixture of all the colours of white light which the glass has not absorbed; and the colours which the glass has absorbed are those which the blue wants of white light, or which, when mixed with this blue, would form white light. In order to determine what these colours are, let us transmit through the blue glass the prismatic spectrum K L (Fig. 54.); or what is the same thing, let the observer place his eye behind the prism B A C, and look through it at the sun, or rather at a circular aperture made in the window-shutter of a dark room. He will then see through the prism the spectrum K L, as far below the aperture as it was above the spot P, when shewn on the screen. Let the blue glass be now interposed between the eye and the prism, and a remarkable spectrum will be seen, deficient in a certain number of its differently coloured rays. A particular thickness absorbs the middle of the red space, the whole of the orange, a great part of the green, a considerable part of the blue, a little of the indigo, and very little of the violet. The yellow space, which has not been much absorbed, has increased in breadth. It occupies part of the space formerly occupied by the *orange* on the one side, and part of the space formerly covered by the *green* on the other. Hence it follows, that the blue glass has absorbed the red light, which, when mixed with the yellow light, constituted *orange;* and has absorbed also the *blue* light, which, when mixed with the *yellow*, constituted the part of the green space next to the *yellow*. We have, therefore, by absorption, decomposed *green* light into *yellow* and *blue*, and *orange* light into *yellow* and *red;* and it consequently follows, that the orange and green rays of the spectrum, though they cannot be decomposed by prismatic refraction, can be decomposed by absorption, and actually consist of two different colours possessing the same degree of refrangibility. *Difference of colour is therefore not a test of difference of refrangibility*, and the conclusion deduced by Newton is no longer admissible as a general truth: 'That to the same degree of refrangibility ever belongs the same colour, and to the same colour ever belongs the same degree of refrangibility.'

"With the view of obtaining a complete analysis of the spectrum, I have examined the spectra produced by various bodies, and the changes which they undergo by absorption when viewed through various coloured media; and I find that the colour of every part of the spectrum may be changed, not only in intensity, but in colour, by the action of particular media; and from these observations, which it would be out of place here to detail, I conclude that the solar spectrum consists

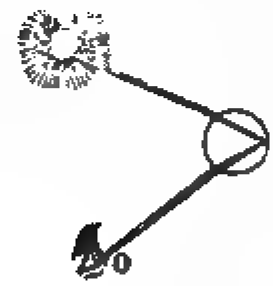
of three spectra of equal lengths, viz., a *red* spectrum, a *yellow* spectrum, and a *blue* spectrum."

PROBLEM LI.

Of the Rainbow; how formed; method of making an artificial one.

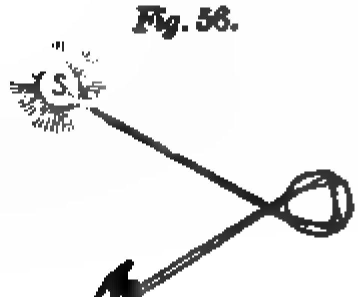
Of all the phenomena of nature, none has excited more the admiration of mankind, in all ages, than the rainbow; but there is none perhaps at present which philosophy can explain in a more satisfactory manner.

Fig. 55.



The rainbow is formed by the solar rays being decomposed into their principal colours, in the small drops of rain, by means of two refractions, which they experience in entering them and issuing from them. In the interior rainbow, which often appears alone, the solar ray enters at the upper part of the drop, is reflected against the bottom, and issues at the lower side. This decomposition may be seen Fig. 55. In the exterior rainbow, the rays enter at the bottom of the drop, experience two reflections, and issue at the upper part. Their progress and decomposition, which produces colours in an order contrary to the former, are represented Fig. 56. Hence the colours of the exterior rainbow appear to be inverted, in regard to those of the first.

Fig. 56.



The manner in which the eye perceives this double series of colours is seen Fig. 57.

Fig. 57.

But the explanation would be incomplete if we did not shew that there is a certain determinate inclination, under which the red rays issue parallel, and as close to each other as possible, while all the rest are divergent; that there is another under which the green rays issue in this manner; and so of the rest. It is by this alone that they can produce an effect on a distant eye.

This explanation of the rainbow is confirmed by a simple experiment. When the sun is very near the horizon, suspend in an apartment a glass globe filled with water, in such a manner as to be illuminated by the sun; and place yourself with your back to that luminary, so that the globe shall be elevated in regard to your eye, about 42 degrees above the horizon. By advancing or retiring a little, you will not fail to meet with the coloured rays, and it will be easily seen that they issue from the bottom of the globe; it will be seen also that the red ray issues from it under the greatest angle with the horizon, and the violet, which is the lowest one, under the least, so that the red must be without the axis, and the violet within it.

Then raise the globe, in regard to your eye, to 54 degrees, or continue to approach it till it be elevated at that angle, and you will meet with the coloured rays issuing from the top of it; first the violet, and then the blue, green, and red, in an order altogether contrary to the preceding. If you cover, in the first case, the upper part of the globe, and in the second the lower part, no colours will be produced; which is a proof of the manner in which they enter it, and issue from it.

The spectacle of an artificial rainbow may be easily obtained; it is seen in the

vapour of a jet of water, when the wind disperses it in minute drops. For this purpose, place yourself in a line between the jet of water and the sun, with your back turned towards the latter. If the sun be at a moderate elevation above the horizon, by advancing towards the jet of water, or receding from it, you will soon find a point from which a rainbow will be seen in the drops that fall down in fine light rain.

If there be not a jet of water in the neighbourhood, you may make one at a very small expense. Nothing will be necessary but to fill your mouth with water, and having turned your back to the sun when at a moderate elevation, to spurt the water into the air as high as possible, and in a direction somewhat oblique to the horizon. The imitation of this phenomenon may be greatly facilitated by employing a syringe, which will scatter the water in very small drops.

If you are desirous of performing the experiment in a manner still easier, fill a very transparent cylindric glass bottle with water, and place it on a table in an upright position; place a lighted candle at the same height, and at the distance from it of 10 or 12 feet, and then walk in a transversal direction between the light and the bottle, keeping your eye at the same elevation. When you have reached a certain point, you will see bundles of coloured rays issuing from one of the sides of the bottle, in the following order: violet, blue, yellow, red; and if you continue to walk transversely, you will meet with a second series, in a contrary order, viz. red, yellow, blue and violet, proceeding from the other side of the bottle. This is exactly what takes place in regard to the drops of rain; and to imitate the phenomenon completely, seven similar bottles might be arranged in such a manner, that the eye being placed in the proper point, one of the seven colours should be seen in each; and seven others might be arranged at some distance, so as to exhibit the same colours in an inverted order.

Two rainbows would still be produced, even if the solar rays were not differently refrangible; but they would be destitute of colour, and would consist only of two circular bands of white or yellowish light.

The rainbow always forms a portion of a circle around the line drawn from the sun, and passing through the eye of the spectator; for this reason, when the sun is elevated above the horizon, the rainbow is less than a semicircle; but when the sun is in the horizon, it is equal to a semicircle.

A rainbow, however, has been seen larger than a semicircle, and which intersected the common rainbow; but this phenomenon was produced by the image of the sun reflected from the calm, smooth surface of a river. The image of the sun, in this case, produced the same effect as if that luminary had been below the horizon.

Dr. Halley has calculated, from the ratio of the different refrangibilities of the sun's rays, that the semi-diameter of the interior rainbow, taken in the middle of its extent, ought to be 41° 10′; and that its breadth, which would be only 1° 45′, if the sun were a point, ought to be 2° 15′ on account of the apparent diameter of that luminary. This apparent diameter is the cause why the colours are not separated from each other with the same distinctness as they would be, if the sun were a luminous point: the radius of the exterior rainbow, taken in the same manner, that is to say in the middle of its extent, is 52° 30′.

This geometrician and astronomer not only calculated the dimensions of that rainbow which actually appears to us in the heavens, but of those also which would be produced if the light of the sun did not issue from the drop of water till after 3, 4, 5, &c., reflections; whereas, in the principal and interior rainbow, it issues after one, and in the second or exterior one, after two. By these calculations it is found that the semi-diameter of the third rainbow, counted from the place of the sun, would be 41°; that of the fourth, 43° 50′; &c. But geometry here goes much farther than nature: for besides the continued weakness of the rays, which would render these

rainbows scarcely perceptible, being towards the sun, they would be lost amidst the splendour of that luminary. If the drops which form the rainbow, instead of being water, were glass, the mean semi-diameter of the interior rainbow would be 22° 52′, and that of the exterior 9° 30′, towards the side opposite to the sun.

PROBLEM LII.

Analogy between Colours and the Tones of Music. Of the Ocular Harpsichord of Father Castel.

As soon as it had been observed that there were seven primitive colours in nature there was some reason to conceive that there might be an analogy between these colours and the tones of music; for the latter form a series of seven in the whole extent of the octave. This observation did not escape Newton, who remarked also that, in the coloured spectrum, the spaces occupied by the violet, indigo, blue, &c., correspond to the divisions of the monochord, which gives the sounds *re, mi, fa, sol, la, si, ut, re.*

Newton on this subject proceeded no farther. But Father Castel, whose visionary scheme is well known, enlarged this idea; and on the above analogy of sounds founded a system, in consequence of which he promised to the eyes, but unfortunately without success, a new pleasure similar to that which the ears experience from a concert.

Father Castel, for reasons of analogy, first changes the order of the colours into the following, viz. blue, green, yellow, orange, red, violet, indigo, and in the last place blue, which forms as it were the octave of the first. These, according to his system, are the colours which correspond to the diatonic octave of our modern music, *ut, re, mi, fa, sol, la, si, ut.* The flats and the sharps gave him no embarrassment; and the chromatic octave divided into its twelve colours, was blue, sea-green, olive-green, yellow, apricot, orange, red, crimson, violet, agate, indigo, blue, which corresponded to *ut, ut,* re, re,* mi, fa, fa,* sol, sol,* la, la,* si, ut.*

Now if a harpsichord be constructed in such a manner, says Father Castel, that on striking the key *ut*, instead of hearing a sound, a blue band shall appear; that on striking *re*, a green one shall be seen, and so on, you will have the required instrument; provided that for the first octave of *ut* a different blue be employed. But what are we to understand by a blue an octave to another? We do not find that Father Castel ever explained himself on this subject in a manner sufficiently clear. He only says that, as there are reckoned to be twelve octaves appreciable by the ear, from the lowest sound to the most acute, there are in like manner twelve octaves of colours, from the darkest blue to the lightest; which gives us reason to believe that since the darkest blue is that which ought to represent the lowest key, the blue corresponding to the octave must be formed of eleven parts of pure blue, and one of white; that the lightest must be formed of one part of blue and eleven parts of white, and so of the rest.

However, Father Castel did not despair of producing by these means an ocular music, as interesting to the eyes as the common music is to well organised ears; and he even thought that a piece of music might be translated into colours for the use of the deaf and dumb. "You may conceive (says he,) what a spectacle will be exhibited by a room covered with rigadoons and minuets, sarabands and passcailles, sonatas and cantatas, and if you choose with the complete representation of an opera? Have your colours well diapasoned, and arrange them on a piece of canvas according to the exact series, combination, and mixture of the tones, the parts and concords of the piece of music which you are desirous to paint, observing all the different values of the notes, minims, crotchets, quavers, syncopes, rests, &c.; and disposing all the parts according to the order of counter-point. It may be readily seen that this is

not impossible, nor even difficult, to any person who has studied the elements of painting, and at any rate that a piece of tapestry of this kind would be equal to those where the colours are applied as it were at hazard in the same manner as they are in marble.

"Such a harpsichord (continues he,) would be an excellent school for painters, who might find in it all the secrets and combinations of the colours, and of that which is called claro-obscuro. But even our harmonical tapestry would be attended with its advantages; for one might contemplate there at leisure what hitherto could be heard only in passing with rapidity, so as to leave little time for reflection. And what pleasure to behold the colours in a disposition truly harmonical, and in that infinite variety of combinations which harmony furnishes! The design alone of a painting excites pleasure. There is certainly a design in a piece of music; but it is not so sensible when the piece is played with rapidity. Here the eye will contemplate it at leisure; it will see the concert, the contrast of all the parts, the effect of the one in opposition to the other, the fugues, imitations, expression, concatenation of the cadences, and progress of the modulation. And can it be believed that those pathetic passages, those grand traits of harmony, those unexpected changes of tone, that always cause suspension, languor, emotions, and a thousand unexpected changes in the soul which abandons itself to them, will lose any of their energy in passing from the ears to the eyes, &c.? It will be curious to see the deaf applauding the same passages as the blind, &c. Green, which corresponds to *re*, will no doubt shew that the tone *re* is rural, agreeable, and pastoral; red, which corresponds to *sol*, will excite the idea of a warlike and terrific tone; blue, which corresponds to *ut*, of a noble, majestic, and celestial tone, &c. It is singular that the colours should have the proper characters ascribed by the ancients to the exact tones which correspond to them, but a great deal might be said, &c.

"A spectacle might be exhibited of all forms human and angelical, animals, birds, reptiles, fishes, quadrupedes, and even geometric figures. By a simple game the whole series of Euclid's Elements might be demonstrated." Father Castel's imagination seems here to conduct him in the straight road to Bedlam.

These passages of Father Castel are so singular, that we could not help quoting them; but unfortuately all his fine promises came to nothing. He had constructed a model of his harpsichord, as he tells us himself, so early as the end of the year 1734, and he spent almost the remainder of his life, till the time of his death, which took place in 1757, in completing his instrument, but without success. This harpsichord, constructed at a great expense, as we are told by the author of his life, neither answered the author's intention, nor the expectation of the public. And indeed if there be any analogy between colours and sounds, they differ in so many other points, that it needs excite no wonder that this project should miscarry.

PROBLEM LIII.

To compose a table representing all the Colours; and to determine their number.

Though Newton has proved the homogeneity of the colours into which the solar rays are decomposed, and the orange, green, and purple produced by this decomposition are no less unalterable, by farther refraction, than the red, yellow, and blue; it is however well known that with the three latter, the three former, and all the other colours of nature, can be imitated: for red combined with yellow, in different proportions, gives all the shades of orange; yellow and blue produce pure greens; red and blue violets, purples and indigoes; in a word the different combinations of these compound colours, give birth to all the rest. On these principles is founded the invention of the chromatic triangle, which serves to represent them.

Fig. 58.

Construct an equilateral triangle, as seen Fig. 58, and divide the two sides adjacent to the vertical angle into 13 equal parts: if parallel lines be then drawn through the points of division, in each side, they will form 91 equal rhombuses.

In the three angular rhombs place the three primitive colours, red, yellow, and blue, having an equal degree of strength, and as we may say of concentration; consequently, between the yellow and blue, there will be left 11 rhomboidal cells, which must be filled up in the following manner: in that nearest the yellow put 11 parts of yellow and one of red; in the next, 10 parts of yellow and 2 of red; &c.; so that in the cell nearest the red, there will be 1 part of yellow and 11 of red: by these means we shall have all the shades of orange, from the one nearest red to that nearest yellow. By filling up, in like manner, the intermediate cells between red and blue, and between blue and yellow, the result will be all the shades of purple, and all those of green, in a similar gradation.

To fill up the other cells, let us take for example those of the third row below red, where there are three cells. The two extreme cells being filled up on the one side with a combination of 10 parts of red and 2 of yellow, and on the other with a combination of 10 parts of red and 2 of blue, the middle cell will be composed of 10 parts of red, 1 of blue, and 1 of yellow.

In the band immediately below, we shall have, for the same reason, in the first cell towards the yellow, 9 parts of red and 3 of yellow; in the next, 9 parts of red, 2 of yellow, and 1 of blue; in the third, 9 parts of red, 1 of yellow, and 2 of blue; in the fourth, 9 parts of red, and 3 of blue: and the case will be similar in regard to the lower bands; but we shall here content ourselves with detailing the colours in the last except one, or that above the band containing the greens, the cells of which must be filled up as follows: In

The 1st on the left,	11 parts yellow	and 1 red.	
The 2d,	10 parts yellow,	1 red,	1 blue.
The 3d,	9 parts yellow,	1 red,	2 blue.
The 4th,	8 parts yellow,	1 red;	3 blue.
The 5th,	7 parts yellow,	1 red,	4 blue.
The 6th,	6 parts yellow,	1 red,	5 blue.
The 7th,	5 parts yellow,	1 red,	6 blue.
The 8th,	4 parts yellow,	1 red,	7 blue.
The 9th,	3 parts yellow,	1 red,	8 blue.
The 10th,	2 parts yellow,	1 red,	9 blue.
The 11th,	1 part yellow,	1 red,	10 blue.
The 12th,	0 part yellow,	1 red,	11 blue.

This band, as may be seen, contains all the greens of the lowest band into which

one part of red has been thrown. In like manner, there will be found in the band parallel to the purples, all the purples with which one part of yellow has been mixed; and in the band parallel and contiguous to the oranges, all the orange colours with one part of blue.

In the central cell of the triangle there are 4 parts of red, 4 of blue, and 4 of yellow.

All these mixtures might be easily made with colours ground exceedingly fine; and if the proper quantities were employed we have no doubt that they would produce all the shades of the different colours. But if all the colours of nature, from the lightest to the darkest, that is from black to white, be required, we shall find for each cell 12 degrees of gradation to white, and 12 others to black. If 91 therefore be multiplied by 24, we shall have 2184 perceptible colours; to which if we add 24 grays, formed by the combination of pure black and white, and white and black, the number of compound colours which we believe to be distinguishable by the senses, will amount to 2218. But we ought not perhaps to consider as real colours, those formed by the pure colours with black; for black only obscures, but does not colour. In this case, the real colours, with their shades, from the darkest to the lightest, ought to be reduced to 1092, which, with a black, and 12 grays, will form 1106 colours.

PROBLEM LIV.

On the Cause of the Blue Colour of the Sky.

This is a very remarkable phenomenon, though little attention is paid to it, as our eyes are so much accustomed to it from our infancy; and it would be difficult to explain it had not Newton's theory respecting light, by teaching us that it is decomposed into seven colours, of different degrees of refrangibility and reflexibility, afforded us the means of discovering the cause.

To explain this phenomenon, we shall observe then, that according to Newton's theory, so well proved by experience, of the seven colours which the solar light produces when decomposed by the prism, the blue, indigo, and violet are those easiest reflected, when they meet with a medium of different density. But whatever may be the transparency of the air, that which surrounds our earth, and which constitutes our atmosphere, contains always a mixture of vapours more or less combined with it: hence it happens that the light of the sun and stars, sent back in a hundred different ways into the atmosphere, must experience in it numberless inflections and reflections. But as the blue, indigo, and violet rays are those chiefly sent back to us, at each of these reflections, from the minute particles of the vapours which they are obliged to pass through, it is necessary that the medium which sends them back should appear to assume a blue tint. This must even be the case if we suppose a homogeneity in the atmosphere: for however homogeneous a transparent medium may be, it necessarily reflects a part of the rays of light which pass through it. But of all these rays, the blue are reflected with the greatest facility; consequently the air, even supposing it homogeneous, would assume a blue, or perhaps a violet colour.

It is for the same reason that the water of the sea appears of a blue colour when very pure, as is the case at a distance from the coasts. When illuminated by the sun, a part of the rays enters the water, and another part is reflected; but the latter is composed chiefly of blue rays, and consequently it must appear blue.

This explanation is confirmed by a curious observation of Dr. Halley. This celebrated philosopher having descended in a diving bell to a considerable depth in the sea, while it was illuminated with the sun, was much surprised to see the back of his hand, which received the direct rays, of a beautiful rose colour, while the lower part, which received the reflected rays, was blue. This indeed is what ought

to take place, if we suppose that the rays reflected by the surface of the sea, as well as by the minute parts of the middle of it, are blue rays. In proportion as the light penetrates to a greater depth, it must be more and more deprived of the blue rays, and consequently the remainder must incline to red.

PROBLEM LV.

Why the Shadows of bodies are sometimes Blue, or Azure coloured, instead of being Black.

It is often observed at sun-rise, during very serene mornings, that the shadows of bodies projected on a white ground, at a small distance, are blue or azure coloured. This phenomenon appears to us to be sufficiently curious to deserve here a place as well as an explanation.

If the shadow of a body exposed to the sun were absolute, it would be perfectly black, since it would be a complete privation of light; but this does not really take place: for to be so, the field of the heavens ought to be absolutely black; whereas it is blue, or azure coloured, and it is so only because it sends back to us chiefly blue rays, as already observed.

The shadow therefore projected by bodies exposed to the sun, is not a pure shadow, but is itself illuminated by that whole part of the sky not occupied by the luminous body. This part of the heaven being blue, the shadow is softened by the blue or azure coloured rays, and consequently must appear of that colour. It is exactly in the same manner that in painting, reflections are tinted with the colour of the surrounding bodies. The shadow which we here examine, is nothing else than a shadow mixed with the reflection of a blue body, and therefore it must participate in that colour.

It is well known that this phenomenon was first observed and explained by Buffon.

But it may here be asked, why are not all shadows blue? In reply to this question, we shall observe, that to produce this effect, the concurrence of several circumstances are necessary. 1st. A very pure sky, and of a very dark blue colour; for if the heavens be interspersed with light clouds, the rays reflected from them, falling on the bluish shadow, will destroy its effect; if the blue be weak, as is often the case, the quantity of the blue rays will not be sufficient to enlighten the shadow. 2d. The light of the sun must be livelier than it usually is when that luminary is near the horizon, in order that the shadows may be full and strong. But these circumstances are rarely united. Besides, the sun must be only at a small elevation above the horizon; for even when at a moderate altitude there is too much splendour in the atmosphere, to allow the blue rays to be sensible. This light renders the shadows less strong, but does not tinge it blue.

PROBLEM LVI.

Experiment on Colours.

Hold before your eyes two glasses of different colours, suppose the one blue and the other red: and having placed yourself at a proper distance from a candle, if you shut one of your eyes, and look at the light with the other, that for example before which the blue glass is held, the light will appear blue. If you next shut this eye and open the other, the flame will appear red; and if you then open them both, you will see it of a bright violet colour.

Every person almost, in our opinion, must have foreseen the success of this experiment; which we have mentioned merely because an oculist of Lyons, M. Janin, thought he could deduce from it a particular consequence; which is, that the retina may perform the part of a concave mirror, and reflect the rays of light, so that each eye forms at a certain distance an aërial image of the object. Both eyes forming each an image afterwards in the same place, the result is a double image, one blue

and the other red, which by their union produce a violet image, in the same manner as when red and blue rays are mixed together. But this explanation will certainly not bear to be examined according to the true principles of optics. How is it possible to conceive that such an image can be formed by the retina? Is it not more probable, and more agreeable to the well known phenomena of vision, that from the two impressions received by the two eyes, there is produced, in the *common sensorium*, or in the place where the optic nerves are united in the brain, one compound impression? In this experiment therefore the same thing must take place, as when a person looks at a candle with one eye, through two glasses, the one red and the other blue. In this case the flame will be seen of a violet colour, and consequently it must have the same appearance in the former.

PROBLEM LVII.

Method of constructing a Photophorus, very convenient to illuminate a table where a person is reading or writing.

Construct a cone of tin-plate, $4\frac{1}{2}$ inches in diameter at the base, and $7\frac{3}{8}$ inches in height, measured on the slant side; which may be easily done by cutting from a circle, of $7\frac{3}{8}$ inches radius, a sector of $109\frac{1}{2}$ degrees, and bending it into the form of a cone. Then through a point in the axis, $2\frac{1}{2}$ inches distant from the summit, cut off the upper part of the cone by a plane inclined to one of its sides at an angle of 45°. The result will be an elongated elliptical section, which must be placed before a candle or other light, as near to it as possible, the plane of the section being vertical, and the greatest diameter in a perpendicular direction. When disposed in this manner, if the flame of the candle or lamp be raised 12 or 13 inches above the plane of the table, you will be astonished to see the vivacity and uniformity of the light which it will project over an extent of 4 or 5 feet in length.

M. Lambert, the inventor of this apparatus, observes that it may be used with great advantage to those who read in bed; for by placing a lamp or taper furnished with this photophorus upon a pretty high stand, at the distance of 5, 6, or 8 feet, from the bed, it will afford a sufficiency of light without any danger. He says he tried this apparatus also in the street, by placing a lamp furnished with it in a window raised 15 feet above the pavement, and that its effect was so great, that at the distance of 60 feet, a bit of straw could be distinguished much better than by moon light, and that writing could be read at the distance of 35 or 40 feet. A few of these machines, placed on each side of a street, and arranged in a diagonal form, would consequently light it much better than any of the means hitherto employed. See *Mémoires de l'Academie de Berlin, ann.* 1770.

PROBLEM LVIII.

The place of an object, such for example as of a piece of paper on a table, being given; and that of a candle destined to throw light upon it; to determine the height at which the candle must be placed, in order that the object may be illuminated the most possible.

That we may exclude from this problem several considerations, which would render the solution of it very difficult, we shall suppose the object destined to be illuminated to be very small, or that it is only required that the middle of it shall be illuminated as much as possible. We shall suppose also that the light is entirely concentrated into one point, where the splendor of all its different parts is united.

But it is well known that the light diffused by a luminous point, over any surface which it illuminates, decreases, the angle being the same, in the inverse ratio of the square of the distance; and that when the angle of inclination varies, it is as the sine of that angle. Hence it follows that it decreases in the compound ratio of the square of the distance taken inversely, and the sine of the angle of incli-

nation taken directly. To solve this problem, then, we must find that height of the luminous point in the given perpendicular, which will render this ratio the greatest possible.

But it will be found that this ratio is greatest when the perpendicular height, and the distance of the object to be illuminated from the bottom of the candle, are to each other as the side of a square is to the diagonal. On this given and invariable distance as hypothenuse, if a right-angled isosceles triangle be therefore described, the side of this triangle will be the height at which, if the flame of the candle be placed, the given point or centre of the paper will be illuminated in the highest degree possible.

On this subject, the following problem, which is of a similar nature, might also be proposed:

Two candles of unequal height, placed at the extremities of a horizontal line, being given; to find in that line a point so situated, that the object placed in it shall be illuminated the most possible?

But we shall not give the solution, that our readers may exercise their own ingenuity in discovering it.

PROBLEM LIX.

On the Proportion which the Light of the Moon bears to that of the Sun.

This is a very curious problem: but it is only within these few years that philosophers began to turn their attention to the principles, and means, which can lead to the solution of it. We are indebted for them to M. Bouguer, who has explained them in his "Treatise on the Gradation of Light;" a work that contains many curious particulars, a few of which we shall here extract.

To obtain this measure of the intensity of light, M. Bouguer sets out with a fact, founded on experience, which is, that the eye judges pretty exactly by habit, whether two similar and equal surfaces are equally illuminated. Nothing then is necessary, but to place at unequal distances two unequal lights, or by means of concave glasses, the focal distances of which are unequal, to make them be unequally dilated, so that the surfaces which are illuminated by them shall appear to be so in an equal degree. The rest depends merely on calculation: for if two lights, one of which is four times nearer than the other, illuminate equally two similar surfaces, it is evident that, as the degrees of illumination of the same light decrease in the inverse ratio of the squares of the distances, we ought to conclude that the splendour of the first light is sixteen times as great as that of the second. In like manner, if a light dilated into a circular space, double in diameter, illuminates as much as another direct light, there is reason to conclude that the former is quadruple the second.

By employing these means, M. Bouguer found that the light of the sun diminished 11664 times was equal to that of a flambeau, which illuminates a surface at the distance of 16 inches; and that the same flambeau illuminating a similar surface, at the distance of 50 feet, gave it the same light as that of the moon, when diminished 64 times. By compounding these two ratios he concludes, that the light of the sun is to that of the moon, at their mean distances and at the same altitude, as 256289 to 1; that is to say 250,000 times greater. From some other experiments, he is even inclined to think, that the light of the moon is only equal to the 300,000th part of that of the sun.

The result of a celebrated experiment, made by Couplet and La Hire, two academicians of Paris, needs therefore excite no surprise. These two philosophers collected the lunar rays by means of the burning mirror at the Observatory, which is 35 inches in diameter, and made the focus fall on the bulb of a thermometer, but no motion was produced in the liquor. And indeed this ought to be the case, for if we

suppose a mirror like the above, which collects the rays that fall on its surface into a space 1200 or 1400 times less, the heat thence resulting will be 1200 or 1400 times greater; but, on account of the dispersion of the rays, it will be sufficient to suppose this light to be a thousand times denser than the direct light, and the heat in proportion. A mirror of this kind then, by collecting the lunar rays, would produce in its focus a heat 1000 times greater than that of the moon. If 300,000 therefore be divided by 1000, we shall have for quotient 300, which expresses the ratio of the direct solar heat to that of the moon thus condensed. But a heat 300 times less than the direct heat of the sun is not capable of producing any effect on the liquor in the thermometer. This fact then is far from being inexplicable, as we are told by the author of the "History of the Progress of the Human Mind in Philosophy,"* for it is a necessary consequence of Bouguer's calculations, which this writer no doubt overlooked.

We shall, in the last place, observe, that Bouguer found, by a mean calculation, that the splendour of the sun, when on the horizon, supposing the sky to be free from clouds or fog, is about 2000 times less than when elevated 66°. The case ought to be the same also with the light of the moon.

PROBLEM LX.

Of certain Optical Illusions.

I. If you take a seal with a cipher engraved on it, and view it through a convex glass of an inch or more focal distance, the cipher or engraving will be seen sunk in the stone as it really is; but if you continue to look at it, without changing your situation, you will soon see it in relief; and by still continuing to look at it, you will see it once more sunk, and then again in relief. Sometimes, after having discontinued to look at it, instead of seeing it sunk, it will appear in relief; it will then appear sunk, and so on. When the side of the light is changed, this generally causes a change in the appearance.

Some have taken a good deal of pains to discover the cause of this illusion; which in our opinion may be explained without much difficulty. When an object is viewed with a lens of a short focal distance, and consequently with one eye, we judge very imperfectly of the distance, and the imagination has a great share in that assigned to the image which we perceive. On the other hand, the position of the shadow can never serve to rectify the judgment formed of it: for if the engraving is hollow, and if the light comes from the right, the shadow is on the right; it is also on the right if the engraving is in relief, and if the light comes from the left. But when an engraved stone is attentively viewed with a magnifying glass, we do not pay attention to the side from which the light proceeds. Here then, every thing, as we may say, is ambiguous and uncertain; consequently it is not surprising that the organ of sight should form an undecisive and continually variable judgment; but we are fully persuaded that an experienced eye will not fall into these variations.

The same phenomenon is not observed when the experiment is performed with a piece of money. The reason of this probably is, that we are accustomed to handle such pieces, and to see the figures on them in relief, which does not permit the mind to form, in consequence of the image painted in the eye, any other idea than that which it always has had on seeing a piece of money, viz. that of figures in relief.

II. If a glass decanter, half filled with water, be presented to a concave mirror, and at a proper distance, that is to say between the centre and the focus, it will be seen inverted before the mirror. But it is very singular, in regard to many persons, though it is not general, that they imagine they see the water in the half of the bottle

* Saverien Histoire du Progres de l' Esprit humain dans les Sciences physiques.

next the neck, which is turned downwards. For our part, we are of opinion that the cause why some think they see the water in this situation, arises from their knowing by experience that if a bottle half filled with water be inverted, the fluid will descend into the lower part, or that next the neck.

Another cause concurs to make us judge in this manner. When a decanter is half full of very pure water, each half is as transparent as the other, and the presence of the water is perceived only by the reflection of the light which takes place at its surface; but in the inverted image this surface reflects the light below, and even with the same force; by which means we are led to conclude that the fluid is at the bottom.

This subject may be farther exemplified as follows:

Take a glass bottle; fill it partly with water, and cork it in the usual way: place this bottle opposite a concave mirror, and beyond its focus, that it may appear reversed: then place yourself still farther distant than the bottle, and this will be seen inverted in the air, and the water, which is really in the lower part of the bottle, will appear to be in the upper. (See Fig. 59. 60.) If the bottle be inverted while it is before the mirror, the image will appear in its natural erect position, and the water will appear in the lower part of the bottle. While it is in this inverted state, uncork the bottle, then while the water is running out the image is filling. But as soon as the bottle is empty, the illusion ceases. The illusion also ceases when the bottle is quite full.

Fig. 59.

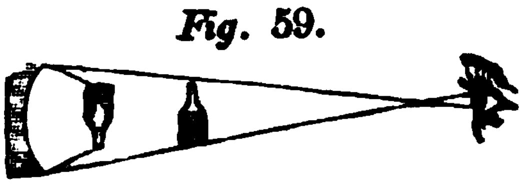
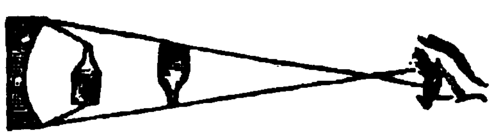


Fig. 60.



The remarkable circumstances in this experiment are, 1st. Not only to see an object where it is not, but also where its image is not. 2d. That of two objects which are really in the same place, as the surface of the bottle and the water it contains, the one is seen in one place, and the other in another, &c. It is conceived that this illusion arises, partly from our not being accustomed to see water suspended in a bottle with the neck downward, and partly from the resemblance there is between the colour of water and the air.

Additional Amusements and Experiments with Concave Mirrors, &c., as described by Mr. Adams, Mr. Jones, &c.

1. Placing yourself before a concave mirror, but farther from it than the centre, you will see an inverted image of yourself, but smaller, in the air between you and the mirror: holding out your hand towards the mirror, the hand of the image will come out towards your hand, and when at the centre of concavity, be of an equal size with it; and you may as it were shake hands with this aërial image. On advancing your hand farther, the hand of the image passes by your hand, and comes between it and your body: on moving your hand towards one side, the hand of the image moves towards the other, the image moving always contrariwise to the object. All this while, the bye-standers see nothing of the image, because none of the reflected rays which form it enter their eyes. To render this effect more surprising, and more vivid, the mirror is often concealed in a box, after the manner as we shall shew presently. In fact, the appearance of the image in the air, between the object and the mirror, has been productive of many agreeable deceptions, which, when exhibited with art and an air of mystery, have been very successful, and the source of emolument to many of our public showmen. In this manner they have exhibited the images of animated and other objects, in such a way, as to surprise the ignorant, and please the scientific, or better informed.

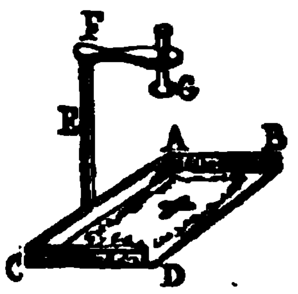
2. Mr. Ferguson mentions two pleasing experiments to be made with a concave mirror, which may be easily tried. If a fire be made in a large room, and a smooth,

well-polished mahogany table be placed at a good distance, near the wall, before a large concave mirror, so that the light of the fire may be reflected from the mirror to its focus on the table; if you stand by the table, you will see nothing but a long beam of light; but if you stand at some distance, as towards the fire, you will see, on the table, an image of the fire, large and erect: and if another person, who knows nothing of the matter before hand, should chance to enter the room, he will be startled at the appearance; for the table will seem to be on fire, and being near the wainscot, to endanger the whole house. For the better deception, there ought to be no light in the room but what proceeds from the fire.

3. If the fire be darkened by a screen, and a large candle be placed at the back of the screen; then a person standing by the candle will see the appearance of a fine large star, or rather planet, on the table, as large as Jupiter or Venus; and if a small wax taper be placed near the candle, it will appear as a satellite to the planet; if the taper be moved round the candle, the satellite will be seen to go round the planet.

4. *The Dioptrical Paradox, or Optical Deception.*

Fig. 61. *No.* 1.

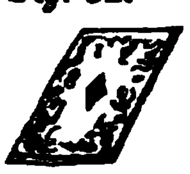


No. 2.

Fig. 61. No. 1. represents the dioptrical paradox. It consists of a mahogany base A B C D, about eight inches square, with a groove, in which slides various coloured prints, or ornamental drawings: and connected with the base are, a pillar E, a horizontal bar F, with a perspective G, which is placed exactly over the centre of the base, and containing a glass of a particular form. The curious and surprising effect of this instrument is, that an ace of diamonds, in the centre of one of the drawings, when placed on the base, shall through the perspective G be actually represented as the ace of clubs; a figure of a cat in another, seen as an owl; a letter A, as an O; and a variety of others equally astonishing.

The principle of this machine is very simple, and is as follows. The glass in the tube G, which produces this change, is somewhat on the principle of the common multiplying glass, and is represented at Fig. 61. No. 2. The only difference is, that its sides are flat, and diverging from its hexagonal base upwards, to a point in the axis of the glass, like a pyramid, each side forming an isosceles triangle. Its distance from the eye is to be so adjusted, that each angular side, by its refractive power on the rays of light coming from the border of the print, and such a portion designedly there placed, will refract to the eye the various parts as one entire figure to be represented; the shape of the glass preventing any appearance of the original figure in the centre, such as the ace of diamonds being seen: so that the ace of clubs being previously and mechanically drawn on the circle of refraction, at six different parts of the border, 1, 2, 3, 4, 5, 6, (Fig. 62.), and artfully disguised there by blending them with it; then the glass in the tube G will change, in appearance, the ace of diamonds into the ace of clubs. And in like manner for the other prints.

Fig. 62.



5. *The Optical Paradox.*

Fig. 63.

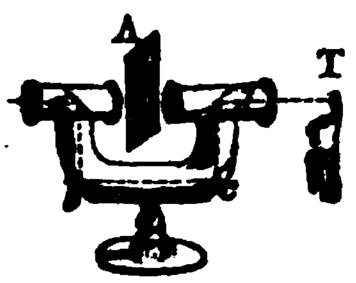


Fig. 63 is a representation of the double perspective, or optical paradox. One of the perspectives of the instrument being placed before the eye, an object will be seen directly through both. A board A, or any opake object, being interposed, will not make the least obstruction to the rays; and the observer will be surprised that he sees through a perspective having the property of penetrating as it were either solid metal or wood.

The artifice in this instrument consists chiefly in four small plane mirrors, *a*, *b*, *c*, *d*, of which *a* and *d* are placed at an angle of 45 degrees in the two perspectives, and *b* and *c* parallel to them in the trunk below; this being so formed as to appear only as a solid handle to the two perspectives. It is hence obvious that, on the principle of catoptrics, the object T, falling on the first mirror *d*, will be reflected down to *c*, thence to *b*, then up to *a*, and so out to the eye, giving the appearance of the straight lineal direction *a d*.

6. *The Endless Gallery.*

Fig. 64.

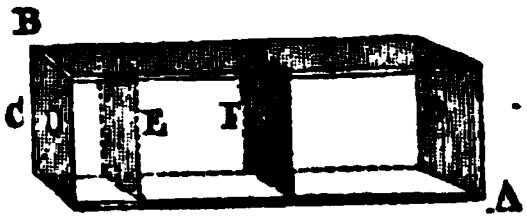


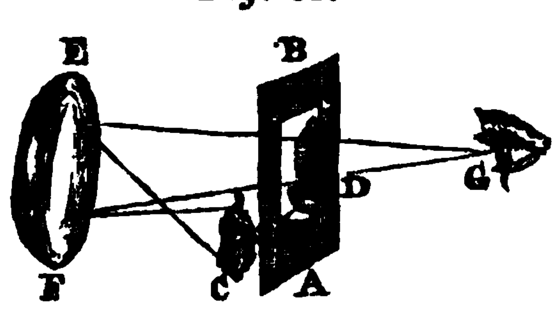
Fig. 64, represents a box, about 18 inches in length, 12 in width, and 9 deep, or any others that are nearly in the same proportions. Against each of its opposite ends A and B within place a plane true-ground glass mirror, as free from veins as possible, of the dimensions nearly equal to the faces, only allowing a small space for a transparent paper, or other cover, at top. From the middle of the mirror C, placed at B, take off neatly a round surface of the silvering, about an inch and half in diameter, against which, in the end of the box, must be cut a hole of the same size, or less. The top of the box should be of glass, covered with gauze, or of oiled transparent paper, to admit as much light as possible into the box. On the two longer sides within must be cut or placed two grooves, at E and F, to receive various drawings or paintings. Indeed many grooves may be cut in the sides, for the reception of a variety of objects. Two paintings or good drawings, of any perspective subject, must be made on the two opposite faces of a pasteboard, such as a forest, gardens, colonnades, &c.; after having cut the blank parts neatly out, place them in the two grooves, E, F, of the box. Take also two other boards, of the same dimensions, painted on one side only with similar subjects, to be placed at the opposite ends C and D; observing that the one which is to be placed against C should have nothing drawn there to prevent the sight, and that the other, for the opposite end D, should also not be very full of figures, that after being neatly cut out, and placed against the glass, it may cover but a small part of it. The top being then closed over with its transparent cover, the instrument is ready for use.

The effect is very striking and entertaining. The eye, being applied to the hole C, will see the various objects drawn on the scenes reflected in a successive and endless manner, by being reflected alternately from each mirror to that which is opposite. As for instance, if they be trees, they will appear an entire grove, very long, seemingly without end; each mirror repeating the objects more faintly, as the reflections are more numerous, and so contributing still more to the illusion.

Ingenuity will suggest a variety of amusing figures, of men, women, &c., to increase the effect. And two mirrors may also be placed on the longer sides, to convey an idea of great breadth, as well as length.

7. *The Real Apparition.*

Fig. 65.



Behind a partition A B, (Fig. 65), place somewhat inclined a concave mirror E F, which must be at least 10 inches in diameter, and its distance equal to three-fourths from its centre. In the partition is cut a square or circular opening, of 7 or 8 inches in diameter, directly opposite to the mirror. Behind this a strong light is so disposed as to illuminate strongly an object placed at C, without shining on the mirror, and without being seen at the opening. Beneath the

aperture, and behind the screen is placed any object at C, which is intended to be represented, but in an inverted position, which may be either a flower, or figure, or picture, &c. Before the partition, and below the aperture, place a flowerpot D, or other pedestal suitable to the object C, so as the top may be even with the bottom of the aperture, and that the eye placed at G may see the flower in the same position as if its stalk came out of the pot. The space between the mirror and the back part of the partition being painted black, to prevent any extraneous light being reflected on the mirror; and indeed the whole disposed so as to be as little enlightened as possible.—Then a person placed at G will perceive the flower, or other object, placed behind the partition, as if standing in the flowerpot or pedestal: but on putting forth his hand to pluck it, he will find that he grasps only at a phantom.

Fig. 66.

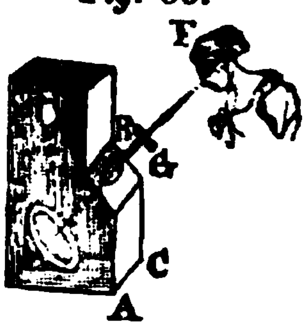


Fig. 66. represents a different position of the mirror and partition, and better adapted for exhibiting effect by various objects. A B C is a thin partition of a room, down to the floor, with an aperture for a good convex lens turned outwards into the room, nearly in a horizontal direction, proper for viewing by the eye of a person standing upright from the floor or footstool. D is a large concave mirror, supported at a proper angle, to reflect upwards through the glass in the partition B, images of objects at E, presented towards the mirror below. A strong light from a lamp, &c. being directed on the object E, and no where else; then to the eye of a spectator at F, in a darkened room, it is truly surprising and admirable to what effect the images are reflected up into the air at G.

It is from this arrangement that a showman, both in London and the country, excited the people to the surprise of wonderful apparitions of various kinds of objects, such as a relative's features for his own, paintings of portraits, plaster figures, flowers, fruit, a sword, dagger, death's head, &c.

The phenomena to be produced by concave mirrors are endless; what have been just described, will be a sufficient specimen of what might be exhibited to elucidate the principles of that curious machine.

PROBLEM LXI.

Is it true that Light is reflected with more Vivacity from Air than from Water?

This assertion is certainly true, provided it be understood in a proper sense, that is as follows: when light tends to pass from air into water, under a certain obliquity, such as 30° for example, the latter reflects fewer rays, than when the light tends, under the same inclination, to pass from the water into air. But what is very singular, if the air were entirely removed, so as to leave a perfect vacuum in its stead, the light, so far from passing with more facility through this vacuum, which could oppose no resistance, would experience more difficulty, and more rays would be reflected in the passage.

We do not know why this has been given in the Philosophical Transactions as a paradoxical novelty; for this kind of phenomenon is a necessary consequence of the law of refraction. When light indeed passes from a rare medium into a denser, as from air into water, the passage is always possible; because the sine of the angle of refraction is less than that of the angle of incidence; that is to say, these sines, in the present case, are in the ratio of 3 to 4. But, on the other hand, when light tends to pass obliquely from water into air, the passage under a certain degree of obliquity is impossible, because the sine of the angle of refraction is always much greater than that of the angle of incidence, their sines, in this case, being in the ratio of 4 to 3. There is therefore a certain obliquity of such a nature, that the sine of the angle of refraction would be much greater than the radius; and this will always happen when the sine of the angle of incidence is greater, however small the excess, than $\frac{3}{4}$ of the

radius, which corresponds to an angle of 48° 36'. But a sine can never exceed radius; consequently it is impossible, in this case, that the ray of light should penetrate the new medium. Thus while light passes from a rare medium into a denser, from air into water for example, under every degree of inclination, there are some rays, viz. all those which form with the refracting substance an angle less than 41° 24', that will not admit of the passage of light from water into air: it is then under the necessity of being reflected, and refraction is changed into reflection. But though light may pass from water into air, under greater angles of inclination, this tendency to be reflected, or this difficulty of proceeding from one medium into another, is continued at all these angles, in such a manner, that fewer rays are reflected when they tend to pass from air into water under an angle of 60°, than when they tend to pass from water into air under the same angle. In the last place, when light tends in a perpendicular direction from water into air, it is more reflected than when it tends to pass in the same direction from air into water.

This truth may be proved by a very simple experiment. Fill a bottle nearly two-thirds with quicksilver, and fill up the other third with water; by which means you will have two parallel surfaces, one of water, and the other of quicksilver. If you then place a luminous object at a mean height between these two surfaces, and the eye on the opposite side at the same height as the object, you will see the object through the bottle, and reflected almost with equal vivacity from the surface of the quicksilver, and from that which separates the water and the air. The air then, in this case, reflects the light with almost as much vivacity as the quicksilver.

Remarks.—1st. We have reason therefore to conclude, that the surface of the water, to beings immersed in that fluid, is a much stronger reflecting mirror, than it is to those beings which are in the air. Fishes see themselves much more distinctly, and clearly, when they swim near the surface of the water, than we see ourselves in the same surface.

2d. Nothing is better calculated than this phenomenon to prove the truth of the reasons assigned by Newton for reflection and refraction. Light passing from a dense fluid into a rarer, is, according to Newton, exactly in the same case as a stone thrown obliquely into the air; if we suppose that the power of gravitation does not act beyond a determinate distance, such for example as 24 feet; for it may be demonstrated, that in this case the deviation of the stone would be exactly the same, and subject to the same law, as that followed by light in refraction. There would also be certain inclinations under which the stone could not pass from this atmosphere of gravity, though there were nothing beyond it capable of resisting it, and even though there were a perfect vacuum.

In this case however we must not say, as a certain celebrated man when explaining the Newtonian philosophy, that a vacuum reflects light: this is only a mode of speaking. To express our ideas correctly, we ought to say that light is sent back with greater force to the dense medium, as the medium beyond it is rarer.

We are far from being satisfied with what is said on this subject in the "Dictionnaire d'Industrie," into which one may be surprised to see optical phenomena introduced; for it is there asserted, that this phenomenon depends on the impenetrability of matter, and the high polish of the reflecting surface. But when light is strongly reflected, during its passage from water into a vacuum, or a space almost free from air, where is the impenetrability of the reflecting substance, since such a space has less impenetrability than air or water? In regard to the polish of the reflecting surface, it is the same, both for the ray which passes from air into water, and that which passes from water into air.

PROBLEM LXII.

Account of a Phenomenon, either not observed, or hitherto neglected by Philosophers.

If you hold your finger in a perpendicular direction very near your eye, that is to say, at the distance of a few inches at most, and look at a candle in such a manner that the edge of your finger shall appear to be very near the flame, you will see the border of the flame coloured red. If you then move the edge of your finger before the flame, so as to suffer only the other border of it to be seen, this border will appear tinged with blue, while the edge of your finger will be coloured red.

If the same experiment be tried with an opake body surrounded by a luminous medium, such for example as the upright bar of a sash window, the colours will appear in a contrary order. When a thread of light only remains between your finger and the bar, the edge of the finger will be tinged with red, and the edge next the bar will be bordered with blue; but when you bring the edge of your finger near the second edge of the bar, so that it shall be entirely concealed, this second edge will be tinged red, and the edge of the finger would doubtless appear to be coloured blue, were it possible that this dark colour could be seen on an obscure and brown ground.

This phenomenon depends no doubt on the different refrangibility of light; but a proper explanation of it has never yet been given.

PROBLEM LXIII.

Of some curious Phenomena in regard to Colours and Vision.

I. When the window is strongly illuminated with the light of the day, look at it steadily and with attention for some minutes, or until your eyes become a little fatigued; if you then shut your eyes, you will see in your eye a representation of the squares which you looked at; but the place of the bars will be luminous and white, while that of the panes will be black and obscure. If you then place your hand before your eyes, in such a manner as absolutely to intercept the remainder of the light which the eye-lids suffer to pass, the phenomenon will be changed; for the squares will then appear luminous, and the bars black: if you remove your hand, the panes will be black again, and the place of the bars luminous.

II. If you look steadily and with attention for some time at a luminous body, such as the sun, when you direct your sight to other objects in a place very much illuminated, you will observe there a black spot; a little less light will make the spot appear blue, and a degree still less will make it become purple: in a place absolutely dark, this spot, which you have at the bottom of your eye, will become luminous.

III. If you look for a long time, and till you are somewhat fatigued, at a printed book through green glasses; on removing the glasses, the paper of the book will appear reddish: but if you look at a book in the same manner, through red glasses; when you lay aside the glasses, the paper of the book will appear greenish.

IV. If you look with attention at a bright red spot on a white ground, as a red wafer on a piece of white paper, you will see, after some time, a blue border around the wafer; if you then turn your eye from the wafer to the white paper, you will see a round spot of delicate green, inclining to blue, which will continue longer according to the time you have looked at the red object, and according as its splendour and brightness have been greater. On directing your eyes to other objects, this impression will gradually become weaker, and at length disappear.

If, instead of a red wafer, you look at a yellow one; on turning your eye to the white ground you will observe a blue spot.

A green wafer on a white ground, viewed in the same manner, will produce in

the eye a spot of a pale purple colour: a blue wafer will produce a spot of a pale red.

In the last place, if a black wafer on a white ground be viewed in the same manner, after looking at it for some time with attention, you will observe a white border form itself around the wafer; and if you then turn your eye to the white ground, you will observe a spot of a brighter white than the ground, and well defined. When you look at a white spot on a black ground, the case will be reversed.

In these experiments, red is opposed to green, and produces it, as green produces red; blue and yellow are also opposed, and produce each other; and the case is the same with black and white, which evidently indicates a constant effect depending on the organization of the eye.

This is what is called the *Accidental colours*, an object first considered by Dr. Jurin, which Buffon afterwards extended, and respecting which he transmitted a memoir in 1743 to the Royal Academy of Sciences. This celebrated man gave no explanation of these phenomena, and only observed that, though certain in regard to the correctness of his experiments, the consequences did not appear to be so well established as to admit of his forming an opinion on the production of these colours. There is reason however to believe that he would have explained the cause, had he not been prevented by other occupations. But this deficiency has been supplied by Dr. Godard of Montpellier; for the explanation which he has given of these phenomena, and several others of the same kind, in the "Journal de Physique" for May and July 1776, seems to be perfectly satisfactory.

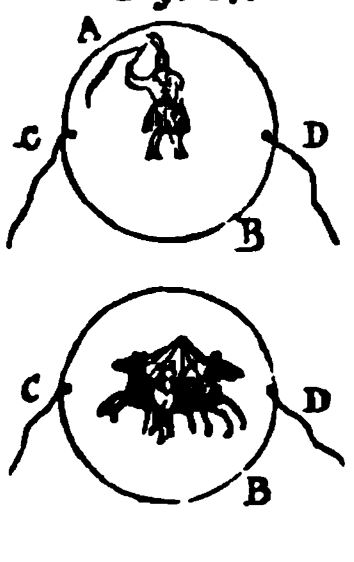
PROBLEM LXIV.

To determine how long the Sensation of Light remains in the Eye.

The following phenomenon, which depends on this duration, is well known. If a fiery stick be moved round in a circular manner, with a motion sufficiently rapid, you will perceive a circle of fire. It is evident that this appearance arises merely from the vibration impressed on the fibres of the retina not being obliterated, when the image of the fiery end of the stick again passes over the same fibres; and therefore, though it is probable that there is only one point of light on the retina, you every moment receive the same sensation as if the luminous point left a continued trace.

But it has been found, by calculating the velocity of a luminous body put in motion, that when it makes its revolution in more than 8 thirds, the string of fire is interrupted; and hence there is reason to conclude, that the impression made on the fibre continues during that interval of time. But it may be asked whether this time is the same for every kind of light, whatever be its intensity? We do not think it is; for a brighter light must excite a livelier and more durable impression.

Fig. 67.



This affection of the eye has been used by Dr. Paris in constructing a toy called the *Thaumatrope*. It is shewn in Fig. 67., where A B is a circle cut out of card, and having two strings C D, fixed to it, by twisting which with the finger and thumb of each hand it may be twirled round rather rapidly. On one side of the cord there is drawn any object, as a chariot, and on the other a charioteer in the attitude of driving,—so that when the cord is twisted round, the charioteer is seen driving the chariot as in the figure,—in consequence of the duration of the impressions of sight on the retina, we see at once what is drawn on both sides of the card.

SUPPLEMENT,

CONTAINING A SHORT ACCOUNT OF THE MOST CURIOUS MICROSCOPICAL OBSERVATIONS.

PHILOSOPHERS were no sooner in possession of the microscope, than they began to employ this wonderful instrument in examining the structure of bodies, which, in consequence of their minuteness, had before eluded their observation. There is scarcely an object in nature to which the microscope has not been applied; and several have exhibited such a spectacle as no one could have ever imagined. What indeed could be more unexpected than the animals or moleculæ (for philosophers are not yet agreed in regard to their animality) which are seen swimming in vinegar, in the infusions of plants, and in the semen of animals? What can be more curious than the mechanism in the organs of the greater part of insects, and particularly those which in general escape our notice; such as the eyes, trunks, feelers, terebræ or augres, &c.? What more worthy of admiration than the composition of the blood, the elements of which we are enabled to perceive by means of the microscope; the texture of the epidermis, the structure of the lichen, that of mouldiness, &c.? We shall here take a view of the principal of these phenomena, and give a short account of the most curious observations of this kind.

I.—*Of the Animals, or Pretended Animals, in Vinegar and the infusions of Plants.*

1st. Leave vinegar exposed for some days to the air, and then place a drop of it on the transparent object-plate of the microscope, whether single or compound: if the object-plate be illuminated from below, you will observe, in this drop of liquor, animals resembling small eels, which are in continual motion. On account of the circumvolutions which they make with their long, slender bodies, they may be justly compared to small serpents.

But it would be wrong, as many simple people have done, to ascribe the acidity of vinegar to the action of these animalcules, whether real or supposed, on the tongue and the organs of taste; for vinegar deprived of them is equally acid, if not more so. These eels indeed, or serpents, are never seen but in vinegar which, having been for some time exposed to the air, is beginning to pass from acidity to putrefaction.

2d. If you infuse pepper, slightly bruised, in pure water for some days, and then expose a drop of it to the microscope, you will behold small animals of another kind, almost without number. They are of a moderately oblong, elliptical form, and are seen in continual motion, going backwards and forwards in all directions; turning aside when they meet each other, or when their passage is stopped by any immoveable mass. Some of them are observed sometimes to lengthen themselves, in order to pass through a narrow space. Certain authors of a lively imagination, it would appear, even pretend to have seen them copulate, and bring forth; but this assertion we are not bound to believe.

If other vegetable bodies be infused in water, you will see animalcules of a different shape. In certain infusions they are of an oval form, with a small bill, and a long tail: in others they have a lengthened shape like lizards: in some they exhibit the appearance of certain caterpillars, or worms, armed with long bristles; and some devour, or seem to devour, their companions.

When the drop in which they swim about, and which to them is like a capacious basin, becomes diminished by the effect of evaporation, they gradually retire towards the middle, where they accumulate themselves, and at length perish, when entirely deprived of moisture. They then appear to be in great distress; writhe their bodies, and endeavour to escape from death, or that state of uneasiness which they experience.

In general, they have a strong aversion to saline or acid liquors. If a small quantity of vitriolic acid be put into a drop of infusion which swarms with these insects, they immediately throw themselves on their backs and expire; sometimes losing their skin, which bursts, and suffers to escape a quantity of small globules that may be often seen through their transparent skin. The case is the same if a little urine be thrown into the infusion.

A question here naturally arises: ought these moveable moleculæ to be considered as animals? On this subject opinions are divided. Buffon thinks they are not animals; and consigns them, as well as spermatic animals, to the class of certain bodies which he calls *organic moleculæ*. But what is meant by the expression organic moleculæ? As this question would require too long discussion, we must refer the reader to the Natural History of that learned and celebrated writer.

Needham also contests the animality of these small bodies, that is to say perfect animality, which consists in feeding, increasing in size, multiplying, and being endowed with spontaneous motion; but he allows them a sort of obscure vitality, and from all his observations he deduces consequences on which he has founded a very singular system. He is of opinion that vegetable matter tends to animalise itself. As the eels produced in flour paste act a conspicuous part in the system of this naturalist, a celebrated writer has omitted no opportunity of ridiculing his ideas, by calling these animals the eels of the jesuit Needham, and representing him as a partisan of spontaneous generation, which has been justly exploded by all the modern philosophers. But ridicule is not reasoning: we are so little acquainted with the boundaries between the vegetable and animal kingdoms, that it would be presuming too much to fix them. But it must be allowed that Needham's ideas on this subject are so obscure, that in our opinion few have been able to comprehend them.

Other naturalists and observers assert the animality of these small beings: for they ask, by what can an animal be better characterised than spontaneity of motion? But these moleculæ, when they meet each other in the course of their movements, retire backwards, not by the effect of a shock, as two elastic bodies would do; but the part which is generally foremost turns aside on the approach of the body that meets it: and sometimes both move a little from their direction, in order to avoid running against each other. They have never yet indeed been seen for certain to copulate, to produce eggs, or even to feed; but the last-mentioned function they may perform, without any apparent act, like the greater part of the other animalcules. The smallness and strange form of these moleculæ can afford no argument against their animality. That of the water polypes is at present no longer doubted, though their form is very extraordinary, and perhaps more so than that of the moving moleculæ of infusions. Why then should animality be refused to the latter?

It might however be replied, in opposition to this supposed similarity, that the polype is seen to increase in size, to regenerate itself, in a way indeed very different from that of the generality of animals, and in particular to feed. The pretended microscopic animals do nothing of the kind, and consequently ought not to be ranked in the same class. But it must be allowed that this subject is still involved in very great obscurity; and therefore prudence requires that we should suspend our opinion respecting it.

II.—*Of Spermatic Animals.*

Of the microscopic discoveries of the last century, none has made a greater noise than that of the moving moleculæ observed in the semen of animals, and which are called *Spermatic animalcules*. This singular discovery was first made and announced by the celebrated Lewenhoek, who observed in the human semen a multitude of small bodies, most of them with very long slender tails, and in continual motion. In size they were much less than the smallest grain of sand, and even so minute in

some seminal liquors, that a hundred thousand, and even a million of them, were not equal to a poppy seed. By another calculation, Lewenhoek has shewn that in the milt of a cod-fish there are more animals of this species than human beings on the whole surface of the earth.

Lewenhoek examined also the prolific liquor of a great many animals, both quadrupeds and birds, and that even of some insects. In all these he observed nearly the same phenomenon; and these researches, since repeated by many other observers, have given rise to a system in regard to generation, which it is unecessary here to explain.

No one however has made more careful, or more correct, observations on this subject than Buffon; and for this reason we shall give a short view of them.

This celebrated naturalist, having procured a considerable quantity of semen, extracted from the seminal vessels of a man who had perished by a violent death, observed in it, when viewed through an excellent microscope, longish filaments, which had a kind of vibratory motion, and which appeared to contain in the inside small bodies. The semen having assumed a little more fluidity, he saw these filaments swell up in some points, and oblong elliptical bodies issue from them; a part of which remained at first attached to the filaments by a very slender long tail. Some time after, when the semen had acquired a still greater degree of fluidity, the filaments disappeared, and nothing remained in the liquor but these oval bodies with tails, by the extremity of which they seemed attached to the fluid, and on which they balanced themselves like a pendulum, having however a progressive motion, though slow, and as it were embarrassed by the adhesion of their tails to the fluid; they exhibited also a sort of heaving motion, which seems to prove that they had not a flat base, but that their transverse section was nearly round. In about twelve or fifteen hours after, the liquor having acquired a still greater degree of fluidity, the small moving moleculæ had lost their tails, and appeared as elliptic bodies, moving with great vivacity. In short, as the matter became attenuated in a greater degree, they divided themselves more and more, so as at length to disappear; or they were precipitated to the bottom of the liquor, and seemed to lose their vitality.

Buffon, while viewing these moving moleculæ, once happened to see them file off like a regiment, seven by seven, or eight by eight, proceeding always in very close bodies towards the same side. Having endeavoured to discover the cause of this appearance, he found that they all proceeded from a mass of filaments accumulated in one corner of the spermatic drop, and which resolved itself successively in this manner into small elongated globules, all without tails. This circumstance reminds us of the singular idea of a naturalist, who observing a similar phenomenon in the semen of a ram, thought he could there see the reason of the peculiar propensity which sheep have to follow each other, when they march together in a flock.

Buffon examined, in like manner, the spermatic liquor of various other animals, such as the bull, the ram, &c., and always discovered the same moleculæ, which at first had tails, and then gradually lost them as the liquor assumed more fluidity. Sometimes they seemed to have no tails, even on their first appearance and formation. In this respect, Buffon's observations differ from those of Lewenhoek, who always describes these animalcules as having tails, with which he says they seem to assist themselves in their movements; and he adds, that they are seen to twist themselves in different directions. Buffon's observations differ also from those of the Dutch naturalist in another respect, as the latter says that he never could discover any trace of these animalcules in the semen or liquor extracted from the ovaria of females; whereas Buffon saw the same moving moleculæ in that liquor, but not so often, and only under certain circumstances.

It appears, from what has been said, that many researches still remain to be made

in regard to the nature of these moving moleculæ; since two observers so celebrated do not agree in all the circumstances of the same fact.

Nothing of this kind is observed in the other animal fluids, such as the blood, lymph, milk, saliva, urine, gall, and chyle; which seems to indicate that these animalcules, or living moleculæ, act a part in generation.

III.—*Of the Animals or Moving Moleculæ in spoiled corn.*

This is another microscopic observation, which may justly be considered as one of the most singular; for if we deduce from it all those consequences which some authors do, it exhibits an instance of a resurrection, repeated, as we may say, at pleasure.

The disease of corn, which produces this phenomenon, is neither smut nor blight, as some authors, for want of a sufficient knowledge in regard to the specific differences of the maladies of grain, have asserted, but what ought properly to be called *abortion*, or *rachitis*. If a grain of corn, in this state, be opened with caution, it will be found filled with a white substance, which readily divides itself into a multitude of small, white elongated bodies, like small eels, swelled up in the middle. While these moleculæ, for we must be allowed as yet to remain neuter in regard to their pretended animality, are in this state of dryness, they exhibit no signs of life; but if moistened with very pure water, they immediately put themselves in motion, and shew every mark of animality. If the fluid drop, in which they are placed, be suffered to dry, they lose their motion; but it may be restored to them at pleasure, even some months after their apparent death, by immersing them in water. Fontana, an Italian naturalist, does not hesitate to consider this phenomenon as a real resurrection. If this circumstance should be verified by repeated observations, and that also of the Peruvian serpent, which may be restored to life by plunging it in the mud, its natural element, several months after it has been suffered to dry at the end of a rope, our ideas respecting animality may be strangely changed. But we must confess that we give very little credit to the latter fact; though Bouguer, who relates it on the authority of Father Gumilla, a Jesuit and a French surgeon, does not entirely disbelieve it. Some other observers, such as Roffredi, pretend to have distinguished, in these eel-formed moleculæ, the aperture of the mouth; that of the female parts of sex, &c. They assert also that they have perceived the motion of the young ones contained in the belly of the mother eel, and that having opened the body, the young were seen to disperse themselves all over the object-plate of the microscope. These observations deserve to be further examined, as a confirmation of them would throw great light on animality.

Note.—It appears from the careful observation of Francis Bauer, Esq., that the molecules in question, the nature of whose existence so puzzled Montucla, are minute parasitical fungi of the genus *uredo*. See an interesting article on the subject in the Penny Magazine for March 31st, 1833, and continued in the No. for May 11th, 1833.

IV.—*On the Movements of the Tremella.*

The tremella is that gelatinous green plant, which forms itself in stagnant water, and which is known to naturalists by the name of *conferva gelatinosa, omnium tenerrima et minima, aquarum limo innascens*. It consists of a number of filaments interwoven through each other, which when considered singly are composed of small parts, about a line in length, united by articulations.

This natural production, when viewed with the naked eye, exhibits nothing remarkable or uncommon; but by means of microscopic observations, two very extraordinary properties have been discovered in it. One is the spontaneous motion with

which these filaments are endowed. If a single one, sufficiently moistened, be placed on the object-plate of the microscope, its extremities are seen to rise and fall alternately, and to move sometimes to the right and sometimes to the left: at the same time, it twists itself in various directions, and without receiving any external impression. Sometimes, instead of appearing extended like a straight line, it forms itself into an oval or irregular curve. If two of them are placed side by side, they become twisted and twined together, and by a sort of imperceptible motion, the one from one side, and the other from the other. This motion has been estimated by Adanson to be about the 400th part of a line per minute.

The other property of this plant is, that it dies and revives, as we may say, several times; for if several filaments, or a mass of tremella, be dried, it entirely loses the faculty above mentioned. It will remain several months in that state of death or sleep; but when immersed in the necessary moisture, it revives, recovers its power of motion, and multiplies as usual.

The Abbé Fontana, a celebrated observer of Parma, does not hesitate, in consequence of these facts, to class the tremella among the number of the Zoophytes; and to consider it as the link which connects the vegetable with the animal kingdom, or the animal with the vegetable; in a word, as an animal or a vegetable endowed with the singular property of being able to die and to revive alternately. But is this a real death, or only a kind of sleep, a suspension of all the faculties in which the life of the plant consists? To answer this question, it would be necessary to know exactly what is the nature of death; a great deal might be said on this subject, were not such disquisitions foreign to the present work.

V.—*Of the Circulation of the Blood.*

Those who are desirous to observe the circulation of the blood by means of the microscope, may easily obtain that satisfaction. The objects employed chiefly for this purpose, are the delicate, transparent member which unites the toes of the frog, and the tail of the tadpole. If this membrane be extended, and fixed on a piece of glass illuminated below, you will observe with great satisfaction the motion of the blood in the vessels with which it is interspersed: you will imagine that you see an archipelago of islands with a rapid current flowing between them.

Take a tadpole, and having wrapped up its body in a piece of thin, moist cloth, place its tail on the object-plate of the microscope, and enlighten it below: you will then see very distinctly the circulation of the blood; which in certain vessels proceeds by a kind of undulations, and in others with an uniform motion. The former are the arteries, in which the blood moves in consequence of the alternate pulsation of the heart; the latter are the veins.

The circulation of the blood may be seen also in the legs and tails of shrimps, by putting these fish into water with a little salt; but their blood is not red. The wings of the locust are also proper for this purpose: in these the observer will see, not without satisfaction, the green globules of their blood carried away by the serosity in which they float. The transparent legs of small spiders, and those of small bugs, will also afford the means of observing the circulation of their blood. The latter exhibit an extraordinary vibration of the vessels, which Mr. Baker says he never saw any where else.

But the most curious of all the spectacles of this kind, is that exhibited by the mesentery of a living frog, applied in particular to the solar microscope, which Mr. Baker tells us he did in company with Dr. Alexander Stuard, physician to the queen. It is impossible to express, says he, the wonderful scene which presented itself to our eyes. We saw at the same moment the blood, which flowed in a prodigious number of vessels, moving in some to one side, and in others to the opposite side. Several of these vessels were magnified to the size of an inch diameter; and the globules of

blood seemed almost as large as grains of pepper, while in some of the vessels, which were much smaller, they could pass only one by one, and were obliged to change their figure into that of an oblong spheroid.

VI.—*Composition of the Blood.*

With the end of a quill, or a very soft brush, take up a small drop of blood just drawn from a vein, and spread it as thin as possible over a bit of talc. If you then apply to your microscope one of the strongest magnifiers, you will distinctly see its globules.

By these means it has been found, that the red globules of the human blood are each composed of six smaller globules, united together; and that when disunited by any cause whatever, they are no longer of a red colour. These red globules are so exceedingly minute, that their diameter is only the 160th part of a line, so that a sphere of a line in diameter would contain 4096000 of them.

VII.—*Of the Skin; its Pores, and Scales.*

If you cut off a small bit of the epidermis by means of a very sharp razor, and place it on the object-plate of the microscope; you will see it covered with a multitude of small scales, so exceedingly minute, that, according to Lewenhoek, a grain of sand would cover two hundred of them; that is to say, in the diameter of a grain of sand there are 14 or 15. These scales are arranged like those on the back of fishes, or like the tiles of a house; that is, each covering the other.

If you are desirous of viewing their form with more convenience, scrape the epidermis with a pen-knife, and put the dust obtained by these means into a drop of water: you will then observe that these scales, in general, have five planes, and that each consists of several strata.

Below these scales are the pores of the epidermis, which when the former are removed may be distinctly perceived, like small holes pierced with an exceedingly fine needle. Lewenhoek counted 120 in the length of a line; so that a line square, 10 of which form an inch, would contain 14400; consequently a square foot would contain 144000000; and as the surface of the human body may be estimated at 14 square feet, it must contain 2016 millions.

Each of these pores corresponds in the skin to an excretory tube, the edge of which is lined with the epidermis. When the epidermis has been detached from the skin, these internal prolongations of the epidermis may be observed in the same manner as we see, in the reverse of a piece of paper, pierced with a blunt needle, the rough edge formed by the surface, which has been torn and turned inwards.

The pores of the skin are more particularly remarkable in the hands and the feet. If you wash your hands well with soap, and look at the palm with a common magnifier, you will see a multitude of furrows, between which the pores are situated. If the body be in a state of perspiration at the time, you will see issuing from these pores a small drop of liquor, which gives to each the appearance of a fountain.

VIII.—*Of the Hair of Animals.*

The hairs of animals, seen through the microscope, appear to be organized bodies, like the other parts; and, by the variety of their texture and conformation, they afford much subject of agreeable observation. In general, they appear to be composed of long, slender, hollow tubes, or of several small hairs covered with a common bark; others, such as those of the Indian deer, are hollow quite through. The bristles of a cat's whiskers, when cut transversely, exhibit the appearance of a medullary part, which occupies the middle, like the pith in a twig of the elder-tree. Those of the hedge-hog contain a real marrow, which is whitish, and formed of radii.

As yet however we are not perfectly certain in regard to the organization of the human hair. Some observers, seeing a white line in the middle, have concluded that it is a vessel which conveys the nutritive juice to the extremity. Others contest this observation, and maintain that it is merely an optical illusion, produced by the convexity of the hair. It appears however that some vessel must be extended lengthwise in the hair, if it be true that blood has been seen to issue at the extremity of the hair cut from persons attacked with that disease called the *Plica Polonica*. But quere, is this observation certain?

IX.—*Singularities in regard to the Eyes of most Insects.*

The greater part of insects have not moveable eyes, which they can cover with eye-lids at pleasure, like other animals. These organs, in the former, are absolutely immoveable; and as they are deprived of that useful covering assigned to others for defending them, nature has supplied this deficiency by forming them of a kind of corneous substance, proper for resisting the shocks to which they might be exposed.

But it is not in this that the great singularity of the eyes of insects consists. We discover by the microscope that these eyes are themselves divided into a prodigious multitude of others much smaller. If we take a common fly, for example, and examine its eyes by the microscope, we shall find that it has on each side of its head a large excrescence, like a flattened hemisphere. This may be perceived without a microscope; but by means of this instrument these hemispheric excrescences will be seen divided into a great number of rhomboids, having in the middle a lenticular convexity, which performs the part of the crystalline humour. Hodierna counted more than 3000 of these rhomboids on one of the eyes of a common fly; M. Puget reckoned 8000 on each eye of another kind of fly, so that there are some of these insects which have 16000 eyes; and there are some which even have a much greater number; for Lewenhoek counted 14000 on each eye of another insect.

These eyes however are not all disposed in the same manner: the dragon fly, for example, besides the two hemispherical excrescences on the sides, has between these two other eminences, the upper and convex surface of which is furnished with a multitude of eyes, directed towards the heavens. The same insect has three also in front, in the form of an obtuse and rounded cone. The case is the same with the fly, but its eyes are less elevated.

It is an agreeable spectacle, says Lewenhoek, to consider this multitude of eyes in insects; for if the observer is placed in a certain manner, the neighbouring objects appear painted on these spherical eminences of a diameter exceedingly small, and by means of the microscope they are seen multiplied, almost as many times as there are eyes, and in such a distinct manner as never can be attained to by art.

A great many more observations might be made in regard to the organs of insects, and their wonderful variety and conformation, but these we shall reserve for another place.

X.—*Of the Mites in Cheese, and other Insects of the same kind.*

If you place on the object-plate of the microscope some of the dust which is formed on the rind and other neighbouring parts of old cheese, it will be seen to swarm with a multitude of small transparent animals, of an oval figure, terminating in a point, and in the form of a snout. These insects are furnished with eight scaly, articulated legs, by means of which they move themselves heavily along, rolling from one side to the other; their head is terminated by an obtuse body in the form of a truncated cone, where the organ through which they feed is apparently situated. Their bodies, particularly the lateral parts, are covered with several long sharp-pointed hairs, and the anus, bordered with hair, is seen in the lower part of the belly.

There are mites of another kind which have only six legs, and which consequently are of a different species.

Others are of a vagabond nature, as the observer calls them, and are found in all places where there are matters proper for their nourishment.

This animal is extremely vivacious; for Lewenhoek says that some of them, which he had attached to a pin before his microscope, lived in that manner eleven weeks.

XI.—*Of the Louse and Flea.*

Both these animals are exceedingly disagreeable, particularly the latter, and do not seem proper for being the subject of microscopic observation; but to the philosopher no object in nature is disagreeable, because deformity is merely relative, and the most hideous animal often exhibits singularities, which serve to make us better acquainted with the infinite variety of the works of the Creator.

If you make a louse fast for a couple of days, and then place it on your hand, you will see it soon attach itself to it, and plunge its trunk into the skin. If viewed in this state by means of a microscope, you will see, through its skin, your blood flowing under the form of a small stream, into its ventricle, or the vessel that supplies its place, and thence distributing itself to the other parts, which will become distended by it.

This animal is one of the most hideous in nature: its head is triangular, and terminates in a sharp point, to which is united its proboscis or sucker. On each side of the head, and at a small distance from its anterior point, are placed two large antennæ, covered with hair; and behind these, towards the two other obtuse angles of the triangle, are the animal's two eyes. The head is united by a short neck to the corslet, which has six legs furnished with hair at the articulations, and with two hooks each at the extremity. The lower part of the belly is almost transparent, and on the sides has a kind of tubercles, the last of which are furnished with two hooks. Dr. Hook, in his "Micrographia," has given the figure of one of these animals, about half a foot in length. Those who see the representation of this insect will not be surprised at the itching on the skin which it occasions to persons, who in consequence of dirtiness are infested with it.

The flea has a great resemblance to the shrimp, as its back is arched in the same manner as the back of that animal. It is covered as it were with a coat of mail, consisting of large scales laid over each other; the hind part is round, and very large in regard to the rest of the body; its head is covered by a single scale, and at the extremity has a kind of three terebræ, by means of which the insect sucks the blood of animals. Six legs, with thighs exceedingly thick, and of which the first pair are remarkably long, enable it to perform all its movements. The great size of the thighs is destined, no doubt, to contain the powerful muscles which are necessary to carry the insect to a height or distance equal to several hundred times its length. Being destined to make such large leaps, it was also necessary that it should be strongly secured against falls to which it might be exposed, and nature has made ample provision against accidents of this kind, by supplying it with scaly armour. Figures of the flea and louse, highly magnified, will be found in the works of Hook and Joblot.

XII.—*Mouldiness.*

Nothing can be more curious than the appearance exhibited by mouldiness, when viewed through the microscope. When seen by the naked eye, one is almost induced to consider it as an irregular tissue of filaments; but the microscope shews that it is nothing else than a small forest of plants, which derive their nourishment from the moist substance, tending towards decomposition, which serves them as a base. The stems of these plants may be plainly distinguished; and sometimes their buds,

some shut and others open. Baron de Munchausen has even done more: when carefully examining these small plants, he observed that they had a great similarity to mushrooms. They are nothing, therefore, but microscopic mushrooms, the tops of which, when they come to maturity, emit an exceedingly fine kind of dust, which is their seed. It is well known that mushrooms spring up in the course of one night; but those of which we here speak, being more rapid, almost in the inverse ratio of their size, grow up in a few hours. Hence the extraordinary progress which mouldiness makes in a very short time.

Another very curious observation of the same kind, made by M. Ahlefeld of Giessen, is as follows: Having seen some stones covered with a sort of dust, he had the curiosity to examine it with a microscope, and found, to his great astonishment, that it consisted of small microscopic mushrooms, raised on very short pedicles, the heads of which, round in the middle, were turned up at the edges. They were striated also from the centre to the circumference, as certain kinds of mushrooms are. He remarked likewise that they contained, above their upper covering, a multitude of small grains, shaped like cherries, somewhat flattened; which in all probability were the seeds. In the last place, he observed, in this forest of mushrooms, several small red insects, which no doubt fed upon them. (See *Act. Leips.* for the year 1739.)

XIII.—*Dust of the Lycoperdon.*

The lycoperdon, or puff-ball, is a plant of the fungus kind, which grows in the form of a tubercle, covered with small grains like shagreen. If pressed with the foot, it bursts, and emits an exceedingly fine kind of dust, which flies off under the appearance of smoke; but commonly a pretty large quantity remains in the half opened cavity of the plant. If some of this dust be placed on the object-plate of the microscope, it appears to consist of perfectly round globules, of an orange colour, the diameter of which is only about the 50th part of a hair; so that each grain of this dust is but the 125000th part of a globule equal in diameter to the breadth of a hair. Some lycoperdons contain browner spherules, attached to a small pedicle. This dust no doubt is the seed of this anomalous plant.

XIV.—*Of the Farina of Flowers.*

It is not long since the utility of this farina in the vegetable economy was known. Before this discovery, it was thought to be nothing else than the excrement of the juices of the flower; but it is shewn by the microscope that this dust is regularly and uniformly organised in each kind of plant. In the mallow, for example, each grain is an opake ball, entirely covered with points. The farina of the tulip, and of most of the lily kind of flowers, has a resemblance to the seeds of cucumbers and melons. That of the poppy resembles a grain of barley, with a longitudinal groove in it.

But we are taught by observation still more; for it is found that this dust or farina is only a capsule, which contains another far more minute; and it is the latter which is the real fecundating dust of plants.

XV.—*Of the rpparent holes in the Leaves of some Plants.*

There are certain plants, the leaves of which appear to be pierced with a multitude of small holes. Of this kind, in particular, is that called by the botanists *hypericum*, and by the vulgar St. John's wort. But if a fragment of one of these leaves be viewed through a microscope, the supposed holes are found to be vesicles, contained in the thickness of the leaf, and covered with an exceedingly thin membrane: in a word, they are the receptacles which contain the essential and aromatic oil peculiar to that plant.

XVI.—*Of the Down of Plants.*

The spectacle exhibited by those plants which have down, such as borage, nettles, &c., is exceedingly curious. When viewed through the microscope, they appear to be so covered with spikes as to excite horror. Those of borage are for the most part bent so as to form an elbow; and, though really very close, they appear by the microscope to be at a considerable distance from each other. Persons who are not previously told what substance they are looking at, will almost be induced to believe that they see the skin of a porcupine.

XVII.—*Of the Sparks struck from a piece of Steel by means of a Flint.*

If sparks struck from a piece of steel by a flint be made to fall on a leaf of paper, they will be found, for the most part, to be globules, formed of small particles of steel, detached by the shock, and fused by the friction. Dr. Hook observed some which were perfectly smooth, and reflected with vivacity the image of a neighbouring window. When in this state, they are susceptible of being attracted by the magnet; but very often they are reduced by the fusion to a kind of scoria, and in that case the magnet has no power over them. The cause of this we shall explain hereafter. This fusion will excite no surprise, when it is known that the bodies most difficult to be liquefied need only, for that purpose, to be reduced to very minute particles.

XVIII.—*Of the Asperities of certain bodies, which appear to be exceedingly sharp and highly polished.*

If a needle, apparently very sharp, be viewed through the microscope, it will seem to have a very blunt, irregular point, much resembling that of a peg broken at the end.

The case is the same with the edge of the best set razor. When viewed through the microscope, it will appear like the back of a penknife, and at certain distances exhibit indentations like the teeth of a saw, but irregular.

If a piece of the highest polished glass be exposed to the microscope, you will be much astonished at its appearance: it will be seen furrowed, and filled with asperities, which reflect the light in an irregular manner, making it assume different colours. The case is the same with the best polished steel.

Art, in this respect, is far inferior to nature; for if works which have been made and polished, as we may say, by the latter, are exposed to the microscope, instead of losing their polish, they appear with greater lustre. When the eyes of a fly, if illuminated by means of a lamp or taper, are viewed through this instrument, each of them exhibits an image of the taper with a precision and vivacity which nothing can equal.

XIX.—*Of Sand seen through the Microscope.*

It is well known that there are some kinds of sand calcareous, and others vitrifiable. The former, seen through the microscope, resembles in a great measure large irregular fragments of rock. The most curious spectacle however is exhibited by the vitreous kind: when it consists of rolled sand, it appears like so many rough diamonds, and sometimes like polished ones. One kind of sand, when seen through the microscope, appears to be an assemblage of diamonds, rubies, and emeralds: another presents the embryos of shells, exceedingly small.

XX.—*Of the Pores of Charcoal.*

Dr. Hook had the curiosity to examine with a microscope the texture of charcoal, which he found to be filled with pores regularly arranged, and passing through its whole length: hence it appears that there is no charcoal into which the air does not

introduce itself. This observer, in the 18th part of an inch, counted 150 of these pores; from which it follows, that in a piece of charcoal, an inch in diameter, there are about 5720000.

On this subject we have been obliged, agreeably to our plan, to be exceedingly brief; but, to supply this deficiency, we shall here point out the principal works which contain micrographic observations, and the authors who have particularly applied themselves to this kind of study. The first we shall mention is Father Bonnani, a Jesuit, author of a book entitled "Ricreazione dell'ochio è della mente," part of which is entirely devoted to this subject. The celebrated Lewenhoek spent almost the whole of his life in the same occupation, and published the observations he made in his "Arcana Naturæ." A great many observations of this kind may be found scattered here and there throughout all the Journals and Memoirs of learned Societies. But few have made so many researches on this subject as M. Joblot, author of a quarto volume, entitled "Description et usages de plusieurs nouveaux Microscopes, &c., avec de nouveaux observations sur un multitude innombrable d'insectes, &c., qui naissent dans les liqueurs, &c. Paris, 1716." He infused in water a great number of different substances, and caused the small animals produced by these infusions to be engraved: to the greater part of them he has even given names, derived from their resemblance to known bodies, or from other circumstances. But we must refer the reader to the work itself, which was republished in 1754, considerably enlarged, under this title: "Observations d'Histoire Naturelle, faites avec le Microscope sur un grand nombre d'Insectes, et sur les Animalcules qui se trouvent dans les liqueurs preparées et non preparées, &c.," 4to, with a great number of plates. Needham, in the year 1750, published his work, called "New Microscopical Observations." Buffon's observations on spermatic moleculæ may be seen in his work on Natural History. We have also Baker's works, entitled "The Microscope made Easy, and Employment for the Microscope." The first part contains a description of the apparatus and the method of using different kinds of microscopes, and the second a very long detail of microscopical observations made on various natural objects. This work was attended with great success, and is exceedingly instructive. The Abbé Spallanzani caused his microscopical observations, in which he several times contradicts Needham, to be printed in Italian; a French translation, entitled "Nouvelles Observations Microscopiques," was published, in octavo, at Paris, in 1769, with notes by the above philosopher. If to these be added various Memoirs by Fontana, Roffredi, Spallanzani, &c., published in the "Journal de Physique," we shall have enumerated all the writings, or at least the principal ones, which have hitherto appeared on this subject.

We shall add to the preceding problems on Optics in Montucla's work, a brief summary of the modern discoveries in this branch of science.

Fixed Lines in the Prismatic Spectrum.

By viewing through a telescope the spectrum formed from a narrow beam of solar light by a fine prism of flint glass, Fraunhofer of Munich discovered that the spectrum was covered throughout by dark lines of different widths, perpendicular to the direction of the length of the spectrum, none of the lines coinciding with the boundary of any of the coloured spaces. There are not less than 600 of these lines. Several of them are sensibly broader than the others, and may be discovered with comparative ease. One is near the outer end of the *red* space; a broad and dark

one is near the middle of the *red*, a strong double line is in the middle of the *orange*, one in the *green* consists of several lines, a very strong one is in the *blue*, one in the *indigo*, and one in the *violet*.

Similar bands are seen in the light of the planets, fixed stars, the electric spark, and coloured flowers, but they are not found in the spectrum formed by the light of a lamp; but in the orange portion of the lamp-light spectrum, there is a line brighter than the rest of the spectrum.

On the Heating Power of the Spectrum.

Dr. Herschel found by experiments that the heating power of the spectrum gradually increased from the violet to the red extremity, and that the thermometer continued to rise beyond the red end, where no light whatever could be perceived. Hence he drew the conclusion *that there are invisible rays in the light of the sun, which produce heat, and which have a less refrangibility than red light.*

Mr. Seebeck however, who has recently experimented on the subject, shews that the place of maximum heat in the spectrum varies with the substance of which the prism is made. Thus with *water* the maximum of heat was in the *yellow* portion of the spectrum; with *sulphuric acid*, it was in the *orange;* with *crown glass*, in the middle of the *red;* and with *flint glass*, (the material which Herschel used) *beyond the red.*

On the Chemical Influence of the Spectrum.

It has long been known that *lunar caustic* becomes black when exposed to the light, and very speedily so when exposed to the light of the sun. When exposed to the light of the spectrum, it is found to become very soon black *beyond* the violet extremity, less readily so in the violet, and so on towards the red end; and when a little blackened by exposure near the violet end, its colour is partially restored by exposure in the red rays.

On the Magnetizing power of the Solar Rays.

Some years ago Dr. Morichini announced that by collecting the violet rays in the focus of a convex lens, and carrying the focus from the middle of one half of a needle to the extremities of that half. and continuing the operation for an hour, the needle acquired perfect polarity. The experiment was repeated sometimes with, and sometimes without, success, by various scientific persons; but the truth of the fact announced has recently been put beyond dispute by some experiments made by our distinguished countrywoman, *Mrs. Somerville.*

Having covered with paper one half of a sewing needle, quite devoid of magnetism, Mrs. S. exposed the uncovered half to the violet rays, and in about two hours the needle had become decidedly magnetized, the exposed end exhibiting north polarity. The indigo rays magnetized a needle with nearly the same facility as the blue ones; and the blue and green produced also a small analogous effect, but the other rays produced no sensible effect whatever.

When the rays were concentrated by means of a lens the effect was produced more speedily; and what is very remarkable, the magnetic effect was produced by exposing the needle half covered with paper to the sun's rays transmitted through *green* glass, or through glass coloured *blue* with cobalt. The light transmitted through a blue or green riband produced the same effect, and when the needles thus covered had hung a day or two in the sun's rays, behind a pane of glass, their exposed ends were north poles, as when the effect was produced by the rays of the spectrum.

These peculiar properties of light have acquired increased importance from the singular and most wonderful application that has been recently made of

some of them to taking drawings and views of matchless accuracy, and of delicacy beyond the reach of all human art.

About twelve months ago it was announced that M. Daguerre, well known for the beautiful and interesting dioramic pictures executed by him, and exhibited in most of the principal cities of Europe, had discovered a mode of fixing the images formed in the Camera Obscura and of producing pictures which exceeded in delicacy any thing that had ever before been seen.

On this announcement reaching England, Mr. Fox Talbot immediately laid before the Royal Society an account of a method which he had practised for some years, and which it was thought might possibly be the same in substance as the process discovered by the ingenious Frenchman. A comparison of the results shewed, however, that the processes of M. Daguerre and Mr. Talbot were by no means identical. Both methods, indeed, gave permanent pictures from images formed in the Camera; but Mr. Talbot's pictures had dark shades where light colours were in the object copied, and lights where in nature the colours were dark. On the contrary, in Daguerre's pictures, light colours were represented by light shades, and dark colours by dark shades; and the gradations of the shades—the translation, so to speak, from *colour in nature* to *shade in the picture*, exhibited a degree of perfection utterly unattainable by art. Still, however, the process of Mr. Talbot has its own useful and independent application in cases where that of M. Daguerre, admirable as it is, does not apply at all. We shall give such an account of both processes as, we trust, will enable an attentive reader to practise either with success; beginning with

PHOTOGENIC DRAWING.

Having pasted a bottle carefully over with paper, dissolve in it a quantity of *nitrate of silver*, (lunar caustic) in distilled water, putting one drachm of the nitrate to four table spoonfuls of water; and taking care that neither the nitrate in its solid state, nor the solution, is exposed to the light of day. Put the bottle away in a dark closet for use. Fill another bottle with a saturated solution of common salt and water; it is not necessary that the water should be distilled. Take a sheet of stout writing paper, such as Bath post, dip it in a solution composed of from ten to twenty parts of pure water to *one* of the saturated solution of salt. Press the wetted paper between sheets of blotting paper, and then dry it at the fire.

In a room from which day-light is carefully excluded, wash over one side of the paper *twice* with the solution of nitrate of silver, using for the purpose a large camel's hair pencil, and dry the paper after each washing.

The paper may now in general be considered as prepared, and it will be found sufficiently sensitive for making photogenic pictures by the direct action of the sun's rays. But if it is intended for taking pictures by means of reflected light, as in the Camera Obscura, it will be necessary to go over the whole process of preparation again: dipping in the salt and water, drying,—washing with the solution of nitrate of silver, and drying again. By proceeding in this way, the sensibility of the paper may be increased almost indefinitely: but it is very difficult to fix images on paper of extreme sensibility, as the darkening process is apt to proceed, whatever process may be adopted for checking it.

Having prepared paper of the requisite sensibility for the object in view, we shall suppose that it is desired to have on the paper a representation of some subject, as shewn in the Camera Obscura.

Having fixed upon the point of view, draw out the slider of the Camera till a distinct image of the object in view is obtained in the focus, or a piece of common paper. Then, in a room from which day-light is shut out, replace the piece of common paper in the Camera with a piece of prepared, highly sensitive photogenic paper (the object glass being covered); and shut down the posterior flap of the

Camera; and removing the instrument to the selected point of view, take off the cover of the object-glass, and allow the image of the object which it is desired to have represented to be formed on the photogenic paper. If the day is bright, a distinct image of the object will be found imprinted on the paper in from 15 to 30 minutes. Then, the cover being put again upon the object-glass, remove the Camera with the paper in it to the dark chamber; and, withdrawing the picture, immerse it immediately in cold water for 15 or 20 minutes. It may be advisable to change the water occasionally, to wash out as much as possible of the nitrate of silver remaining unchanged on the paper. Dry the paper at the fire, and make a solution of one table spoonful of the saturated solution of salt and water, and three or four spoonfuls of pure water,—adding the bulk of two pins' heads of iodide of potassium. Soak the paper in the solution, and dry it at the fire, and in all ordinary cases the image will be found to be fixed, being altogether insensible to the further action of light.

To make an accurate Drawing of any object, by means of Solar Light.

Take a piece of prepared paper large enough for the object in view, and in a darkened room place the object on the side of the paper which has been washed with the nitrate of silver, and place over the object a piece of good window glass.

It may be well to place the paper on a book, and to keep the object in accurate contact with the paper, by pressure on the edge of the glass; for absolute contrast is essential to the perfection of the picture.

The arrangements may be more conveniently made by means of a small drawing board, with a cushioned back-board to lay the paper and the object upon, and a pane of glass adapted to the board to lay upon the object; and the whole can be fastened into the frame by the cross-sticks at the back of the cushioned board, as the paper is fastened into the frame for drawing.

Let the object under the glass be suddenly placed in the direct rays of the sun, and in a few minutes a perfect image of it will be formed on the paper; the parts of the paper uncovered will be black, or nearly so; those parts of the object through which the solar rays pass most freely will be darkest in the picture, and such parts as totally exclude the rays of light will be white; the other parts of different degrees of shade, depending on the transparency of different parts of the object.

It may be stated, generally, with respect to colour, that lights in the object give darks in the picture, and *vice versa*.

If the picture be *fixed* as directed above, *it* may be used, as the object was, to produce another picture in which lights, shades, and direction will be again reversed; but this second picture, is always less distinct than the first one, because all the imperfections of the paper through which the rays penetrate to the prepared paper beneath, are represented in the transferred picture. Copies of engravings may be taken in the same way, by laying the face of the engraving on the prepared side of the photogenic paper; but here again both lights, shadows, and positions will be reversed, and the want of uniformity in the texture of the paper on which the engraving is made will impair the distinctness of the photogenic copy; and if this copy be fixed and used to obtain a reverse, the indistinctness in the reverse will be still further increased from the same cause. But both the copies from the original, and the transfer from the copy, are often very beautiful. Feathers, finely veined leaves, and many grasses, form very interesting photogenic pictures; and the use of the art in preserving pictures, *fac-similes* of rare and delicate plants, &c., is very obvious.

A pleasing application of the art, to persons who draw with confidence and taste, and are desirous of multiplying copies of their sketches, may be called

Photogenic Etching.

Smoke one side of a piece of window glass over the flame of a candle, and

with a pointed implement make a drawing of any object on the smoked side of the glass.

Lay the clean side of this glass on a piece of prepared paper, and expose the drawing for a short time to the sun, when a perfect copy of the drawing will be imprinted on the paper, the part hid from the sun by the soot being the white ground of the copy.

This smoked glass, with the drawing upon it, may be used to take any number of copies in succession; but the pictures, as they are taken, must be kept from the light till they are fixed.

It may again be noticed, that any imperfection in the contact of the glass and the paper injures the distinctness of the impression.

Many other applications of the process will readily suggest themselves to persons who may be inclined to practise it.

We now proceed to give directions for producing pictures with the aid of the Camera, by the more delicate process of M. Daguerre; availing ourselves chiefly of the official account published by the discoverer, in compliance with an agreement made with the French government, which has settled an annuity of 6000 francs on M. Daguerre, and one of 4000 francs on his associate, M. Neipce, as a recompence for making the particulars of the discovery known for the benefit of the public at large.

DAGUERREOTYPE.

The pictures are formed on thin plates of the purest silver plated on copper; the copper being of sufficient thickness to maintain perfect smoothness and flatness in the silver plate; but the thickness of both ought not to exceed that of a stout card, and the size of the plate will depend on that of the Camera.

Powder the surface of the plate with the finest pumice stone; then laying the plate on several folds of paper, take some cotton dipped in a little olive oil, rub the plate gently, rounding the strokes. The pumice stone and cotton must be changed several times. When the plate is well polished, it must be cleaned by powdering it over again with pumice stone, rounding and crossing the strokes to obtain a flat surface.

Roll up a little pledget of cotton, and moisten it with a diluted solution of nitric acid (one part of acid to eight of distilled water), applying the cotton to the mouth of the phial containing the solution, so that the centre of the cotton only may be slightly wetted; and with the pledget so moistened rub the surface of the plate equally. Change the cotton and rub on, rounding the strokes till the acid is perfectly spread, and forms a thin film on the surface. Again powder with pumice, and clean with fresh cotton, rubbing as before, but very slightly.

Put the plate with the silver side upwards in a wire frame, and to the copper side apply the flame of a spirit lamp, the flame playing upon and touching the copper as the lamp is carried round. Continuing this for about five minutes, a strong white coating will be formed on the surface of the silver. Withdraw the lamp, and cool the plate *suddenly* by placing it on a cold substance, as a marble table.

When perfectly cooled, polish it again with dry pumice stone and cotton, repeated several times. Repeat the operations with the acid; and polishing afterwards with dry pumice stone and cotton *thrice;* taking care not to breathe upon the plate, or to touch it with the fingers, or even with the cotton on which the fingers have rested.

Put the plate into a frame, and invert it, so that the silver face may be downward on the top of a box, at the bottom of which is a quantity of *iodine* broken into small pieces, and contained in a little dish. Let the plate remain in this position, till the

condensation of the vapour of the iodine has covered the surface with a fine coating of a yellow gold colour. The time for effecting this may vary from five minutes to half an hour, according to the operation; the operator examines from time to time how the process is proceeding. This process is conducted in a darkened apartment, and the examinations are made by a little light admitted sideways, not from the roof. Lifting the plate with the frame with both hands, and turning it up quickly, the operator sees at a glance the true colour of the coating; very little light sufficing for the purpose. If too pale, the plate is instantly replaced; if the gold tint is passed, the coating is useless, and the whole of the operations must be gone over *de novo.*

Having previously adjusted the Camera to focus, place the plate in the Camera; and placing the Camera in front of the landscape, uncover the lens, and allow the light to form a picture of what is before the Camera on the coated surface of the silver; all light except that from the object glass being rigidly excluded. The lens of the Camera ought to be *periscopic.*

The time which the plate must remain in the Camera depends entirely on the intensity of the light of the objects whose image is to be depicted. At Paris it may vary from three to thirty minutes.

When it is conceived that the plate has remained long enough, the Camera with the plate in it must be removed to a darkened chamber; the only light admissible being that of a taper.

The plate removed, nothing whatever will be perceived upon its surface; its appearance is an absolute blank.

Put it into a box, the face inclining forwards 45 degrees from the perpendicular; and at the bottom of the box place a cup containing mercury. Putting on the lid of the box, place a spirit lamp below the mercury, till it is raised to a temperature of 60° centrigrade. Withdraw the lamp immediately, and after continuing to rise some time, the thermometer, by which the heat of the mercury is measured, will begin to fall. When it falls to 45° centrigrade, withdraw the plate.

A plate of glass is placed in this mercurialising box in front of the plate; so that by means of a feeble taper the operator can see the gradual development of the picture under the influence of the mercurial vapour.

The impress of nature is on the plate when it is removed from the Camera, but it is invisible; and it is not till after several minutes' exposure to the mercurial vapour that the faint traces of objects begin to appear.

After the mercurialising process has been completed, the next object is to fix the image.

Plunge the plate into a plate of common water, and withdraw it immediately, the surface merely requiring to be moistened. Then plunge it into a saturated solution of common salt; or, which is better, into a saturated solution of hyposulphate of soda, moving the plate about in the solution by means of a hook of copper wire. When the yellow colour is quite gone, lift up the plate with both hands, taking care not to touch the drawing; and plunge it again into a trough of pure water.

Lay the plate immediately upon an inclined plane; pour over it, in a stream, hot but not boiling water, to carry off what may remain of the saline solution.

If any drops of water remain on the drawing, they must be *blown* off; for by drying they would leave stains on the drawing.

It remains only now to place the plate in a square of strong pasteboard, covered by a glass; and to frame the whole in wood; and if all the operations have been successfully conducted, a production will be the result which, with respect to delicacy and faithfulness, is unapproachable by any art previously known.

On the Inflexion and Diffraction of Light.

To observe the action of bodies on light passing near them, let a lens of short focus be fixed in a window shutter, and a beam of sun light be transmitted through the lens. This light will diverge from the focus of the lens and form a circular image of light on the opposite wall. The shadows of all bodies held in this light will be found to be surrounded with three coloured fringes. The *first*, reckoning from the shadow, will be, violet, indigo, pale blue, green, yellow, red. The *second*, blue, yellow, *red*; and *the third*, pale blue, pale yellow, pale red.

These fringes may be conveniently examined by means of a lens, and they present various interesting phenomena, according as the light is the direct solar ray, or the different primitive portions of the spectrum; and even according to the shape of the aperture through which the light is admitted.

Of the Colours of Thin Plates.

The thinnest transparent film that can be generally met with will both reflect and transmit light which is *white* or *colourless*; but if the thickness be diminished to a very great degree, the reflected and transmitted light are both *coloured.*

A soap bubble is a familiar and beautiful illustration of the colours produced by reflection and refraction from and through thin plates. The colours of the oxidated film on glass which has been long exposed to the weather, is another example; and if a piece of sealing wax be stuck to a plate of mica, and detached with a jerk, extremely thin filaments will adhere to the wax, and they will exhibit the most brilliant colours by reflected light.

If we blow a soap bubble, and cover it with a clear glass to protect it from currents of air; after it has grown thin by standing awhile, a great many concentric coloured rings will be observed round the top of it. As the bubble grows thinner the rings will dilate; the central spot will become in succession white, bluish, and then black, after which from the extreme thinness of the black part, the bubble will burst.

Of the Colours of Thick Plates.

The colours of thick plates may be seen with a candle held before the eye, ten or twelve feet from a pane of crown glass in a window on which has been a fine deposition of moisture or of dust.

But these colours may be seen to great advantage by means of two equally thick plates of glass placed near to and above each other, and nearly, but not quite, parallel.

If a ray of condensed light, subtending about 2°, fall nearly perpendicularly on the upper plate, and the eye be placed behind the plate, several reflected images will be seen in a row, besides the direct one. The field will be seen crossed by numerous beautiful bands of colour; the central bands consist of blackish and whitish stripes, and the exterior of brilliant bands of *red* and *green* light; the direction of the bands being parallel to the common section of the inclined planes.

On the Colours of Grooved Surfaces.

It has long been known that the beautiful play of colours exhibited by mother of pearl and some other substances is derived from their surfaces being covered with minute grooves; and the late ingenious Mr. Barton, of the Royal Mint, made a beautiful application of this property in the production of what he very appropriately called *Iris* ornaments. By means of a delicate machine he was enabled to cut, with a diamond or polished steel, parallel grooves at the distance of from the 2000th to the

10000th part of an inch apart; and the light reflected from the finely grooved surface exhibited the most beautiful prismatic colours. They were formed into buttons for dress coats for gentlemen, and into articles of ornament for ladies, arranged in patterns.

In forming the buttons, patterns were cut in steel, which was afterwards hardened and used as a *die* to stamp the impressions on buttons made of brass. In sun-light, gas-light, or even brilliant lamp or candle-light, the brilliancy of the colours of these ornaments was scarcely surpassed by the flashes of the diamond.

Perhaps it was because these very beautiful articles were soon supplied at a *cheap* rate, that they have ceased to be much used in fashionable life; they are, however, very elegant; and, as a branch of Optics, the phenomena which they exhibit form a most interesting object of contemplation.

On the Absorption of Light.

All bodies absorb light. On the summits of high mountains, where light from celestial objects has to pass through a thinner stratum of air, a much greater number of stars are visible to the eye than on the plains below; and through great depths of matter objects become almost invisible. The absorptive power of air is finely displayed in the colour of the morning and evening clouds; and that of water, in the red colour of even the meridian sun, when seen from a diving bell at a great depth in the sea.

These appearances are caused by the absorption of one class of rays in passing through the air or the water, while the rest make their way, either directly or by reflection, to the eye.

Charcoal, in its ordinary state, is the most absorbent of all bodies; but in some particular states of combination—in gas or in flame—as forming the essential constituent of the diamond, it is very transparent. Metals are transparent in a state of solution; and silver and gold, when beaten very thin, transmit light, the former blue, the latter green.

Some clouds absorb blue rays, and transmit red; others absorb all rays in equal proportions, and exhibit the sun through them perfectly white. The image of the sun, as seen through diluted *ink*, is also quite white.

The absorbing power of different bodies is variously modified by heat and other circumstances. Pure phosphorus, which is of a slightly yellow colour, transmits freely almost all the coloured rays. When melted, and gradually cooled, it absorbs all the colours of the spectrum at thicknesses at which it formerly transmitted them.

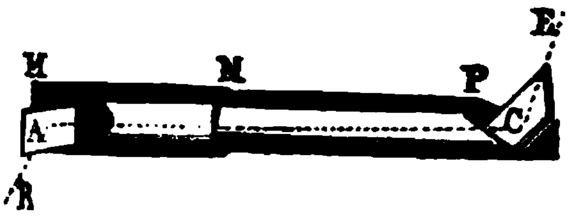
On the Polarisation of Light.

When light emitted from the sun or any self-luminous body, is reflected from the surface, or transmitted through the surface, of any homogeneous uncrystallized body, the reflected or transmitted light continues the same when we turn round the body; so that the light falls upon it at the same angle, or the different *sides* of the rays from the same paper lie with reference to the angle of incidence. Such light is called *common light.*

But a kind of light has been discovered which exhibits different properties with respect to the angle of incidence, according as the reflecting or transmitting surface is presented to one *side* or another of the incident ray. Such light is called *polarised light.*

Whenever this light is obtained it must have previously existed in the form of common light; from which it may be obtained *by reflection* from the surface of transparent and opake bodies, by transmission through plates or planes of uncrystallized bodies, or by transmission through bodies regularly crystallized, and possessing the property of double refraction.

Fig. 68.



To explain the difference between common and polarised light, let A (Fig. 68.), be a plate of glass so placed at the end of a tube M N, that a ray of light incident at A, may be reflected along the axis of the tube M N. At the end of another smaller tube N P, which can turn round within M N, place a similar piece of glass capable of reflecting a ray A C, to the eye at E.

Let a ray of light, R A, fall upon the vertical plate of glass A, at an angle of 56°, and incline the plate A to the axis A C, so that the ray may be reflected along A C, and from C again at an angle of 56° to E. Then, when the first reflection is horizontal, and the second vertical, or when R A C is a horizontal plane, and A C F a vertical one, the ray C E will be so weak as to be scarcely visible. But if we turn the tube N P within M N without moving A or M N, or altering the inclination of C to A C, the ray C E will become stronger and stronger, till it has been turned round 90°, or that A C E is in a horizontal plane as well as R A C, when the light of the beam C E attains its maximum—continuing to turn the tube it will become fainter and fainter, till, after being turned 90° more, when the plane A C E is in the opposite vertical C E, it will again be invisible; at the next 90° the brightness of the ray will be at its maximum, and on completing the revolution it will vanish again.

This experiment shews that when either the upper or the under side of the ray is nearest C, the plate is incapable of reflecting it; but that when the right or the left side is nearest the reflecting plate, the plate reflects the ray as if it were common light, and in intermediate positions, intermediate portions of light are reflected. The ray A C has therefore properties different from common light, and we hence conclude that a ray of common light, as R A, reflected from glass at an angle of 56°, becomes polarized by the reflection. If the original beam R A has considerable intensity, the reflected pencil C E does not wholly vanish, and the part remaining visible is coloured.

This branch of Optics is fertile in striking phenomena; but it is of such extent that we must, in this place, content ourselves with referring to works in which room has admitted its being treated in requisite detail. Dr. Brewster's Volume in Lardner's Cyclopædia, and the article *Optics*, apparently by the same author, in the Library of Useful Knowledge, may be consulted with advantage.

PART FIFTH.

CONTAINING EVERY THING MOST CURIOUS IN REGARD TO ACOUSTICS AND MUSIC.

The ancients seem to have considered sounds under no other point of view than that of Music; that is to say, as affecting the ear in an agreeable manner. It is even very doubtful whether they were acquainted with any thing more than melody, and whether they had any art similar to what we call composition. The moderns, however, by studying the philosophy of sounds, have made many discoveries in this department, so much neglected by the ancients; and hence has arisen a new science, distinguished by the name of Acoustics. Acoustics have for their object the nature of sounds, considered in general, both in a mathematical and a philosophical view. This science therefore comprehends music, which considers the ratios of sounds, so far as they are agreeable to the ear, either by their succession, which constitutes melody, or by their simultaneity, which forms harmony. We shall here give an account of every thing most curious and interesting in regard to this science.

ARTICLE I.

Definition of Sound; how diffused and transmitted to our organs of hearing; experiments on this subject; different ways of producing Sound.

Sound is nothing else but the vibration of the particles of air, occasioned either by some sudden agitation of a certain mass of the atmosphere violently compressed or expanded; or by the communication of the vibration of the minute parts of hard and elastic bodies.

These are the two best known ways of producing sound. The explosion of a pistol, or any other kind of fire arms, produces a report or sound, because the air or elastic fluid contained in the gunpowder, being suddenly dilated, compresses the external air with great violence: the latter, in consequence of its elasticity, re-acts on the surrounding atmosphere, and produces in its moleculæ an oscillatory motion, which occasions the sound, and which extends to a greater or less distance, according to the intensity of the cause that gave rise to it.

To form a proper idea of this phenomenon, let us conceive a series of springs, all maintaining each other in equilibrium, and that the first is suddenly compressed in a violent manner by some shock, or other cause. By making an effort to recover its former situation, it will compress the one next to it, the latter will compress the third; and the same thing will take place to the last, or at least to a very great distance; for the second will be somewhat less compressed than the first, the third a little less than the second, and so on; so that, at a certain distance, the compression will be almost insensible, and at length it will totally cease. But each of these springs, in recovering itself, will pass a little beyond the point of equilibrium, and this will occasion, throughout the whole series put in motion, a vibration, which will continue for a longer or shorter time, and at length cease. Hence it happens that no sound is instantaneous, but always continues, more or less, according to circumstances.

The other method of producing sound, is to excite, in an elastic body, vibrations sufficiently rapid to occasion, in the surrounding parts of the air, a similar motion. Thus, an extended string, when struck, emits a sound, and its oscillations, that is to say, its motion backward and forward, may be distinctly seen. The elastic parts of the air, struck by the string during the time it is vibrating, are themselves put into a state of vibration, and communicate this motion to the neighbouring ones. Such is the mechanism by which a bell produces its sound: when struck, its vibrations are sensible to the hand which touches it.

Should these facts be doubted, the following experiments will exhibit the truth of them in the clearest point of view.

Experiment 1.

Half fill a vessel, such as a drinking glass, with water; and having made it fast, moisten your finger a little, and move it round the edge. By these means a sound will be produced, and at the same time you will see the water tremble, and form undulations so as to throw up small drops. What but the vibration of the particles of the water can produce in it such a motion?

Experiment 2.

If a bell be suspended in the receiver of an air-pump, so as not to touch any part of the machine; it will be found, on the bell being made to sound, that as the air is evacuated and becomes rarer, the sound grows weaker and weaker, and that it ceases entirely when as complete a vacuum as possible has been effected. If the air be gradually re-admitted, the sound will be revived, as we may say, and will increase in proportion as the air contained in the machine approaches towards the same state as that of the atmosphere.

From these two experiments it results, that sound considered in the sonorous bodies, is nothing else than rapid vibrations of their minute parts: that air is the vehicle of it; and that it is transmitted so much the better when the air by its density is itself susceptible of a similar motion.

In regard to the manner in which sound affects the mind, we must first observe that at the interior entrance of the ear, which contains the different parts of the organ of hearing, there is a membrane extended like that of a drum, and which on that account is called the *tympanum*. It is very probable that the vibrations of the air, produced by the sonorous body, excite vibrations in this membrane; that these produce similar ones in the air with which the internal cavity of the ear is filled; and that the sound is increased by the peculiar construction of the parts, and the circumvolutions both of the semicircular canals and of the helix: hence there is occasioned in the nerves that cover the helix, a motion which is transmitted to the brain, and by which the mind receives the perception of sound. Here however we must stop; for it is not possible to ascertain how the motion of the nerves can affect the mind; but it is sufficient for us to know, by experience, that the nerves are as it were the mediators between our spiritual part, and the external and sensible objects.

Sound always ceases when the vibrations of the sonorous body cease, or become too weak. This is proved also by experiment, for when the vibrations of a sonorous body are damped by any soft body, the sound seems suddenly to cease. In a piano-forte, therefore, the quills are furnished with bits of cloth, that by touching the strings when they fall down, they may damp their vibrations. On the other hand, when the sonorous body, by its nature, is capable of continuing its vibrations for a considerable time, as is the case with a large bell, the sound may be heard for a long time after.

ARTICLE II.

On the Velocity of Sound; experiments for determining it; method of measuring distances by it.

Light is transmitted from one place to another with incredible velocity; but this is not the case with sound: the velocity of sound is very moderate, and may be measured in the following manner.

Let a cannon be placed at the distance of several thousand yards, and let an observer, with a pendulum that vibrates seconds, or rather half seconds, put the pendulum in motion as soon as he sees the flash, and then count the number of seconds or half seconds which elapse between that period and the moment when he hears the explosion. It is evident that, if the moment when the flash is seen be considered as the signal of the explosion, nothing will be necessary, to obtain the number of yards which the sound has passed over in a second, but to divide the number of the yards between the place of observation and the cannon, by the number of seconds or half seconds which have been counted.

Now the moment when the flash is perceived, whatever be the distance, may be considered as the real moment of the explosion; for so great is the velocity of light, that it employs scarcely a second to traverse 60000 leagues.*

By similar experiments the members of the Royal Academy of Sciences found, that sound moved at the rate of 1172 Parisian feet in a second. Gassendus makes its velocity to be 1473 feet in a second; Mersenne 1474; Duhamel, in the History of the Academy of Sciences, 1338; Newton 968; and Derham, in whose measure Flamsteed and Halley concurred, 1142. Though it is difficult to determine among so many authorities, the last estimate, viz. 1142 per second, has been generally adopted in this country.

Recent experiments on the velocity of sound, made with all the advantages of modern science, give results differing considerably from that of Derham; and from the close agreement which they present, they seem entitled to great confidence.

The results being reduced to the temperature of freezing, Arago and others found the velocity per second, in English feet, to be 1086·1; Professor Moll and assistants, 1089·42; Dr. O. Gregory 1088·05; Myrbach 1092·1; and Goldingham, at Madras, mean of two results 1084·9.

It may therefore be stated, in round numbers, that, in dry air, at the freezing temperature, sound travels at the rate of 1090 feet in a second.

"That sounds, of all pitches and of every quality, travel with equal speed, we have a convincing proof in the performance of a rapid piece of music by a band at a distance. Were there the slightest difference of velocity in the sounds of different notes they could not reach our ears in the same precise order, and at the exact intervals of time, in which they are played; nor would the component notes of a harmony, in which several sounds of different pitches concur, arrive at once."—(Sir J. HERSCHEL; *Sound*, Encyc. Metrop.)

It is to be observed that, according to Derham's experiments, the temperature of the air, whether dry or moist, cold or hot, causes no variation in the velocity of sound. This philosopher had often an opportunity of seeing the flash and hearing

* The velocity of the particles and rays of light is truly astonishing, as it amounts to nearly two hundred thousand miles in a second of time, which is nearly a million times greater than the velocity of a cannon-ball. It has been found, by repeated experiments, that when the earth is exactly between Jupiter and the sun, his satellites are seen eclipsed 8¼ minutes sooner than they could be according to the tables; but when the earth is nearly in the opposite point of its orbit, these eclipses happen about 8¼ minutes later than the tables predict them: hence it is certain that the motion of light is not instantaneous, but that it takes up about 16½ minutes of time in passing over a space equal to the diameter of the earth's orbit, which is at least 190 millions of miles in length, or moves at the rate of nearly 200000 miles per second. Hence therefore light takes about 8¼ minutes in passing from the sun to the earth.

the report of cannon fired at Blackheath, nine or ten miles distant, from Upminster, the place of his residence; but whatever might be the state of the weather, he always counted the same number of half seconds, between the moment of seeing the flash and that of hearing the report, unless any wind blew from either of these places; in which case the number of the seconds varied from 111 to 112. It may be readily conceived, that if the wind impelled the fluid put into a state or vibration, towards the place of the observer, the vibrations would reach him sooner than if the fluid had been at rest, or had been impelled in a contrary direction.

But notwithstanding what Derham has said, we can hardly be persuaded that the velocity of sound is not affected by the temperature of the air; for when the air is heated, and consequently more rarefied or elastic, the vibrations must be more rapid: observations on this subject ought to be carefully repeated.

A remarkable instance of the effect of a low temperature and a dry atmosphere in facilitating the transmission of sound, is recorded in the account of Sir E. Parry's voyage, in which he wintered at Port Bowen. On a particular occasion there was found no difficulty in making a man hear, at the distance of a mile, directions which were given in an ordinary tone of voice.

An inaccessible distance then may be measured by means of sound. For this purpose provide a pendulum that swings half seconds, which may be done by suspending from a thread a ball of lead, half an inch in diameter, in such a manner, that there shall be exactly 9$\frac{25}{32}$ inches, or 9$\frac{3}{4}$ between the centre of the ball and the point of suspension: then the moment you perceive the flash of a cannon, or musket, let go the pendulum, and count how many vibrations it makes till the instant when you hear the report: if you then multiply this number by 571 feet, you will have the distance of the place where the musket or cannon was fired.

We here suppose the weather to be calm, or that the wind blows only in a transversal direction; for if the wind blows towards the observer from the place where the cannon or gun is fired, and if it be violent, as many times twelve feet as there have been counted half seconds must be added to the distance found; and in the contrary case, that is to say, if the wind blows from the observer, towards the quarter where the explosion is made, they must be subtracted. It is well known that a violent wind makes the air move at the rate of about twenty-four feet per second, which is nearly the 48th part of the velocity of sound. If the wind be moderate, a 96th may be added or subtracted; and if it be weak, but sensible, a 192nd: but this correction, especially in the latter case, seems to be superfluous; for can we ever flatter ourselves that we have not erred a 192d part in the measuring of time?

This method may be employed to determine the distance of ships at sea, or in a harbour, when they fire guns, provided the flash can be seen, and the explosion heard. During a storm also, the distance of a thunder cloud may be determined in the same manner. But as a pendulum is not always to be obtained, its place may be supplied by observing the beats of the pulse, for when in its usual state, each interval between the pulsations is almost equal to a second; but when quick and elevated, each pulsation is equal to only two thirds of a second.

ARTICLE III.

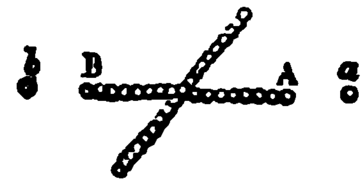
How Sounds may be propagated in every direction without confusion.

This is a very singular phenomenon in the propagation of sounds; for if several persons speak at the same time, or play on instruments, their different sounds are heard simultaneously, or all together, either by one person or by several persons, without being confounded in passing through the same place in different directions, or without damping each other. Let us endeavour to account for this phenomenon.

The cause no doubt is to be found in the property of elastic bodies. For let us conceive a series of globules equally elastic, and all contiguous, and let us suppose

that a globule is impelled with any velocity whatever against the first of the series: it is well known that in a very short time the motion will be transmitted to the other extremity, so that the last globule will have the same motion communicated to it asif it had been itself immediately impelled. Now if two globules with unequal velocities impel at the same time the two extremities of the series, the globule *a*, for example, the extremity A, and the globule *b* the extremity B (Fig. 1.), it is certain, from the well known properties of elastic bodies, that the globules *a* and *b*, after being a moment at rest, will be repelled, making an exchange of velocity, as if they had been immediately impelled against each other.

Fig. 1.



If we suppose a second series of globules, intersecting the former in a transversal direction, the motion of this second series will be transmitted by means of the common globule, from one end to the other of this series, in the same manner as if it had been alone. The case will be the same if two, three, four, or more series cross the first one, either in the same point or in different points. The particular motion communicated to the beginning of each series will be transmitted to the other end, as if that series were alone.

This comparison may serve to shew how several sounds may be transmitted in all directions, by the help of the same medium; but it must be allowed that there are some small differences.

For, in the first place, we must not conceive the air, which is the vehicle of sound, to be composed of elastic globules, disposed in such regular series as those here supposed; each particle of air is no doubt in contact with several others at the same time, and its motion is thereby communicated in every direction. Hence it happens that the sound, which would reach to a very great distance almost without diminution, if communicated as here supposed, experiences a considerable decrease, in proportion as it recedes from the body which produced it. Though the movement by which sound is transmitted be more complex, there is reason to believe that it is reduced, in the last instance, to something similar to what has been here described.

The second difference arises from the particles of air by which the organ of hearing is immediately affected, not having a movement of translation, like the last globule of the series, which proceeds with a greater or less velocity, in consequence of the shock that impels the other extremity of the series. But the movement in the air consists merely of an undulation or vibration, which, in consequence of the elasticity of its aërian particles, is transmitted to the extremity of the series, such as it was received at the other. It must be observed that the sonorous body communicates to the air, which it touches, vibrations isochronous with those which it experiences itself; and that the same vibrations are transmitted from the one end to the other of the series, and always with the same velocity: for we are taught by experience that a grave sound, *cæteris paribus*, does not employ more time than an acute one to pass through a determinate space.

ARTICLE IV.

Of Echoes; how produced; account of the most remarkable echoes, and of some phenomena respecting them.

Echoes are well known; but however common this phenomenon may be, it must be allowed that the manner in which it is produced is still involved in considerable obscurity, and that the explanation given of it does not sufficiently account for all the circumstances attending it.

All philosophers almost have ascribed the formation of echoes to a reflection of sound, similar to that experienced by light, when it falls on a polished body. But, as D'Alembert observes, this explanation is false; for if it were not, a polished surface

would be necessary to the production of an echo; and it is well known that this is not the case. Echoes indeed are frequently heard opposite to old walls, which are far from being polished; near huge masses of rock, and in the neighbourhood of forests, and even of clouds. This reflection of sound therefore is not of the same nature as that of light.

It is evident however, that the formation of an echo can be ascribed only to the repercussion of sound; for echoes are never heard but when sound is intercepted, and made to rebound by one or more obstacles. The most probable manner in which this takes place, is as follows.

For the sake of illustration, we shall resume our comparison of the aërian moleculæ to a series of elastic globules. If a series of elastic globules then be infinite, it may readily be conceived, that the vibrations communicated to one end, will be always propagated in the same direction, and continually recede; but if the end of the series rest against any fixed point, the last globule will re-act on the whole series, and communicate to it, in the contrary direction, the same motion as it would have communicated to the rest of the series, if it had not rested against a fixed point. This ought indeed to be the case whether the obstacle be in a line with the series, or oblique to it, provided the last globule be kept back by the neighbouring ones; only with this difference, that the retrograde motion will be stronger in the latter case, according as the obliquity is less. If the aërian and sonorous moleculæ then rest against any point at one end; and if the obstacle be at such a distance from the origin of the motion, that the direct and repercussive motion shall not make themselves sensible at the same instant, the ear will distinguish the one from the other, and there will be an echo.

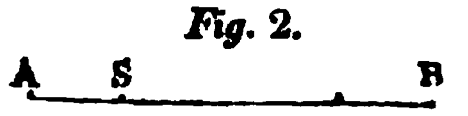
But we are taught by experience, that the ear does not distinguish the succession of two sounds, unless there be between them the interval of at least one-twelfth of a second: for during the most rapid movement of instrumental music, each measure of which cannot be estimated at less than a second,* twelve notes are the utmost that can be comprehended in a measure, to render the succession of sounds distinguishable; consequently the obstacle which reflects the sound must be at such a distance, that the reverberated sound shall not succeed the direct sound till after one-twelfth of a second; and as sound moves at the rate of about 1142 feet in a second, and consequently about 95 feet in the twelfth of a second, it thence follows that, to render the reverberated sound distinguishable from the direct sound, the obstacle must be at the distance at least of about 48 feet.

There are single and compound echoes. In the former only one repetition of the sound is heard; in the latter there are 2, 3, 4, 5, &c., repetitions. We are even told of echoes that can repeat the same word 40 or 50 times.

Single echoes are those where there is only one obstacle: for the sound being impelled backwards, will continue its course in the same direction without returning; but double, triple, or quadruple echoes may be produced different ways. If we suppose, for example, several walls one behind the other, the remotest being the highest; and if each be so disposed as to produce an echo; as many repetitions of the same sound as there are obstacles will be heard.

Another way in which these numerous repetitions may be produced, is as follows:

Fig. 2.



Let us suppose two obstacles, A and B (Fig. 2.) opposite to each other, and the productive cause of the sound to be placed between them, in the point S; the sound propagated in the direction from S to A, after returning from A to S, will be driven back by the obstacle B, and again return to S; having then traversed the space S A, it will experience a new repercussion, which

* If a piece of music, consisting of 60 measures, were executed in a minute, this, in our opinion, would be a rapidity of which there are few instances in the art.

will carry it to S, after it has struck the obstacle B; and this would be continued *in infinitum* if the sound did not always become weaker. On the other hand, since the sound is propagated as easily from S to B as from S to A, it will at first be sent back also from B towards S; having then passed over the space SA, it will be repelled from A towards S; then again from B towards S, after having traversed the distance SB, and so on in succession, till the sound dies entirely away.

The sound therefore produced in S will be heard after times, which may be expressed by 2SA; 2SB, 2SB + 2SA; 4SA + 2SB; 4SB + 2SA; 4SA + 4SB; 6SA + 4SB; 6SB + 4SA; 6SA + 6SB, &c.; which will form a repetition of the sound after equal intervals, when SA is equal to SB, and even when SB is double SA; but when SA is a third, for example, of SB, this remarkable circumstance will take place, that after the first repetition, there will be a kind of double silence; three repetitions will then follow, at equal intervals; there will then be a silence double one of these intervals; then three repetitions after intervals equal to the former; and so on till the sound is quite extinguished. The different ratios of the distances SA, SB, will also give rise to different irregularities in the succession of these sounds, which we have thought it our duty to notice, as being possible, though we do not know that they have been ever observed.

There are some echoes that repeat several words in succession; but this is not astonishing, and must always be the case when a person is at such a distance from the echo, that there is sufficient time to pronounce several words before the repetition of the first has reached the ear.

There are some echoes which have been much celebrated on account of their singularity, or of the number of times that they repeat the same word. Misson, in his Description of Italy, speaks of an echo at the Villa Simonetta, which repeated the same word 40 times.

At Woodstock in Oxfordshire, there is an echo which repeats the same sound 50 times.*

The description of an echo still more singular near Roseneath, some miles distant from Glasgow, may be found in the Philosophical Transactions for the year 1698. If a person, placed at the proper distance, plays 8 or 10 notes of an air with a trumpet, the echo faithfully repeats them, but a third lower; after a short silence another repetition is heard in a tone still lower; and another short silence is followed by a third repetition, in a tone a third lower.

A similar phenomenon is perceived in certain halls; where, if a person stands in a certain position, and pronounces a few words with a low voice, they are heard only by another person standing in a determinate place. Muschenbroeck speaks of a hall of this kind in the castle of Cleves; and most of those who have visited the Observatory at Paris have experienced a similar phenomenon in the hall on the first story.

Philosophers unanimously agree in ascribing this phenomenon to the reflection of the sonorous rays; which, after diverging from the mouth of the speaker, are reflected in such a manner as to unite in another point. But it may be readily conceived, say they, that as the sound by this union is concentrated in that point, a person whose ear is placed very near will hear it, though it cannot be heard by those who are at a distance.

We do not know whether the hall in the castle of Cleves, of which Muschenbroeck speaks, is elliptical, and whether the two points where the speaker and the person who listens ought to be placed are the two foci; but in regard to the hall in the Observatory at Paris, this explanation is entirely void of foundation. For,

1st. The echoing hall, or as it is called the *Hall of Secrets*, is not at all elliptical;

* This seems to be a mistake: the echo at Woodstock, according to Dr. Plat, repeats in the day time very distinctly 17 syllables, and in the night time 20.—*Nat. Hist. of Oxf.* chap. i. p. 7.

it is an octagon, the walls of which at a certain height are arched with what are called in architecture *cloister arches*; that is to say, by portions of a cylinder which, in meeting, form re-entering angles, that continue those formed by the sides of the octagonal plan.

2d. The person who speaks does not stand at a moderate distance from the wall, as ought to be the case in order to make the voice proceed from one of the foci of the supposed ellipsis: he applies his mouth to one of the re-entering angles, very near the wall, and the person whose ear is nearly at the same distance from the wall, on the side diametrically opposite, hears the one who speaks on the other side, even when he does so with a very low voice.

It is therefore evident that, in this case, there is no reflection of the voice according to the laws of catoptrics; but the re-entering angle continued along the arch, from one side of the hall to the other, forms a sort of canal, which contains the voice, and transmits it to the other side. This phenomenon is entirely similar to that of a very long tube, to the end of which if a person applies his mouth and speaks, even with a low voice, he will be heard by a person at the other end.

The Memoirs of the Academy of Sciences, for the year 1692, speak of a very remarkable echo in the court of a gentleman's seat called Le Genetay, in the neighbourhood of Rouen. It is attended with this singular phenomenon, that a person who sings or speaks in a low tone, does not hear the repetition of the echo, but only his own voice; while those who listen hear only the repetition of the echo, but with surprising variations; for the echo seems sometimes to approach and sometimes to recede, and at length ceases when the person who speaks removes to some distance in a certain direction. Sometimes only one voice is heard, sometimes several, and sometimes one is heard on the right, and another on the left. An explanation of all these phenomena, deduced from the semicircular form of the court, may be seen in the above collection.

ARTICLE V.

Experiments respecting the vibrations of Musical Strings, which form the basis of the theory of Music.

If a string of metal or catgut, such as is used for musical instruments, made fast at one of its extremities, be extended in a horizontal direction over a fixed bridge; and if a weight be suspended from the other extremity, so as to stretch it; this string, when struck, will emit a sound produced by reciprocal vibrations, which are sensible to the sight.

If the part of the string made to vibrate be shortened, and reduced to one half of its length, any person who has a musical ear will perceive, that the new sound is the octave of the former, that is to say twice as sharp.

If the vibrating part of the string be reduced to two thirds of its original length, the sound it emits will be the fifth of the first.

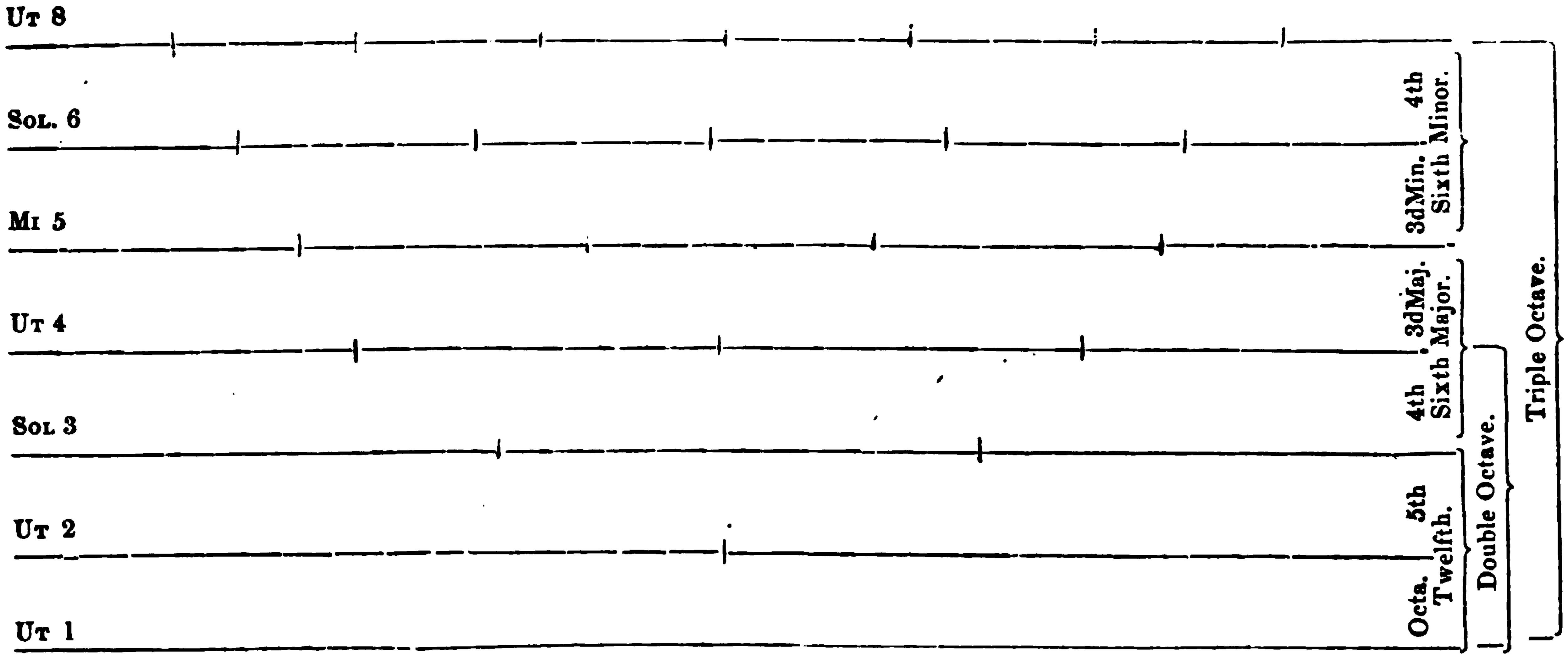
If the length be reduced to three fourths, it will give the fourth of the first.

If it be reduced to $\frac{4}{5}$, it will give the third major; if to $\frac{5}{6}$, the third minor. If reduced to $\frac{8}{9}$, it will give what is called the tone major; if to $\frac{9}{10}$, the tone minor; and if to $\frac{15}{16}$, the semi-tone, or that which in the gamut is between *mi* and *fa* or *si* and *sol*.

The same results will be obtained if a string be fastened at both ends, and $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of it, be successively intercepted by means of a moveable bridge.

As this subject will be better understood if the reader has a clear idea of the relation of the sounds in the diatonic progression, we shall here insert the following table.

Ingenious Manner in which Rameau expresses the relation of the Sounds in the Diatonic Progression.



It may here be seen that if these seven lines represent seven strings of equal length, the order of the principal harmonic concords will be determined by the following numbers:—Thus

			4 to 5	denotes	the third major.	3 to 5	denotes	the sixth major
1 to 2	denotes	the octave.	5 to 6	—	the third minor.	5 to 8	—	the sixth minor
2 to 3	—	the fifth.	6 to 8	—	the fourth.	1 to 4	—	the double octave.
3 to 4	—	the fourth.	1 to 3	—	the twelfth.	1 to 8	—	the triple octave.

Such is the result of a determinate degree of tension given to a string, when the length of it has been made to vary. Let us now suppose that the length of the string is constantly the same, but that its degree of tension is varied. The following is what we are taught by experiment on this subject.

If a weight be suspended at one end of a string of a determinate length, made fast by the other, and if the tone it emits be fixed; when another weight quadruple of the first has been applied, the tone will be the octave of the former; if the weight be nine times as heavy, the tone will be the octave of the fifth; and if it be only a fourth part of the first the tone will be the octave below. Nothing more is necessary to prove that the effect produced, by successively reducing a string to one half, $\frac{2}{3}$, $\frac{3}{4}$, &c., will be produced also by suspending from it in succession weights in the ratio of 4, $\frac{9}{4}$, $\frac{16}{9}$, &c.; that is to say, the squares of the weights, or the degrees of tension, must be reciprocally as the squares of the lengths proper for emitting the same tones.

Pythagoras, we are told, was led to this discovery by the following circumstance. Harmonious sounds proceeding from the hammers striking on an anvil in a smith's shop happening one day to reach his ear, while walking in the street, he entered the shop, and found, by weighing the hammers which had occasioned these sounds, that the one which gave the octave was exactly the half of that which produced the lowest tone; that the one which produced the fifth, was two thirds of it; and that the one which produced the third major, was four fifths. When he returned home, meditating on this phenomenon, he extended a string, and after successively shortening it to one half, two thirds, and four fifths, perceived that it emitted sounds which were the octave, the fifth, and the third major of the tone emitted by the whole string. He then suspended weights from it, and found that those which gave the octave, the fifth, and the third major, ought to be respectively as 4, $\frac{9}{4}$, $\frac{25}{16}$, to that which emitted the principal tone; that is to say, in the inverse ratio of the squares of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$.

Whatever may be the degree of credit due to this anecdote, which is appreciated as it deserves in the History of the Mathematics, such were the first facts that enabled mathematicians to subject the musical intervals to calculation. The sum of what the moderns have added to them, is as follows:

It can be demonstrated at present by the principles of mechanics:

1st. That if a string of a uniform diameter, extended by the same weights, be lengthened or shortened, the velocity of its vibrations, in these two states, will be in the inverse ratio of the lengths. If this string then be reduced to one half of its length, its vibrations will have a double velocity; that is to say, it will make two vibrations for one which it made before; if it be reduced to two thirds, it will make three vibrations for two which it made before. When a string therefore performs two vibrations, while another performs one, the tones emitted by these strings will be octaves to each other; when one vibrates three times while another vibrates twice, the one will be the fifth to the other, and so on.

2d. The velocity of the vibrations performed by a string, of a determinate length, and distended by different weights, is as the square roots of the stretching weights: quadruple weights therefore will produce double velocity, and consequently double the number of vibrations in the same time; a noncuple weight will produce vibrations of triple velocity, or a triple number in the same time.

3d. If two strings, differing both in length and in weight, be stretched by different weights, the velocities of their vibrations will be as the square roots of the distending weights divided by the lengths and the weights of the strings: thus, if the string A, stretched by a weight of 6 pounds, weigh six grains, and be a foot in length; while the string B, stretched by a weight of 10 pounds, weighs five grains, and is half a foot in length; the velocity of the vibrations of the former will be to that of the

vibrations of the latter, as the square root of $6 \times 6 \times 1$ to that of $5 \times 10 \times \frac{1}{2}$, that is, as the square root of 36, which is 6, to that of 25, or 5: the first therefore will perform 6 vibrations while the second performs 5.

From these discoveries it follows, that the acuteness or gravity of sounds, is merely the effect of the greater or less frequency of the vibrations of the string which produces them; for since we know by experience, on the one hand, that a string when shortened, if subject to the same degree of tension, emits a more elevated tone; and on the other, both by theory and experience, that its vibrations are more frequent the shorter it is, it is evident that it is only the greater frequency of the vibrations that can produce the effect of elevating the tone.

It thence results also, that a double number of vibrations produces the octave of the tone produced by the single number; that a triple number produces the octave of the fifth; a quadruple number, the double octave; a quintuple, the third major above the double octave, &c.; and if we descend to ratios less simple, three vibrations for two will produce the concord of fifth; four for three, that of the fourth, &c.

The ratios of tones therefore may be expressed, either by the lengths of the equally stretched strings which produce them, or by the ratio of the number of the vibrations performed by these strings; thus, if the principal tone be denoted by 1, the octave above is expressed mathematically by $\frac{1}{2}$, or by 2; the fifth by $\frac{2}{3}$ or $\frac{3}{2}$; the third major by $\frac{4}{5}$ or $\frac{5}{4}$, &c. In the first case, the respective lengths of the strings are denoted; in the second, the respective numbers of vibrations. In calculation, the results will be the same, whichever method of denomination be adopted.

PROBLEM.

To determine the number of the vibrations made by a string, of a given length and size, and stretched by a given weight; or, in other words, the number of the vibrations which form any tone assigned.

Hitherto we have considered only the ratios of the number of the vibrations, performed by strings which give the different concords; but a more curious, and far more difficult problem, is, to find the real number of the vibrations performed by a string which gives a certain determinate tone; for it may be readily conceived that their velocity will not admit of their being counted. Geometry, however, with the help of mechanics, has found means to resolve this question, and the rule is as follows:

Divide the stretching weight by that of the string; multiply the quotient by the length of the pendulum that vibrates seconds, which at London is $39\frac{1}{8}$ inches, or $469\frac{1}{2}$ lines, and divide the product by the length of the string from the fixed point to the bridge; extract the square root of this new quotient, and multiply it by the ratio of the circumference of the circle to the diameter, viz. $3\frac{1}{7}$ nearly, or the fraction $\frac{355}{113}$, in decimals 3·1416 nearly; the product will be the number of the vibrations performed by the string in the course of a second.

Let a string of a foot and a half in length, for example, and weighing 8 grains, be stretched by a weight of 4 pounds Troy weight, or 23040 grains: the quotient of 23040 divided by 8 is 2880; and as the length of the pendulum which swings seconds is $469\frac{1}{2}$ lines, the product of 2880 by this number will be 1352160; if this product be divided by 216, the lines in a foot and a half, we shall have 6260, the square root of which will be 79·1201: this number multiplied by $\frac{355}{113}$ or 3·1416, gives 248·563, which is the number of the vibrations made by the above string in the course of a second.

A very ingenious method, invented by M. Sauveur, for finding the number of these vibrations, may be seen in the Memoirs of the Academy of Sciences for 1700.

Having remarked, when two organ pipes, very low, and having tones very near to each other, were sounded at the same time, that a series of pulsations or beats was heard in the sounds; by reflecting on the cause of this phenomenon, he found that these beats arose from the periodical meeting of the coincident vibrations of the two pipes. Hence he concluded, that if the number of these pulsations, which took place in a second, could be ascertained by a stop watch, and if it were possible also to determine, by the nature of the consonance of the two pipes, the ratio of the vibrations which they made in the same time, he should be able to acertain the real number of the vibrations made by each.

We shall here suppose, for example that two organ pipes are exactly tuned, the one to *mi* flat, and the other to *mi*, as it is well known that the interval between these two tones is a semi-tone minor, expressed by the ratio of 24 to 25, the higher pipe will perform 25 vibrations while the lower performs only 24; so that at each 25th vibration of the former or 24th of the latter, there will be a pulsation; if 6 pulsations therefore are observed in the course of one second, we ought to conclude that 24 vibrations of the one and 25 of the other are performed in the 10th part of a second: and consequently that the one performs 240 vibrations, and the other 250 in the course of a second.

M. Saveur made experiments according to this idea, and found that an open organ-pipe, 5 feet in length, makes 100 vibrations per second; consequently one of 4 feet, which gives the triple octave below, and the lowest sound perceptible to the ear, would make only 12$\frac{1}{2}$: on the other hand, a pipe of one inch less $\frac{1}{16}$, being the shortest the sound of which can be distinguished, will give in a second 6400 vibrations. The limits therefore of the slowest and quickest vibrations, appreciable by the ear, are according to M. Sauveur 12$\frac{1}{2}$ and 6400.

We shall not enlarge farther on these details, but proceed to a very curious phenomenon respecting strings in a state of vibration.

Make fast a string at both its extremities, and by means of a bridge divide it into aliquot parts, for example 3 on the one side, and 1 on the other; and put the larger part, that is to say the $\frac{3}{4}$, in a state of vibration; if the bridge absolutely intercepts all communication from the one part to the other, these $\frac{3}{4}$ of the string, as is well known, will give the tone of the fourth of the whole string; if $\frac{4}{5}$ be intercepted, the tone will be the third major.

But if the bridge only prevents the whole of the string from vibrating, without intercepting the communication of motion from the one part to the other, the greater part will then emit only the same sound as the less; and the $\frac{3}{4}$ of the string, which in the former case gave the fourth of the whole string, will give only the double octave, which is the tone proper to the fourth of the string. The case is the same if this fourth be struck: its vibrations, by being communicated to the other three fourths, will make them sound, but in such a manner as to give only this double octave.

The following reason, which may be rendered plain by an experiment, is assigned for this phenomenon: when the bridge absolutely intercepts all communication between the two parts of the string, the whole of the largest part vibrates together; and if it be $\frac{3}{4}$ of the whole string, it makes, agreeably to the general law, 4 vibrations in the time that the whole string would make 3: its sound therefore is the fourth of the whole string.

But in the second case, the larger part of the string divides itself into as many portions as the number of times it contains the less, which in the present example is 3, and each of these portions, as well as the fourth, performs its particular vibrations: at the points of division, as B, C, D (Fig. 3.), there are established fixed points, between which the portions of the string A B, B C, C D, D E, each vibrate separately, forming alternate bendings in a contrary direction, as if these parts were alone and invariably fixed at their extremities.

Fig. 3.

This explanation is founded on a fact which M. Saveur rendered sensible to the eyes in the presence of the Royal Academy of Sciences. ("Hist. de l'Acad. année 1700.") On the points C and D (Fig. 3.), he placed small bits of paper; and having put the small part of the string A B in a state of vibration, the vibrations being communicated to the remaining part B E, the spectators saw, with astonishment, the small bits of paper placed on the points C and D remain motionless, while those placed on the other parts of the string were thrown down.

If the part A B of the string, instead of being exactly an aliquot part of the remainder B E, be for example $\frac{2}{5}$ of it, the whole string A E will divide itself into 7 portions, of which A B will contain two, and each of these portions will vibrate separately, and emit only that sound which belongs to the $\frac{1}{7}$ of the string.

If the parts A B and B E be incommensurable, they will emit a sound absolutely discordant, and which almost immediately ceases on account of the impossibility of bendings and invariable points of rest being established.

ARTICLE VI.

Method of adding, subtracting, multiplying, and dividing Concords.

It is necesary for those who wish to understand the theory of music, to know what concords result from two or more concords, either when added or subtracted, &c., by each other. For this reason we shall give the following rules.

PROBLEM I.

To add one concord to another.

Express the two concords by the fractions which represent them, and then multiply these two fractions together; that is to say, first the numerators, and then the denominators; the number thence produced will express the concord resulting from the sum of the two concords given.

Example 1.—*Let it be required to add the fourth and fifth together.*

The expression for the fifth is $\frac{2}{3}$; and that for the fourth $\frac{3}{4}$; the product of these two is $\frac{6}{12} = \frac{1}{2}$, being the expression for the octave. It is indeed well known that the octave is composed of a fifth and a fourth.

Example 2.—*What is the concord arising from the addition of the third major and the third minor?*

The expression for the third major is $\frac{4}{5}$, and that of the third minor is $\frac{5}{6}$, the product of which is $\frac{20}{30}$ or $\frac{2}{3}$, which expresses the fifth; and this concord indeed is composed of a third major and a third minor.

Example 3.—*What is the concord produced by the addition of two tones major?*

A tone major is expressed by $\frac{8}{9}$, consequently, to add two tones major, $\frac{8}{9}$ must be multiplied by $\frac{8}{9}$. The product $\frac{64}{81}$ is a fraction less than $\frac{64}{80}$ or $\frac{4}{5}$, which expresses the third major; hence it follows, that the concord expressed by $\frac{64}{81}$ is greater than the third major; and consequently two tones major are more than a third major, or form a third major false by excess.

On the other hand, by adding two tones minor, which are each expressed by $\frac{9}{10}$, it will be found that their sum $\frac{81}{100}$ is greater than $\frac{80}{100}$ or $\frac{4}{5}$, which denotes the third major: two tones minor therefore, added together, make more than a third major. This third indeed is composed of a tone major and a tone minor, as may be proved by adding together the concords $\frac{8}{9}$ and $\frac{9}{10}$, which make $\frac{72}{90} = \frac{8}{10}$ or $\frac{4}{5}$.

It might be proved, in like manner, that two semi-tones major make more than a

tone major, and two semi-tones minor less than even a tone minor; and in the last place, that a semi-tone major and a semi-tone minor make exactly a tone minor.

PROBLEM II.

To subtract one concord from another.

Instead of multiplying together the fractions which express the given concords, they must here be divided; or invert that which expresses the concord to be subtracted from the other, and then multiply them together as before: the product will give a fraction expressing the quotient, or concord required.

Example 1.—*What is the concord which results from the fifth subtracted from the octave?*

The expression of the octave is $\frac{1}{2}$, that of the fifth $\frac{2}{3}$, which inverted gives $\frac{3}{2}$; and if $\frac{1}{2}$ be multiplied by $\frac{3}{2}$, we shall have $\frac{3}{4}$, which expresses the fourth.

Example 2.—*What is the difference between the tone major and the tone minor?*

The tone major is expressed by $\frac{8}{9}$, and the tone minor by $\frac{9}{10}$, which when inverted gives $\frac{10}{9}$; the product of $\frac{8}{9}$ by $\frac{10}{9}$ is $\frac{80}{81}$, which expresses the difference between the tone major and the tone minor: this is what is called the *great comma*.

PROBLEM III.

To double a concord, or to multiply it any number of times at pleasure.

In this case, nothing is necessary but to raise the terms of the fraction, which expresses the given concord, to the power denoted by the number of times it is to be multiplied; that is, to the square if it is to be doubled, to the cube if to be tripled, and so on.

Thus, the concord arising from the tone major tripled is $\frac{512}{729}$: for as the expression of the tone major is $\frac{8}{9}$, we shall have $8 \times 8 \times 8 = 512$, and $9 \times 9 \times 9 = 729$. This concord $\frac{512}{729}$ corresponds to the interval between *ut* and a *fa* higher than *fa* sharp of the gamut.

PROBLEM IV.

To divide one concord by any number at pleasure, or to find a concord which shall be the half, third, &c. of a given concord.

To answer this problem, take the fraction which expresses the given concord, and extract that root of it which is denoted by the determinate divisor: that is to say, the square root, if the concord is to be divided into two; the cube root, if it is to be divided into three, &c.; and this root will express the concord required.

Example.—As the octave is expressed by $\frac{1}{2}$, if the square root of it be extracted it will give $\frac{7}{10}$ nearly; but $\frac{7}{10}$ is less than $\frac{3}{4}$, and greater than $\frac{2}{3}$; consequently the middle of the octave is between the fourth and the fifth, or very near *fa* sharp.

ARTICLE VII.

Of the resonance of sonorous bodies; the fundamental principle of harmony and melody; with some other harmonical phenomena.

Experiment 1.

If you listen to the sound of a bell, especially when very grave, however indifferent your ear may be, you will easily distinguish, besides the principal sound, several others more acute; but if you have an ear accustomed to appreciate the musical intervals, you will perceive that one of these sounds is the twelfth, or fifth

above the octave, and another the seventeenth major, or third major above the double octave. If your ear be exceedingly delicate, you will distinguish also its octave, its double, and even its triple octave: the latter indeed are somewhat more difficult to be heard, because the octaves are almost confounded with the fundamental sound, in consequence of that natural sensation which makes us confound the octave with unison.

The same effect will be perceived if the bow of a violoncello be strongly rubbed against one of its large strings, or the string of a trumpet-marine.

In short, if you have an experienced ear, you will be able to distinguish these different sounds, either in the resonance of a string, or in that of any other sonorous body, and even in the voice.

Another method of making this experiment.

Suspend a pair of tongs by a woollen or cotton cord, or any other kind of small string, and twisting the extremities of it around the fore finger of each hand, put these two fingers into your ears. If the lower part of the tongs be then struck, you will first hear a loud and grave sound, like that of a large bell at a distance; and this tone will be accompanied by several others more acute; among which, when they begin to die away, you will distinguish the twelfth and the seventeenth of the lowest tone.

The truth of this phenomenon, in regard to the multiplicity of sounds, is confirmed by another experiment, mentioned by Rameau, in his "Harmonical Generation." If you take, says he, those stops of the organ called *bourdon*, *prestant* or *flute*, *nazard* and tierce, which form the octave, the twelfth and seventeenth major of the *bourdon*, and if you draw out in succession each of the other stops, while the *bourdon* alone is sounding, you will hear their sounds successively mixed with each other; you may even distinguish them while they are all sounding together; but if you prelude for a moment, by way of amusement, on the same set of keys, and then return to the single key first touched, you will think you hear only one tone, that of the bourdon, the gravest of all which corresponds to the sound of the whole system.

Remark.—This experiment, respecting the resonance of bodies, is not new. It was known to Dr. Wallis, and to Mersenne, who speak of it in their works; but it appeared to them a simple phenomenon, with the consequences of which they were entirely unacquainted. Rameau first discovered its use in deducing from it all the rules of musical composition, which before had been founded on mere sentiment, and on experience, incapable of serving as a guide in all cases, and of accounting for every effect. It forms the basis of his theory of fundamental bass, a system which has been opposed with much declamation, but which however most musicians seem at present to have adopted.

All his harmony then is multiple, and composed of sounds which would be produced by the aliquot parts of the sonorous body $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and we might add $\frac{1}{7}$, $\frac{1}{8}$, &c. But the weakness of these sounds, which go on always decreasing in strength, renders it difficult to distinguish them. Rameau, however, says that he could distinguish very plainly the sound expressed by $\frac{1}{7}$, which is the double octave of a sound divided nearly into two equal parts, being the interval between *la* and *si* flat below the first octave: he calls it a lost sound, and totally excludes it from harmony. It would indeed be singularly discordant with all the sounds given by the fundamental tone.

We must however observe that the celebrated Tartini, in regard to this sound, was not of the same opinion as Rameau. Instead of calling it a lost sound, he maintains that it may be employed in melody as well as harmony; he distinguishes it by the name of the *seventh consonant*. But we shall leave it to musicians to appreciate this idea of Tartini, whose celebrity in composition, as well as execution,

required a refutation of a different kind, from that to be found at the end of a work printed in 1767, entitled "Histoire de la Musique."

Experiment 2.

If you tune several strings to the octave, to the twelfth, and to the seventeenth major, of the determinate sound emitted by another string, both ascending and descending; as often as you make that which gives the determinate sound to resound strongly, and with continuance, you will immediately see all the rest put themselves in a state of vibration: you will even hear those sound which are tuned lower, if care be taken to damp suddenly, by means of a soft body, the sound of the former.

Most persons have heard the glasses on a table sound, when a person near them has been singing with a strong and a loud voice. The strings of an instrument though not touched, are often heard to sound, in consequence of the same cause, especially after swelling notes long continued.

This phenomenon arises, no doubt, from the vibrations of the air being communicated to the string, or to the sonorous body, elevated to the above tones: for it may be easily conceived that the vibrations of strings, tuned to unison or to the octave, or to the twelfth, &c., of that put in motion, are disposed to recommence regularly, and at the same time as those of that string, one vibration corresponding to another, in the case of unison; two to one, in case of the octave; or three to one in that of the twelfth: the small impulsions therefore of the vibrating air, produced by the string put in motion, will always concur to increase those movements, at first insensible, which they have occasioned in the other strings; because they will take place in the same direction, and will at length render them sensible. Thus a gentle breadth of air, continued always in the same direction, is at length able to elevate the waters of the ocean. But when the strings in question are stretched in such a manner, that their vibrations can have no correspondence with those of the string which is struck, they will in this case be sometimes assisted and sometimes opposed, and the small movement which can be communicated to them, will be annihilated as soon as produced, consequently they will remain at rest.

Question.—Do the sounds heard with the principal sound derive their source immediately from the sonorous body, or do they reside only in the air or the organ?

It is very probable that the principal sound is the only one that derives its origin immediately from the vibrations of the sonorous body. Philosophers of eminence have endeavoured to discover whether, independently of the total vibrations made by the body, there are not also partial vibrations; but hitherto they have been able to observe only simple vibrations. Besides, how can it be conceived that the whole of a string should be in vibration, and that during its motion it should divide itself into two or three parts that perform also their distinct vibrations?

It must then be said that these harmonical sounds of octave, twelfth, seventeenth, &c., are in the air or the organ—both suppositions are probable,—for since a determinate sound has the property of putting into a state of vibration bodies disposed to give its octave, its twelfth, &c., we must allow that this sound may put in motion the particles of the air susceptible of vibrations of double, triple, quadruple, and quintuple velocity. What, however, appears most probable in this respect is, that these vibrations exist only in the ear: it seems indeed to be proved, by the anatomy of this organ, that sound is transmitted to the soul only by the vibrations of those nervous fibres which cover the interior part of the ear; and as they are of different lengths, there are always some of them which perform their vibrations isochronous to those of a given sound. But, at the same time, and in consequence of the property above mentioned, this sound must put in motion those fibres which are susceptible of isochronous vibrations, and even those which can make vibrations of double,

triple, quadruple, &c., velocity. Such, in our opinion, is the most probable explanation that can be given of this singular phenomenon.

Experiment 3.

For this experiment we are indebted to the celebrated Tartini of Padua. If you draw from two instruments, at the same time, any two sounds whatever, you will hear in the air a third, which will be the more perceptible the nearer your ear is placed to the middle of the distance between the two instruments. Let us suppose then, for example, two sounds which succeed each other in the order of consonances, as the octave and the twelfth, the double octave and the seventeenth major, &c.; the sound resulting, says Tartini, will be the octave of the principal sound.

This experiment was repeated in France with the same success, as we are assured by M. Serres, in his "Principes de l'Harmonie," printed in 1753; but with this exception, that M. Serres found the latter sound to be lower by an octave. As the octaves are easily confounded, this difference needs excite no surprise. We must however here observe, that the celebrated musician of Padua established on this phenomenon a system of harmony and composition; but it does not seem to have met with so favourable a reception as that of Rameau.

ARTICLE VIII.

Of the different systems of Music; the Grecian and the Modern, together with their peculiarities.

I.—*Of the Grecian Music.*

During the infancy of music among the Greeks, their lyre had four strings, the sounds of which would have corresponded to *si, ut, re, mi;* but they afterwards added other three *fa, sol, la.* The first diatonic scale therefore of the Greeks, translated into our musical language, was *si, ut, re, mi, fa, sol, la,* and was composed of two tetrachords, or systems of four sounds, *si, ut, re, mi; mi, fa, sol, la;* in which the last of the one and the first of the other were common, and on this account they were called *conjunct tetrachords.*

We must here observe that, however singular this disposition of sounds may appear to those who are acquainted only with the modern diatonic order, it is no less natural and agreeable to the rules of harmony; for Rameau has shewn that it is nothing else than a chant, the fundamental bass of which would be, *sol, ut, sol, ut, fa, ut, fa.* It possesses also the advantage of having only one altered interval, viz. the third minor from *re* to *fa,* which, instead of being in the ratio of 5 to 6, is in that of 27 to 32; which is somewhat less, and consequently too low by a *comma* of from 80 to 81.

But this perfection in the Grecian gamut was counter-balanced by two great imperfections, viz. 1st. that it did not complete the octave; 2d. that it did not terminate by a rest, which leaves to the ear that kind of uneasiness resulting from a song begun and not finished. It could neither ascend to *si,* nor descend to *la;* and therefore the musicians who, to complete the octave, added the latter note below, considered it to be foreign, as we may say, and gave it the name of *proslambanomenos.*

For this reason they endeavoured to discover another remedy for this defect, and Pythagoras, as is said, proposed the succession of sounds *mi, fa, sol, la; si, ut, re, mi,* composed, as it appears, of two *disjunct tetrachords.* This diatonic scale is almost the same as ours, with this difference, that ours begins and ends with the tonic note, while the former begins and ends with the mediante, or third major. This termination, almost reprobated at present, was very common among the Greeks, and is still so in the chants or vocal music of our churches.

But here, in consequence of the harmonic generation, the values of the sounds and

intervals are not the same as in the first scale. In the first, the interval from *sol* to *la* was a tone minor, in the second it is a tone major. In the last place, according to this second arrangement there are three intervals altered or false, viz. the tierce major, from *fa* to *la*, too high; the tierce minor, from *la* to *ut*, too low; and the fifth, from *la* to *mi*, too high. These are the same faults as those of our diatonic scale; but the temperament corrects them.

To these sounds the Greeks afterwards added a conjunct tetrachord descending, *si*, *ut*, *re*, *mi*, and another ascending, *mi*, *fa*, *sol*, *la*; by which they nearly supplied all the wants of melody, so far as it was confined to one tone. Ptolemy speaks of a combination, by means of which they joined the second primitive tetrachord to the first, lowering the *si* a semi-tone, which made *si* flat, *ut*, *re*, *mi*. This, no doubt, answered the purpose when they passed from the tone of *ut* to that of its lower fifth *fa*; a transition common in the Grecian music, as well as in our church music: for in that case a *si* flat is required. Plutarch also speaks of a combination where the two last tetrachords were disjoined, by raising the *fa* a semi-tone, and that no doubt of its lower octave. Who does not here perceive our *fa* ♯, which is necessary when we pass from the tone of *ut* to that of its upper fifth *sol*? The strings, which corresponded to *si* flat and *fa* sharp, were no doubt merely added, and not substituted in the room of *si* and *fa*.

It is well known that in the Grecian music there were three genera, viz. the diatonic, chromatic, and enharmonic. What has been hitherto said relates only to the diatonic.

What characterises the enharmonic is, that it employs, either ascending or descending, several semi-tones in succession. The chromatic gamut of the Greeks was *si*, *ut*, *ut* sharp, *mi*, *fa*, *fa* sharp, *la*. This disposition, by which they passed immediately from *ut* sharp to *mi*, omitting the *re*, must no doubt appear very strange; but it is certain that this was the gamut employed by the Greeks in the chromatic genus. It is however not known whether the Greeks had considerable pieces of music of this kind, or whether, like us, they employed it only in very short passages of cantatas; for we also have a chromatic kind, though in a different acceptation. This transition from semi-tones to semi-tones, is less natural than the diatonic succession; but it has more energy to express certain peculiar sensations: the Italians therefore, who are great colorists in music, make frequent use of it in their airs.

In regard to the enharmonic music of the Greeks, though considered by the ancients as the most perfect kind, it is to us still an enigma. To give some idea of it, let us assume the sign ✻, as that of the enharmonic diesis or sharp, which raises the note a quarter of a tone: the enharmonic scale then was *si*, *si* ✻, *ut*, *mi*, *mi* ✻, *fa*, *la*, where it appears that, after two-fourths of a tone, from *si* to *ut*, or from *mi* to *fa*, they proceeded to *mi* or *la*. It can hardly be conceived how there could be ears so well exercised as to appreciate fourths of a tone, and even if we suppose that there were, what modulation could they make with these sounds? It is however very certain that this kind of music was long held in high estimation in Greece; but on account of its difficulty it was at length abandoned, so that not even a fragment of Grecian music in the enharmonic kind has been handed down to us; nor any in chromatic, though we have some in the diatonic.

We must however here observe, that this enharmonic music of the Greeks is not perhaps so remote from nature as has been hitherto supposed; for does not Tartini, in proposing the use of his consonant seventh, which is nearly a mean sound between *la* and *si* flat, pretend that this intonation, *la*, *si*^{bb}, *si*^{b}, *re*, *re*, *si*^{b}, *si*^{bb}, *la*, is not only supportable, but highly agreeable. Tartini does more; for he assigns to this succession the sounds of its bass, *fa*, *ut*, *sol*, *sol*, *ut*, *fa*, marking *ut* with this sign b^7, which signifies consonant seventh. If this pretension of Tartini should find partizans, may we not say that the enharmonic music of the Greeks has been revived?

It now remains that we should say a few words respecting the modes of the Grecian music. However obscure this matter may be, if we can believe the author of "Histoire des Mathematiques," who founds his ideas on certain tables of Ptolemy, these modes are nothing else than the tones of our music, and he gives the following comparison.

The Dorian being taken hypothetically for the mode of *ut*, these modes, some lower than the Dorian and others higher, were:

The Hypodorian ......corresponding to	*sol*
The Hypophrygian	*la* flat.
The Hypophrygian acutior............	*la*
The Hypolydian or *Hypo-ælian*	*si* flat.
The Hypolydian acutior	*si*
The Dorian	*ut*
The Iastian or *Ionian*	*ut* sharp.
The Phrygian........................	*re*
The Æolian	*re* sharp.
The Lydian	*mi*
The Hyperdorian	*fa*
The Hyperiastian or *Mixolydian*	*fa* sharp.
The Hypermixolydian	*sol* the replicate of the first.

But this question might be asked: If the difference of the Grecian modes consisted in the greater or less height of the tone of the modulation, how can we explain what is told us of the characters of these different modes, some of which excited fury, others appeased it, &c.? There is reason therefore to think that they depended on something more; and it is not improbable, that besides differences of tone, there was a character of modulation peculiar to each. The Phrygian, for example, which originated among a hardy and warlike people of that name, had a masculine and warlike character, while the Lydian, which was derived from a soft and effeminate people, had an analogous character, and consequently was proper for calming the transports excited by the former.

As we have here said enough respecting the Grecian music, we shall now proceed to the modern.

II.—*Of the Modern Music.*

Every person acquainted with music knows, that the gamut, or diatonic scale of the moderns, is represented by these sounds *ut*, *re*, *mi*, *fa*, *sol*, *la*, *si*, *ut*, which complete the whole extent of the octave;* and, we shall add, that from its generation, as explained by Rameau, it follows that between *ut* and *re* there is a tone major; from *re* to *mi*, a tone minor; from *mi* to *fa* a semi-tone major; from *fa* to *sol* a tone major, as well as from *sol* to *la*; from *la* to *si* a tone minor, and from *si* to *ut* a semitone major.

Hence it is concluded, that in this scale there are three intervals which are not entirely just, viz., the third minor from *re* to *fa*, and indeed, being composed of a tone minor and a semi-tone major, it is only in the ratio of 27 to 32, which is somewhat less, viz. an 80th, than that of 5 to 6, the just ratio of the sounds which compose the third minor.

In like manner, the third major, from *fa* to *la*, is too high, being composed of two tones major, whereas it ought to be composed of a tone major and a tone minor, to

* Of the seven notes in the French scale *ut*, *re*, *mi*, *fa*, *sol*, *la*, *si*, four only are generally used among us, as *mi*, *fa*, *sol*, *la*, which are applied to the scale in this order, *fa*, *sol*, *la*, *fa*, *sol*, *la*, *mi*, *fa*, and express the natural series from C. It is of little consequence however which method be used; the principles still remain the same.

be exactly in the ratio of 4 to 5. In the last place, the third minor, from *la* to *ut*, is also altered, and for the same reason as that from *re* to *fa*.

If this disposition of tones major and minor were arbitrary, they might no doubt be arranged in such a manner that fewer intervals should be altered; it would be sufficient for this purpose, to make the tone from *ut* to *re* minor, and that from *re* to *mi* major; the tone from *sol* to *la* might also be made minor, and that from *la* to *si* major. For it will be found, that by this method there would be no more than a single third altered; whereas, according to the other disposition, there are three. This circumstance has given rise to disputes among the musicians respecting the distribution of the tones minor and major; some being desirous, for example, that there should be a tone major between *ut* and *re*, and others a tone minor. The harmonic generation of the diatonic scale, as explained by Rameau, will not however allow this disposition, but only the former, which is that indicated by nature; and notwithstanding its imperfections, which the temperament corrects in the execution, it is preferable to the first of the Grecian scales, a scale very deficient, as it did not comprehend the whole extent of the octave; it is superior also to the second, *mi, fa, sol*, &c., ascribed to Pythagoras, because its desinence is more perfect, and conveys to the ear a rest, which is not in that of Pythagoras, on account of its fall on the tonic note, announced and preceded by the note *si*, the third of the fifth *sol*, the effect of which is so striking to our musical ears, that it has been distinguished by the name of the *sensible note*.

Two modes, properly so called, are known in music, the characters of which are exceedingly striking to ears possessed of any musical sensibility: these are the *major mode* and the *minor mode*. The major mode is, when in the diatonic scale the third of the tonic note is major; such is the third from *ut* to *mi*. The above gamut, or diatonic scale, therefore is in the major mode.

But if the third of the tonic note be minor, it indicates the minor mode. This mode has its scale as well as the major. Thus, for example, if we assume *la* as the tonic note, the scale of the minor mode ascending will be *la, si, ut, re, mi, fa, sol* ♯, *la*. We here make use of the term ascending, because it is a singularity of the minor mode, that its scale descending, is different from what it is ascending; and indeed in descending we ought to say *la, sol, fa, mi, re, ut, si, la*. If the tone were in *ut*, the ascending scale would be *ut, re, mi* ♭, *fa, sol, la* ♭, *si, ut*, and descending *ut si* ♭, *la* ♭, *sol, fa, mi* ♭, *re, ut*. Hence the reason why, in airs in the minor mode, we so often find, without the tone being changed, accidental *flats* or *sharps*, or *naturals*, which soon destroy their effect, or that of those which are in the clef. This is one of those singularities, of the necessity of which the ear made musicians sensible: the cause of it however, which depends on the progress of the fundamental bass, was first explained by Rameau.

To these two modes shall we add a third, proposed by M. de Blainville, under the name of the *mixed mode*, the generation and properties of which he explains in his History of Music? His scale is *mi, fa, sol, la, si, ut, re, mi*. We shall here only observe, that musicians do not seem to have given a very favourable reception to this new mode, and we confess that we are not sufficiently versed in these matters to be able to decide whether they are right or wrong.

But however this may be, the character of the major mode is sprightliness and gaiety; while in the minor mode there is something gloomy and sad, which renders it peculiarly fitted for expressions of that kind.

The modern music has its genera as well as the ancient. The diatonic is the most common; and is that most agreeable to what is pointed out by nature; but the moderns have their chromatic also, and even in certain respects their enharmonic, though in a sense somewhat different from that assigned to these words by the ancients.

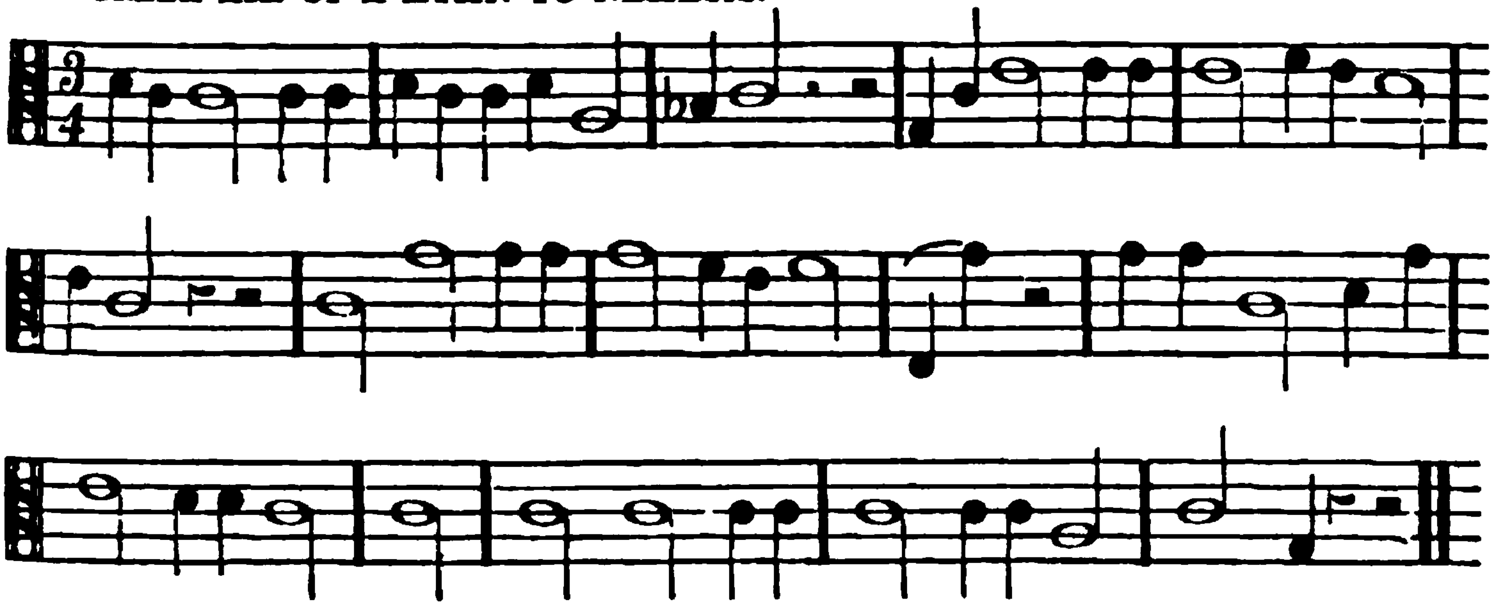
The modulation is chromatic when several semi-tones are passed over in succession, as if we should say *fa*, *mi*, *mi* ♭, *re*, or *sol*, *fa*♯, *fa mi*. It is very rare to have more than three or four semi-tones following each other in this manner; yet in an air of the second act of *la Zingara*, or the Gypsey, an Italian *intermede*, there is a whole lower octave almost from *ut* to *re* in consecutive semi-tones. It is the longest chromatic passage with which we are acquainted.

Rameau finds the origin of this progression in the nature of the fundamental bass, which, instead of proceeding from fifth to fifth, which is its natural movement, proceeds from third to third. But it must here be remarked, that in the first passage from *mi* to *mi* ♭, there ought strictly to be only a semi-tone minor, and from *mi* ♭ to *re* a semi-tone major; but the temperament and constitution of most instruments, by confounding the *re*♯ with *mi* ♭, divide into equal parts the interval from *re* to *mi*, and the ear is affected by them exactly in the same manner, especially by means of the accompaniment.

There are two enharmonic genera, the one called the *diatonic enharmonic*, and the other the *chromatic enharmonic*, but they are very rarely employed by musicians. These genera are not so called because quarters of a tone are employed in them, as in the ancient enharmonic; but because, from the progress of the fundamental bass, there result sounds, which, though taken one for the other, really differ a quarter of a tone, called by the ancients enharmonic, or are in the ratio of 125 to 128. In the diatonic enharmonic, the fundamental bass goes on alternately by fifths and thirds, and in the chromatic enharmonic it goes on alternately by third major and minor. This progression introduces, both into the melody and the harmony, sounds which, belonging neither to the principal tone nor its relatives, convey astonishment to the ear, and affect it in a harsh and extraordinary manner, but which are proper for certain terrible and violent expressions. It was for this reason that Rameau employed the diatonic enharmonic in the trio of the Fates, in his opera of *Hippolitus and Aricia*; and though he was not able to get it executed, he was firmly persuaded that it would have produced a powerful effect, had he found performers disposed to fall into his ideas, so that he suffered it to remain in the partition which was printed. He mentions, as a piece of the enharmonic kind, a scene of the Italian opera of *Coriolano*, beginning with these words, *O iniqui Marmi!* which he says is admirable. Specimens of this genus are to be found also in two of his own pieces for the harpsichord, the *Triumphante* and the *Enharmonique*, and he did not despair of being able to employ the chromatic enharmonic at least in symphonies. And why indeed might he not have done so, since Locatelli, in his first concertos, employed this genus, leaving the flats and sharps to exist, and distinguishing for example the *re* ♯ from *mi* ♭. This, says a modern historian of music, M. de Blainville, is a piece truly infernal, which throws the soul into a violent state of apprehension and terror.

We cannot terminate this article better than by giving a few specimens of the music of different nations. For this purpose we have given, on the opposite page, Grecian, Persian, Chinese, Armenian and Tartar airs, which will serve to form an idea of the modulation that characterises the music of these people.

GREEK AIR OF A HYMN TO NEMESIS.



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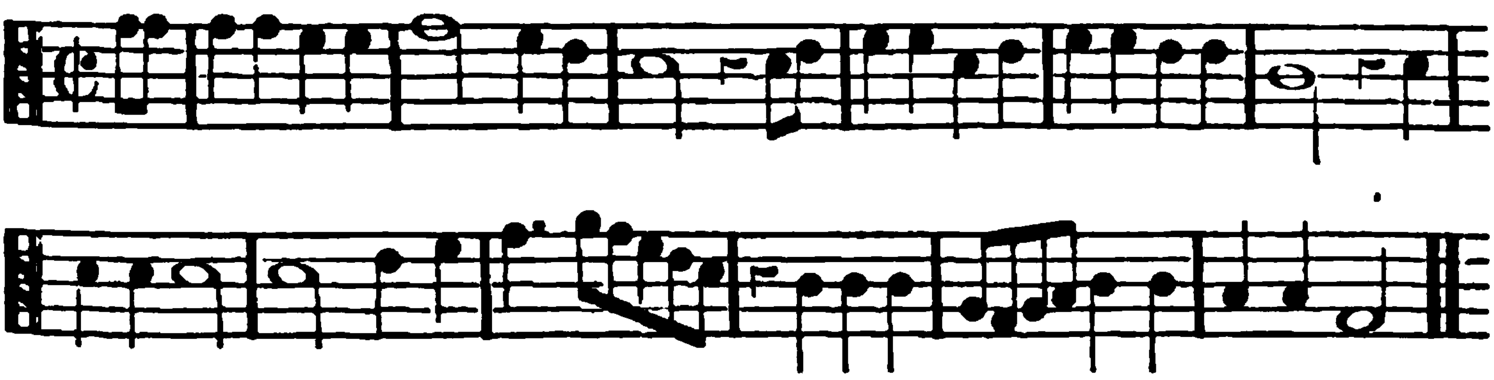
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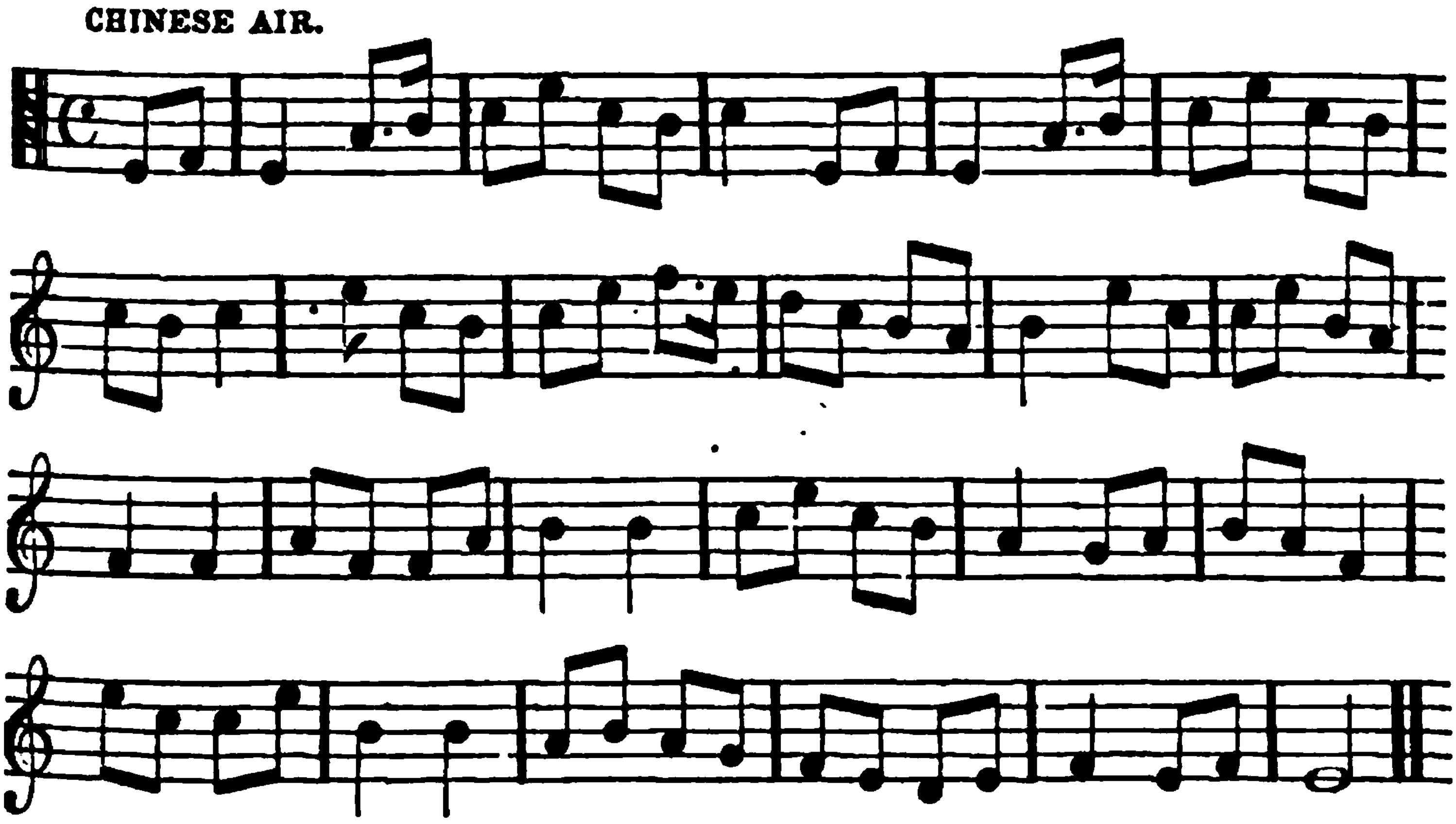
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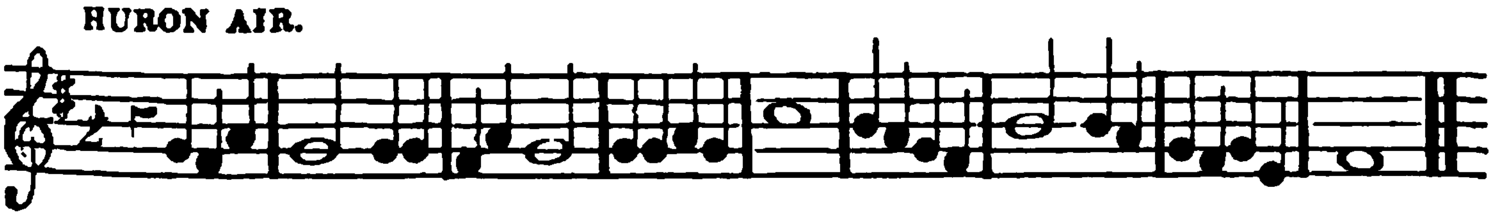

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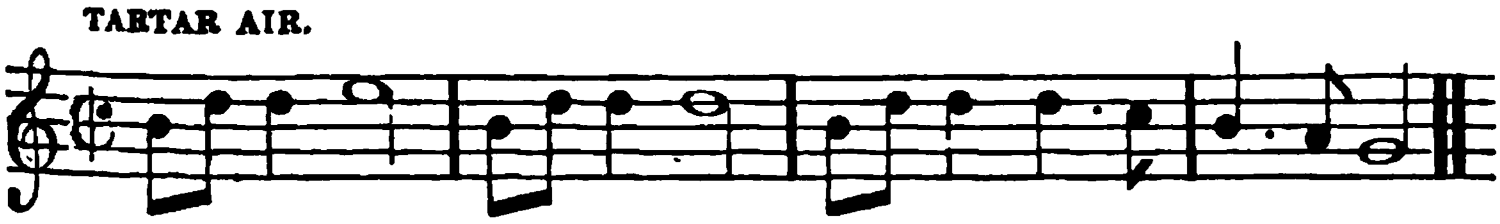

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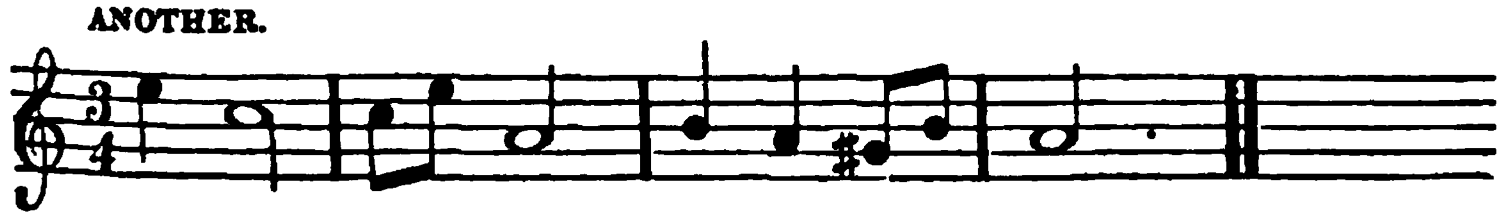


ARTICLE IX.

Musical Paradoxes.

I.—*It is impossible to intonate justly the following intervals,* sol, ut, la, re, sol; *that is to say, the interval between* sol *and* ut *ascending, that from* ut *to* la *redescending from third minor, then ascending from fourth to* re, *and that between* re *and* sol *descending from fifth, and to make the second* sol *in unison with the first.*

It will be found indeed by calculation, that if the first *sol* be represented by 1, the *ut*, ascending from fourth, will be $\frac{3}{4}$; consequently the *la*, descending from third minor, will be $\frac{9}{10}$; the *re* above them will be $\frac{27}{40}$; and in the last place the *sol*, descending from fifth, will be $\frac{81}{80}$. But the sound represented by $\frac{81}{80}$, is lower than that represented by 1, therefore the last *sol* is lower than the first.

But how comes it that experience is contrary to this calculation? In answer to this question we shall observe, that the difference arises merely from the remembrance of the first tone *sol*. If the ear however were not affected by this tone, and if the performer's whole attention were directed to the just intonation of the above intervals, it is evident that he would end with a lower *sol*. It therefore often happens that a voice, without an accompaniment, after having chanted a long air, in which several tones are passed through, remains, in ending, higher or lower than the tone by which it began.

This arises from the necessary alteration of some intervals in the diatonic scale. In the preceding example, from *la* to *ut*, there is only a third minor in the ratio of 27 to 32, and not of 5 to 6; but it is the latter which is intonated if the voice be true and well exercised; consequently the person who chants, lowers by a comma more than is necessary, and therefore it is not astonishing that the last *sol* should always be lower, by a comma, than the first.

II.—*In instruments constructed with keys, such as the harpsichord, it is impossible that the thirds and the fifths should be both just.*

This may be easily demonstrated in the following manner.—Let there be a series of tones, fifths to each other ascending, as *ut, sol, re, la, mi;* if *ut* be denoted by 1, *sol* will be $\frac{2}{3}$, *re* $\frac{4}{9}$, *la* $\frac{8}{27}$, *mi* $\frac{16}{81}$: this *mi* ought to form the third major with the double octave of *ut* or $\frac{1}{4}$, that is to say they ought to be in the ratio of 1 to $\frac{4}{5}$, or of 5 to 4, or of 80 to 64; but this is not the case, for $\frac{1}{4}$ and $\frac{16}{81}$ are to each other as 81 to 64: this *mi* therefore does not form the third major with the double octave of *ut;* or if both are lowered from the double octave, *ut* and *mi* are not thirds to each other, if *mi* is a just fifth to *la*.

In instruments with keys then, such as the harpsicord, however well tuned, all the intervals, the octaves excepted, are either false or altered. This necessarily follows from the manner in which that instrument is tuned; for when all the *ut's* are made octaves to each other, as they ought to be, the *sol* is made the fifth to *ut*, *re* the fifth to *sol*, and the octave is lowered, because it is too high; *la* is then made the fifth to *re*, thus lowered, and *mi* the fifth to *la*, and this *mi* is lowered from octave. By continuing in this manner to ascend twice from fifth, and then to descend from octave, the series of sounds *si*, *fa*, *ut*, *sol* ♯, *re* ♯, *la* ♯, *mi* ♯, *si* ♯, are obtained. But the latter *si* ♯, which ought at most to be in unison with the *ut*, the octave of the first, is found to be higher; for calculation shews that it is expressed by $\frac{262144}{531441}$, which is less than $\frac{1}{2}$ the value of the octave of *ut:* this renders necessary what is called temperament, which consists in lowering gently and equally all the fifths, so that the latter *si* ♯, is found to be exactly the octave of the first *ut*. Such at least is the method taught by Rameau, and it is no doubt the most rational. But whatever may be the method employed, it always consists in rejecting, in a more or less equal

manner from the notes of the octave, this excess of *si* ♯ above *ut*, which cannot be done without altering, in some measure, the fifths, thirds, &c.

We have just seen that the *si* ♯, given by the progression of fifths, is higher than *ut*; but if the following progression of thirds be employed, *ut*, *mi*, *sol* ♯, *si* ♯, this *si* ♯ will be very different from the former; for it will be found that it is expressed by $\frac{64}{125}$, while the octave of *ut* is $\frac{1}{2}$. But $\frac{1}{2}$ is less than $\frac{64}{125}$, consequently this *si* ♯ is below *ut* expressed by $\frac{1}{2}$, and the interval of these two sounds is expressed by the ratio of 128 to 125, which is the fourth of the enharmonic tone.

III.—*A lower note, for example* re, *affected by a sharp, is not the same thing as the higher note* mi, *affected by a flat; and the case is the same with other notes which are a whole tone distant from each other.*

The sharps are generally given by the major mode, and even by the minor, provided the sub-tonic note is not distant from the tonic more than a semi-tone major, as the *si* is from *ut*, in the tone of *ut*; then, as from *re* to *mi* there is a tone minor, which is composed of a semi-tone major and a semi-tone minor, if we take away a semi-tone major, by which *re* ♯ ought to be lower than *mi*, the remainder will be a semi-tone minor, by which the same *re* ♯ ought to be higher than *re*. If the distance between the notes were a tone major, the sharp would raise the lower note by an interval equal to a semi-tone minor, plus a comma of 80 to 81, which is a mean semi-tone between the major and the minor.

The note therefore is raised by the sharp only a mean semi-tone, or a semi-tone minor.

Flats are generally introduced in modulation by the minor mode, when it is necessary to lower the note a third, so that it shall form with the tonic a third minor: *mi* flat therefore ought to form with *ut* a third minor; consequently if from the third major *ut mi*, which is $\frac{4}{5}$, we take the third minor, which is $\frac{5}{6}$, the remainder $\frac{24}{25}$ is the quantity which expresses how much the flat lowers the *mi* below the natural tone: *mi* flat then is higher than *re* sharp.

In practice however the one is taken for the other, especially in instruments constructed with keys: the flat in these is lowered, and the sharps gradually raised, till they coincide with each other; and we do not know whether practice would gain much by making a distinction between them.

ARTICLE X.

On the cause of the pleasure arising from music—The effects of it on man and on animals.

It has often been asked, why two sounds, which form to each other the fifth and the third, excite pleasure, while the ear experiences a disagreeable sensation by hearing sounds which are no more than a tone or a semi-tone distant from each other. Though it is difficult to answer this question, the following observations may tend to throw some light on it.

Pleasure, we are told, arises from the perception of relations, as may be proved by various examples taken from the arts. The pleasure therefore derived from music, consists in the perception of the relations of sounds. But are these relations sufficiently simple for the soul to perceive and distinguish their order? Sounds will please when heard together in a certain order; but on the other hand, they will displease if their relations are too complex, or if they are absolutely destitute of order.

This reasoning will be sufficiently proved by an enumeration of the known concords. In unison, the vibrations of two sounds continually coincide throughout the whole time of their duration; this is the simplest kind of relation. Unison also is the first concord. In the octave, the two sounds, of which it is composed, perform their vibrations in such a manner, that two of the one are completed in the same time as

2 c 2

one of the other. Thus unison is succeeded by the octave. It is so natural to man, that he who, through some defect in his voice, cannot reach a sound too grave or too acute, falls into the higher or lower octave.

When the vibrations of two sounds are performed in such a manner, that three of the one correspond to one of the other, these give the simplest relation, next to those above mentioned. Who does not know, that the concord most agreeable to the ear is the twelfth, or the octave of the fifth? In this respect it even surpasses the fifth, the ratio of which, a little more compounded, is that of 2 to 3.

Next to the fifth is the double octave of the third, or the seventeenth major, which is expressed by the ratio of 1 to 3. This concord therefore, next to the twelfth, is the most agreeable; and if it be lowered from the double octave, to obtain the third, it will still be in consonance; the ratio of 4 to 5, by which it is then expressed, being very simple.

In the last place the fourth, expressed by $\frac{3}{4}$, the third minor, expressed by $\frac{5}{6}$, and the sixths, both major and minor, expressed by $\frac{3}{5}$ and $\frac{5}{8}$, are concords, and for the same reason.

But it appears that all the other sounds, after these relations, are too complex for the soul to perceive their order: of this kind are the intervals called the tone major and the tone minor, expressed by $\frac{8}{9}$ and $\frac{9}{10}$, and much more so the semi-tones major and minor, expressed by $\frac{15}{16}$ and $\frac{24}{25}$. Such also are the concords of third and fifth, however little they may be altered; for the third major, raised a comma, is expressed by $\frac{64}{81}$; and the fifth diminished by the same quantity, has for its expression $\frac{27}{40}$: in the last place, the tritone, as from *ut* to *fa* ♯, is one of the most disagreeable discords, and is expressed by $\frac{18}{25}$.

The following very strong objection however may be made to this reasoning. How can the pleasure arising from concords consist in the perception of relations, since the soul often does not know whether such relations exist between the sounds? The most ignorant person is no less pleased with a harmonious concert than he who has calculated the relation of all its parts: what has hitherto been said may therefore be more ingenious than solid.

We cannot help acknowledging that we are rather inclined to think so; and it appears to us that the celebrated experiment on the resonance of sonorous bodies, may serve to account, in a still more plausible manner, for the pleasure arising from concords; because, as every sound degenerates into mere noise, when not accompanied by its twelfth and its seventeenth major, besides its octaves, is it not evident that, when we combine any sound with its twelfth or its seventeenth major, or with both at the same time, we only imitate the process of nature, by giving to that sound, in a fuller and more sensible manner, the accompaniment which nature itself gives it, and which cannot fail to please the ear on account of the habit it has acquired of hearing them together? This is so agreeable to truth, that there are only two primitive concords, the twelfth and the seventeenth major; and that the rest, as the fifth, the third major, the fourth, and the sixth, are derived from them. We know also that these two primitive concords are the most perfect of all, and that they form the most agreeable accompaniment that can be given to any sound; though on the harpsichord, for example, to facilitate execution, the third major and the fifth itself, which with the octave form what is called perfect harmony, are substituted in their stead. But this harmony is perfect only by representation, and the most perfect of all would be that in which the twelfth and the seventeenth were combined with the fundamental sound and its octaves. Rameau therefore adopted it as often as he could in his chorusses, and particularly in his Pygmalion. We might enlarge farther on this idea, but what has been already said will be sufficient for every intelligent reader.

Some very extraordinary things are related in regard to the effects produced by

the music of the ancients, which, on account of their singularity, we shall here mention. We shall then examine them more minutely, and shew that, in this respect, the modern music is not inferior to the ancient.

Agamemnon, it is said, when he set out on the expedition against Troy, being desirous to secure the fidelity of his wife, left her under the care of a Dorian musician, who, by the effect of his airs, rendered fruitless, for a long time, the attempts of Ægisthus to obtain her affection; but that Prince having discovered the cause of her resistance, got the musician put to death, after which he triumphed without difficulty over the virtue of Clytemnestra.

We are told also that, at a later period, Pythagoras composed songs or airs capable of curing the most violent passions, and of recalling men to the paths of virtue and moderation: while the physician prescribes draughts for curing bodily diseases, an able musician might therefore prescribe an air for rooting out a vicious passion.

The story of Timotheus, the director of the music of Alexander the Great, is well known.—One day, while the prince was at table, Timotheus performed an air in the Phrygian mode, which made such an impression on him that, being already heated with wine, he flew to his arms, and was going to attack his guests, had not Timotheus immediately changed the style of his performance to the Sub-Phrygian. This mode calmed the impetuous fury of the monarch, who resumed his place at table. This was the same Timotheus who, at Sparta, experienced the humiliation of seeing publicly suppressed four strings which he had added to his lyre. The severe Spartans thought that this innovation would tend to effeminate their manners, by introducing a more extensive and more variegated kind of music. This at any rate proves that the Greeks were convinced that music had a peculiar influence on manners; and that it was the duty of government to keep a watchful eye over that art.

Who indeed can doubt that music is capable of producing such an effect? Let us only interrogate ourselves, and examine what have been our sensations on hearing a majestic or warlike piece of music, or a tender and pathetic air sung or played with expression. Who does not feel that the latter tends as much to melt the soul, and dispose it to pleasure, as the former to rouse and exalt it? Several facts in regard to the modern music place it on a level in this respect with the ancient.

The modern music indeed has also had its Timotheus, who could excite or calm, at his pleasure, the most impetuous emotions. Henry III. king of France, says "Le Journal de Sancy," having a concert on occasion of the marriage of the Duke de Joyeuse, Claudin le Jeune, a celebrated musician of that period, executed certain airs, which had such an effect on a young nobleman, that he drew his sword, and challenged every one near him to combat; but Claudin, equally prudent as Timotheus, instantly changed to an air, apparently Sub-Phrygian, which appeased the furious youth.

But, what shall we say of Stradella, the celebrated composer, whose music made the daggers drop from the hands of his assassins? Stradella having carried off the mistress of a Venetian musician, and retired with her to Rome, the Venetian hired three desperadoes to assassinate him; but fortunately for Stradella, they had an ear sensible to harmony. These assassins, while waiting for a favourable opportunity to execute their purpose, entered the church of St. John de Lateran, during the performance of an Oratorio composed by the person whom they intended to destroy, and were so affected by the music, that they abandoned their design, and even waited on the musician to forewarn him of his danger. Stradella, however, was not always so fortunate; other assassins, who apparently had no ear for music, stabbed him some time after at Genoa: this event took place about the year 1670.

Every person almost has heard that music is a cure for the bite of the tarantula. This cure, which was formerly considered as certain, has by some been contested; but, however this may be, Father Schott, in his "Musurgia Curiosa," gives the

tarantula air, which appears to be very dull, as well as that employed by the Sicilian fishermen to entice the thunny fish into their nets.

Various anecdotes are related respecting persons whose lives have been preserved, by music affecting a sort of revolution in their constitutions. A woman being attacked for several months with the vapours, and confined to her apartment, had resolved to starve herself to death: she was however prevailed on, but not without difficulty, to see a representation of the *Servo Padrona*, at the conclusion of which she found herself almost cured, and, renouncing her melancholy resolution, was entirely restored to health by a few more representations of the like kind.

There is a celebrated air in Switzerland, called *Ranz des Vaches*, which had such an extraordinary effect on the Swiss troops in the French service, that they always fell into a deep melancholy when they heard it: Louis XIV. therefore forbade it ever to be played in France, under the pain of a severe penalty. We are told also of a Scotch air (*Lochaber no more*) which has a similar effect on the natives of Scotland.

Most animals, and even insects, are not insensible to the pleasure of music. There are few musicians perhaps who have not seen spiders suspend themselves by their threads in order to be near the instruments. We have several times had that satisfaction. We have seen a dog who, at an adagio of a sonata by Sennaliez, never failed to shew signs of attention, and some peculiar sensation by howling.

The most singular fact however is that mentioned by Bonnet, in his History of Music. This author relates that an officer, being shut up in the Bastile, had permission to carry with him a lute, on which he was an excellent performer: but he had scarcely made use of it for three or four days, when the mice issuing from their holes, and the spiders suspending themselves from the ceiling by their threads, assembled around him to participate in his melody. His aversion to these animals made their visit at first disagreeable, and induced him to lay aside his recreation; but he was soon so accustomed to them, that they became a source of amusement. We are informed by the same author, that he saw, in 1688, at the country seat of Lord Portland, the English ambassador in Holland, a gallery in a stable, employed, as he was told, for giving a concert once a week to the horses, which seemed to be much affected by the music. This, it must be allowed, was carrying attention to horses to a very great length. But it is not improbable that this anecdote was told to Bonnet by some person, in order to make game of him.

ARTICLE XI.

Of the properties of certain Instruments, and particularly Wind Instruments.

I. We are perfectly well acquainted with the manner in which stringed instruments emit their sounds; but erroneous ideas were long entertained in regard to wind instruments, such as the flute; for the sound was ascribed to the interior surface of the tube. The celebrated Euler first rectified this error, and it results from his researches:

1st. That the sound produced by a flute, is nothing else than that of the cylinder of air contained in it.

2d. That the weight of the atmosphere which compresses it, acts the part of a stretching weight.

3d. That the sound of this cylinder of air, is exactly the same as that which would be produced by a spring of the same mass and length, extended by a weight equal to that which compresses the base of the cylinder.

This fact is confirmed by experiment and calculation: for Euler found that a cylinder of air, of 7½ Rhinlandish feet, at a time when the barometer is at a mean height, must give C—*sol—ut;* and such is nearly the length of the open pipe of an organ which emits that sound. The reason of its being made generally 8 feet is, be-

cause that length is required at those times when the weight of the atmosphere is greater.

Since the weight of the atmosphere produces, in regard to the sounding cylinder of air, the same effect as that produced by the weight which stretches a string, the more that weight is increased, the more will the sound be elevated; it is therefore observed that during serene warm weather, the tone of wind instruments is raised; and that during cold and stormy weather it is lowered. These instruments also emit a higher sound, in proportion as they are heated; because the mass of the cylinder of heated air becoming less, while the weight of the atmosphere remains unchanged, the case is exactly the same as if a string should become less, and be still stretched by the same weight: every body knows that such a string would emit a higher tone.

But as stringed instruments must become lower, because the elasticity of the strings insensibly decreases, it thence follows that wind and stringed instruments, however well tuned they may be to each other, soon become discordant: for this reason the Italians never admit the former into their orchestras.

II. A very singular phenomenon is observed in regard to wind instruments, such as the flute and huntsman's horn: with a flute, for example, when all the holes are stopped, if you blow faintly into the mouth aperture, a certain tone will be produced; if you blow a little stronger, the tone instantly rises to the octave; and by blowing successively with more force, you will produce the twelfth, or fifth above the octave; then the double octave or seventeenth major.

The cause of this effect is the division of the cylinder of air contained in the instrument: when you breathe into the flute gently, the whole column resounds, and it emits the lowest tone; but if you endeavour, by a stronger inspiration, to make it perform quicker vibrations, it divides itself into two parts, which perform their vibrations separately, and which consequently must give the octave; a still stronger inspiration makes the column divide itself into three portions, which give the twelfth, &c.

III. It remains for us to speak of the trumpet marine. This instrument is only a monochord of a singular construction, being composed of three boards that form a triangular body. It has a very long neck, and one thick string, mounted on a bridge, which is firm on the one side, and tremulous on the other. It is struck by a bow with one hand, and with the other the string is stopped or pressed on the neck by means of the thumb, applied to the divisions indicated for the different tones. The trembling of the bridge, when the string is struck, makes it imitate the sound of the trumpet; and this it does to such perfection that it is scarcely possible to distinguish the one from the other. Hence it had its name; but whereas, in the common stringed instruments the tone becomes lower as the part of the string struck is longer, the case here is the contrary; for if the half of the string, for example, gives *ut*, the two thirds give the *sol* above, and the three fourths give the octave.

M. Saveur first assigned the reason of this singularity, and proved it in a sensible manner, by shewing that when the string, by the gentle application of the finger, is divided into two parts which are to each other as 1 to 2, whatever part be touched, the greater immediately divides itself into two equal portions, which consequently perform their vibrations in the same time, and give the same sound as the less. But the less being the third of the whole, and the two thirds of the half, it must give the fifth or *sol* when the half gives *ut*. In like manner, the three fourths of the string divide themselves into three portions, each equal to the remaining fourth, and as they perform their vibrations separately they must emit the same sound, which can be only the octave of the half. The case is the same with the other sounds of the trumpet marine, which may be easily explained on the same principle.

ARTICLE XII.

Of a fixed Sound; method of preserving and transmitting it.

Before the effects of the temperature of the air on sound, and on the instruments by which it is produced, were known, this would not have formed the subject of a question, but to the few possessed of an ear exceedingly fine and delicate, and in which the remembrance of a tone is perfect: to others no doubt would remain that a flute, not altered, would always give the same tone. Such an opinion however would be erroneous, and if the means of transmitting to St. Domingo, for example, or to Quito, or only to posterity, the exact pitch of our opera were required, to solve this problem would be attended with more difficulty than might at first be imagined.

Notwithstanding what may be generally said in this respect, we shall here begin by a sort of paradox. It is every where said that the degree of the tone varies according to the weight of the atmosphere, or the height of the barometer. This we can by no means admit: and we flatter ourselves that we can prove the contrary.

It has been demonstrated by the formulæ of Euler, and no one entertains any doubt in regard to their truth, that if G represents the weight which compresses the column of air in a flute, L the length of that column, and W its weight, the number of the vibrations it makes will be expressed by $\sqrt{\frac{G}{WL}}$, that is to say, will be in the compound ratio of the square root of G, or the compressing weight taken directly, and the product of the length by the weight taken inversely. Let us suppose then that the length of the column of air put in vibration is invariable, and that the gravity of the atmosphere only, or G, is variable, as well as the weight of the vibrating column. In this case we shall have the number of the vibrations proportional to the expression $\sqrt{\frac{G}{W}}$. But the density of any stratum of air being proportional to the whole weight of that part of the atmosphere immediately above it, it thence follows that W, which in equal lengths is as the density, is as G. The fraction $\frac{G}{W}$ therefore is constantly the same, when difference of heat does not alter the density. The square root of $\frac{G}{W}$ then is always the same; consequently there will be no variation in the number of the vibrations, or in the tone, at whatever height in the atmosphere the instrument may be situated, or whatever be the gravity of the air, provided its temperature has not changed.

This reasoning, in our opinion, is unanswerable; and if the gravity of the air has hitherto been reckoned among those causes which alter the tone of wind instruments, it is because it has been implicitly believed that the weight of the column of air put in vibration is invariable. It is however evident that under the same temperature it must be more or less dense, according to the greater or less density of the atmosphere; since it has a communication with the surrounding stratum of air, the density of which is proportional to that gravity. But the gravity in equal volumes is proportional to the density; therefore, &c.

Nothing then remains to be considered but the temperature of the air, which is the only cause that can produce variations in the tone of a wind instrument. But whatever may be the degree of heat or of cold, the tone might be fixed in the following manner. For this purpose provide an instrument, such as a German flute, the cylinder of air in which can be lengthened or shortened by moving the joints closer to or farther from each other; and have another so constructed, as to remain invariable, and which ought to be preserved in the same temperature, such as 54 degrees

of Fahrenheit's thermometer. The first flute being at the same degree of temperature, bring them both into perfect unison, and then heat the first to 74° degrees of Fahrenheit, which will necessarily communicate to the cylinder of air contained in it the same degree of heat, and lengthen it by the quantity necessary to restore perfect unison: it is evident that if this elongation were divided into twenty parts, each of them would represent the quantity by which the flute ought to be lengthened for each degree of Fahrenheit's thermometer.

But it may be readily conceived that the quantity of this elongation, which at most would be but a few lines, could not be divided into so many parts; and therefore it ought to be executed by the motion of a screw, that is to say one of the joints of the instrument should be screwed into the other; for it would then be easy to make this elongation correspond to a whole revolution, and hence it might be divided into a great number of equal parts.

By these means the opera at Lima, if required, where the heat frequently rises to 110° of Fahrenheit's thermometer, might be made to have exactly the same pitch or tone as at Paris. But this is sufficient on a subject the utility of which would not be worth the trouble necessary for attaining to such a degree of precision.

ARTICLE XIII.

Singular application of Music to a question in Mechanics.

This question was formerly proposed by Borelli; and though we do not think that it can at present be a subject of controversy, it has occasioned some difference of opinion among a certain class of mechanicians.

Fasten a string at one end to a fixed point; and having stretched it over a kind of bridge, suspend from it a weight, such as 10 pounds for example.

Now, if instead of the fixed point, which maintains the string in its place in opposition to the action of the weight, a weight equal to the former be substituted, will the string in both cases be equally stretched?

We have no doubt that every well informed mechanician will readily believe that in both cases the tension will be the same; and this necessarily follows from the principle of equality between action and re-action. According to this principle, the immoveable point, which in the first case counteracts the weight suspended from the other end of the string, opposes to it a resistance exactly equal to the action which it exercises: if a weight equal to the former be therefore substituted instead of the fixed point, every thing remains equal in regard to the tension experienced by the parts of the string, and which tends to separate them.

But music furnishes us with a method of proving this truth to the reason, by means of the sense of hearing; for as the tone is not altered while the tension remains the same, nothing is necessary but to make the following experiment. Take two strings of the same metal, and the same size, and having fastened one of them by one end to a fixed point, stretch it over a bridge, so as to intercept between it and the fixed point a determinate length, such as a foot for example; and suspend from the other end of it a given weight, such as ten pounds. Then extend the second string over two bridges, a foot distant from each other, and suspend from each extremity of it a weight of ten pounds; if the tone of these two strings be the same, there will be reason to conclude that the tension also is the same. We do not know whether this experiment was ever made; but we will venture to assert that it will decide in favour of equality of tension.

This ingenious application of music to mechanics, is the invention of Diderot, who proposed it in his "Mémoires sur différentes sujets de Mathematique et de Physique," printed at Paris, in octavo, in the year 1748.

ARTICLE XIV.

Some singular considerations in regard to the Flats and Sharps, and to their progression on their different tones.

Those in the least acquainted with music know that, according to the different keys employed in modulation, a certain number of sharps or flats are required; because in the major mode, the diatonic scale, with whatever tone we begin, must be similar to that of *ut*, which is the simplest of all, as it has neither sharp nor flat. These flats or sharps have a singular progress, which deserves to be observed; it is even susceptible of a sort of analysis, and as we may say algebraic calculation.

To give some idea of it, we shall first remark, that a flat may and ought to be considered as a negative sharp, since its effect is to lower the note a semi-tone; whereas the sharp raises it the same quantity. This consideration alone may serve to determine all the sharps and flats of the different tones.

It may be readily seen that when a melody in *ut* major is raised a fifth, or brought to the tone of *sol*, a sharp is required on the *fa*. It may therefore be thence concluded, that this modulation, lowered a fifth or brought to *fa*, will require a flat; and indeed one is required on the *si*.

It hence follows also, that if the air be raised another fifth, that is to say to *re*, one sharp more will be required; and this is the reason why two are necessary. But to raise two fifths, and then descend an octave to approach the primitive tone, is to rise only one tone; consequently to raise the air one tone, two sharps must be added. The tone of *re* indeed requires two sharps, and for the same reason the tone of *mi* requires four.

The tone of *fa* requires one flat, and that of *mi* requires four sharps; therefore, when an air is raised a semi-tone, five flats must be added; for, a flat being a negative sharp, it is evident that such a number of flats must be added to the four sharps of *mi*, as shall efface these four sharps, and leave one flat remaining; which cannot be done but by five flats; for, according to the language of analysis, $-5x$ must be added to $4x$, to leave as remainder $-x$. For the same reason, if the modulation be lowered a semi-tone, five sharps must be added: thus, as the tone of *ut* has neither sharps nor flats, five sharps will be found necessary for *si*, which is indeed the case. If the modulation be still lowered a tone, to be in *la*, we must add two flats, in the same manner as two sharps are added when we rise a tone. But five sharps plus two flats, is the same thing as five sharps minus two sharps, or three sharps. We still find therefore, by this method, that the tone of *la* requires three sharps.

But, before we proceed farther, it will be necessary to observe, that all the chromatic tones, that is to say all those inserted between the tones of the natural diatonic scale, may be considered as sharps or flats; for it is evident that *ut* ♯ or *re* ♭ are the same thing. It is very singular however, that according as this note is considered an inferior one affected by a sharp, or a superior one affected by a flat, the number of sharps required by the tone of the first, *ut* ♯, for example, and that of the flats required by the tone of the second, *re* ♭, always make 12; which evidently arises from the division of the octave into 12 semi-tones: therefore, since *re* ♭, as above shewn, requires five flats, if, instead of this tone, we consider it as *ut*, seven sharps will be required; but for the facility of execution it is much better, in the present case, to consider this tone as *re* ♭, than *ut* ♯.

This change therefore ought always to be made when the number of the sharps exceeds six; so that, since ten sharps, for example, would be found in the tone of *la* ♯, we must call it *si* ♭, and we shall have for that tone two flats, because two flats are the complement of ten sharps. On the other hand, in following the progression of the semi-tones descending, if we should find a greater number of sharps than 12, we ought to reject 12, and the remainder will be that of the tone proposed:

for example, as *ut* has neither sharp nor flat, we have five sharps for the lower tone *si*; ten sharps for the semi-tone below *la* ♯; fifteen sharps for the still lower semi-tone *la*: if twelve sharps therefore be rejected, there will remain three, which are indeed the number of sharps necessary in the tone of A—*mi*—*la*.

The tone of *sol* ♯ ought to have 8 or 4 flats, if we call it *la* ♭.

The tone of *sol* will have 13 sharps, from which if 12 be deducted, one sharp will remain, as is well known.

The tone of *fa* ♯ will have 6 sharps, or 6 flats, if we call it *sol*♭.

The tone *fa* ought to have 6 flats plus 5 sharps; that is to say, 1 flat, as the 5 sharps destroy the same number of flats.

That of *mi* will have 1 flat plus 5 sharps; that is, 4 sharps, as the flat destroys one of them.

That of *re* ♯ will have 9 sharps or 3 flats, if it be considered as *mi* ♭.

That of *re* will have 14 sharps; that is to say 2, by rejecting 12, or 3 flats plus 5 sharps = 2 sharps.

That of *ut* will have 7 sharps, or 5 flats, if we call it *re* ♭.

In the last place, the tone *ut* natural will have 12 sharps; that is to say none, or 5 flats plus 5 sharps, which destroy each other.

The very same results would be obtained in ascending by semi-tone after semi-tone from *ut*, and adding 5 flats for each; taking care to reject 12 when they exceed that number. Our readers, by way of amusement, may make the calculation. By calculating the number of the semi-tones, either ascending or descending, we might in like manner find that of the sharps or flats of any tone given.

Let us take, for example, that of *fa* ♯: from *ut* ascending there are six semi-tones, and six times 5 flats makes 30 flats; from which if we deduct 24, a multiple of 12, the remainder will be 6: *sol* ♭ therefore will have 6 flats.

The same *fa* ♯ is 6 tones lower than *ut*; consequently there must be six times 5 or 30 sharps; from which if 24 be deducted, 6 sharps will remain, as we have found by another method.

The tone of *sol* is 5 semi-tones lower than *ut*; consequently there must be five times 5, or 25 sharps; from which if 24 be deducted, there will remain only one sharp.

As the same tone is 7 semi-tones higher than *ut*, there must be seven times 5 or 35 flats; from which if 24 be deducted, the remainder will be 11 flats, that is to say one sharp.

This progression appeared to us so curious as to be worthy of this notice; but in order that it may be exhibited under a clearer and more favourable point of view, we shall form it into a table, which will at any rate be useful to those who are beginning to play on the harpsichord. For this purpose we shall present each chromatic note as flattened or sharpened, and on the left of the former we shall mark the sharps it requires, and the flats on the right of the latter.

0 sharp		*ut* *		0 flats.	1 sharp		*sol* *		
7 sharps	*ut* ♯	or	*re*♭ *	5 flats.	8 sharps	*sol* ♯	or	*la* *	4 flats.
2 sharps		*re* *			3 sharps		*la* *		
9 sharps	*re* ♯	or	*mi*♭	3 flats.	10 sharps	*la* ♯	or	*si*♭ *	2 flats.
4 sharps		*mi* *			5 sharps		*si* *		
11 sharps		*fa* *		1 flat.	0 sharp		*ut* *		0 flat.
6 sharps	*fa* ♯	or	*sol*♭ *	6 flats.					

Of these tones, we have marked those usually employed with a *; for it may be easily conceived, that by employing *re* ♯ under this form, we should have 9 sharps, which would give two notes with double sharps, viz. *fa* ♯♯, *ut* ♯♯; so that the gamut

would be *re*♯, *mi*♯ or *fa*, *fa*♯♯ or *sol*, *sol*♯, *la*♯, *si*♯ or *ut*, *ut*♯♯ or *re*, *re*♯; which it would be exceedingly difficult to execute: but by taking *mi*♭, instead of *re*♯, we have only 3 flats, which renders the gamut much simpler, as it then becomes

mi♭, *fa*, *sal*, *la*♭, *si*♯, *ut*, *re*, *mi*♭.

We are almost inclined to ask pardon of our readers for having amused them with this frivolous speculation; but we hope the title of our work will plead our excuse.

ARTICLE XV.

Method of improving Barrel-instruments, and of making them fit to execute airs of every kind.

The mechanism of that instrument called the barrel organ, is well known. It consists of a great number of pipes, graduated according to the tones and semi-tones of the octave, or at least those semi-tones which the progress of modulation in general requires. But these pipes never sound except when the wind of a bellows, kept in continual action, is made to penetrate to them by means of a valve. This valve is shut by a spring, and opened when necessary by a small lever, raised by spikes implanted in a wooden cylinder, which is put in motion by a crank. The crank serves also to move the bellows, which must continually furnish the air, destined to produce the different sounds by its introduction into the pipes.

But in order that the subject of this article may be properly comprehended, it will be necessary that the reader should have a perfect idea of the manner in which the notes are arranged on the cylinder.

The different small levers, which must be raised to produce the different tones, being placed at a certain distance from each other, that of half an inch for example, circular lines are traced out at that distance on the cylinder. One of these lines is intended for receiving the spikes that produce the sound *ut*, the next for those that sound *ut*♯, the next for those that give *re*, and so on. There are as many lines of this kind as there are pipes; but it may be easily conceived that the duration of an air or tune can not exceed one revolution of the cylinder.

Let us suppose then that the air consists of twelve measures. Each of these circumferences is divided into twelve equal parts at least, by twelve lines drawn parallel to the axis of the cylinder; and if we suppose that the shortest note of the air is a quaver, and that the air is in triple time, denoted by $\frac{3}{4}$, each interval must be divided into six equal portions; because, in this case, a measure will contain six quavers. Let us now suppose that the first notes of the air are *la*, *ut*, *si*, *re*, *ut*, *mi*, *re*, &c., all equal notes, and all simple crotchets. At the beginning of the line for receiving the *la*, and of the first measure, a spike must be placed of such a construction as to keep raised up during the third of a measure the small lever that makes the *la* sound; then, in the line destined for the *ut*, at the end of the second division or beginning of the third, a spike similar to the first must be fixed in the cylinder; and in the line destined for the *si*, another of the same kind must be placed: it is evident that, when the cylinder begins to turn, the first spike will make *la* sound during the third of a measure. The second, as the first third of the measure is elapsed, will catch the lever and make *ut* sound; and the third will in like manner make *si* sound during the last third. The instrument therefore will say *la*, *ut*, *si*, &c.

If, instead of three crotchets, there were six quavers, which in this measure are the first long, the second short, the third long, and so on alternately, which are called dotted quavers, it may be easily perceived, that, after the spikes of the first, third, and fifth notes have been fixed in the respective places of the division where they ought to be, nothing will be necessary but to take care that the spike of the first quaver, which in this time ought to be equal to a quaver and a half, shall have its head constructed in such manner as to raise the lever during one part and a half of the six

divisions into which the measure is divided; which may be done by giving it a tail behind of the necessary length. In regard to the short quavers, the spikes representing them ought to be removed back half a division, and to be formed in such a manner as to keep the lever corresponding to them raised up only during the revolution of a semi-division of the cylinder. By these examples it may be easily seen what must be done in the other cases, that is, when the notes have other values.

Were the cylinder immoveable in the direction of its axis, only one air could be performed; but as the spikes move the small levers merely by touching them beneath in a very narrow space, such as the breadth of a line at most, which is a mechanism that may be easily conceived, it will be readily seen that, by giving to the cylinder the small lateral motion of a line, none of the spikes can communicate motion to the levers. Another line therefore, to receive spikes arranged so as to produce a different air, may be drawn close to each of the first set of lines, and the number of the different sets of lines may be six or seven, according to the interval between the first lines, which is the same as that between the middle of one pipe and the middle of the neighbouring one: by these means, if the cylinder be moved a little in the direction of its axis, the air may be changed.

Such is the mechanism of the hand or barrel organ, and other instruments constructed on the same principle; but it may be easily seen that they are attended with this inconvenience, that they can perform only a very small number of airs. But as a series of five, six, eight, or a dozen of tunes, is soon exhausted, it might be a matter of some importance to discover a method by which they might be changed at pleasure.

We agree in opinion with Diderot, who has given some observations on this subject, in the work above quoted, that this purpose might be answered by constructing the cylinder in the following manner. Let it be composed of a piece of solid wood, covered with a very hard cushion, and let the whole be pushed into a hollow cylinder, of about a line in thickness. On this inner cylinder draw the lines destined to receive the spikes, placed at the proper intervals for producing the different tones; and let holes be pierced in these lines at certain distances, six for example in each division of the measure if it be triple time, or eight in the measure if it be common time, denoted by C: we here suppose that no air is to be set that has shorter notes than plain quavers. Twelve holes per measure will be required in the first case, and sixteen in the second, if the air contains semi-quavers.

It may now be readily conceived that on a cylinder of this kind any air whatever might be set; nothing will be necessary for this purpose, but to thrust into the holes of the exterior cylinder spikes of the proper length, taking care to arrange them as above explained; they will be sufficiently firm in their places in consequence of the elasticity of the cushion*, strongly compressed between the inner cylinder and the hollow outer one. When the air is to be changed, the spikes may be drawn out, and put into a box divided into small cells, in the same manner as printing types when distributed in the cases. The interior cylinder may then be made to revolve a little, in order to separate the holes in the cushion from those in the exterior cylinder, and a new air may then be set with as much facility as the former.

We shall not examine, with Diderot, all the advantages of such an instrument, because it must be allowed that it never can be of much utility, and will have no value in the eyes of the musician. It is however certain that it would be agreeable, for those who possess such instruments, to be able to give more variety to the airs they are capable of performing; and this end would be answered by the construction here described.

* Might not cork be employed instead of the cushion here proposed?

ARTICLE XVI.

Of some Musical Instruments, or Machines, remarkable for their singularity of construction.

At the head of all these musical instruments, or machines, we ought doubtless to place the organ; the extent and variety of the tones of which would excite much more admiration, were it not so common in our churches; for, besides the artifice necessary to produce the tones by means of keys, what ingenuity must have been required to contrive mechanism for giving that variety of character to the tones, which is obtained by means of the different stops, such as those called the voice stop, flute stop, &c.? A complete description therefore of an organ, and of its construction, would be sufficient to occupy a large volume.

The ancients had hydraulic organs, that is, organs the sound of which was occasioned by air produced by the motion of water. These machines were invented by Ctesibius of Alexandria, and his scholar Hero. From the description of these hydraulic organs given by Vitruvius, in the tenth book of his Architecture, Perrault constructed one which he deposited in the King's Library, where the Royal Academy of Sciences held their sittings. This instrument indeed is not to be compared to the modern organs; but it is evident that the mechanism of it has served as a basis for that of ours. St. Jerome speaks with enthusiasm of an organ which had twelve pair of bellows, and which could be heard at the distance of a mile. It thence appears that the method employed by Ctesibius, to produce air to fill the wind box, was soon laid aside, for one more simple; that is, for a pair of bellows.

The performer on the *tambour de basque*, and the automaton flute-player of Vaucanson, which were exhibited and seen with admiration in most parts of Europe, in the year 1749, may be classed among the most curious musical machines ever invented. We shall not however say any thing of the former of these machines, because the latter appears to have been far more complex.

The automaton flute-player performed several airs on the flute, with the precision and correctness of the most expert musician. It held the flute in the usual manner, and produced the tone by means of its mouth; while its fingers, applied on the holes, produced the different notes. It is well known how the fingers might be raised by spikes fixed in a cylinder, so as to produce these sounds; but it is difficult to conceive how that part could be executed which is performed by the tongue, and without which the music would be very defective. Vaucanson indeed confesses that this motion in his machine was that which cost him the greatest labour. Those desirous of farther information on this subject may consult a small work, in quarto, which Vaucanson published respecting these machines.

A very convenient instrument for composers was invented some years ago in Germany: it consists of a harpsichord which, by certain machinery added to it, notes down any air or piece of music, while a person is playing it. This is a great advantage to composers, as it enables them, when hurried away by the fervour of their imagination, to preserve what has successively received from their fingers a fleeting existence, and what otherwise it would often be impossible for them to remember. A description of this machine may be found in the Memoirs of the Academy of Berlin, for the year 1773.

ARTICLE XVII.

Of a new instrument called the Harmonica.

This new instrument was invented in America, by Dr. Franklin, who gave a description of it to Father Beccaria, which the latter published in his works, printed in 1773.

It is well known that when the finger, a little moistened, is rubbed against the

edge of a drinking glass, a sweet sound is produced; and that the tone varies according to the form, size, and thickness of the glass. The tone may be raised or lowered also by putting into the glass a greater or less quantity of water. Dr. Franklin says that an Irishman, named Puckeridge, first conceived the idea, about twenty years before that time, of constructing an instrument with several glasses of this kind, adjusted to the various tones, and fixed to a stand in such a manner, that different airs could be played upon them. Mr. Puckeridge having been afterwards burnt in his house along with this instrument, Mr. Delaval constructed another of the same kind, with glasses better chosen, which he applied to the like purpose. Dr. Franklin, hearing this instrument, was so delighted with the sweetness of its tones, that he endeavoured to improve it; and the result of his researches was the instrument which we are now going to describe.

Cause to be blown on purpose glasses of different sizes, and of a form nearly hemispherical, having each in the middle an open neck. The thickness of the glass, near the edge, should be at most one tenth of an inch, and ought to increase gradually to the neck, which in the largest glasses should be an inch in length, and an inch and a half in breadth in the inside. In regard to the dimensions of the glasses themselves, the largest may be about nine inches in diameter at the mouth, and the least three inches, each glass decreasing in size a quarter of an inch. It will be proper to have five or six of the same diameter, in order that they may be more easily tuned to the proper tones; for a very slight difference will be sufficient to make them vary a tone, and even a third.

When these arrangements are made, try the different glasses, in order to form of them a series of three or four chromatic octaves. To elevate the tone, the edge towards the neck ought to be ground, trying them every moment, for if they be raised too high, it will afterwards be impossible to lower them.

When the glasses have been thus graduated, they must be arranged on a common axis. For this purpose, put a cork stopper very closely into the neck of each, so as to project from it about half an inch; then make a hole of a proper size in all these corks, and thrust into them an iron axis, but not with too much force, otherwise the necks might burst. Care must also be taken to place the glasses in such a manner, that their edges may be about an inch distant from each other, which is nearly the distance between the middle of the keys of a harpsichord.

To one of the extremities of the axis affix a wheel of about eighteen inches in diameter, loaded with a weight of from twenty to twenty-five pounds, that it may retain for some time the motion communicated to it. This wheel, which must be turned by the same mechanism as that employed to turn a spinning wheel, communicates, as it revolves, its motion to the axis, which rests in two collars, one at the extremity, and the other at some distance from the wheel. The whole may be fitted into a box of the proper form, placed on a frame supported by four feet. The glasses corresponding to the seven tones of the diatonic octave, may be painted of the seven prismatic colours in their natural order, that the different tones to which they correspond may be more readily distinguished.

The person who plays on this instrument, is seated before the row of glasses, as if before the keys of a harpsichord; the glasses are slightly moistened, and the wheel being made to revolve, communicates the same motion to the glasses: the fingers are then applied to the edges of the glasses, and the different sounds are by these means produced. It may be easily seen that different parts can be executed with this instrument, as with the harpsichord.

About fourteen or fifteen years ago, an English lady at Paris performed, it is said, exceedingly well on this instrument. The sounds it emits are remarkably sweet, and would be very proper as an accompaniment to certain tender and pathetic airs. It is attended with one advantage, which is, that the sounds can be maintained or

prolonged, and made to swell at pleasure; and the instrument, when once tuned, never requires to be altered. It afforded great satisfaction to many amateurs; but we have heard that the sound, on account of its great sweetness, became at last somewhat insipid, and for this reason perhaps it is now laid aside, and confined to cabinets, among other musical curiosities.

A few years ago, Dr. Chladni, who has made various researches respecting the theory of sound, and the vibrations of sonorous bodies,* invented a new kind of instrument of this kind, to which he gave the name of *euphon*. This instrument has some resemblance to a small writing desk, and contains in the inside 40 glass tubes of different colours, of the thickness of the barrel of a quill, and about sixteen inches in length. They are wetted with water by means of a sponge, and stroked with the fingers in the direction of their length; so that the increase of the tone depends merely on the stronger or weaker pressure, and the slower or quicker movement of the fingers. In the back part there is a perpendicular sounding board, through which the tubes pass. In sweetness of sound, this instrument approaches near to harmonica; but seems to be attended with advantages which the other does not possess.

1st. It is simpler, both in regard to its construction, and the movement necessary to produce the sound; as neither turning nor stopping is required, but merely the motion of the finger.

2d. It produces its sound more speedily; so that as soon as touched the tone may be made as full as the instrument is capable of giving it: whereas in the harmonica the tones, and particularly the lower ones, must be made to increase gradually.

3d. It has more distinctness in quick passages, because the tones do not resound so long as in the harmonica, where the sound of one low tone is often heard when you wish only to hear the following one.

4th. The unison is purer than is generally the case in the harmonica; where it is difficult to have perfect glasses, which in every part give like tones with mathematical exactness. It is however as difficult to be tuned as the harmonica.

5th. It does not affect the nerves of the performer; for a person scarcely feels a weak agitation in the fingers; whereas in the harmonica, particularly in concords of the lower notes, the agitation extends to the arms, and even through the whole body of the performer.

6th. The expense of this instrument will be much less than that of the harmonica.

7th. When one of the tubes breaks, or any other part is deranged, it can be easily repaired: whereas when one of the glasses of the harmonica breaks, it requires much time, and is difficult to procure another capable of giving the same tone as the former, and which will correspond sufficiently with the rest.

For farther particulars respecting this instrument, and the history of its invention, see "The Philosophical Magazine," No. 8, or vol. ii. p. 391.

ARTICLE XVIII.

Of some singular ideas in regard to Music.

1st. One perhaps would scarcely believe it possible for a person to compose an air, though entirely ignorant of music, or at least of composition. This secret, however, was published a few years ago in a small work entitled "Le Jeu Dez Harmonique," or "Ludus Melothedicus," containing various calculations, by means of which any person, even ignorant of music, may compose minuets, with the accompaniment of a bass. 8vo. Paris 1757. In this work, the author shows how a minuet

* He published a work on this subject entitled, 'Entdeckungen uber die Theorie des Klanges." Leipsic, 1787. 4to

and its bass may be composed, according to the points thrown with two dice, by means of certain tables.

This author gives a method also of performing the same thing by means of a pack of cards. We do not remember the title of this work; and we confess that we ought to attach no more importance to it than the author does himself.

We shall therefore content ourselves with having mentioned works to which the reader may have recourse for information respecting this kind of amusement, the combination of which must have cost more labour than the subject deserved. We shall however observe, that this author published another work, entitled "Invention d'une Manufacture et Fabrique de Vers au petit metier," &c. 8vo. 1759, in which he taught a method of answering, in Latin verse, by means of two dice and certain tables, any question proposed. This, it must be confessed, was expending much labour to little purpose.

2d. A physician of Lorrain, some years ago, published a small treatise, in which he employed music in determining the state of the pulse. He represented the beats of a regular pulse by minuet time, and those of the other kinds of pulse by different measures, more or less accelerated. If this method of medical practice should be introduced, it will be a curious spectacle to see a disciple of Hippocrates feeling the pulse of his patient by the sound of an instrument, and trying airs analogous by their time to the motion of his pulse, in order to discover its quality. If all other diseases should baffle the physician's skill, there is reason to believe that low spirits will not be able to withstand such a practice.

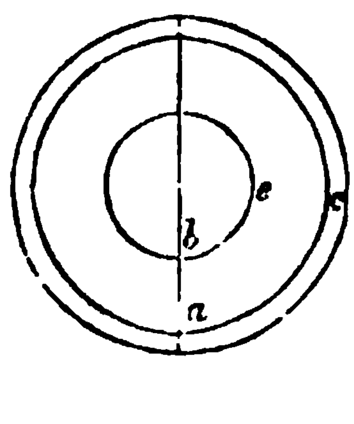
ARTICLE XIX.

On the Figures formed by Sand and other light substances on vibrating surfaces.

Dr. Chladni of Wittenberg, by his experiments on vibrating surfaces, published in 1787, opened a new field in this department of science, viz., the consideration of the curves formed by sand and other light bodies, on surfaces put into a state of motion. As this subject is curious, and seems worthy of farther research, we shall present the reader with a few observations on the method of repeating these experiments, taken from Gren's Journal of Natural Philosophy, vol. iii.*

Vibration figures, as they are called, are produced on vibrating surfaces, because some parts of these surfaces are at rest, and others in motion. The surfaces fittest for being made to vibrate, are panes of glass; though the experiments will succeed equally well with plates of metal, or pieces of board, a line or two in thickness. If the surface of any of these bodies be strewed over with substances easily put in motion; such for example as fine sand; these, during the vibration of the body, will remain on the parts at rest, and be thrown from the parts in motion, so as to form mathematical figures. To produce such figures, nothing is necessary but to know the method of bringing that part of the surface, which you wish not to vibrate, into a state of rest; and of putting in motion that which you wish to vibrate; on this depends the whole expertness of producing vibration figures.

Fig. 4.



Those who have never tried these experiments might imagine, that to produce Fig. 4. it would be necessary to damp, in particular, every point of the part to be kept at rest, viz., the two concentric circles and the diameter, and to put in motion every part intended to vibrate. This however is not the case; for you need damp only the points *a* and *b*, and cause to vibrate one part *c*, at the edge of the plate; for the motion is soon communicated to the other parts,

* See also Phil. Mag. No. 12.

which you wish to vibrate, and the required figure will in this manner be produced.

Fig. 5.

Fig. 6.

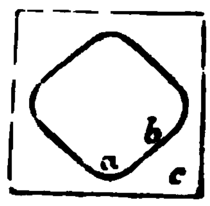


Fig. 7.

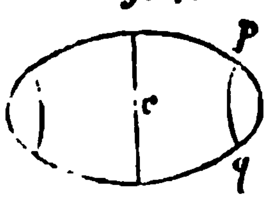


Fig. 8.

Fig. 9.

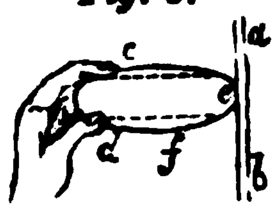
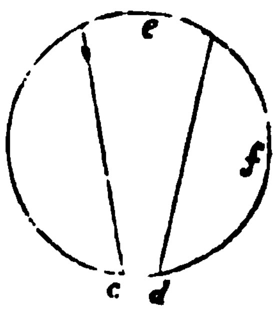


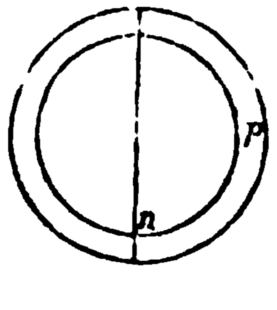
Fig. 10.



The damping may be best effected by laying hold of the place to be damped between two fingers, or by supporting it only by one finger. This will be more clearly comprehended by turning to Fig. 5., where the hand is represented in that position necessary to hold the plate. In order to produce Fig. 6. you must hold the plate horizontally, placing the thumb above at *a*, with the second finger directly below it; and besides this, you must support the point *b* on the under side of the plate. If the bow of a violin be then rubbed against the plate at *c*, you will produce on the glass the figure which is delineated Fig. 6. When the point to be supported or damped lies too near the centre of the plate, you may rest it on a cork, not too broad at the end, brought into contact with the glass in such a manner, as to supply the place of the finger. It is convenient also, when you wish to damp several points at the circumference of the glass, to place your thumb on the cork, and to use the rest of your fingers for touching the parts which you wish to keep at rest. For example, if you wish to produce Fig. 7. on an elliptic plate, the larger axis of which is to the less as 4 to 3, you must place the cork under *c*, the centre of the plate; put your thumb upon this point, and then damp the two points of the edge *p* and *q*, as may be seen Fig. 8., and make the plate to vibrate by rubbing the violin bow against it at *r*. There is still another convenient method of damping several points at the edge, when large plates are employed. Fig. 9. represents a strong square bit of metal *a b*, a line in circumference, which is screwed to the edge of the table, or made fast in any other manner; and a notch, about as broad as the edge of the plate, is cut into one side of it with a file. You then hold the plate resting against this bit of metal, by two or more fingers when requisite, as at *c* and *d*; by which means the edge of the plate will be damped in three points *d*, *c*, *e*; and in this manner, by putting the vibration at *f*, you can produce Fig. 10. In cases of necessity you may use the edge of a table, instead of the bit of metal; but it will not answer the purpose so well.

To produce the vibration at any required place, a common violin bow, rubbed with rosin, is the most proper instrument to be employed. The hair must not be too slack, because it is sometimes necessary to press pretty hard on the plate, in order to produce the tone sooner.

Fig. 11.



When you wish to produce any particular figure, you must first form it in idea on the plate, in order that you may be able to determine where a line at rest, and where a vibrating part, will occur. The greatest rest will always be where two or more lines intersect each other, and such places must in particular be damped. For example, in Fig. 11. you must damp the part *n*, and stroke with the bow in *p*. Fig. 12. may be produced with no less ease, if you hold the plate at *r*, and stroke with the bow at *f*. The strongest vibration seems

Fig. 12.

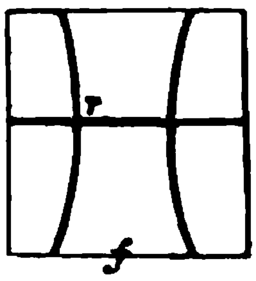


Fig. 13.

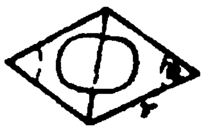
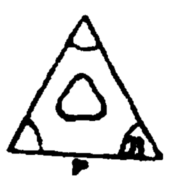


Fig. 14.



always to be in that part of the edge which is bounded by a curve: for example, in Fig. 8. and Fig. 2. at *n*. To produce these figures, therefore, you must rub with the bow at *n*, and not at *r*.

You must however damp, not only those points where two lines intersect each other, but endeavour to support at least one which is suited to that figure, and to no other. For example, when you support *a* and *b*, Fig. 4., and rub with the bow at *c*, Fig. 11. also may be produced; because both these figures have these two points at rest. To produce Fig. 4., you must support with one finger the part *e*, and rub with the bow in *c*; but Fig. 11. cannot be produced in this manner, because it has not the point *e* at rest.

One of the greatest difficulties in producing the figures, is to determine before-hand the vibrating and resting points which belong to a certain figure, and to no other. Hence, when one is not able to damp those points which distinguish one figure from another, if the violin bow be rubbed against the plate, several hollow tones are heard, without the sand forming itself as expected. You must therefore acquire by experience a readiness, in being able to search out among these tones that which belongs to the required figure, and to produce it on the plate by rubbing the bow against it. When you have acquired sufficient expertness in this respect, you can determine before-hand, with a considerable degree of certainty, the figures to be produced, and even the most difficult. It may be easily conceived, that you must not forget what part of the plate, and in what manner, you damped; and you may mark these points by making a scratch on the plate with a bit of flint.

When the plate has acquired the proper vibration, you must endeavour to keep it in that state for some seconds; which can be best done by rubbing the bow against it several times in succession. By these means the sand will be formed much more accurately.

Any sort of glass may be employed for these experiments, provided its surface be smooth; otherwise the sand will fall into the hollow parts, or be thrown about in an irregular manner. Common glass plates, when cut with a stone, are very sharp on the edge, and would soon destroy the hair of the violin bow: on this account the edge must be rendered somewhat smooth, by means of a file, or a piece of coarse hard free-stone.

You must endeavour to procure such plates as are pretty uniform in thickness; and you ought to have them of different sizes; such as circular ones of from four to twelve inches in diameter. You must not employ sand too fine, but rather that which is somewhat coarse. The plate must be equally strewed with it, and not too thick; as the lines will then be exceedingly fine, and the figures will acquire a better defined appearance.

PART SIXTH.

CONTAINING THE EASIEST AND MOST CURIOUS PROBLEMS, AS WELL AS THE MOST INTERESTING TRUTHS, IN ASTRONOMY AND GEOGRAPHY, BOTH MATHEMATICAL AND PHYSICAL.

Of all the parts of the mathematics, none are better calculated to excite curiosity than astronomy and its different branches. Nothing indeed can be a stronger proof of the power and dignity of the human mind, than its having been able to raise itself to such abstract knowledge as to discover the causes of the phenomena exhibited by the revolution of the heavenly bodies; the real construction of the universe; the respective distances of the bodies which compose it, &c. At all times therefore this study has been considered as one of the sublimest efforts of genius; and Ovid himself, though a poet, never expresses his thoughts on this subject but with a sort of enthusiasm. Thus, when speaking of the erect posture of man, he says:

Cunctaque cum spectent animalia cætera terram,
Os homini sublime dedit, cœlumque tueri
Jussit, et erectos in sidera tollere vultus.

Metamorph. Lib. I.

In another place, speaking of astronomers, he says:

Felices animæ! quibus hæc cognoscere primis,
Inque domos superas scandere cura fuit.
Credibile est illos pariter vitiisque, jocisque,
Altius humanis exeruisse caput.
Non venus aut vinum sublimia pectora fregit,
Officiumve fori, militiæve labor;
Nec levis ambitio perfusaque gloria fuco,
Magnarumve fames sollicitavit opum.
Admovere oculis distantia sidera nostris,
Ætheraque ingenio supposuere suo.

If astronomy at that period excited admiration, what ought it not do at present, when the knowledge of this science is far more extensive and certain than that of the ancients, who, as we may say, were acquainted only with the rudiments of it! How great would have been the enthusiasm of the poet, how sublime his expressions, had he foreseen only a part of the discoveries which the sagacity of the moderns has enabled them to make with the assistance of the telescope! The moons which surround Jupiter and Saturn; the singular ring that accompanies the latter; the rotation of the sun and planets around their axes; the various motions of the earth; its immense distance from the sun; the still more incredible distance of the fixed stars; the regular course of the comets; the discovery of new planets and comets; and in the last place, the arrangement of all the celestial bodies, and their laws of motion, now as fully demonstrated as the truths of geometry. With much more reason would he have called those who have ascended to these astronomical truths, and who have placed them beyond all doubt, privileged beings, and of an order superior to human nature.

CHAPTER. I.

ELEMENTARY PROBLEMS IN ASTRONOMY AND GEOGRAPHY.

PROBLEM I.

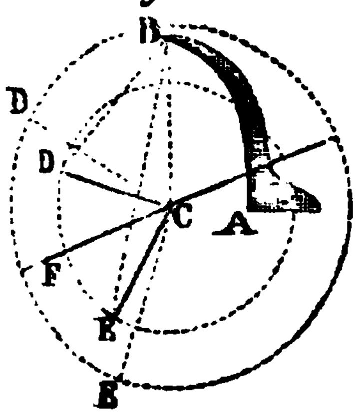
To find the Meridian Line of any place.

The determination of the meridian line, is certainly the basis of every operation, both in astronomy and geography; for which reason we shall make it the first problem relating to this subject.

There are several methods of determining this line, which we shall here describe.

I.—On any horizontal plane, fix obliquely, and in a firm manner, a spike or sharp pointed piece of iron, with the point uppermost, as A B, Fig 1. Then provide a double square, that is to say, two squares joined together so as to form an angle, and by its means find, on the horizontal plane, the point C, corresponding in a perpendicular direction with the summit of the style. From this point describe several concentric circles, and mark, in the forenoon, where the summit of the shadow touches them. Do the same thing in the afternoon; and the two points D and E being thus determined in the same circle, divide into two equal parts the arc intercepted between them. If a straight line be then drawn through the centre, and this point of bisection, it will be the meridian line required.

Fig. 1.



By taking two points in one of the other circles, and repeating the same operation; if the two lines coincide, it will be a proof, or at least afford a strong presumption, that the operation has been accurately performed; if they do not coincide, some error must have arisen; and therefore it will be necessary to recommence the operation with more care.

Two observations, the least distant from noon, ought in general to be preferred; both because the sun is then more brilliant and the shadow better defined, and because the change in the sun's declination is less; for this operation supposes that the sun neither recedes from nor approaches to the equator, at least in a sensible manner, during the interval between the two observations.

In short, provided these two observations have been made between 9 o'clock in the morning and 3 in the afternoon, even if the sun be near the equator, the meridian found by this method will be sufficiently exact, in the latitude of from 45 to 60 degrees; for we have found that in the latitude of Paris, and making the most unfavourable suppositions, the quantity which such a meridian may err will not be above 20″. If it be required with perfect exactness, nothing is necessary but to make choice of a time when the sun is either in one of the tropics, particularly that of Cancer, or very near it, so that in the interval between the two operations his declination may not have sensibly changed.

We are well aware that, for the nice purposes of astronomy, something more precise will be necessary; but the object of this work is merely to give the simplest and most curious operations in this science. The following however is a second method of finding the meridian by means of the pole star.

II.—To determine the meridian line in this manner, it will be necessary to wait till the pole star, which we here suppose to be known, has reached the meridian. But this will be the case when that star and the first in the tail of the Great Bear, or the one nearest the square of the constellation, are together in the same line perpendicular to the horizon; for about the year 1700 these two stars passed over

the meridian exactly at the same time; so that when the star in the Great Bear was below the pole, the polar star was above it; but though this is not precisely the case at present, these stars, as we shall here shew, may be still employed in obtaining an approximate meridian.

Having suspended a plumb line in a motionless state, wait till the pole star, and that in the Great Bear above described, are together concealed by the thread; and at that moment suspend a second plumb line, in such a manner that it shall hide the former and the two stars. These two threads will then comprehend between them a plane which will be that of the meridian; and if the two points on the ground, corresponding to the extremities of the two plumb lines, be joined by a straight line, you will have the direction of the meridian.

The time at which the pole star, or any other star, passes the meridian on a given day, may be found in the following manner:

Subtract the right ascension of the sun from the right ascension of the star (increased by 24 hours if necessary), and the remainder is the apparent time of the star's passing the meridian. The sun's right ascension, as well as that of any star likely to be used in the ordinary procedure of astronomy, may be found in the Nautical Almanac, or White's Ephemeris, one of which no English astronomer will be without.

To trace out a meridian line by means of the pole star, find as above the time when the star is on the meridian; and about six hours before that time, the star will be at its greatest elongation *east* of the meridian: and about six hours after at its greatest elongation west of the meridian.

This greatest elongation may be found by adding together the *log. cosine* of the declination, and the *log. secant* of the latitude, and the sum (rejecting 10) is the *log. sine* of the greatest elongation.

Thus on Feb. 10th, 1839, in lat. 51° 29 N., to find the greatest elongation of the pole star one hour :—

Declination	88° 27′ 24″	cosine	8·430279
Latitude	51° 29′ 0″	secant	10·205692
Required elongation	2° 28′ 43″	sine	18·635971

Now for some time, about the greatest elongation, the azimuth does not sensibly change; there is therefore sufficient time to note from a given place the direction of the star at the time, and to draw on the ground a line in that direction.

From this line let an angle be laid off equal to the greatest elongation computed as above; towards the *west* of the star is east of the meridian, but towards the east of the star is west of the meridian; and the line forming this angle with the line drawn in the direction of the star, will be the meridian line.

But the object may be effected with much more neatness by a theodolite; for, having adjusted the instrument, and directed the telescope to the star at the time of its greatest elongation, it is only requisite to turn round the instrument in azimuth 2° 28′ 43″, or such other angle as may result from the computation, and then any object on the horizon bisected by the cross wires of the telescope will be in a meridian line from the place of the instrument.

PROBLEM II.

To find the Latitude of any place.

The latitude of any place on the earth is its distance from the equator; and is measured by an arc of the celestial meridian, intercepted between the zenith of the place and the equator; for this arc is similar to that comprehended on the earth between the place and the terrestrial equator. This is equal to the elevation of the

pole, which is the arc of the meridian intercepted between the pole and the horizon. To those therefore who live under the equator, the poles are in the horizon; and if there were inhabitants at either pole, the equator would be in their horizon.

The latitude of any place on the earth may be easily found by various methods.

1st. By the meridian altitude of the sun on any given day. For if the sun's declination for that day, when the sun is in any of the northern signs, and the given place in the northern hemisphere, be subtracted from the altitude, the remainder will be the elevation of the equator, the complement of which is the elevation of the pole, or the latitude. If the sun be in any of the southern signs, it may be readily seen that, to find the elevation of the equator, the declination must be added.

2d. If the meridian altitude of one of the circum-polar stars, which do not set, be taken twice in the course of the same night, namely, once when directly above the pole, and again when exactly below it; and if from each of these altitudes the refraction be subtracted; the mean between these two altitudes will be that of the pole, or the latitude. Or, take any two altitudes of such a star at the interval of $11^{h}\ 58^{m}$ of time, correcting them by subtracting the refractions as before; then the mean between them will be the height of the pole, or the latitude of the place.

3d. Look, in some catalogue of the fixed stars, for the distance of any star from the equator, that is to say its declination; then take its meridian altitude, and by adding or subtracting the declination, you will have the elevation of the equator, the complement of which, as before said, is the latitude.

PROBLEM III.

To find the Longitude of any place on the earth.

The longitude of any place, or the second element of its geographical position, is the distance of its meridian from a certain meridian, which by common consent is considered as the first. This first meridian was formerly supposed to be that passing through the island of Ferro, the most eastern of the Canaries. But the meridian of the observatory of Paris is now used by the French, and that of the Royal Observatory of Greenwich by the English.

Formerly the longitude was reckoned, from west to east, throughout the whole circumference of the equator; but at present it is almost the general practice to reckon both ways from the first meridian, or the meridian accounted as such; that is to say east and west, so that the longitude according to this method can never exceed 180 degrees: and in the tables it is marked whether it be east or west. We shall now proceed to shew in what manner the longitude is determined.

If two terrestrial meridians, distant from each other 15°, for example, be supposed to be continued to the heavens; it is evident that they will intercept, in the equator and all its parallels, arcs of the same number of degrees. It may be readily seen also that the sun will arrive first at the more eastern meridian, and that he will then have to pass over 15° in the equator, or the parallel which he describes that day during his diurnal rotation, before he arrives at the more western meridian. But to pass over 15° the sun requires one hour, since he employs 24 hours to pass over 360°; hence it follows, that when it is noon at the more eastern places it will be only 11 o'clock in the morning at the more western. If the distance of the meridians of the two places be greater or less, the difference of the hours will be greater or less, in the proportion of one hour for 15°; and consequently of 4 minutes for a degree, 4 seconds for a minute, and so on.

Thus it is seen, that to determine the longitude of a place, nothing is necessary but to know what hour it is there, when it is a certain hour in another place situated under the first meridian, or the distance of which from the first meridian is known; for if this difference of time be changed into degrees and parts of a degree, allowing

15° for one hour of time, one degree for 4 minutes, and so on, then the longitude of the proposed place will be obtained.

To find this difference of hours, the usual method is to employ the observation of some celestial phenomenon that happens exactly at the same moment to every place on the earth, such for example as eclipses of the moon. Two observers stationed at two places, the difference of the longitude of which is required, observe, by means of a well-regulated clock, the moments when the shadow successively reaches several remarkable spots on the moon's disc; they then compare their observations, and by the difference of the time which they reckoned when the shadow reached the same spot, they determine, as above explained, the difference of the longitude of the two places.

Let us suppose, by way of example, that an observer at London found, by observation, that the shadow reached the spot called Tycho at 1h. 45m. 50s. in the morning; and that another stationed at a place A made a similar observation at 24m. 30s. after midnight: the difference of this time is 1h. 21m. 20s., which, reduced to degrees and minutes of the equator, gives 20° 20′. This is the difference of longitude: and as it was later at London when the phenomenon was observed, than at the place A, it thence follows that the place A is situated 20° 20′ farther west than London.

As eclipses of the moon are very rare, and as it is difficult to observe with precision when the shadow comes into contact with the moon's disc, so as to determine the commencement of the eclipse, and also the exact period when the shadow reaches any particular spot, the modern astronomers make use of the immersions, that is to say the eclipses, of Jupiter's satellites, and particularly those of the first, which, as it moves very fast, experiences frequent eclipses that end in a few seconds. The case is the same with the emersion or return of light to the satellite, which takes place very quickly. For the sake of illustration we shall suppose that an observer, stationed at the place A, observes an immersion of the first satellite to have happened on a certain day at 4h. 55m. in the morning; and another stationed at a place B at 3h. 25m. The difference being 1h. 30m. it gives 22° 30′ for the difference of longitude. We may therefore conclude that the place A is farther to the east than B, since the inhabitants at the former reckoned an hour more at the time of the phenomenon.

Remark.—These observations of the satellites, which, since the discovery of those of Jupiter, have been often repeated in every part of the globe, have in some measure made an entire reformation in geography; for the position in longitude of almost all places was determined merely by itinerary distances very incorrectly measured; so that in general the longitudes were counted much greater than they really were. Towards the end of the seventeenth century there were more than 25° to be cut off from the longitude assigned to the old continent from the western ocean to the eastern coast of Asia.

This method, so evident and demonstrative, was however criticised by the celebrated Isaac Vossius, who preferred the itinerary results of travellers, or the estimated distances of navigators; but by this he only proved that, though he possessed a great deal of erudition badly digested, he had a weak judgment, and was totally unacquainted with the elements of astronomy.

A knowledge of the latitude and longitude of the different places of the earth, is of so much importance to astronomers, geographers, &c., that we think it our duty to give a table of those of the principal places of the earth. This table, which is very extensive, contains the position of the most considerable towns both in England and in France, as well as of the greater part of the capitals and remarkable places in

every quarter of the globe; the whole founded on the latest astronomical observations, and the best combinations of distances and positions.

The reader must observe, that the longitude is reckoned from the meridian of Greenwich, both east and west. When east it is denoted by the letter E, and when west by the letter W. In regard to the latitude, it is distinguished, in the same manner, by the letters N and S, which denote north and south.

A TABLE,

CONTAINING THE LATITUDES AND LONGITUDES OF THE CHIEF TOWNS AND MOST REMARKABLE PLACES OF THE EARTH.

Names of Places.	Lat.	Long.
Abbeville, France	50° 7′ N	1° 50′ E
Aberdeen, Scotland	57 9 N	2 6 W
Abo, Finland	60 27 N	22 17 E
Acapulco, America	16 50 N	99 49 W
Acheen, Sumatra	5 36 N	95 19 E
Adrianople, Turkey	41 3 N	27 8 E
Agra, India	27 13 N	78 17 E
Aleppo, Syria	36 11 N	37 10 E
Alexandretta, Syria	36 45 N	36 15 E
Alexandria, Egypt	31 13 N	29 55 E
Algiers, Algiers	36 49 N	3 5 E
Alicant, Spain	38 12 N	0 29 W
Altona, Germany	53 33 N	9 57 E
Altorf, Germany	47 45 N	9 34 E
Amiens, France	49 54 N	2 18 E
Amboyna I. India	3 40 S	128 15 E
Amsterdam, Holland	52 22 N	4 53 E
Anabona I. Ethiopia	1 25 S	5 45 E
Ancona, Italy	43 38 N	13 29 E
Andrew's St., Scotland	56 20 N	2 50 W
Angers, France	47 28 N	0 28 W
Angouleme, France	45 39 N	0 9 E
Annapolis Royal, Nova Scotia	44 45 N	65 46 W
Antigua, Caribbee	17 4 N	61 55 W
Antibes, France	43 35 N	7 8 E
Antiochetta, Syria	36 6 N	32 20 E
Antwerp, Flanders	51 13 N	4 24 E
Archangel, Russia	64 34 N	40 43 E
Arcot, India	12 54 N	79 22 E
Arles, France	43 41 N	4 38 E
Arras, France	50 18 N	2 46 E
Ascension I., Brazil	7 57 S	13 59 W
Astracan, Siberia	46 21 N	48 8 E
Athens, Greece	37 58 N	23 46 E
Auch, France	43 39 N	0 40 E
Augustine St., Florida	29 48 N	81 18 W
Augsburg, Germany	48 22 N	10 55 E
Avignon, France	43 57 N	4 48 E
Avranches, France	48 41 N	1 21 W
Aurillac, France	44 55 N	2 32 E
Auxerre, France	47 48 N	3 34 E
Awatcha, Kamtschatka	52° 52′ N	158° 47′ E
Azoph, Crimea	47 0 N	39 14 E
Bagdad, Mesopotamia	33 20 N	44 25 E
Bahama I., America	26 43 N	78 56 W
Baldivia, Chili	39 50 S	73 34 W
Bale, Switzerland	47 34 N	7 35 E
Bangalore, India	12 58 N	77 33 E
Bantry Bay, Ireland	51 34 N	10 10 W
Barcelona, Spain	41 22 N	2 10 E
Bassora, Arabia	30 32 N	44 46 E
Batavia, Java I.	6 9 S	106 52 E
Bayeux, France	49 17 N	0 42 W
Bayonne, France	43 29 N	1 28 W
Beechy Head, England	50 44 N	0 15 E
Belfast, Ireland	54 35 N	5 57 W
Bencoolen, Sumatra I.	3 48 S	102 0 E
Belgrade, Turkey	44 43 N	20 10 E
Bender, Turkey	46 51 N	29 46 E
Bergen Castle, Norway	60 24 N	5 20 E
Berlin, Germany	52 32 N	13 22 E
Bermuda, Bahama I.	32 22 N	64 30 W
Berne, Switzerland	46 57 N	7 26 E
Berwick Tweed, Eng.	55 46 N	2 0 W
Besançon, France	47 14 N	6 3 E
Bezieres, France	43 21 N	3 13 E
Bilboa, Spain	43 26 N	3 18 W
Blois, France	47 35 N	1 20 E
Bologna, Italy	44 30 N	11 21 E
Bolkereskoy, Kamtschatka	52 54 N	156 50 E
Bombay, India	18 54 N	72 50 E
Borneo, Borneo I.	4 55 N	114 55 E
Boston, England	53 10 N	0 25 E
Boston, America	42 22 N	70 59 W
Botany Bay, N. Holland	34 0 S	151 14 E
Boulogne, France	50 44 N	1 37 E
Bourdeaux, France	44 50 N	0 34 W
Bourges, France	47 5 N	2 24 E
Bremen, Germany	53 5 N	8 48 E
Breslaw, Silesia	51 6 N	17 2 E
Brest, France	48 23 N	4 29 W
Bridge Town, Barbad.	13 5 N	59 41 W

Names of Places.	Lat.	Long.	Names of Places.	Lat.	Long.
Bristol Cathedral, Eng.	51° 27′ N	2° 35′ W	Conception Is, Chili	36° 49′ S	73 5 W
Bruges, Flanders	51 13 N	3 14 E	Congo R. Ent., Congo	6 10 S	11 15 E
Brussels, Flanders	50 51 N	4 22 E	Constance, Switzerland	47 36 N	9 8 E
Buchan-ness, Scotland	57 30 N	1 47 W	Constantinople, Turkey	41 1 N	28 55 E
Bucharest, Wallachia	44 27 N	26 8 E	Copenhagen, Denmark	55 41 N	12 40 E
Buda, Turkey	47 30 N	19 2 E	Cordova, Spain	37 52 N	4 46 W
Buenos Ayres, Brazil	34 37 S	58 24 W	Corfu, Vido I., Turkey	39 38 N	19 56 E
Cadiz Observ., Spain	36 32 N	6 17 W	Corinth, Greece	37 58 N	23 28 E
Caen, France	49 11 N	0 22 W	Cork, Ireland	51 52 N	8 16 W
Caffa, Crimea	45 6 N	35 13 E	Coutances, France	49 3 N	1 26 W
Cagliari, Sardinia	39 13 N	9 6 E	Cowes, Isle of Wight	50 45 N	1 19 W
Cairo, Egypt	30 2 N	31 19 E	Cracow, Poland	50 4 N	19 57 E
Calais, France	50 58 N	1 51 E	Cremsmunster, Germ.	48 3 N	14 8 E
Calcutta, India	22 34 N	88 26 E	Cuddalore, India	11 43 N	79 48 E
Calicut, India	11 15 N	76 5 E	Curaçoa, West Indies	12 8 N	69 0 W
Callao, Peru	12 4 S	77 4 W	Dabul, India	17 45 N	72 53 E
Camboida, India	10 0 N	104 10 E	Dantzic, Poland	54 21 N	18 38 E
Cambray, France	50 11 N	3 14 E	Dartmouth, England	50 17 N	3 35 W
Cambridge, England	52 13 N	0 6 E	Deseada, Caribbees	16 20 N	61 2 W
Canary I., Canaries	28 10 N	15 31 W	Dieppe, France	49 56 N	1 5 E
Candy, Ceylon	7 23 N	80 47 E	Dijon, France	47 19 N	5 2 E
Canterbury, England	51 17 N	1 5 E	Dillengen, Germany	48 34 N	10 30 E
Cape Comorin, India	8 5 N	77 44 E	Dol, France	44 33 N	1 45 W
Cape Finisterre, Spain	42 54 N	9 16 W	Dole, France	47 7 N	5 30 E
Cape François, St. Domingo	19 57 N	71 22 W	Domingo St., Antilles	18 30 N	69 49 W
			Dordrecht, Netherlands	51 49 N	4 40 E
Cape Town, Caffraria	33 55 S	18 20 E	Dover, England	51 8 N	1 19 E
Cape Ortegal, B. of Bis.	43 47 N	7 56 W	Dresden, Saxony	51 3 N	13 43 E
C. St. Lucas, California	22 52 N	109 50 W	Drontheim, Norway	63 26 N	10 23 E
Cape Verd, Negroland	14 44 N	17 32 W	Dublin Obs., Ireland	53 23 N	6 20 W
Caracas, S. America	10 31 N	67 5 W	Dunbar, Scotland	55 58 N	2 36 W
Carcassone, France	43 13 N	2 21 E	Dundee, Scotland	56 25 N	3 2 W
Carlescrona, Sweden	56 7 N	15 33 E	Dungeness, England	50 55 N	0 58 E
Carlisle, England	54 44 N	2 46 W	Dunkirk, France	51 2 N	2 23 E
Carthagena, Spain	37 36 N	1 0 W	Durazzo, Turkey	41 19 N	19 27 E
Carthagena, S. America	10 25 N	75 30 W	Edinburgh Obs., Scot.	55 57 N	3 11 W
Casan, Russia	55 48 N	49 21 E	Elba, P. Torrey, Italy	42 49 N	10 20 E
Cassel, Germany	51 19 N	9 35 E	Elbing, Poland	54 8 N	19 22 E
Castres, France	43 37 N	2 15 E	Elsinore, Denmark	56 2 N	12 38 E
Cayenne I., S. America	4 56 N	52 15 W	Embden, Germany	53 22 N	7 11 E
Cephalonia I., Turkey	38 27 N	20 33 E	Enchuysen, Holland	52 42 N	5 18 E
Cette Light H., France	43 24 N	3 41 E	Ephesus, Natolia	37 50 N	27 37 E
Ceuta, Barbary	35 54 N	5 17 W	Erfurth, Germany	50 59 N	11 2 E
Chalons-sur-Marne, Fr.	48 57 N	4 22 E	Erivan, Armenia	40 20 N	44 35 E
Châlons-sur-Saône, Fr.	46 47 N	4 51 E	Erzerum, Armenia	39 57 N	48 36 E
Chandernagor, Bengal	22 51 N	88 29 E	Eustatia, Caribbees	17 20 N	63 5 W
Charlestown Light, Carolina	32 43 N	79 46 W	Faenza, Italy	44 17 N	11 21 E
			Falmouth, Pend. Cas.,	50 8 N	5 2 W
Chartres, France	48 27 N	1 29 E	Fernambouc, Brazil	8 3 S	34 54 W
Cherbourg, France	49 38 N	1 37 W	Ferrara, Italy	44 50 N	11 36 E
Chester, England	53 11 N	2 53 W	Ferro I., Canaries	27 50 N	17 58 W
Christiana, Norway	59 55 N	10 48 E	Finisterre C., France	42 54 N	9 16 W
Christianstadt, Sweden	56 1 N	14 9 E	Fladstrandt, Denmark	57 27 N	10 33 E
Civita Vecchia, Italy	42 5 N	11 45 E	Florence, Italy	43 47 N	11 16 E
Clagenfurth, Carinhia	46 37 N	14 20 E	Flushing, Holland	51 27 N	3 35 E
Clermont-Ferrand, Fr.	45 47 N	3 5 E	Forbisher's Straits, Greenland	62 5 N	47 18 W
Cochin, India	9 57 N	76 29 E			
Colchester, England	51 53 N	0 54 E	Formoso I. N. p. China	25 11 N	121 56 E
Collioure, France	42 32 N	3 5 E	——— ———S. p. E. do.	21 54 N	121 5 E
Cologne, Germany	50 55 N	6 55 E	Frankfort on the Mayn, Germany	50 7 N	8 36 E
Compiegne, France	49 25 N	2 54 E			

Names of Places.	Lat.	Long.
Frankfort on the Oder. Germany	52° 22′ N	14° 33′ E
Frederickstadt, Norway	59 12 N	11 1 E
Frejus, France	43 26 N	6 44 E
Gallipoli, Turkey	40 26 N	26 37 E
Gambia R. mouth, Negroland	16 30 N	13 30 W
Geneva, Switzerland	46 12 N	6 9 E
Genoa, Italy	44 25 N	8 58 E
Ghent, Netherlands	51 3 N	3 44 E
Gibraltar, Spain	36 7 N	5 22 W
Glasgow, Scotland	55 52 N	4 16 W
Gloucester Cath. Eng.	51 52 N	2 14 W
Gluckstadt, Holstein	53 48 N	9 27 E
Goa, India	15 29 N	73 53 E
Gombroon, Persia	27 18 N	56 12 E
Good Hope C., Africa	34 23 S	18 24 E
Gottenburg, Sweden	57 41 N	11 54 E
Gottingen, Germany	51 32 N	9 56 E
Granville, France	48 50 N	1 36 W
Gratz, Styria	47 4 N	15 27 E
Greenwich, England	51 29 N	0 0
Grenoble, France	45 12 N	5 44 E
Guadaloupe, Caribbee	15 59 N	61 45 W
Guernsey, St. P. Eng.	49 26 N	2 33 W
Hague, Holland	52 5 N	4 19 E
Halifax, Nova Scotia	44 44 N	63 36 W
Halle, Saxony	51 29 N	11 58 E
Hamburgh, Germany	53 33 N	9 59 E
Harlem, Holland	52 23 N	4 38 E
Harwich, England	51 47 N	1 17 E
Hastings, England	50 52 N	0 35 E
Havannah, Cuba I.,	23 9 N	82 23 W
Havre de Grace, France	49 29 N	0 7 E
Helena St. I., Africa	15 55 S	5 43 W
Holy Head, Wales	53 19 N	4 30 W
Horn Cape, S. America	55 58 S	67 21 W
Hull, England	53 45 N	0 16 W
Hydrabad, India	17 12 N	78 51 E
Jafnapatan, C. Ceylon I	9 45 N	80 9 E
Jago, St. Cape Verd I.	14 53 N	23 32 W
Jamaica, Kingston, W.I.	18 0 N	76 42 W
Jassey, Moldavia	47 8 N	27 30 E
Java Head, Java I.	6 48 S	105 11 E
Jeddo, Japan	36 29 N	140 0 E
Jena, Germany	50 56 N	11 37 E
Jersey I. St. Aubin, Eng.	49 13 N	2 11 W
Jerusalem, Palestine	31 48 N	35 20 E
Jeniseik, Russ. Tartary	58 27 N	91 59 E
Ingolstadt, Germany	48 46 N	11 25 E
Inspruc, Tyrol	47 16 N	11 24 E
Inverness, Scotland	57 31 N	4 12 W
Joppa, Syria	32 2 N	34 53 E
Ipswich, England	52 3 N	1 9 E
Ismail, Turkey	45 21 N	28 50 E
Ispahan, Persia	32 25 N	51 50 E
Juan Fernandez I., Chili	33 40 S	78 58 W
Judda, Arabia	21 29 N	39 15 E
Ivica I., Spain	38 53 N	1 29 E
Kilda St. I., Scotland	57 49 N	8 26 W
Kinsale, Ireland	51 41 N	8 28 W

Names of Places.	Lat.	Long.
Konigsberg, Prussia	54° 42′ N	21° 29′ E
Lancaster Steeple, Eng.	54 3 N	2 48 W
Landau, France	49 12 N	8 7 E
Land's End, England	50 4 N	5 42 W
Landscrona, Sweden	55 52 N	12 50 E
Langres, France	47 52 N	5 20 E
Lausanne, Switzerland	46 31 N	6 45 E
Leeds, England	53 48 N	1 34 W
Leghorn, Italy	43 33 N	10 17 E
Leipsic, Germany	51 20 N	12 22 E
Leostoff, England	52 29 N	1 44 E
Lepanto, Turkey	38 16 N	22 1 E
Leyden, Holland	52 9 N	4 29 E
Liverpool, St. Paul's, England	53 25 N	2 59 W
Liege, Germany	50 39 N	5 32 E
Lima, Peru	12 3 S	76 57 W
Limerick, Ireland	52 36 N	8 31 W
Lisbon Obs., Portugal	38 42 N	9 8 W
Lizard Light, England	49 58 N	5 1 W
London, St. Paul's Ch.	51 31 N	0 8 W
Londonderry, Ireland	54 59 N	7 15 W
Loretto, Italy	43 27 N	13 35 E
Louisburg, Cape Breton	45 54 N	59 55 W
Louvain, Netherlands	50 53 N	4 42 E
Lubeck, Germany	53 51 N	10 41 E
Lucia, St. I. Caribbee	13 37 N	60 30 W
Lucca, Italy	43 54 N	10 34 E
Lunden Tower, Sweden	55 43 N	13 13 E
Luxembourg, Netherlds.	49 38 N	6 10 E
Lynn, Old Tower, Eng.	52 47 N	0 25 E
Macao, China	22 11 N	113 31 E
Macassar, Celebes I.	5 9 S	119 39 E
Madras, India	13 4 N	80 16 E
Madrid, Spain	40 25 N	3 42 W
Madura, India	9 54 N	78 18 E
Mahon Port, Minorca	39 51 N	4 18 E
Malacca Fort, India	2 12 N	102 15 E
Malta I., Val. Obs., Italy	35 53 N	14 31 E
Manchester, England	53 24 N	2 20 W
Manilla, Luconia I.	14 36 N	120 58 E
Mantua, Italy	45 9 N	10 48 E
Marseilles Obs., France	43 18 N	5 22 E
Martinico I., F. Royal, West Indies	14 36 N	61 6 W
Masulipatam, India	16 11 N	81 13 E
Mauritius I., Pt. Lewis, Africa	20 10 S	57 28 E
Meaco, Japan	35 24 N	153 30 E
Meaux, France	48 58 N	2 53 E
Mecca, Arabia	21 18 N	40 15 E
Mechlin, Netherlands	51 2 N	4 28 E
Memel, Courland	55 42 N	21 8 E
Messina Light, Sicily	38 11 N	15 35 E
Metz, France	49 7 N	6 10 E
Mexico, Mexico	19 26 N	99 5 W
Milan Obs., Italy	45 28 N	9 12 E
Mocha, Arabia	13 20 N	43 20 E
Modena, Italy	44 34 N	11 12 E
Montpelier Obs., France	43 36 N	3 53 E
Montreal, Canada	45 31 N	73 35 W

Names of Places.	Lat.	Long.
Mosambique Harbour, Zangue	15° 1′ S	40° 47′ E
Moscow, Russia	55 46 N	37 33 E
Munich, Germany	48 8 N	11 34 E
Munster, Germany	51 58 N	7 36 E
Namur, Netherlands	50 28 N	4 51 E
Nangasaki, Japan	32 44 N	129 52 E
Nankin, China	32 5 N	118 7 E
Nantes, France	47 13 N	1 33 W
Naples, Italy	40 50 N	14 16 E
Narbonne, France	43 11 N	3 0 E
Narva, Livonia	59 23 N	28 14 E
Naze, Norway	57 58 N	7 3 E
Negapatnam Port, India	10 45 N	79 55 E
Nevis, I., S. Pt. Caribb.	17 5 N	62 33 W
Newcastle, England	55 0 N	1 36 W
Nice, Italy	43 41 N	7 17 E
Nieuport, Flanders	51 08 N	2 45 E
Nombre de Dios, S. A.	9 36 N	79 35 W
Nootka Sound, America	49 36 N	126 37 W
Noyon, France	49 35 N	3 1 E
Nuremberg, Germany	49 27 N	11 4 E
Ochotsk, Tartary	59 20 N	143 14 E
Olinda, Brazil	8 3 S	34 54 W
Olmutz, Moravia	49 34 N	17 9 E
Oneglia. Italy	43 55 N	8 4 E
Oporto Bar, Portugal	41 9 N	8 37 W
Oran, Barbary	35 44 N	0 39 W
Orenburg, Astracan	51 46 N	55 5 E
Orleans, N. Louisiana	29 58 N	90 11 W
Orleans, France	47 54 N	1 55 E
Ormus I. N. end, Pers.	27 7 N	56 37 E
Ostend, Flanders	51 14 N	2 55 E
Oxford, England	51 46 N	1 16 W
Padua Obs., Italy	45 24 N	11 52 E
Palermo Obs. Sicily	38 7 N	13 22 E
Palicaud, India	10 50 N	76 50 E
Pampeluna, Spain	42 50 N	1 41 W
Panama, Mexico	8 59 N	79 27 W
Para, South America	1 28 S	48 40 W
Paris Obs. France	48 50 N	2 20 E
Parma, Italy	44 38 N	10 27 E
Passau, Germany	48 36 N	13 25 E
Patmos I., Natolia	37 30 N	26 40 E
Pavia, Italy	45 11 N	9 10 E
Pegu, India	17 40 N	96 12 E
Pekin Obs., China	39 54 N	116 28 E
Perpignan, France	42 42 N	2 54 E
Petersburgh, Russia	59 56 N	30 19 E
Philadelphia, America	39 57 N	75 11 W
Pico I. Peak, Azores	38 28 N	28 33 W
Pisa Obs., Italy	43 43 N	10 24 E
Plymouth, N. ch. Eng.	50 22 N	4 7 W
Pondicherry, India	11 56 N	79 54 E
Port Mahon, Minorca I.	39 52 N	4 18 E
Porto Bello, New Spain	9 34 N	79 43 W
Port Royal, Jamaica	17 58 N	76 52 W
Port Royal, Martinico	14 36 N	61 6 W
Portsmouth Obs., Eng.	50 48 N	1 6 W
Prague, Bohemia	50 5 N	14 25 E
Presburg, Hungary	48 8 N	17 11 E

Names of Places.	Lat.	Long.
Quebec, Canada	46° 47′ N	71° 10′ W
Quiloa, Zanguebar	8 41 S	39 47 E
Quimper, France	47 58 N	4 6 W
Quito, Peru	0 13 S	78 45 W
Ragusa, Dalmatia	42 39 N	18 6 E
Ramsgate, England	51 20 N	1 25 E
Ratisbon, Germany	49 1 N	12 4 E
Ravenna, Italy	44 25 N	12 11 E
Rennes, France	48 7 N	1 41 W
Rheims, France	49 16 N	4 6 E
Revel, Livonia	59 27 N	24 35 E
Riga, Livonia	56 57 N	24 8 E
Rimini, Italy	44 4 N	12 33 E
Rio Janeiro, Rat. I., Brazil	22 53 S	43 12 W
Rochelle, France	46 9 N	1 10 W
Rochester, England	51 23 N	0 30 E
Rome College. Italy	41 54 N	12 30 E
Rotterdam, Holland	51 56 N	4 29 E
Rouen, France	49 26 N	1 6 E
Rye, England	51 3 N	0 45 E
Saffia, Barbary	32 20 N	9 5 W
Saint-Flour, France	45 2 N	3 0 E
Saint-Malo, France	48 39 N	2 1 W
Saint-Omer, France	50 45 N	2 15 E
Salerno, Italy	40 40 N	14 35 E
Sallee, Barbary	34 5 N	6 43 W
Salonica, Turkey	40 38 N	22 56 E
Saragossa. Spain	41 38 N	1 42 W
Scanderoon, Syria	36 35 N	36 15 E
Schamaki, Persia	40 27 N	36 45 E
Scilly Isles, St. Mary's, England	49 54 N	6 17 W
Selinginsk, Russ. Tart.	51 6 N	106 39 E
Senegal R. ent. Negro.	15 53 N	16 31 W
Senlis, France	49 12 N	2 35 E
Sens, France	48 12 N	3 17 E
Seringapatam, India	12 25 N	76 42 E
Seville, Spain	37 24 N	5 36 W
Sheerness Staff, Eng.	51 11 N	0 44 E
Siam, India	14 21 N	100 50 E
Sienna, Italy	43 22 N	11 10 E
Sierra Leone, Guinea	8 31 N	13 18 W
Shields, England	55 2 N	1 20 W
Skalpolt, Iceland	64 0 N	16 0 W
Smyrna, Natolia	38 25 N	27 6 E
Socotra I., Africa	12 30 N	54 10 E
Soissons, France	49 23 N	3 20 E
Southampton Spire, En.	50 54 N	1 24 W
Spoletto, Italy	42 45 N	12 36 E
Start Point, England	50 13 N	3 38 W
Stettin, Pomerania	53 26 N	14 46 E
Stockholm, Sweden	59 21 N	18 3 E
Stockton, England	54 34 N	1 16 W
Stralsund, Germany	54 19 N	13 32 E
Strasburgh, France	48 34 N	7 51 E
Stromness, Orkneys	58 56 N	3 31 W
Stuttgard, Germany	48 46 N	9 11 E
Sunderland, England	54 55 N	1 15 W
Surat, India	21 4 N	72 51 E
Surinam, ent. S. America	6 25 N	55 40 W

Names of Places.	Lat.	Long.	Names of Places.	Lat.	Long.
Swansea, Wales	51° 37′ N	3° 56′ W	Valparaiso, Chili	33° 0′ N	71° 38′ W
Syracuse Light, Sicily	37 3 N	15 16 E	Vannes, France	47 39 N	2 45 W
Tangier, Barbary	35 42 N	5 50 W	Venice St Mark's, Italy	45 26 N	12 21 E
Tarento, Italy	40 35 N	17 29 E	Vera Cruz, New Spain	19 12 N	96 9 W
Tauris, Persia	38 10 N	46 37 E	Verona Obs. Italy	45 26 N	11 1 E
Teffis, Georgia	41 43 N	62 40 E	Versailles, France	48 48 N	2 7 E
Tellicherry, India	11 44 N	75 36 E	Vienna, Germany	48 13 N	16 23 E
Temeswar, Hungary	44 47 N	29 0 E	Vigo, Spain	42 13 N	8 33 W
Teneriff Peak, Canaries	28 17 N	16 40 W	Vilna, Poland	54 41 N	25 17 E
Tetuan, Barbary	35 50 N	5 20 W	Upsal, Sweden	59 52 N	17 39 E
Tinmouth, England	55 3 N	1 18 W	Uraniburg, Denmark	55 55 N	12 43 E
Thessalonica, Greece	48 38 N	22 56 E	Urbino, Italy	43 44 N	12 37 E
Tobago I., N.E. point, Caribbees	11 10 N	60 27 W	Wardhus, Lapland	70 23 N	31 7 E
			Warsaw, Poland	52 14 N	21 3 E
Tobolsk, Siberia	58 12 N	68 6 E	Waterford, Ireland	52 13 N	7 10 W
Toledo, Spain	39 52 N	4 11 W	Wells, England	51 11 N	10 12 W
Tonsberg, Norway	59 23 N	10 12 E	Wexford Harb., Ireland	52 22 N	6 19 W
Torbay, England	50 26 N	3 31 W	Weymouth, England	50 37 N	2 22 W
Tornea, Sweden	65 51 N	24 12 E	Whitby, England	54 28 N	0 36 W
Toulon, France	43 7 N	5 56 E	Whitehaven, W. mill, E	54 33 N	3 35 W
Toulouse, France	43 36 N	1 26 E	Wicklow Light, Ireland	52 38 N	6 0 W
Tours, France	47 24 N	0 42 E	Wittenberg, Saxony	51 53 N	12 46 E
Trente, Italy	46 6 N	11 4 E	Wurtzburg, Franconia	49 46 N	9 55 E
Trieste, Carniola	45 38 N	13 47 E	Wybourg, Finland	60 43 N	28 46 E
Trincomalee, Ceylon I.	8 33 N	81 22 W	Yarmouth, England	52 46 N	1 41 E
Tripoli, Syria	34 26 N	35 51 E	Yellow River, China	34 3 N	120 0 E
Tripoli, Barbary	32 54 N	13 12 E	Ylo, Peru	17 36 S	71 10 W
Truxilla, Peru	8 6 S	79 3 W	York New, Bat. Amer.	40 42 N	73 59 W
Tunis, Barbary	36 48 N	10 11 E	Youghal, Ireland	51 55 N	7 48 W
Turin, Piaz. Cast., Italy	45 4 N	7 40 E	Zagrab, Croatia	45 49 N	16 5 E
Tyrnau, Hungary	48 23 N	17 35 E	Zante I. Town, Italy	37 47 N	20 55 E
Valencia, Spain	36 29 N	0 23 W	Zara, Dalmatia	44 2 N	15 10 E
Valladolid, Spain	41 42 N	5 34 W	Zurich, Switzerland	47 23 N	8 31 E

PROBLEM IV.

To find what o'clock it is at any place of the earth, when it is a certain hour at another.

As the earth makes one revolution on its axis in the course of a common day, or 24 hours, every point of the equator will describe the whole circle of 360 degrees in that period: and therefore if 360 be divided by 24, the quotient 15 will be the number of degrees that correspond to one hour of time. Hence it is evident that two places which are 15 degrees of longitude distant from each other, will differ one hour in their computation of time, one of them making it earlier or later according as it is situated to the east or west of the other. To determine this problem therefore, find by the preceding table the difference of longitude of the two places, which may be done by subtracting the longitude of the one from that of the other if they are both east or both west of Greenwich, or by adding them if the one is east and the other west, and then change the sum or difference into time: this time added to or subtracted from the hour at one of the given places, will give for result the hour at the other. If Greenwich be one of the places proposed, the difference of longitude will be found in the last column to the right in the preceding table.

Multiply the degrees by 4 for minutes of time, and the miles by 4 for seconds of time, or find the hours and minutes corresponding to the given degrees and minutes in the subjoined table, which will greatly facilitate operations of this kind.

Now let it be proposed to find what o'clock it is at Cayenne, when it is noon at London. The difference of longitude, or of meridians, between London and Cayenne, is 52° 7′; which converted into time, gives 3 hours 28 minutes 28 seconds; and as Cayenne lies to the west of London, if 3h. 28m. 28s. be subtracted from 12 hours, the remainder will be 8 hours 31 minutes 32 seconds: hence it appears that when it is noon at London, it is only 8h. 31m. 32s. in the morning at Cayenne; consequently when it is noon at Cayenne, it is 3h. 28m. 28s. in the afternoon at London.

When it is noon at London, required the hour at Pekin? The difference of meridians between London and Pekin is 116° 36′, which is equal in time to 7 hours 46 minutes 24 seconds. But as Pekin lies to the east of London, these 7h. 46m. 24s. must be added to 12 hours: and hence it is evident that when it is noon at London, it is 7h. 46m. 24s. in the evening at Pekin. On the other hand, to find what o'clock it is at London when it is noon at Pekin, these 7h. 46m. 24s. must be subtracted from 12 hours, and the result will be 4h. 13m. 36s. in the morning.

When the two given places are both to the west of London, to find their difference of meridians, the longitude of the one must be subtracted from that of the other. If Madrid and Mexico, for instance, be proposed; as the longitude of the first is 3° 42′, and that of the second 99° 5′, if the former be subtracted from the latter, the remainder 95° 23′ will be their difference of longitude, which, changed into time, gives 6 hours 21 minutes 32 seconds. Hence, when it is noon at Madrid, it is 5h. 38m. 28s. in the morning at Mexico.

If one of the proposed places lies to the east and the other to the west of London, the longitude of the one must be added to that of the other, in order to have their difference of longitude; and the sum must then be converted into time, and added or subtracted as before.

By way of example we shall take Constantinople and Mexico, the former of which lies to the east of London. The longitude of Constantinople is 28° 55′, and that of Mexico 99° 5′, which added give for difference of longitude 171°= in time to 8h. 32m. When it is noon therefore at Constantinople, it is only 3h. 28m. in the morning at Mexico; and when it is noon at the latter, it is 8h. 32m. in the evening at Constantinople.

A table for changing degrees and minutes into hours, minutes, and seconds; or the contrary.

D	H M	D	H M	D	H M	D	H M	D	H M
M	M S	M	M S	M	M S	M	M S	M	M S
1	0 4	37	2 28	73	4 52	109	7 16	145	9 40
2	0 8	38	2 32	74	4 56	110	7 20	146	9 44
3	0 12	39	2 36	75	5 0	111	7 24	147	9 48
4	0 16	40	2 40	76	5 4	112	7 28	148	9 52
5	0 20	41	2 44	77	5 8	113	7 32	149	9 56
6	0 24	42	2 48	78	5 12	114	7 36	150	10 0
7	0 28	43	2 52	79	5 16	115	7 40	151	10 4
8	0 32	44	2 56	80	5 20	116	7 44	152	10 8
9	0 36	45	3 0	81	5 24	117	7 48	153	10 12
10	0 40	46	3 4	82	5 28	118	7 52	154	10 16
11	0 44	47	3 8	83	5 32	119	7 56	155	10 20
12	0 48	48	3 12	84	5 36	120	8 0	156	10 24
13	0 52	49	3 16	85	5 40	121	8 4	157	10 28
14	0 56	50	3 20	86	5 44	122	8 8	158	10 32
15	1 0	51	3 24	87	5 48	123	8 12	159	10 36
16	1 4	52	3 28	88	5 52	124	8 16	160	10 40
17	1 8	53	3 32	89	5 56	125	8 20	161	10 44
18	1 12	54	3 36	90	6 0	126	8 24	162	10 48
19	1 16	55	3 40	91	6 4	127	8 28	163	10 52
20	1 20	56	3 44	92	6 8	128	8 32	164	10 56
21	1 24	57	3 48	93	6 12	129	8 36	165	11 0
22	1 28	58	3 52	94	6 16	130	8 40	166	11 4
23	1 32	59	3 56	95	6 20	131	8 44	167	11 8
24	1 36	60	4 0	96	6 24	132	8 48	168	11 12
25	1 40	61	4 4	97	6 28	133	8 52	169	11 16
26	1 44	62	4 8	98	6 32	134	8 56	170	11 20
27	1 48	63	4 12	99	6 36	135	9 0	171	11 24
28	1 52	64	4 16	100	6 40	136	9 4	172	11 28
29	1 56	65	4 20	101	6 44	137	9 8	173	11 32
30	2 0	66	4 24	102	6 48	138	9 12	174	11 36
31	2 4	67	4 28	103	6 52	139	9 16	175	11 40
32	2 8	68	4 32	104	6 56	140	9 20	176	11 44
33	2 12	69	4 36	105	7 0	141	9 24	177	11 48
34	2 16	70	4 40	106	7 4	142	9 28	178	11 52
35	2 20	71	4 44	107	7 8	143	9 32	179	11 56
36	2 24	72	4 48	108	7 12	144	9 36	180	12 0

In the above table the narrow columns contain degrees or minutes, and the broad ones hours and minutes, or minutes and seconds. Thus, if 4 in the first narrow column represent degrees, the 16 opposite to it in the broad column will be minutes; and if 4 represent minutes, the 16 will be seconds. If it be required to change $4^{\circ}\ 20'$ into time; opposite to 4 will be found 16, which, in this case is minutes, and opposite to $20'$ stands 1 minute 20 seconds, which added to 16 minutes, gives 17 minutes 20 seconds, the time answering to $4^{\circ}\ 20'$.

PROBLEM V.

How two men may be born on the same day, die at the same moment, and yet the one may have lived a day, or even two days more than the other.

It is well known to all navigators, that if a ship sails round the world, going from east to west, those on board when they return will count a day less than the inhabitants of the country. The cause of this is, that the vessel, following the course of the sun, has the days longer, and in the whole number of the days reckoned, during the voyage, there is necessarily one revolution of the sun less.

On the other hand, if the ship proceeds round the world from west to east, as it goes to meet the sun, the days are shorter, and during the whole circumnavigation, the people on board necessarily count one revolution of the sun more.

Let us now suppose that there are two twins, one of whom embarks on board a vessel which sails round the world from east to west, and that the other has remained at home. When the ship returns, the inhabitants will reckon Thursday, while those on board the vessel will reckon only Wednesday; and the twin who embarked will have a day less in his life. Consequently if they should die the same day, one of them would count a day older than the other, though they were born at the same hour.

But let us next suppose that, while the one circumnavigates the globe from east to west, the other goes round it from west to east, and that on the same day they return to port, where the inhabitants reckon Thursday, for example: in this case, the former will count Wednesday, and the latter Friday, so that there will be two days' difference in their ages.

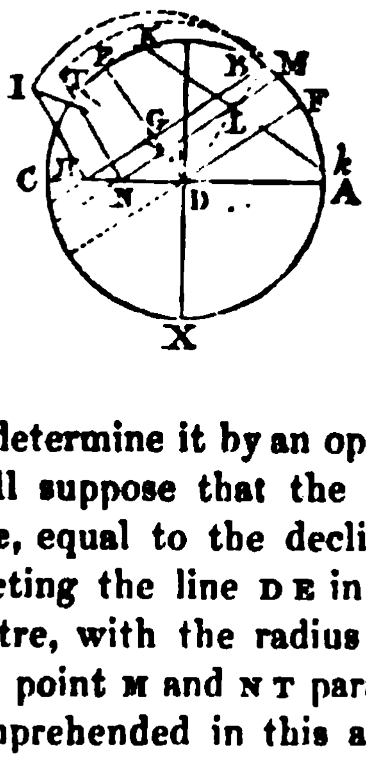
In short, it is evident that the one is as old as the other; the only difference is, that in the course of their voyage the one has had the days longer and the other shorter.

If the latter returned on a Wednesday and the former on a Friday, the former would count the day of his arrival Thursday: next day would be Thursday to the inhabitants, and the day after would be a Thursday to those who arrived in the second vessel; which, notwithstanding the popular proverb, would give three Thursdays in one week.

PROBLEM VI.

To find the length of any given day in any proposed latitude.

Fig. 2.



Let the circle A B C X (Fig. 2.), represent a meridian, and A C the horizon. Assume the arc C E, equal to the elevation of the pole of the proposed place, for example London, which is 51° 31′; and having drawn D E, draw D F perpendicular to it, or make the arc A F equal to the complement of C E, and draw F D: it is here evident that E D will represent the circle of 6 hours, and D F the equator.

After this is done, find by the Ephemeris the sun's declination, when in the proposed degree of the ecliptic, or determine it by an operation which we shall shew how to perform hereafter. We shall suppose that the declination is north: assume the arc F M toward the arctic pole, equal to the declination, and through the point M draw M N parallel to F D, meeting the line D E in O, and the horizon A C in N. Then from the point O, as a centre, with the radius O M, describe an arc of a circle M T, comprehended between the point M and N T parallel to D E. Having measured the number of the degrees comprehended in this arc, which may be easily done by means of a protractor, and having changed them into time, at the rate of 1 hour to 15 degrees, &c., the double of the result will be the length of the day.

Thus, if the length of the day at London, at the time when the sun has attained to his greatest northern declination, be required; as the greatest declination is 23° 28′, make F B equal to 23° 28′, and the arc B I will be found to be 123° 6′, which corresponds to 8h. 12m., and this doubled gives 16h. 24m., as the length of the day.

If you have no table of the sun's declination for each degree of the ecliptic, this deficiency may be supplied in the following manner. Find the number of degrees which the sun is distant from the nearest solstice, whether he has not yet reached it, or has passed it. We shall suppose that he is in the 23d degree of Taurus. The nearest solstice is that of Cancer, from which the sun, according to this supposition,

is distant 37°. Draw the line B D representing a quarter of the ecliptic; and having assumed, from the point B, the arcs B K and B k, each equal to 37°, draw K k, intersecting B D in L: if M N be then drawn through the point L, it will give the position of the parallel required.

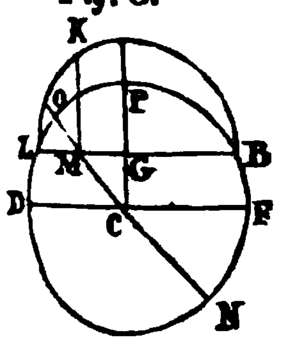
All these things may be found much more correctly by trigonometrical calculation. Thus, in the case in hand, add together the *log. tangents* of the latitude and declination, and the sum, rejecting 10 from the index, is the *log. sine* of an arc; which, added to or taken from 90°, according as the latitude and declination are of the same or contrary denominations, and the sum or remainder reduced to time, the result is half the length of the day.

PROBLEM VII.

The longest day in any place being given, to find the Latitude.

This problem is the converse of the preceding, and may be solved without much difficulty; for the longest day, in all places of the northern hemisphere, always happens when the sun has just entered the sign Cancer. Let F D (Fig. 3.) then represent the celestial equator, or rather its diameter, and B L that of the tropic of Cancer. On the latter describe a circle B K L; and having assumed the arc B K equal to the number of degrees corresponding to half the length of the given day, at the rate of 15° for one hour, draw K M perpendicular to B L; if the diameter N M O be then drawn through the point M, the angle P C O will be the elevation of the pole, or latitude of the place.

Fig. 3.



The same Trigonometrically.

From the sum of the *log. cotangent* of the declination, and the *log. sine* of half the length of the day diminished by twelve hours, deduct 10, and the remainder is the *log. tangent* of the latitude.

PROBLEM VIII.

The Latitude of a place being given, to find the Climate in which it is situated.

In astronomy, the name climate is given to an interval, on the surface of the earth, comprehended between two parallels under which the difference of the longest days is half an hour: thus the days in summer, under the parallel, whether north or south, distant from the equator 8° 25′, being 12h. 30m, this interval, or the zone comprehended between the equator and that parallel, is called the first climate.

The limits of the different climates may therefore be easily determined, by finding in what latitudes the days are 12½ hours, 13, 13½, 14, &c. The following is a table of all these climates.

Climates.		Latitude.			Latitude.		Climates.		Latitude.			Latitude.	
I	from	0°	0′	to	8°	25′	XIII	from	58°	29′	to	59°	58′
II	....	8	25	....	16	25	XIV	....	59	58	....	61	18
III	....	16	25	....	23	50	XV	...	61	18	....	62	25
IV	....	23	50	....	30	20	XVI	....	62	25	....	63	22
V	....	30	20	....	36	28	XVII	....	63	22	....	64	6
VI	....	36	28	....	41	22	XVIII	....	64	6	...	64	49
VII	....	41	22	....	45	29	XIX	....	64	49	....	65	21
VIII	....	45	29	....	49	21	XX	....	65	21	....	65	47
IX	....	49	21	....	51	28	XXI	....	65	47	....	66	6
X	....	51	28	....	54	27	XXII	....	66	6	....	66	20
XI	....	54	27	....	56	37	XXIII	....	66	20	....	66	28
XII	....	56	37	....	58	29	XXIV	....	66	28	....	66	31

As the longest day at the polar circle is 24 hours, and at the pole 6 months, there are supposed to be six climates between that circle and the pole.

Climates.		Latitude.		Latitude.	Climates.		Latitude.		Latitude.
XXV	from	66° 31′	to	67° 30′	XXVIII	from	73° 20′	to	78° 20′
XXVI	..	67 30	....	69 30	XXIX	..	78 20	....	84 0
XXVII	..	69 30	....	73 20	XXX	..	84 0	....	90 0

Now if it be asked in what climate London is, it may be easily replied that it is in the tenth; its latitude being 51° 31′, and its longest day 16h 34m.

Remark.—The idea of climates belongs to the ancient astronomy; but the modern pays no attention to this division, which in a great measure is destitute of correctness, in consequence of the refraction; for if the refraction be taken into account, as it ought to be, whatever Ozanam may say, it will be found that, under the polar circle, the longest day, instead of 24, will be several times 24 hours; for as the horizontal refraction elevates the centre of the sun 32′ at least, the centre of that luminary ought consequently never to set between the 9th of June and the 3d or 4th of July; and the upper limb, from the 6th of June to the 6th of July; this makes a complete month, during which the sun would never be out of sight.

PROBLEM IX.

To measure a degree of a great circle of the earth, and even the earth itself.

The rotundity of the earth, that is to say its being a globe, or of a form approaching very near to one, is proved by a number of astronomical phenomena; but we think it needless to enumerate these proofs, which must be known by those who are in the least acquainted with the principles of philosophy and the mathematics.

We shall here then suppose that the earth is perfectly spherical, as it apparently is; and shall begin our reasoning on that hypothesis.

What is called a degree of the meridian on the earth, is nothing else than the distance between two observers, the distance between whose zeniths is equal to a degree, or the geometrical distance between two places lying under the same meridian, the latitudes of which, or their elevation of the pole, differ a degree. Hence, if a person proceeds along a meridian of the earth, measuring the way he travels, he will have passed over a degree when he finds a degree of difference between the latitude of the place which he left, and that at which he has arrived; or when any star near the zenith of his first station has approached or receded a degree.

Nothing then is necessary but to make choice of two places, situated under the same meridian, the distance and latitudes of which are exactly known; for if the less latitude be taken from the greater, the remainder will be the arc of the meridian comprehended between the two places; and thus it will be known that a certain number of degrees and minutes correspond to a certain number of toises, or yards, or feet, &c. All then that remains to be done, is to make use of the following proportion. As the given number of degrees and minutes, is to the given number of toises, yards, or feet, so is one degree to a fourth term, which will be the toises, yards, or feet corresponding to a degree.

But as the stations chosen may not lie exactly under the same meridian, but nearly so, as Paris and Amiens, the meridional distance between their two parallels must be measured geometrically; and when this distance, as well as the difference of latitude of the two places, is known, the number of toises, yards, or feet corresponding to a degree may be found by a proportion similar to the preceding.

This was the method employed by Picard to determine the length of a terrestrial degree, or the meridian in the neighbourhood of Paris. By a series of trigonometrical

operations, he measured the distance between the pavilion of Malvoisine, to the south of Paris, as far as the steeple of Amiens, reducing it to the meridian, and found it to be 78907 toises. He found also, by astronomical obervations, that the cathedral of Amiens was 1° 22′ 58″ farther north than the pavilion of Malvoisine. By making this proportion then: As 1° 22′ 58″ are to one degree, so are 78907 toises to 57057, he concluded that a degree was equal to 57057 toises.

Picard's measurement having been since rectified in some points, it has been found that this degree is equal to 57070 toises.

Corollaries.—I. Thus, if we suppose the earth spherical, its circumference will be 20545200 French toises = 24881·8 English miles.

II. Its diameter will easily be found by making use of the following proportion: as the circumference of the circle is to its diameter, or as 314159 is to 100000, so is the above number to a fourth term, which is 6530196 toises = the diameter of the earth = 7920·12 English miles.

III. If we suppose its surface to be as smooth as that of the sea during a calm, its superficial content will be found to be 134164182859200 square toises = 197063856 English square miles. The rule for obtaining this result is: Multiply the circumference by half the radius, and then quadruple the product; or still shorter, multiply the circumference by the diameter.

IV. To find the solidity: Multiply the superficial content, above found, by a third of the radius, which will give 146019735041736067200 cubic toises = 260124289920 English cubic miles.

Remark.—The operation performed by Picard between Paris and Amiens, was afterwards continued throughout the whole extent of the kingdom, both north and south; that is to say, from Dunkirk, where the elevation of the pole is 51° 2′ 27″, to Collioure, the latitude of which is 42° 31′ 16″: the distance therefore between the parallels of these two places is 8° 31′ 11″. But it was found at the same time, by measurement, that the distance between these parallels was 486058 toises, which gives for a mean degree in the whole extent of France 57051 toises; and by corrections made afterwards, this number was reduced to 57038.

During this operation care was also taken to determine the distance of the first meridian, which in France is that of the observatory of Paris, from the principal places between which it passes.

The meridian of France continued, enters Spain, leaving Gironne on the east, at the distance of about ¼ of a degree; passes two or three thousand toises to the east of Barcelona, traverses very nearly the island of Majorca, to the east of that city, and then enters Africa, about seven minutes of a degree west of Algiers. But we shall not follow its course farther through unknown nations and countries, and shall only observe that it issues from Africa in the kingdom of Ardra. The astronomers of France have, since the above, repeated the measurement of the said arc through the country, with no great difference; from whence they have deduced the length of the meridional quadrant, which has been assumed as the standard of the new universal measures. Also several degrees of the meridian through England are now measuring by Colonel Mudge, and Colonel Colby, of the Royal Artillery, under the auspices of the Master General and Board of Ordnance; and an arc exceeding in amplitude all others that have yet been measured, is in course of execution in India.

PROBLEM X.

Of the real figure of the Earth.

We have already said that the rotundity of the earth is proved by various astronomical and physical phenomena; but these phenomena do not prove that it is a perfect

sphere. Accurate methods for measuring it were no sooner employed, than doubts began to be entertained respecting its perfect sphericity. In fact, it is now demonstrated that our habitation is flattened or depressed towards the poles, and elevated about the equator; that is to say, the section of it through its axis, instead of being a circle, is a figure approaching very near to an ellipse, the less axis of which is the axis of the earth, or the distance from the one pole to the other, and the greater the diameter of the equator. Newton and Huygens first established this truth, on physical reasoning deduced from the centrifugal force and rotation of the earth; and it has since been confirmed by astronomical observations.

The manner in which Newton and Huygens reasoned, was as follows. If we suppose the earth originally spherical and motionless, it would be a globe, the greater part of the surface of which would be covered with water. But it is at present demonstrated, that the earth has a rotary motion around its axis, and every one knows that the effect of circular motion is to make the revolving bodies recede from the centre of motion: thus the waters under the equator will lose a part of their gravity, and therefore they must rise to a greater height, to regain by that elevation the force necessary to counterbalance the lateral columns, extended to other points of the earth, where the centrifugal force, which counterbalances their gravity, is less, and acts in a less direct manner. The waters of the ocean then must rise under the equator as soon as the earth, supposed to be at first motionless, assumes a rotary motion round its axis: the parts near the equator will rise a little less, and those in the neighbourhood of the poles will sink down; for the polar column, as it experiences no centrifugal force, will be the heaviest of all. This reasoning cannot be weakened, but by supposing that the nucleus of the earth is of an elongated form; or by supposing a singular contexture in its interior parts, expressly adapted for producing that effect; but this is altogether improbable.

The philosophers however on the continent persisted a long time in refusing to admit this truth. Their principal arguments against it were founded on the measurement of the degrees of the meridian made in France; by which it appeared that a degree was less in the northern part of the kingdom than in the southern, and hence they concluded that the figure of the earth was a spheroid elongated at the poles. If the earth, said they, were perfectly spherical, by advancing uniformly under the same meridian, the elevation of the pole would be uniformly changed. Thus, in advancing from Paris, for example, towards the north 57070 toises, the elevation of the pole would vary a degree; and to make the elevation of the pole increase another degree, it would be necessary to advance towards the north 57070 toises more; and so on throughout the whole circumference of a meridian.

If, in proportion as we proceed northwards, it is found necessary to travel farther than the above number of toises before the latitude is changed one degree, there is reason to conclude that the earth is not spherical, but that it is less curved or more flattened towards the north, and that the curvature decreases the nearer we approach the pole, which is the property of an ellipsis having its poles at the extremities of its less axis. In the contrary case, it would be a proof that the curvature of the earth decreased towards the equator; which is the property of a body formed by the revolution of an ellipsis around its greater axis.

But it was believed in France at first, that the degrees of the meridian were found to increase the more they approached the south. The degree measured in the neighbourhood of Collioure, the austral boundary of the meridian, appeared to be equal to 57192 toises, while that in the neighbourhood of Dunkirk, which was the most northern, seemed to be only 56954. There was reason therefore to conclude that the earth was an elongated spheroid, or formed by the revolution of an ellipsis around its greater axis.

The partisans of the Newtonian philosophy, at that time too little known in France.

replied, that these observations proved nothing, because the above difference, being so inconsiderable, could be ascribed only to the errors unavoidable in such operations. As 19 toises correspond to about a second, the 238 toises of difference would amount only to about 12 seconds; an error which might have arisen from various causes: they even asserted that this difference might be on the opposite side.

To decide the contest, it was then proposed to measure two degrees as far distant from each other as possible, one under the equator, and the other as near the pole as the cold of the polar regions would admit. For this purpose, Maupertius, Camus, and Clairaut, were dispatched by the king, in the year 1735, to measure a degree of the meridian at the bottom of the Gulf of Bothnia, under the arctic polar circle; and Bouguer, Godin, and Condamine were sent to the neighbourhood of the equator, where they measured, not only a degree of the meridian, but almost three. It resulted from these operations, performed with the utmost care and attention, that a degree near the polar circle was equal to 57422 toises, and that a degree near the equator contained 56750, which gives a difference of 672 toises, and therefore too considerable to be ascribed to the errors unavoidable in the necessary observations. Since that time it has been generally admitted that the earth is flattened towards the poles, as Newton and Huygens asserted. We shall here add that the measurements formerly made in France having been repeated, it was found that the degree goes on increasing from south to north, as ought to be the case if the earth be an oblate spheroid.

This truth has been since confirmed by other measurements of the meridian, made in different parts of the earth. The Abbé de la Caille having measured a degree at the Cape of Good Hope, that is under the latitude of about 33° south, found it to be 57037 toises; and in 1755, Fathers Mairé and Boscovich, two Jesuits, having measured a degree in Italy, in latitude 43°, found it to be 56979: it is therefore certain that the degrees of the terrestrial meridian go on increasing from the equator towards the poles, and that the earth has the form of an oblate spheroid.

Other operations of the same kind for measuring a degree of the terrestrial meridian have been since undertaken at different times, as by the Abbé Liesganig in Germany, near Vienna; by Father Beccaria in Lombardy; and by Messrs. Mason and Dixon, members of the Royal Society of London, in North America; and again more lately by Mechain and De Lambre in France. They all confirm the diminution of the terrestrial degrees as they approach the equator, though with inequalities difficult to be reconciled with a regular figure. But it may here be asked, why should the earth have a figure perfectly regular?

It is, indeed, impossible to determine with perfect accuracy the proportion between the axis of the earth and its diameter at the equator: it has been proved that the former is shorter, but to find their exact ratio would require observations which can be made only at the pole. However the most probable ratio is that of 177 to 178.

Consequently, if this ratio be admitted, the axis of the earth, from the one pole to the other, will be 6525376 toises, and the diameter of the equator 6562242.

In the last place, the difference between the distance of any point of the equator on a level with the sea, to the centre of the earth, and the distance of the pole from the same centre, will be 18433 toises, or about 22 English miles.

Since Montucla wrote the above, however, the French astronomers Mechain and De Lambre, in 1799, completed their measurement of the meridian, from Dunkirk in France, to near Barcelona in Spain, an extent of almost 10 degrees; from whence it has been more accurately deduced, that the flattening of the earth at the poles is only the 334th part, the ratio of the axes being that of 334 to 333; that the polar axis is 7899¾ English miles, the equatorial diameter 7923½ miles, their half difference only 11⅞ miles, which is the height of the equator more than at the pole, from the

centre; the mean diameter 7911$\frac{3}{5}$ miles, the mean circumference 24873$\frac{3}{5}$ miles, the greatest or equatorial circumference 24892$\frac{2}{5}$ miles, the least or meridional circle 24855 miles, and the difference of the two 37$\frac{2}{5}$ miles.

Corollaries.—I. From what has been said, several curious truths may be deduced. The first is, that all bodies, except those placed under the equator and the poles, do not tend to the centre of the earth; for a circle is the only figure in which all the lines perpendicular to its circumference tend to the same point. In other figures, the curves of which are continually varying, as is the case with the meridians of the earth, the lines perpendicular to the circumference all pass through different points of the axis.

II. The elevation of the waters under the equator, and their depression under the poles, being the effect of the earth's rotation around its axis, it may be readily conceived that if this rotary motion should be accelerated, the elevation of the waters under the equator would increase; and as the solid part of the earth has assumed, since its creation, a consistence which will not suffer it to give way to such an elevation, the rising of the waters might become so great, that all the countries lying under the equator would be inundated; and in that case the polar seas, if not very deep, would be converted into dry land.

On the other hand, if the diurnal motion of the earth should be annihilated, or become slower, the waters accumulated, and now sustained under the equator, by the centrifugal force, would fall back towards the poles, and overwhelm all the polar parts of the earth: new islands and new continents would be formed in the torrid zone by the sinking down of the waters, which would leave new tracts of land uncovered.

Remark.—We cannot help here remarking one advantage which France, and all countries near the mean latitude of about 45 degrees, would in this case enjoy. If such a catastrophe should take place, these countries would be sheltered from the inundation, because the spheroid, which is the real figure of the earth at present, and the globe or less oblate spheroid into which it would be changed, would have their intersection about the 45th degree; consequently the sea would not be altered in that latitude.

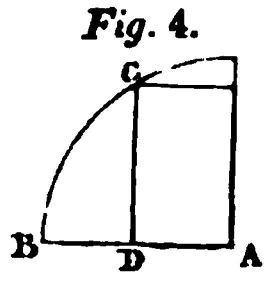
PROBLEM XI.

To determine the length of a degree on any given parallel of latitude.

As the difference between the greater and less diameter of the earth does not amount to the 300th part, in this and the following problems we shall consider it as absolutely spherical; especially as the solution of these problems, if we supposed the earth to be a spheroid, would be attended with difficulties inconsistent with the plan of this work.

Let it be proposed then to determine how many miles or yards are equal to a degree on the parallel passing through London; that is to say under the latitude of 51 degrees 31 minutes This problem may be solved either geometrically or by calculation, according to the following methods.

Fig. 4.



1st. Draw any straight line A B (Fig. 4.), and divide it into 23 equal parts, because a degree of the equator contains 69·14 miles, or about 23 leagues. Then from the point A as a centre, with the distance A B, describe the arc B C, equal to 51° 31; and from the point C draw C D perpendicular to A B: the part A D will indicate the number of leagues contained in a degree on the parallel of 51° 31.

2d. This however may be found much more correctly by trigonometrical calculation; for which purpose nothing is necessary but to make use of the following proportions: viz.

Rad. : cosine lat. : : the miles in a degree of the equator : to the miles in a degree of the parallel.

The above is worked by logarithms in the following manner:

As Radius............................	10·0000000
is to the cosine of the latitude 51° 31′....	9·7939907
So is the number of miles contained in a degree of the meridian, viz. 69·14....	1·8397294
to a fourth term....................	1·6337201

which in the table of logarithms will be found answering to 43·025 miles, as before. A degree therefore on the parallel of London contains nearly 43 miles, or about 75643 yards.

The demonstration of this rule is easy, if it be recollected that the circumferences of two circles, or degrees of these circles, are to each other in the ratio of their radii. But the radius of the parallel of London is the cosine of the latitude; whereas the radius of the earth, or of the equator, is the real radius or sine of 90°, and hence the above rule.

3d. If the circumference of the earth at the given parallel be required, nothing is necessary but to multiply the degree found as above by 360: thus as a degree on the parallel of London is equal to 43 miles, if this number be multiplied by 360, we shall have 15480 miles, for the whole circumference of the circle of that parallel.

The following table, which shews the number of miles contained in a degree on every parallel, from the equator to the pole, is computed on the supposition that the length of the degrees of the equator are equal to those of the meridian, at the medium latitude of 45°, which length is nearly $69\frac{1}{18}$ English miles.

Deg. of Lat.	Eng. miles.	Deg. of Lat.	Eng. miles.	Deg. of Lat.	Eng. miles.	Deg. of Lat	Eng. miles.	Deg. of Lat.	Eng. miles.	Deg. of Lat.	Eng. miles
0	69·07	16	66·31	31	59·13	46	47·93	61	33·45	76	16·70
1	69·06	17	65·98	32	58·51	47	47·06	62	32·40	77	15·52
2	69·03	18	65·62	33	57·87	48	46·16	63	31·33	78	14·35
3	68·97	19	65·24	34	57·20	49	45 26	64	30·24	79	13·17
4	68·90	20	64·84	35	56·51	50	44·35	65	29·15	80	11·98
5	68·81	21	64·42	36	55·81	51	43·42	66	28·06	81	10·79
6	68·62	22	63·97	37	55·10	52	42·48	67	26·96	82	9·59
7	68·48	23	63·51	38	54·37	53	41·53	68	25·85	83	8·41
8	68·31	24	63·03	39	53·62	54	40·56	69	24·73	84	7·21
9	68·15	25	62·53	40	52 86	55	39·58	70	23·60	85	6·00
10	67·95	26	62·02	41	52·07	56	38·58	71	22·47	86	4·81
11	67·73	27	61·48	42	51·27	57	37·58	72	21·32	87	3·61
12	67·48	28	60·93	43	50·46	58	36·57	73	20·17	88	2·41
13	67·21	29	60·35	44	49·63	59	37·54	74	19·02	89	1·21
14	66·95	30	59·75	45	48·78	60	34·50	75	17·86	90	0·00
15	66·65										

PROBLEM XII.

Given the latitude and longitude of any two places on the earth, to find the distance between them.

We must here observe, that the distance of any two places on the surface of the earth, ought to be the arc of the great circle intercepted between them. The distance therefore of any two places, lying under the same parallel, is not the arc of that parallel intercepted between them, but an arc of a great circle, having the same extremities as that arc; for on the surface of a sphere, it is the shortest way from one point to another, as a straight line is upon a plane surface.

This being premised, it may be readily seen that this problem is susceptible

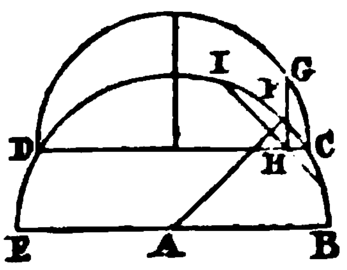
of several cases; for the two places proposed may lie under the same meridian; that is to say have the same longitude, but different latitudes; or they may have the same latitude; that is, lie under the equator or under the same parallel; or in the last place, their longitudes and latitudes may be both different: there is also a subdivision into two cases, viz. one where the two places are in the same hemisphere, and another where one is in the northern and the other in the southern hemisphere. But we shall confine ourselves to the solution of the only case which is attended with any difficulty.

For it is evident that if the two places are under the same meridian, the arc which measures their distance is their difference of latitude, provided they are in the same hemisphere, or the sum of these latitudes if they are in different hemispheres. Nothing then is necessary but to reduce this arc into leagues, miles, or yards, and the result will be the distance of the two places in similar parts.

If the places lie under the equator, the amplitude of the arc which separates them may be determined with equal ease; and can then be reduced into leagues, miles, &c.

Let us suppose then, which is the only case attended with difficulty, that the places differ both in longitude and latitude, as London and Constantinople, the former of which is 28° 53′ farther west than the latter, and 10° 31′ farther north. If we conceive a great circle passing through these two cities, the arc comprehended between them will be found by the following construction.

Fig. 5.

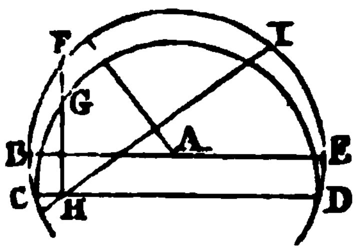


From A as a centre (Fig. 5.), with any opening of the compasses taken at pleasure, describe the semicircle B C D E, representing the meridian of London. Take the arc B F equal to 51° 31′, which is the latitude of London, in order to find its place in F, and draw the radius A F.

In the same semi-circle, if the arcs B C and E D be taken each equal to 41°, the latitude of Constantinople, the line C D will be the parallel of Constantinople, the place of which must be found in the following manner.

On C D as a diameter, describe the semi-circle C G D; and in the circumference of it take the arc C G equal to the difference of longitude between London and Constantinople, that is 28° 53′; then from the point G draw G H, perpendicular to C D, to have in H the projection of the place of Constantinople; and from the point H draw H I, perpendicular to A F and terminated at I by the arc B C D E: if the arc F I be measured, it will give the distance required in degrees and minutes. In this case it is about 22 degrees.*

Fig. 6.



If one of the places be on the other side of the equator, as the city of Fernambouc in Brazil is in regard to London, being in 7° 30′ of south latitude, the arc B C must be assumed on the other side of the diameter B E, (Fig. 6.), equal to the latitude of the second place given, which is here 7° 30′; and as the difference of longitude between London and Fernambouc is 35° 5′, it will be necessary to make the arc C G = 35° 5′. By these means the arc F I will be found to be equal to about 66° †, which reduced into miles of 69·07 to a degree, gives 4558 miles for the distance between London and the above city of Brazil.

This problem may be solved trigonometrically thus: If the latitudes are both north or both south, either add them both to 90°, or subtract them both from 90°; if one is north and the other south, add either to 90° and subtract the other. Call the results l and l', and the difference long. L.

Then call half the sum of l and l' arc 1, and add together (in logarithms) twice

* Calculation by spherical trigonometry gives 22 deg. 23 min.

† Trigonometrical calculation gives 66 deg. 15 min.

the cosine of half L, and the sines of l and l', and half the sum, rejecting tens, will be the sine of arc 2: add together the sines of the sum and difference of arcs 1 and 2, and half the sum will be the sine of half the required distance.

Remark.—When the distance between the two places is not very considerable, as is the case with Lyons and Geneva, the latter being only 36' farther north than the former, and more to the east by 6 minutes of time, which is equal to 1° 30' under the equator, the calculation may be greatly shortened.

For this purpose, take the mean latitude of the two places, which in this instance is 46° 4', and find by the preceding problem the extent of a degree on the parallel passing through that latitude, which will be $= 47 \cdot 922$ miles. The difference of longitude between these places is 1° 30', which on that parallel, allowing 47·922 miles to a degree, gives 71·88 miles, and the miles corresponding to the difference of latitude are 41·44.

If we therefore suppose a right angled triangle, one of the sides of which adjacent to the right angle is 41·44 miles, and the other 71·88, by squaring these two numbers, adding them together, and extracting the square root of the sum, we shall have the hypothenuse equal to 82·97 miles; which will be the distance, in a straight line, between Lyons and Geneva.

As this is the proper place for making known the measures employed by different nations, in measuring itinerary distances, it will doubtless be gratifying to our readers to find here a table of them, especially as it is difficult to collect them: for the same reason we have added some of the itinerary measures of the ancients, the whole expressed in English feet.

TABLE OF ITINERARY MEASURES,

ANCIENT AND MODERN.

	Feet.
Ancient Greece.	
The Olympic stadium	604
A smaller stadium	482
The least stadium	322
Egypt.	
The schœnus	19421
Persia.	
The parasang or farsang	14499
Roman Empire.	
The mile (*milliare*)	4833
Judea.	
The rast or stadium	486
The berath or mile	3640
Ancient Gaul.	
The league (*leug*)	7249
Germany.	
The rast or league	14498
The mile 12½ to a degree	28995
The same 15 to a degree	24292
Arabia.	
The mile	6929
France.	
The mile of 1000 French toises	6392
The small league of 30 to a degree	12159
The mean league of 25 to a degree	14594
The great or marine league of 20 to a degree	18238
Sweden.	
The mile	35050
Denmark.	
The mile	25123
England.	
The mile	5280
Scotland.	
The mile	7332

Ireland.

	Feet.
The mile	6724

Spain.

The league (*legale of* 5000 *vares*)	13724
The common league 17½ to a degree	20846

Italy.

The Roman mile	4909
The Lombard mile	5425
The Venetian mile	6341

Poland.

The league	18223

Russia.

	Feet.
The ancient werst	4193
The modern werst	3497

Turkey.

The agash	16211

India.

The little coss	8579
The great coss	9857
The gau of the Malabar coast	38356
The nari or nali of the same	5753

China.

The present li	1885
The pu, equal to 10 lis	18857

These values are extracted from a work by Danville, entitled *Traité des Mesures itinéraires anciennes et modernes*, Paris, 1768, 8vo., in which this subject is treated with great erudition and sagacity; so that, amidst the uncertainty which prevails in regard to the precise relation between these measures and ours, the evaluations given by Danville may be considered as the most probable, and the best founded. We have deviated therefore in many points from those given by Christiani, in his book *Delle Misure d'ogne genere antiche è moderne*. This work is valuable in some respects; but the subject is far from being examined there in so profound a manner as it has been by Danville.

PROBLEM XIII.

To represent the terrestrial Globe in plano.

A map, which represents the whole superficies of the terrestrial globe on a flat surface, is called a planisphere, or general map of the world.

A map of this kind is generally represented in two hemispheres; because the artificial globe, which represents the globe of the earth, cannot be all seen at one view: hence, when delineated in plano, it is necessary to divide it into two halves, each of which is called a hemisphere. It may be thus represented in three ways.

The first is to represent it as divided by the plane of the meridian into two hemispheres, one eastern the other western. This method is that generally used for a map of the world, because it exhibits the old continent in the one hemisphere, and the whole of the new in the other.

The second is to represent it as divided by the equator into two hemispheres, the one northern and the other southern. This representation is in some cases attended with advantage, because the disposition of the most northern and most southern countries are better seen. Some maps of this kind have been published, in which the tracts pursued by our modern navigators, and all the discoveries made by them in the South Seas, are accurately delineated.

The third method is to exhibit the globe of the earth as divided by the horizon into two hemispheres, the upper and lower, according to the position of each.

Under certain circumstances, this form has its advantages also. The disposition of the different parts of the earth, in regard to the proposed place, are better seen, and a great many geographical problems can be solved by it with much greater facility.

Father Chrysologue of Gy, in Franche-Comté, published some years ago two he-

mispheres of this kind, the centre of one of which was occupied by Paris; and he added an explanation of the different uses to which they might be applied.

Two methods may be employed in these representations.

According to one of them, the globe is supposed to be seen by the eye placed without it; and such as it would appear at an infinite distance.

According to the other, each hemisphere is supposed to be viewed on the concave side; as if the eye were placed at the end of the central diameter, or at the pole of the opposite hemisphere; and it is conceived to be projected on the plane of its base. Hence arise the different properties of these representations, which we shall here describe.

I. When the globe is represented as seen on the convex side, and divided into two hemispheres by the plane of the first meridian, the eye is supposed to be at an infinite distance, opposite to the point where the equator is intersected by the 90th meridian. All the meridians are then represented by ellipses, the first excepted, which is represented by a circle, and the 90th which becomes a straight line: the parallels of latitude also are represented by straight lines. This representation is attended with one great fault, viz., that the parts near the first meridian are very much contracted, on account of the obliquity under which they present themselves.

When the hemispheres are represented by the second method, that is to say as seen on the concave side, and projected on the plane of the meridian, the contrary is the case. It is supposed, in regard to the eastern hemisphere, that the eye is placed at the extremity of the diameter which passes through the place where the equator and the 90th meridian intersect each other. In this case there is more of equality between the distances of the meridians; and even the parts of the earth represented in the middle of the map lie somewhat closer than those towards the edges. Besides, all the meridians and parallels are represented by arcs of a circle, which is very convenient in *constructing* the map. It is attended however with this inconvenience, that the parts of the earth have an appearance different from what they have when seen from without. Asia for example is seen on the left, and Europe on the right; but this may be easily remedied by a counter-impression.

II. If a projection of the earth on the plane of the equator be required, the eye according to the first method may be supposed at an infinite distance in the axis produced: the pole will then occupy the centre of the map; the parallels will be concentric circles, and the meridians straight lines. But it is attended with this inconvenience, that the parts of the earth near the equator will be very much contracted.

For this reason it will be better to have recourse to the second method, which supposes the northern hemisphere to be seen by an eye placed at the south pole, and *vice versa*: as there is here an inversion of the relative position of the places, it may be remedied in like manner by a counter-impression.

III. If the eye be supposed in the zenith of any determinate place, as of London for example, and at an infinite distance, we shall have on the plane of the horizon a representation of the terrestrial hemisphere, the pole of which is occupied by London, and which is of the third kind. But this representation will still be attended with the inconvenience of the places near the horizon being too much crowded.

This defect however may be remedied by employing the second method, or by supposing the above hemisphere to be seen through the horizon by an eye placed in the pole of the lower hemisphere: the different meridians will then be represented by arcs of a circle, as will also the parallels: the circles representing the distance

from the proposed place, to all other places of the earth, will be straight lines. The inversion of position may be remedied as in the preceding cases.

The numerous uses to which this particular kind of projection can be applied, may be seen in a work published by Father Chrysologue in 1774, and which was intended as an explanation of his double map of the world, already mentioned.

Various other projections of the globe might be conceived: and by supposing the eye in some other point than the pole of the hemisphere, more equality might be preserved between the parts lying near to the centre and the edges of the projection; but this would be attended with other inconveniences, viz., that the circles on the surface of the sphere or globe would not be represented by circles or straight lines, which would render a description of them difficult. It is therefore better to adhere to the projection where the eye is supposed to be in the pole of the hemisphere opposite to that intended to be represented; whether the terrestrial globe, as in common maps, is to be projected on the plane of the first meridian, or whether it be required to project it on the plane of the equator, or on that of the horizon of any determinate place.

A series of six maps of the stars, prepared under the direction of J. W. Lubbock, Esq., has recently been published by the Society for the diffusion of Useful Knowledge. If we conceive the celestial sphere to be inscribed in a cube, touching at the poles, and at four equi-distant points on the equinoctial; then if lines be drawn from the centre of the sphere through the stars as depicted on its surface, these lines continued will, on the faces of the squares, give the projected places of the stars. This is the construction which Mr. Lubbock has adopted: and it is certainly attended with many and great advantages.

PROBLEM XIV.

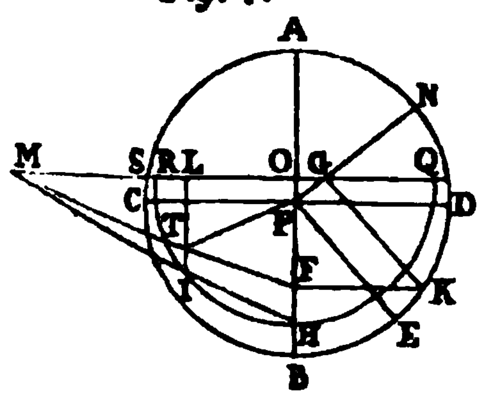
The latitude and longitude of two places, London and Cayenne for example, being given; to find with what point of the horizon the line drawn from the one to the other corresponds; or what angle the azimuth circle, drawn from the former of these places through the other, makes with the meridian.

The solution of this problem is attended with very little difficulty, if spherical trigonometry be employed, as it is reduced to the following: the two sides of a spherical triangle and the included angle being given, to find one of the other two angles. But for want of trigonometrical tables, which I had lost with all my baggage in consequence of shipwreck, I found myself obliged, on a certain occasion, to solve this problem by a simple geometrical construction, which I shall here describe. I cannot however help mentioning the singular circumstance which conducted me to it.

Being at the island of Socotora, near Madagascar, on board a vessel belonging to the East India Company, which had touched there, I formed an acquaintance with a devout Mussulman, one of the richest and most respectable inhabitants of the island. As he soon learned, by the astronomical observations which he saw me make, that I was an astronomer, he requested me to determine, in his chamber, the exact direction of Mecca; that he might turn himself towards that venerable place when he repeated his prayers. I at first hesitated on account of the object; but the good Iahia (that was his name) begged with so much earnestness, that I was not able to refuse. Having neither charts nor globes, and knowing only the latitude and longitude of the two places, I had recourse to a graphic construction on a pretty large scale. I determined the angle of position which Mecca formed with the above island; and traced out, on the floor of his oratory, the line in the direction of which he ought to look, in order to be turned towards Mecca. Words can hardly express how much the good Iahia was gratified by my compliance with his wishes; and I have no doubt, if still alive, that he offers up grateful prayers to his prophet for my

conversion. But let us return to our problem, in which we shall take, by way of example, London and Cayenne.

Fig. 7.



To resolve it by a geometrical construction, describe a circle to represent the horizon of London, which we shall suppose to be in the centre P: the larger this circle is, the more correct will the operation be. Draw the two diameters A B and C D, cutting each other at right angles: and having assumed D N, equal to the distance of London from the pole, draw the radius N P, and P E perpendicular to it, which will represent a radius of the equator: make the arc E K equal to the distance of the second place from the equator, which, in regard to Cayenne, is 4° 56′; draw also K F and K G perpendicular to the radii P B and P N; and from the point G draw G O perpendicular to the diameter A B, and continue it on both sides: if from O as a centre, with the radius G K, a semicircle R H Q be then described on the line R O Q, the points R and Q will necessarily fall within the circle; because P G being greater than P O, we shall have, on the other hand, G K or O R less than O S.

Having described the semicircle R H Q, assume the arc H I equal to the difference of the longitudes of the two places, that is towards the side C, which we here suppose to represent the west, and towards the south if the second place lies to the west of London and farther south, which is the case in the proposed example; for Cayenne is situated to the west of London, and lies much nearer the equator. Hence it may be readily seen what ought to be done, if the second place lay farther north, or to the east, &c. The arc H I then having been taken equal to 52° 11′, draw I L perpendicular to the diameter R Q; and draw H I, till it meet, in M, that diameter continued: if M F be then drawn, which will cut L I in T, the point T will represent the projection of Cayenne on the horizon of London; and consequently, by drawing the line P T, the angle T P A will be that formed by the azimuth of London passing through Cayenne.

It will be found, by this operation, that the line of position of Cayenne, in regard to London, makes with the meridian an angle of 61° 48′, consequently Cayenne bears from London south-west by west ½ west nearly.

It must however be allowed that this problem can be solved mechanically, by means of a globe, with much more ease and convenience; for nothing more is necessary than to rectify the globe for the latitude of London, to screw fast the quadrant of altitude to that point, and then to turn it till the edge of it corresponds with Cayenne: if the number of degrees intercepted between it and the meridian be then counted on the horizon, you will have the angle it forms with the meridian. But as a globe may not always be at hand, nor tables of sines and tangents to solve it trigonometrically, this want may be supplied by the graphic construction above described.

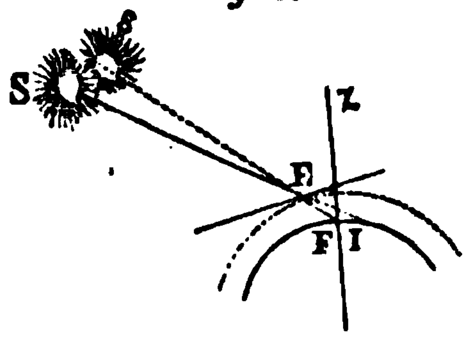
THEOREM.

The heavenly bodies are never seen in the place where they really are: thus, for example, the whole face of the sun is seen above the horizon, after he is actually set.

Though this has the appearance of a paradox, it is a truth acknowledged by all astronomers, and which philosophers explain in the following manner.

The earth is surrounded by a stratum of a fluid much denser than that which fills the expanse of the celestial regions. A small portion of the terrestrial globe enveloped by this stratum, commonly called the atmosphere, is seen represented Fig. 8. If the sun then be in S, a central ray S E, when it reaches the atmosphere, instead

Fig. 8.



of continuing its course in a straight line, is refracted towards the perpendicular, and assumes the direction E F. A spectator at F, must consequently see the sun in the line F E; and as we always judge the object to be in the direct continuation of the ray by which the eye is affected, the spectator at F sees the centre of the sun at *s*, a little nearer the zenith than he really is; and this deviation is greater, the nearer the body is to the horizon, because the ray then falls with more obliquity on the surface of the atmospheric fluid.

Astronomers have found that when the body is on the horizon, this refraction is about 33 minutes; therefore when the upper limb of the sun is in the horizontal line, so that if there were no atmosphere he would seem only beginning to peep over the horizon, he appears to be elevated 33 minutes; and as the apparent diameter of the sun is less than 33 minutes, his lower limb will appear to touch the horizon. Thus the sun is risen in appearance, though he is not really so, and even when he is entirely below the horizon. Hence follow several curious consequences, which deserve to be remarked.

I. More than one half of the celestial sphere is always seen; though in every treatise on the globes it is supposed that we see only the half; for besides the upper hemisphere, we see also a band round the horizon of about 33 minutes in breadth, which belongs to the lower hemisphere.

II. The days are every where longer, and the nights shorter, than they ought to be, according to the latitude of the place; for the apparent rising of the sun precedes the real rising, and the apparent setting follows the real setting; therefore, though the quantity of day and night ought to be equally balanced at the end of the year, the former exceeds the latter in a considerable degree.

III. The effect of refraction, above described, serves also to account for another astronomical paradox, which is as follows.

The moon may be seen eclipsed even totally and centrally, when the sun is above the horizon.

A total and central eclipse of the moon cannot take place but when the sun and moon are directly opposite to each other. We here suppose that the reader is acquainted with the causes of these phenomena, an explanation of which may be found in every elementary work on astronomy. When the centre of the moon therefore, at the time of a total eclipse, is in the rational horizon, the centre of the sun ought to be in the opposite point; but by the effect of refraction these points are raised 33 minutes above the horizon. The apparent semi-diameter of the sun and moon being only about 15 minutes, the lower limbs of both will appear elevated about 18 minutes.—Such is the explanation of a phenomenon which must take place at every central eclipse of the moon; for there is always some place of the earth where the moon is on the horizon at the middle of the eclipse.

IV. Refraction enables us to explain also a very common phenomenon, viz., the apparent elliptical form of the sun and moon, when on the horizon; for the lower limb of the sun corresponding, we shall suppose, with the rational horizon, is elevated 33 minutes by the effect of refraction; but the upper limb being really elevated 30 minutes, (which is nearly the apparent diameter of that luminary at its mean distances,) is elevated in appearance by refraction no more than 28 minutes above its real altitude; the vertical diameter therefore will appear shortened by the difference between 33 and 28, that is to say 5 minutes; for if the refraction of the upper limb were equal to that of the lower, the vertical diameter would be neither lengthened nor shortened. The apparent vertical diameter will thus be reduced to about 28 minutes.

But there ought to be no sensible decrease in the horizontal diameter; for the extremities of this diameter are carried only a little higher in the two vertical circles passing through them, and which, as they meet in the zenith, are sensibly parallel. The vertical diameter then being contracted, while the horizontal diameter remains the same, the result must be, that the discs of the sun and moon will apparently have an elliptical form, or appear shorter in the vertical direction than in the horizontal.

V. There is always more than one half of the earth enlightened by a central illumination; that is to say by an illumination, the centre of which is visible; for if there were no refraction, the centre of the sun would not be seen till it corresponded with the plane of the rational horizon; but as the refraction raises it about 33 minutes, it will begin to appear when it is in the plane of a circle parallel to the rational horizon, and 33 minutes below it.

There is therefore a central illumination for the whole hemisphere, plus the zone comprehended between that hemisphere and a parallel distant from it 33 minutes; and there is a complete illumination from the whole disc of the sun to the same hemisphere, and the zone comprehended between the border of it, and a parallel about 16 minutes farther below the horizon.

What Ozanam therefore, or his continuator, endeavours to demonstrate after Deschales with so much labour and tediousness, (see "Recreations Mathematiques" vol. II. p. 277, edit. of 1750,) is absolutely false: because no allowance is made for refraction.

PROBLEM XV.

To determine, without astronomical tables, whether there will be an Eclipse at any new or full moon given.

Though the calculation of eclipses, and particularly those of the sun, is exceedingly laborious; those which took place in any given year of the 18th century, that is between 1700 and 1801, may be found, without much difficulty, by the following operation. The method of finding those of the present or 19th century, will be shewn in the additional remark to this problem.

For the New Moons.

Find the complete number of lunations between the new moon proposed, and the 8th of January 1701, according to the Gregorian calendar, and multiply that number by 7361; to the product add 33890, and divide the sum by 43200, without paying any regard to the quotient. If the remainder after the division, or the difference between that remainder and the divisor, be less than 4060, there will be an eclipse, and consequently an eclipse of the sun.

Example.—It is required to find whether there was an eclipse of the sun on the first of April 1764. Between the 8th of January 1701, and the 1st of April 1764, there were 782 complete lunations; if this number then be multiplied by 7361, the product will be 5756302; to which adding 33890, we shall have 5790192; and this sum divided by 43200 will leave for remainder 1392: this number being less than 4060, shews that on the 1st of April 1764 there was an eclipse of the sun, which was indeed the case; and this eclipse was annular to a part of Europe.

For the Full Moons.

Find the number of complete lunations between that which began on the 8th of January 1701, and the conjunction which precedes the full moon proposed: multiply this number by 7361; and having added to the product 37326, divide the sum by 43200: if the remainder after the division, or the difference between the remainder and the divisor, be less than 2800, it will shew that an eclipse of the moon took place at that time.

Example.—Let it be required to find whether there was an eclipse at the full moon which took place on the 13th of December 1769. Between the 8th of January 1701, and the 28th of November 1769, the day of the new moon preceding the 13th of December, there were 852 complete lunations: the product of this number by 7361 is 6271572; to which if we add 37326, the sum will be 6308898. But this sum divided by 43200, leaves for remainder 1698, which being less than 2800, shews that there was an eclipse of the moon on the 13th of December 1769, as indeed may be seen by the almanacs for that year.

Remark.—To determine the number of lunations, which have elapsed between the 8th of January 1701, and any proposed day, the following method, which is attended with very little difficulty, may be employed. Diminish by unity the number of years above 1700, and multiply the remainder by 365; to the product add the number of bissextiles between 1700 and the given year, and the result will be the number of days from the 8th of January 1701 to the 8th of January of the proposed year. Then add the number of days from the 8th of January of the given year to the day of the new moon proposed, or to that which precedes the full moon proposed; and having doubled the sum, divide it by 59, the quotient will be the number of lunations required.

Let us propose, by way of example, the 13th of December 1769, the day of full moon. The preceding new moon fell on the 28th of November. If 69 be diminished by unity, the remainder is 68; which, multiplied by 365, gives 24820. As in that interval there were 17 bissextiles, we must add 17, which will give 24837. Lastly, the number of days from January 8th to November 28th 1769 was 309, which added to the above sum make 25146. This number doubled is 50292; which divided by 59 gives for quotient 852. The number of complete lunations therefore, between the 8th of January 1701 and the full moon December 13th 1769, was 852.

Additional Remarks.—This easy method of finding eclipses was invented by M. de la Hire, a celebrated French astronomer; but as it will require some alteration to make it answer for the present century, we shall first explain the principles on which it is founded, and then shew how this alteration is to be made.

1st. In regard to the full moons, we shall suppose that the sun is at present in the ascending node, and the moon in the descending: the former during the period of a lunation will move from his node 30 degrees 40 minutes 15 seconds; which expressed in quarters of a minute are equal to 7361. Hence M. de la Hire multiplies this number by that of the complete lunations, between the new moon on the 8th of January 1701, and the full moon proposed; and the product necessarily gives all the movements which the sun has made during that time, to recede from the one node and to approach the other.

2d. The sun at the time of the full moon, in the month of January 1701, was distant from his node 155 degrees 31 minutes 30 seconds, which, expressed in quarters of a minute, give 37326: hence, according to M. de la Hire, this number must be added to the product of 7361 multiplied by the lunations.

3d. The two nodes of the lunar orbit are distant from each other 180 degrees, or 10800 minutes; which multiplied by 4, give 43200: the distance therefore of the one node from the other is represented by 43200.

4th. To obtain the true distance of the sun from the node 43200 must be subtracted from the sum mentioned in the example, viz. 6308898, as many times as possible; and hence, according to M. de la Hire, this sum must be divided by 43200, neglecting the quotient.

5th. The remainder after the last division gives the true distance of the sun from his node, which we have hitherto supposed to be the ascending node; that is to say, the node by which the moon passes from the southern to the northern side of the

ecliptic. If this remainder does not exceed 2800, there will be an eclipse, or at least it will be possible; because the sun will not be distant from his node 11 degrees 40 minutes. For 11 degrees 40 minutes are equal to 700 minutes; and 700 minutes multiplied by 4, give 2800 quarters of a minute.

6th. There may be an eclipse though the remainder after the last division exceeds 2800; but in that case the difference between this remainder and the divisor will be less than 2800. The reason of this is, that the sun is necessarily distant from one of the two nodes less than 11 degrees 40 minutes. The one node indeed being distant from the other only 43200 quarters of a minute, and as the sun cannot recede from the one node without approaching the other, if the difference between the remainder after the division, and the divisor 43200, does not exceed 2800, there will necessarily be one of the nodes from which the sun will not be distant 11 degrees 40 minutes.

But it may here be objected, as the sun during the time of a lunation does not pass over 30 degrees of the ecliptic from west to east, why have we asserted that if he be at present in the ascending node, he will remove from it, in the course of a lunation, 30 degrees 40 minutes 15 seconds?

This objection will not appear of much consequence, but to those who imagine that the nodes which the lunar orbit forms with the solar are fixed and immoveable. This is not the case; these nodes have a periodical motion; that is, they pass through the 12 signs of the zodiac in the course of almost 19 years, not from west to east, as the sun, but from east to west: at the end of a lunation then the sun must be 30 degrees 40 minutes 15 seconds distant from the node he has quitted; because he not only moves from his node, but his node moves from him.

In regard to new moons, the only difference in the operation is, that 33890 is added to the product of the lunations by 7361, instead of 37326. At the time of the new moon in January 1701, the sun was distant from his node 141 degrees 12 minutes 30 seconds; which expressed in quarters of a minute are equal to 33890. For an eclipse of the sun, therefore, 33890 must be added to the product of the lunations by 7361.

It is to be observed also, that for solar eclipses, the remainder must be less than 4060; which represents the quarters of a minute contained in 16 degrees 55 minutes. A solar eclipse indeed is not impossible but when the sun and moon are at a greater distance from their nodes than 16 degrees 55 minutes: the remainder and divisor therefore must not be compared with 2800, as for eclipses of the moon, but with 4060.

To apply the above rules to the present century.

It is evident, from what has been said, that to find, by the above method, the eclipses of the sun and moon in the present century, nothing will be necessary but to substitute, for the sun's distance from the node at the time of the new and full moon in the month of January 1701, the same distance at the time of the new and full moon in the month of January 1801, and to count the lunations between the new moon in January 1801, that is the 14th, and the time proposed. But the sun's distance from the node at the time of the new moon on the 14th day of January 1801, was 280° 56′ 44″, and his distance from the node at the time of the full moon on the 29th of January 1801 was 297° 15′ 11″. The former of these reduced to quarters of a minute gives 67427, and the latter reduced in the same manner gives 71341.

Example 1st.—Let it be required to find whether there will be an eclipse at the full moon on the 18th of March 1802. Between the 14th of January 1801 and the 3d of March 1802, the day of the new moon preceding the 18th of March, there will be 14 complete lunations. The product of this number by 7361 is 103054; to which if we add 71341, the sum will be 174395. But this sum divided by 43200, leaves for remainder 1595; which, being less than 2800, shews that there will be an eclipse of the moon on the 18th of March 1802

Example 2d.—It is required to find whether there will be an eclipse of the sun on the 3d of March 1802. Between the 14th of January 1801, and the 3d of March 1802, there will be 14 complete lunations; if this number be multiplied by 7361, the product will be 103054, to which adding 67427, we shall have 170481; and this sum divided by 43200, will leave for remainder 40881: this number is not less than 4060, but its difference from 43200, which is 2319, is less than 4060; we may conclude therefore that there will be an eclipse of the sun on the 3d of March 1802.

Eclipses of the Sun and Moon during the nineteenth century.

To gratify the curiosity of the reader, we shall here give a table of the eclipses, both of the sun and moon, which will take place in the course of the present century; with the different circumstances attending them, such as the time of the middle of the eclipse, and its extent; and, in regard to eclipses of the moon, how many digits will be eclipsed, &c.

We must however observe, that as this table is extracted from an immense labour,* undertaken for another purpose, perfect exactness must not be expected, either in extent or time, and particularly in regard to the eclipses of the sun, since it is well known that a solar eclipse, on account of the moon's parallax, varies in quantity according to the place of the earth; that an eclipse, for example, which is central and total to the regions of the southern hemisphere, may be only partial and small to the northern. The author therefore, to whom we allude, was satisfied with indicating, rather than calculating, these eclipses; and left the more exact determinations to astronomers.

To render this table however more generally useful, we shall add the following explanation. The hour marked indicates the middle of the eclipse in true time; ½ signifies one half, ¼ one fourth of an hour, morn. morning, aft. afternoon. The quantity of the eclipse is expressed in digits and divisions of a digit. A digit is one twelfth part of the diameter of the luminary eclipsed. Six digits are equal to one half of the disc; four digits to one third, &c. When an eclipse is marked 0 digits, the meaning is that it is less than a quarter, or ¼ of a digit. When the moon is within a minute of a degree or less of the centre, the eclipse is marked central; when within two minutes, almost central. The duration of eclipses is nearly proportioned to their greatness; a total lunar eclipse will continue at least 3½ hours, and at most 4 hours and some minutes; a partial eclipse, which exceeds six digits, may continue 2½ or 3¼ hours; eclipses of between three and six digits, are of 2 or 3 hours' duration; those of two digits will last about 1½ hour; those of one digit about 1 hour; and those of half a digit about ¾ of an hour. The time therefore of the middle of an eclipse, and its duration, being given, its beginning and end may be nearly ascertained by the following rule: viz., subtract its semi-duration from the time given, and the remainder will be the hour of the beginning; add the same quantity, and the sum will be the time of the end. A lunar eclipse must begin and end every where at the same time; with this difference, that so many hours must be added or subtracted as the one place is to the eastward or westward of the other. Thus, an eclipse that begins about 4¼ hours P. M. at Greenwich observatory, will begin about 12 P. M. at Pekin, as the latter is 7 hours 46 minutes eastward of the former.

In regard to solar eclipses, they are dated from the time of the conjunction of the sun and moon. Though this date be sensibly different from that of the middle of the eclipse; yet this difference will never amount to two hours, and may be nearly found by the following rules:—1st. In the morning a solar eclipse must always happen sooner, and in the evening later, than the time of the conjunction.

* This labour is a table of the solar and lunar eclipses since the commencement of the Christian æra, to the year 1900, inserted in "L'Art de verifier les Dates," by the Abbe Pingere, a celebrated astronomer, and member of the Royal Academy of Sciences.

2d. The nearer the sun is to the horizon, the more sensible will be the difference. 3d. The acceleration in the morning will be great in proportion to the elevation of the sun at mid-day, three months before, and the retardation in the evening will be great in proportion to the sun's elevation three months after, the time proposed. It thence follows, 1st. That the difference must be greatest in the torrid zone; and 2d. That the greatest difference in the other latitudes must happen in the evening of the vernal, and in the morning of the autumnal, equinox; for the greatest meridian altitudes are observed three months before and after these seasons.

The parts of the world where the eclipse is visible, are marked. If there be no limitation, the whole or the greater part of Europe or Asia must be understood. Particular divisions of these quarters are denoted by the letters E. W. N. and S., that is, East, West, &c. When an eclipse is said to be visible in E. or W. of Europe, &c., the meaning is, that it is visible in all the parts of the region specified, where the sun is sufficiently elevated above the horizon at the time of conjunction. When it is marked as visible N. or S. of any particular region, all places in every other direction are excluded. The terms small and great for the most part refer to the eclipses, and not to the places where they are visible. The latitude of those places is marked in which an eclipse is central. South latitude is indicated by the letter S., and North latitude by N., which is frequently omitted. An 0, or cipher, denotes North latitude.

The course of a central eclipse is ofttimes pointed out by three numbers. The first and third shew the latitude in which the eclipse is central in the planes of the 5th and 155th meridians; the second, included in crotchets, gives the latitude in which it is central at mid-day. The place where an eclipse is central at mid-day, may be easily found, when the time of the true conjunction at Paris is known. The interval between the true conjunction as given, and mid-day, nearly shews how many hours and minutes the required place is east or west of the meridian of Paris.

It is to be observed also, that the limits of eclipses are fixed to be the tropic of Cancer in Africa, and the northern extremity of Lapland; and from 5° to 6° N. lat. in Asia to the Polar circle. In longitude, the limits are the 5th and the 155th meridians, supposing the 20th to pass through Paris.

The first and third numbers above mentioned, do not always express the latitude, under the 5th and 155th meridians. Sometimes an eclipse begins before the sun has risen upon the former, and ends after it has gone down on the latter meridian. In these cases, the first number denotes the latitude in which the eclipse is central at sun-rising; and the next the latitude in which it is central at sun-set. The number included in crotchets is omitted when there is no meridian within the limits prescribed, under which the time of mid-day coincides with the middle of the eclipse. It is to be observed also, that a number is sometimes added to point out the increase or decrease of an eclipse.

A single character or number indicates the latitude in which an eclipse is central in Europe or Africa at sun-set; and towards the eastern extremity of Asia at sun-rising. An asterisk * denotes that the course of a central eclipse extends many degrees beyond the equator. A dagger † indicates that its course is beyond the pole; and the excess is sometimes added to 90. Thus 94 intimates that the eclipse referred to is central 4° beyond the pole. The sign + affixed to pen. is used to express that the penumbra is deep or strong.

An eclipse is visible from 32° to 64° north; and as far south of the place where it is central.

LIST OF ECLIPSES,

FROM THE BEGINNING TO THE END OF THE PRESENT CENTURY.

1801.—Eclipse of the moon, total, March 30th, 5⅓ morn. cent. Of the sun, April 13th, 4⅓ morn. Europe N.E. Asia N. dim. from W. to E. Of the sun, September 8th, 6 morn. Asia N.E. small. Of the moon, total, September 22d, 7⅓ morn.

1802.—Of the moon, March 19th, 11½ morn. 5 dig. Of the sun, August 28th, 7½ morn. Eur. Afr. Asia, cent. 69 (59) 23 an. Of the moon, partial, September 11th, 11 aft. 9 digits.

1803.—Of the sun, August 17th, 8½ morn. great part of Eur. S. Afr. Asia, S. cent. 26 (12) * an.

1804.—Of the moon, partial, January 26th, 9¼ aft. Of the sun, February 11th, 11⅔ morn. Eur. Afr. Asia, W. cent. 25 (32) 64. Of the moon, partial, July 22d, 5½ aft. 10¾ dig.

1805.—Of the moon, total, January 15th, 9 morn. Of the sun, June 26th, 11 aft. part of Asia, N.E. Of the moon, total, July 11th, 9 aft.

1806.—Of the moon, partial, January 5th, 0 morn. 9 dig. Of the sun, June 16th, 4 aft. Eur. Afr. W. cent. 31 – 16 tot. Of the moon, partial, June 30th, 10 aft. pen. Of the sun, December 10th, 2⅓ morn. small, Asia, S.E.

1807.—Of the moon, partial, May 21st, 5⅓ aft. 1⅓ dig. Of the sun, June 6th, 5⅓ morn. small, Asia S.E. Of the moon, partial, November 15th, 8⅓ morn. 3 dig. Of the sun, November 29th, merid. all Eur. Afr. Asia, W. cent. 18 (13) 9 — 25.

1808.—Of the moon, total, May 10th, 8 morn. Of the moon, total, November 3d, 9 morn. Of the sun, November 18th, 3 morn. great part of Asia N. incr. from W. to E.

1809.—Of the moon, partial, April 30th, 1 morn. 10 dig. Of the moon, partial, October 23d, 9⅓ morn. 9⅓ dig.

1810.—Of the sun, April 4th, 2 morn. Asia, S.E. cent. * 10 an.

1811.—Of the moon, partial, March 10th, 6⅓ morn. 5 dig. Of the moon, partial, September 2d, 11 aft. 7 dig.

1812.—Of the moon, total, February 27th, 6 morn. almost cent. Of the moon, total, August 22d, 3 aft.

1813.—Of the sun, February 1st, 9 morn. Eur. Afr. Asia, cent. 32 – 24 (26) 55 an. Of the moon, partial, February 15th, 9 morn. 7⅓ dig. Of the moon, partial, August 12th, 3¼ morn. 4⅓ dig.

1814.—Of the sun, January 21st, 2⅓ aft. Eur. S.E. Afr. cent. * 10 an. Of the sun, July 17th, 7 morn. Eur. S. Afr. E. Asia, S. cent. 14 — 33 (31) 5 tot. Of the moon, partial, December 26th, 11⅓ aft. 6 dig.

1815.—Of the moon, total, June 21st, 6⅓ aft. 12¼ dig. Of the sun, July 7th, 0 morn. Eur. and Asia, N. cent. 62 † tot. Of the moon, partial, December 16th, 1¼ aft.

1816.—Of the moon, total, June 10th, 1⅓ morn. Of the sun, November 19th, 10⅓ morn. Eur. Afr. Asia, W. cent. 59 (38) 33 — 37 tot. Of the moon, partial, December 4th, 9 aft. 7¾ dig.

1817.—Of the sun, May 16th, 7 morn. Asia, S. cent. * (7) 12 — 7 an. Of the moon, partial, May 3d. 3⅓ aft. pen. +. Of the sun, November 9th, 2⅓ morn. Asia, E. cent. 26 — 5 S. tot.

1818.—Of the moon, partial, April 21st, 0⅓ morn. 5¾ dig. Of the sun, May 5th, 7⅓ morn. Eur. Afr. Asia, cent. 13 (51) 60 — 53 an. Of the moon, partial, October 14th, 6 morn. 2 dig.

1819.—Of the moon, total, April 10th, 1⅓ aft. Of the sun, April 24th, merid. N. of Eur. and of Asia, dim. from W. to E. Of the sun, September 19th, 1 aft. Eur. N.E. small. Of the moon, total, October 3d, 3⅓ aft.

1820.—Of the moon partial, March 29th, 7 aft. 6 dig. Of the sun, September 7th, 2 aft. Eur. Afr. Asia, W. cent. 62 — 29 an. Of the moon, partial, September 22d, 7 morn. 10 dig.

1821.—Of the sun, March 4th, 6 morn. Asia, S.E. cent. * (7 S.) 24 tot.

1822.—Of the moon, partial, February 6th, 5¼ morn. 4¼ dig. Of the moon partial, Aug. 3d, 0¼ morn. 9 dig.

1823.—Of the moon total, January 26th, 5¼ aft. Of the sun, February 11th, 3 morn. great part of Asia N. small. Of the sun, July 8th, 6¼ morn. Eur. and Asia, N. Of the moon, total, July 23d, 3½ morn.

1824.—Of the moon partial, January 16th, 9 morn. 9 dig. Of the sun, June 26th, 11¼ aft. Asia, E. cent. 27 — 41 tot. Of the moon, partial, July 11th, 4½ morn. 1 dig. Of the sun, December 20th, 11 morn. Indies, S. small.

1825.—Of the moon partial, June 1st, 0¼ morn. Of the sun, June 16th, 0¼ aft. Afr. small cent. * (0)*. Of the moon partial, November 25th, 4¼ aft. 2¼ dig.

1826.—Of the moon, total, May 21st, 3½ aft. Of the moon, total, November 14th, 4¼ aft. Of the sun, November 29th, 11¼ morn. Eur. Afr. Asia, W.

1827.—Of the sun, April 26, 3¼ morn. Eur. N.E. Asia, N. cent. 49 (81) 84 an. Of the moon partial, May 11th, 8¾ morn. 11½ dig. Of the moon partial, November 3d, 5 aft. 10 dig.

1828.—Of the sun, April 14th, 9¼ morn. small part of Eur. S.E. Afr. Asia, cent. 2 S. (18) 29 — 26. Of the sun, October 9th, 0¼ morn. Asia S.E. cent. 7 * an.

1829.—Of the moon, partial, March 20th, 2 aft. 4 dig. Of the moon, partial, September 13th, 7 morn. 5¾ dig. Of the sun, September 28th, 2¼ morn. Asia, E. cent. 59 — 40 an.

1830.—Of the sun, February 23d, 5 morn. Asia, N. dim. from W. to E. Of the moon, total, March 9th, 2 aft. Of the moon, total, September 2d, 11 aft. cent.

1831.—Of the moon, partial, February 26th, 5 aft. 8 dig. Of the moon, partial, August 23d, 10¼ morn. 6 dig.

1832.—Of the sun, July 27th, 2¼ aft. Eur. S. Afr. Asia, S.E. cent. 23 N. 3 S. tot.

1833.—Of the moon, partial, January 6th, 8 morn. 5¾ dig. Of the moon, partial, July 2d, 1 morn. 10½ dig. Of the sun, July 17th, 7 morn. Eur. Afr. E. Asia, N. cent. 83 (80) 73 tot. Of the moon, total, December 26th, 10 aft.

1834.—Of the moon, total, June 21st, 8¼ morn. Of the moon, partial, December 16th, 5½ morn. 8 dig.

1835.—Of the sun, May 27th, 1½ aft. small part of Eur. Afr. Asia, S.W. cent. 7 — 8 — 3 S. an. Of the moon partial, June 10th, 11 aft. 0¼ dig. Of the sun, November 20th, 11 morn. small part of Eur. S.W. Afr. small part of Asia, S.W. cent. 4 (11 S.) * tot.

1836.—Of the moon, partial, May 1st, 8¼ morn. 4½ dig. Of the sun, May 15th, 2¼ aft. Eur. Afr. Asia, W. cent. 53 — 54 — 44 an. Of the moon partial, October 24th, 1¾ aft. 1¼ dig.

1837.—Of the moon, total, April 20th, 9 aft. Of the sun, May 4th, 7¼ aft. small part of Eur. N. great part of Asia, N.E. Of the moon total, October 13th, 11¼ aft.

1838.—Of the moon, partial, April 10th, 2¼ morn. 7 dig. Of the moon, partial, April 10th, 2¼ morn. 7 dig. Of the moon, partial, October 3d, 3 aft. 0¾ dig.

1839.—Of the sun, March 15th, 2¼ aft. Eur. S. Afr. Asia, S.W. cent 17 — 26 tot. Of the sun, September 7th, 10¼ aft. extrem. of Asia, E. cent. 37. an.

1840.—Of the moon, partial, February 17th, 2 aft. 4¼ dig. Of the sun, March 4th, 4 morn. cent. 16 (37) 48. Of the moon, partial, August 13th, 7¼ morn. 7½ dig.

1841.—Of the moon total, February 6th, 2¼ morn. Of the sun, February 21st, 11 morn. almost all Eur. N. Asia, N.W. dim. from W. to E. Of the sun, July 18th, 2 aft. great part of Eur. N.E. Asia, N.W. incr. from W. to E. Of the moon, total, August 2d, 10 morn.

1842.—Of the moon, partial, January 26th, 6 aft. 9 dig. Of the sun, July 8th, 7 morn. Eur. Afr. Asia, cent. 35 — 50 (49) 21 tot. Of the moon, partial, July 22d, 11 morn. 3 dig.

1843—.Of the moon, partial, June 12th, 8 morn. pen. Of the moon, partial, December 7th, 0¾ morn. 2¼ dig. Of the sun, December 21st, 5¼ morn. Asia, cent. 25 (8) 21 tot.

1844.—Of the moon, total, May 31st, 11¼ aft. Of the moon, total, November 25th, 0¼ morn.

1845.—Of the sun, May 6th, 10¾ morn. almost all Eur. N.W. Asia, N.W. cent. 90 (98) † an. Of the moon, total, May 21st, 4¼ aft. 12¾ dig. Of the moon, partial, November 14th, 1 morn. 10¼ dig.

1846.—Of the sun, April 25th, 5¼ aft. Eur. and Afr. W. cent. 28 — 26. Of the sun, October 20th, 8¾ morn. Europe S.W. Afr. Asia, S.W. cent. (18 S.)* an.

1847.—Of the moon, partial, March 31st, 9¼ aft. 2¾ dig. Of the sun, September 24th, 3 aft. 4¼ dig. Of the sun, October 9th, 9¼ morn. Eur. Afr. Asia, cent. 58 (31) 16 — 17 an.

1848.—Of the moon total, March 19, 9¼ aft. Of the moon total, September 13th, 6¼ morn. Of the sun, September 27th, 10 morn. Eur. N.E. Asia, N.

1849.—Of the sun, February 23d, 1¼ morn. Asia, E. cent. 31 — 28 — 32 an. Of the moon partial, March 9th, 1 morn, 8¼ dig. Of the moon partial, September 2d, 5¼ aft. 7 dig.

1850.—Of the sun, February 12th, 6¼ morn. Asia, S.E. cent. * (11 S.) 17 N. an. Of the sun, August 7th, 10 aft. extrem. of Asia, E. cent. 14 tot.

1851.—Of the moon partial, January 17th, 5 aft. 5¼ dig. Of the moon, partial, July 13th, 7½ morn. 8¼ dig. Of the sun, July 28th, 2¼ aft. Eur. Afr. Asia, W. cent. 70 — 39 tot.

1852.—Of the moon, total, January 7th, 6¼ morn. Of the moon total, July 1st, 3¾ aft. Of the sun, December 11th, 4 morn. Asia, E. cent. 59 (36) 35 tot. Of the moon, partial, December 26th, 1 aft. 8 dig.

1853.—Of the moon, partial, June 21st, 6 morn. 2½ dig.

1854. Of the moon, partial, May 12th, 4 aft. 3 dig. Of the moon, partial, November 4th, 9¼ aft. 1 dig.

1855.—Of the moon, total, May 2d, 4¼ morn. Of the sun, May 16th, 2¼ morn. great part of Asia, N. dim. from W. to E. Of the moon, total, October 25th, 8 morn.

1856.—Of the moon, partial, April 20th, 9¼ morn. 8½ dig. Of the sun, September 29th, 4 morn. Asia, N. cent. 84 (67) 66 an. Of the moon, partial, October 13th 11¼ aft. 11¼ dig.

1857.—Of the sun, September 18th, 6 morn. Eur. and Afr. E. Asia, S. cent. 40 (12) 12 S. an.

1858.—Of the moon partial, February 27th, 10½ aft. 4 dig. Of the sun, March 15th, 0¼ aft. Eur. Afr. Asia, W. cent. (40) 68. Of the moon, partial, August 24th, 2¼ aft. 5½ dig.

1859.—Of the moon, total, February 17th, 11 morn. Of the sun, July 29th 9¾ aft. small, Asia, N.E. Of the moon total, August 13th, 4½ aft.

1860.—Of the moon, partial, February 7th, 2¾ morn. 9½ dig. Of the sun, July 18th, 2 aft. Eur. Afr. Asia, W. cent. 49 — 16 tot. Of the moon, partial, August 1st, 5½ aft. 4¾ dig.

1861.—Of the sun, January 11th, 3½ morn. small, Asia S.W. Of the sun, July 8th, 2 morn. Asia, S.E. cent.* 9 an. Of the moon, partial, December 17th, 8¼ morn. 2 dig.—Of the sun, December 31st, 2½ aft. all Eur. Afr. cent. 17 — 36 tot.

1862.—Of the moon total, June 12th, 6¾ morn. Of the moon, total, December 6th, 8 morn. Of the sun, December 21st, 5¼ morn. great part of Asia, N.

1863.—Of the sun, May 17th, 5 aft. great part of Eur. N. Of the moon, total, June 2d, 0 morn. Of the moon, partial, November 25th, 9 morn. 11 dig.

1864.—Of the sun, May 6th, 0¾ morn. Asia, S.E. cent. 6 — 23.

1865.—Of the moon, partial, April 11th, 5 morn. 1½ dig. Of the moon, partial, October 4th, 11 aft. 3¾ dig. Of the sun, October 19th, 5 aft. extrem. of Eur. and of Afr. W. cent. 16 an.

1866.—Of the sun, March 16th, 10 aft. small, Asia, N.E. Of the moon, total, March 31st, 5 morn. Of the moon, total, September 24th, 2¾ aft. Of the sun, October 8th, 5¼ aft. Eur. W. dim. from N. to S.

1867.—Of the sun, March 6th, 10 morn. Eur. Afr. Asia, cent. 31 (45) 69 an. Of the moon, partial, March 20th, 9 morn. 9¼ dig. Of the moon, partial, September 14th, 1 morn. 8 dig.

1868.—Of the sun, February 23d, 2½ aft. Eur. S. Afr. Asia, S.W. cent. 9 — 21 an. Of the sun, August 18th, 5¼ morn. Eur. S.E. Afr. Asia, S. cent. 14 — 18 (11) 0 tot.

1869.—Of the moon, partial, January 28th, 1¾ morn. 5¼ dig. Of the moon, partial, July 23d, 2 aft. 6¾ dig. Of the sun, August 7th, 10 aft. Asia, N.E. cent. 46 tot.

1870.—Of the moon, total, January 17th, 3 aft. Of the moon, total, July 12th, 11 aft. Of the sun, December 22d, 0¾ aft. Europe, Africa, Asia, W. cent. (36) 49, total.

1871.—Of the moon, partial, January 6th, 9¼ aft. 8 dig. Of the sun, June 18th, 2¾ morn. Asia, S.E. small. Of the moon, partial, July 2d, 1¼ aft. 4 dig. Of the sun, December 12th, 4¼ morn. Asia, S. cent. 17 * total.

1872.—Of the moon, partial, May 22d, 11¼ aft. 1¼ dig. Of the sun, June 6th, 3¼ morn. Asia, cent. 8 (42) 43 an. Of the moon, partial, November 15th, 5¾ morn. 0¼ dig.

1873.—Of the moon, total, May 12th, 11¼ morn. Of the sun, May 26th, 9¼ morn. great part of Europe N.W. Africa W. Asia N. dim. from W. to E. Of the moon, total, November 4th, 4¼ aft.

1874.—Of the moon, partial, May 1st, 4¼ aft. 9¾ dig. Of the sun, Oct. 10th, 11¼ morn. Europe, Africa, Asia, W. cent. 82 (74) 55 an. Of the moon, partial, October 25th, 8 morn. 12 dig.

1875.—Of the sun, April 6th, 7 morn. Asia, S.E. cent. * (1) 21 total. Of the sun, September 29th, 1¼ aft. small part of Europe S.W. Africa, Asia, S.W. cent. 13 (10) 13 S. an.

1876.—Of the moon, partial, March 10th, 6¼ morn. 3¼ dig. Of the moon, partial, September 3d, 9½ aft. 4 dig.

1877.—Of the moon, total, February 27th, 7¼ aft. Of the sun, March 15th, 3 morn. great part of Asia N. dim. from W. to E. Of the sun, August 9th, 5 morn. Asia, N.E. small. Of the moon, total, August 23d, 11¼ aft. almost cent.

1878.—Of the moon, partial, February 17th, 11¼ morn. 9¼ dig. Of the sun, July 29th, 9¼ aft. extremity of Asia, E. cent. 52 total. Of the moon, partial, August 13th, 0¼ morn. 6¼ dig.

1879.—Of the sun, January 22d, merid. small, Asia, S.W. cent. * 7 an. Of the sun, July 19th, 9 morn. Europe S. Africa, Asia, S.W. cent. 8 — 16 (12) * an. Of the moon, partial, December 28th, 4¼ aft. 1¾ dig.

1880.—Of the sun, January 11th, 11 aft. Asia, E. cent. 16 total. Of the moon, total, June 22d, 2 aft. 12¾ dig. Of the moon, total, December 16th, 4 aft. Of the sun, December 31st, 2 aft. Europe, Africa, dim. from N. to S.

1881.—Of the sun, May 28th, 0 morn. Asia, N. dim. from W. to E. Of the moon, total, June 12th, 7½ morn. Of the moon, partial, December 5th, 5¼ aft. 11¼ dig.

1882.—Of the sun, May 17th, 8 morn. Europe, S.E. Africa, Asia, cent. 10 (38) 42 — 26 total. Of the sun November 11th, 0 morn. Asia, S.E. cent. 2 * an.

1883.—Of the moon, partial, April 22d, merid. 0¼ dig. Of the moon, partial, October 16th, 7½ morn. 3 dig. Of the sun, October 31st, 0¾ morn. Asia, E. cent. 46 an.

1884.—Of the sun, March 27th, 6 morn. small, great part of Europe N.E. dim. in Asia, from W. to E. Of the moon, total, April 10th, merid. Of the moon, total, October 4th, 10¾ aft. Of the sun, October 19th, 1 morn. Asia, N.

1885.—Of the moon, partial, March 30th, 5 aft. 10 dig. Of the moon, partial, September 24th, 8¾ morn. 9 dig.

1886.—Of the sun, August 29th, 1½ aft. extremity of Europe, S.W. Africa, cent. 6 (4) * total.

1887.—Of the moon, partial, February 8th, 10½ morn. 5½ dig. Of the moon, partial, August 3d, 9 aft. 5 dig. Of the sun, August 19th, 6 morn. Europe and Africa, E. Asia, cent. 54 — 62 (54) 29 total.

1888.—Of the moon, total, January 28th, 11½ aft. Of the moon, total, July 23d, 6 morn. almost central.

1889.—Of the moon, partial, January 17th, 5¼ morn, 8½ dig. Of the moon, partial, July 12th, 9 aft. 5¾ dig. Of the sun, December 22d, 1 aft. Asia, S.W. cent. * 5 total.

1890.—Of the moon, partial, June 3d, 6 morn. 0½ dig. Of the sun, June 17th, 10 morn. Europe, Africa, Asia, cent. 25 (38) 19 an. Of the moon, partial, November 26th, 2 aft. 0½ dig.

1891.—Of the moon, total, May 23d, 7 aft. Of the sun, June 6th, 4¾ aft. great part of Europe, N. cent. † Of the moon, total. November 16th, 0¾ morn.

1892.—Of the moon, partial, May 11th, 11½ aft. 11½ dig. Of the moon, total, November 4th, 4½ aft. 12½ dig.

1893.—Of the sun, April 16th, 3 aft Europe, S. Africa, cent. 20 — 18 total.

1894.—Of the moon, partial, March 21st, 2½ aft. 3 dig. Of the sun, April 6th, 4½ morn. Europe, N.E. Asia, cent. 10 (43) 8. Of the moon, partial, September 15th, 4¾ morn. 2¼ dig. Of the sun, September 29th, 5½ morn. Africa, E. small.

1895.—Of the moon, total, March 11th, 4 morn. Of the sun, March 26th, 10 morn. almost all Europe, N.W. Asia, N. dim. from W. to E. Of the sun, August 20th, 0¾ aft. Asia, N. small. Of the moon, total, September 4th, 6 morn.

1896.—Of the moon, partial, February 28th, 8 aft. 10 dig. Of the sun, August 9th, 4½ morn. Europe, E. Asia, cent. 60 — 68 (59) 49 total. Of the moon, partial, August 23d, 7 morn. 8 dig.

1897.—No eclipse.

1898.—Of the moon, partial, January 8th, 0¾ morn. 1¼ dig. Of the sun, January 22d, 8 morn. Europe, E. Africa, E. all Asia, cent. 11 — 5 (10) 44 total. Of the moon, partial, July 3d, 9¾ aft, 11 dig. Of the moon, total, December 27th, 12 aft.

1899.—Of the sun, January 11th, 11 aft. extremity of Asia, E. dim, from N. to S. Of the sun, June 8th, 7 morn. Europe, W. and N. Asia, N. Of the moon, total, June 23d, 2½ aft. Of the moon, partial, December 17th, 1¼ morn. 11½ dig.

1900.—Of the sun, May 28th, 3½ aft. Europe, Africa, cent. 45 — 26 total. Of the moon, partial, June 13th, 4 morn. pen. +. Of the sun, November 22d, 8 morn. small eclipse, Africa, cent. 3 S. * an.

PROBLEM XVI.

To observe an Eclipse of the Moon.

To observe an eclipse of the moon, in such a manner as to be useful to geography and astronomy, it will be necessary, in the first place, to have a clock or watch that indicates seconds, and which you are certain is so well constructed as to go in an uniform manner: it ought to be regulated some days before by means of a meridian, if you have one traced out, or by some of the methods employed for that purpose by

astronomers; and you must ascertain how much it goes fast or slow in 24 hours; that the difference may be taken into account at the time of the observation.

You ought to be provided also with a refracting or reflecting telescope, some feet in length; for the longer it is, the more certain you will be of discerning exactly the moment of the phases of the eclipse; and if you are desirous of observing the quantity of the eclipse, it should be furnished with a micrometer.

When you find the moment of the eclipse approaching, which may be always known either by a common almanac, or the Ephemerides published by the astronomers in different parts of Europe, you must carefully remark the instant when the shadow of the earth touches the moon's disc. It is necessary here to mention, that there will always be some uncertainty on account of the penumbra; because it is not a thick black shadow which first covers the moon's disc, but an imperfect one that thickens by degrees. This arises from the sun's disc being gradually occulted from the moon; and hence it is difficult to fix with exactness the real limits of the shadow, and the penumbra. Here, as in many other cases, observers are enabled by habit to distinguish this boundary; or at least prevented from falling into any great error.

When you are certain that the real shadow has touched the moon's disc, the time must be noted down; that is to say, the hour, minute, and second, at which it happened.

In this manner you must follow the shadow on the moon's disc, and remark at what hour, minute, and second the shadow reaches the most remarkable spots: all this likewise must be noted down.

If the eclipse is not total, the shadow, after having covered part of the lunar disc, will decrease. You must therefore observe in like manner the moment when the shadow leaves the different spots it before covered, and the time when the disc of the moon ceases to be touched by the shadow, which will be the end of the eclipse.

If the eclipse is total, so that the moon's disc remains some time in the shadow, you must note down the time when it is totally eclipsed, as well as that when it begins to be illuminated, and the moments when the shadow leaves the different spots.

When this is done, if the time of the commencement of the eclipse be subtracted from that of the end, the remainder will be its duration; and if half the duration be added to the time of commencement, the result will be the middle.

To facilitate these operations, astronomers have given certain names to most of the spots with which the moon's disc is covered. The usual denominations are those of Langrenus, who distinguished the greater part of them by the names of astronomers and philosophers who were his contemporaries, or who had flourished before his time. Some others have been since added; but there was no room for the most celebrated of the moderns, such as Huygens, Descartes, Newton, and Cassini. Hevelius, far more judicious in our opinion, gave to these spots names taken from the most remarkable places of the earth: in this manner he calls the highest mountain of the moon, Mount Sinai, &c. This however is a matter of indifference, provided there be no confusion. We have here subjoined a representation of the moon, by means of which and the following catalogue they can be easily known, on comparing the numbers in the latter with those in the former.

1 Grimaldi.
2 Gallileo.
3 Aristarchus.
4 Kepler.
5 Gassendi.
6 Schikard.
7 Harpalus.
8 Heraclides.
9 Lansberg.
10 Rheinhold.
11 Copernicus.
12 Helicon.
13 Capuanus.
14 Bullialdi.
15 Eratosthenes.
16 Timocharis.
17 Plato.
18 Archimedes.
19 Isle of the middle Bay.
20 Pittacus.
21 Tycho.
22 Eudoxus.
23 Aristotle.
24 Manilius.
25 Menelaus.
26 Hermes.
27 Posidonius.
28 Dionysius.
29 Pliny.
30 Catherina, Cyrillus, Theopilus.
31 Fracastorius.
32 The acute promontory.
33 Messala.
34 Promontory of dreams.
35 Proclus.
36 Cleomedes.
37 Snellius and Furnerius.
38 Petau.
39 Langrenus.
40 Taruntius.

A Sea of humours.
B Sea of clouds.
C Sea of rain.
D Sea of nectar.
E Sea of tranquillity.
F Sea of serenity.
G Sea of fecundity
H Sea of crises.

PROBLEM XVII.

To observe an Eclipse of the Sun.

1st. The same precautions, in regard to the measuring of time, must be employed in this case, as in that of lunar eclipses; that is to say, care must be taken to regulate a good clock by the sun on the day before, or even on the day of the eclipse.

2d. A good telescope must be provided, of at least three or four feet in length; which must be directed towards the sun on a convenient supporter. If you are then desirous to look at the sun without the telescope, you must employ a piece of smoked glass, or rather two pieces, the smoked sides of which are turned towards each other; but are prevented from coming into contact by means of a small diaphragm cut from a card placed between them. These two bits of glass may be then cemented at the edges, so as to make them adhere. By means of these glasses interposed between the eye and the telescope, you may then view the sun without any danger to the sight.

About the time when the eclipse ought to commence, you must carefully observe the moment when the solar disc begins to be touched by the disc of the moon: this period will be the commencement of the eclipse. If there are any spots on the solar disc, you must observe the time when the moon's disc reaches them, and also when it again permits them to appear; in the last place, you must observe, with all the attention possible, the instant when the moon's disc ceases to touch the solar disc, which will be the end of the eclipse.

But if, instead of observing in this manner, you are desirous of making an observation susceptible of being seen by a great number of persons at the same time, affix to your telescope, on the side of the eye-glass, an apparatus to support a piece of very straight pasteboard at the distance of some feet. This pasteboard ought to be perpendicular to the axis of the telescope, and if it be not sufficently white, you must paste to it a sheet of white paper. Make the end of the telescope, which contains the object glass, to pass through the window-shutter of a darkened room, or one rendered considerably obscure; and if the axis of the telescope be directed to the sun, the image of that luminary will be painted on the paper, and of a larger size according as the paper is at a greater distance. It is necessary here to remark, that before you begin to observe, a circle of a convenient size must be delineated on it; so that, by moving it nearer to or farther from the telescope, the image of the sun may be exactly comprehended within it. The space contained within this circle must be divided by twelve other concentric circles, equally distant from each other; so that the diameter of the largest may be divided into 24 equal parts, each of which will represent a semi-digit.

It may now be readily conceived, that if a little before the commencement of the eclipse you look with attention at the image of the sun, you will see the moment when it begins to be obscured by the entrance of the moon's body; and that you may in like manner observe the end of it, and also its extent.

It must not however be expected that the same exactness can be attained by employing this method, as by the former; especially if you are furnished with a long telescope, and a good micrometer,

In observing the great solar eclipse of May 15th, 1836, two gentlemen residing at Greenwich, one observing the sun's image on paper in a dark chamber, and the other looking directly at the sun, found that the time at which they observed the contact of the moon with the spots on the sun, were in several instances identical.

Remarks.—There are partial eclipses of the sun, that is to say, eclipses in which only a part of the solar disc seems to be covered, and these are most common. Others are total and annular.

Total eclipses take place when the centre of the moon passes over that of the sun, or nearly so; and when the apparent diameter of the moon is equal to that of the sun, or greater. In the latter case, the total eclipse may be what is called *cum morâ;* that is to say, with duration of darkness: of this kind was the famous eclipse of 1706.

During eclipses which are total and *cum morâ*, so great darkness prevails, that the stars are seen in the same manner as at night, and particularly Mercury and Venus. But what excites a sort of terror, is the dismal appearance which all nature assumes during the last moments of the light. Animals, struck with fear, retire therefore to their habitations, sending forth loud cries; the nocturnal birds issue from their holes; the flowers contract their leaves; a coldness is felt, and the dew falls; but as the moon has suffered a few rays of the solar light to escape, all is again illumination; day instantly returns, and with more brightness than when the weather is cloudy.

Some eclipses, as already said, are really annular: they take place when the eclipse is very near being central, while the apparent diameter of the moon is less than that of the sun; which may be the case if the moon at the time of the eclipse is at her greatest distance from the earth, and the sun at his nearest distance to it. The eclipse of the sun, on the 1st of April 1764, was of this kind to a part of Europe, and also that of May 1836.

During eclipses of this kind, when the sun is entirely eclipsed, a luminous circle of a silver colour, and as broad as the twelfth part of the diameter of the sun or moon, is often observed around the former; it is effaced as soon as the smallest part of the sun begins to shine: it appears more lively towards the sun's limb, and decreases in brilliancy the further it is distant. Some are inclined to believe that this circle is formed by the luminous atmosphere with which the sun is surrounded; others have conjectured that it is produced by the refraction of his rays in the atmosphere of the moon; and some have ascribed it to the diffraction of the light. Those who are desirous of farther information on this subject, may consult the Memoirs of the Academy of Sciences, for the years 1715 and 1748.

On various occasions, persons who have witnessed the formation and dissolution of the *annulus* in annular eclipses, have recorded certain singular appearances as having been noticed by them; and the eclipse above referred to of May 15th, 1836, being annular in the north of England and south of Scotland, FRANCIS BAILY, of London, well known for his devotion to astronomical science, went to Jedburgh, in Roxburgshire, over which the central line of the moon's umbra passed on that occasion, to observe the rare phenomenon, and to see what truth there might be in these reported singular appearances.

It is needless to say that he was provided with all needful instruments; and he got the error of his chronometers from the neighbouring observatory of Sir T. Brisbane, at Makerston. He was fortunate also in meeting at Jedburgh with an able assistant in Mr. Veitch, of that place, a most ingenious mechanic, a self-taught maker of telescopes, and enthusiastically attached to scientific pursuits. The day was uncommonly favourable.

Mr. Baily, in his account of the eclipse, published in the 10th volume of the Memoirs of the Royal Astronomical Society, says, "The diminution of light, during the existence of the annulus, was not so great as was generally expected; being little more than what might be caused by a temporary cloud passing over the sun; the light however was of a peculiar kind, somewhat resembling that produced by the sun shining through a mist. The thermometer in the shade fell only 3 or 4 degrees; it was 61° during the time of the annulus. About twenty minutes before the formation of the annulus, *Venus* was seen with the naked eye; and a few minutes afterwards I found it impossible to fire gunpowder with the concentrated rays of the sun

through a lens three inches in diameter. The same lens had no effect on the bulb of a thermometer during the existence of the annulus."

After some further remarks, Mr. Baily goes on to say:—"I shall now proceed to detail the singular appearances which occurred at the formation and dissolution of the annulus."—"When the last portion of the moon's disc was about to enter on the face of the sun, I prepared myself to observe the formation of the annulus. I was in expectation of meeting with something extraordinary; but imagined that it would be momentary only, and consequently that it would not interrupt the noting of the time of its occurrence. In this however I was deceived, as the following facts will shew. For when the cusps of the sun were about 40° asunder, a row of lucid points, like a string of bright beads, irregular in size and distance from each other, *suddenly* formed round that part of the circumference of the moon that was about to enter, or which might be considered as having just entered, on the sun's disc. Its formation indeed was so rapid, that it presented the appearance of having been caused by the ignition of a fine train of gunpowder. This I intended to note as the correct time of the formation of the annulus; expecting every moment to see the thread of light completed round the moon; and attributing this serrated appearance (as others had done before me) to the lunar mountains, although the remaining part of the moon's circumference was comparatively smooth and circular, as seen through a telescope. My surprise however was great on finding that these luminous points, as well as the dark intervening spaces, increased in magnitude, some of the contiguous ones appearing to run into each other like drops of water; for the rapidity of the change was so great, and the singularity of the appearance so fascinating and attractive, that the mind was for the moment distracted, and lost in the contemplation of the scene, so as to be unable to attend to every minute occurrence. Finally, as the moon pursued her course, these dark intervening spaces (which, at their origin, had the appearance of lunar mountains in high relief, and which still continued attached to the sun's border) were stretched out into long, black, thick, parallel lines, joining the limbs of the sun and moon; when, all at once, they *suddenly* gave way, and left the circumference of the sun and moon in those points, as in the rest, comparatively smooth and circular; and the moon perceptively advanced on the face of the sun.

"The appearances here recorded passed off in less time than it has taken me now to describe them, but they were so extraordinary and so rapid that all idea of time was lost, except by the recollection afterwards of what had passed; for I was so riveted to the scene, that I could not take my eye away from the telescope, to note down any thing during the progress of the phenomenon. I estimate, however, that the whole lasted about six or eight seconds, or perhaps ten at the utmost.

"After the formation of the annulus thus described, the moon preserved its usual circular outline during its progress across the sun's disk, till its opposite limb again approached the border of the sun, and the annulus was about to be dissolved. When, all at once (the limb of the moon being at some distance from the edge of the sun) a number of long, black, thick, parallel lines, exactly similar in appearance to the former ones above mentioned, *suddenly darted forward* from the moon, and joined the two limbs as before: and the same phenomena were repeated, but in inverse order. For, as these dark lines got shorter, the intervening bright parts assumed a more circular and irregular shape, and at length terminated in a fine curved line of bright heads (as at the commencement) till they ultimately vanished, and the annulus consequently became wholly dissolved."

Mr. Baily says that he shall not attempt to account for this phenomenon: but he says the lines "were as plain, as distinct, and as well defined, as the open fingers of the human hand held up to the light."

It appears that, in a total eclipse April 22, 1715, Dr. Halley noticed similar pheno-

mena, as did M. Cassini May 22, 1724; Mr. Ellicot, June 16th, 1810, at total eclipses which they observed.

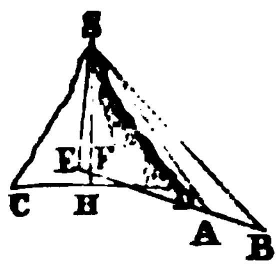
An annular eclipse of the sun was observed by Mr. S. Webber, April 3, 1791, and precisely similar phenomena were noticed; and like phenomena had been noticed by M. Nicolai and Professor Moll of Leyden. It would appear also that similar phenomena have been noticed in the transits of Venus over the sun's disc.

PROBLEM XVIII.

To measure the Height of Mountains.

The height of a mountain may be measured by the common rules of geometry: for if we suppose C S D (Fig. 10), to be a mountain, the perpendicular height of which is required, the following method can be employed. If the nature of the adjacent ground will admit, measure a horizontal line A B, in the same vertical plane as the summit S of the mountain. The greater the extent of this line, the more correct will be the result. At the two stations A and B, measure the angles S A E and S B E, which are the apparent heights of the summit S, above the horizon, when seen from A and B. It will then be easy, by means of plane trigonometry, to find, in the right-angled triangle S E A, the side E A, as well as the perpendicular S E, or the elevation of the summit S above A E continued.

Fig. 10.



Now let us suppose the vertical line S F H to be drawn, intersecting B E in F. As, in dimensions of this kind, the angle E S F, formed by the vertical line and the perpendicular S E, will for the most part be exceedingly small, and much below one degree, the lines S E and S F may be considered as equal.* On the other hand, the line F H, comprehended between the line A E and the spherical surface C A, is evidently the quantity by which the real level is lower than the apparent level, in an extent such as A F, or more correctly in a mean length between A F and B F: for this reason take the mean length between A E and B E, which differ very little from A F and B F; and in the table of differences between the apparent and real levels, find the height corresponding to that mean distance: if this height be then added to the height S E or S F, already found, you will have S H for the corrected height of the mountain, above the spherical surface, where the points A and B are situated.

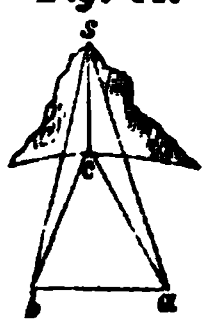
If it be known how much this surface is higher than the level of the sea, it will be known also how much the summit S of the mountain is elevated above the same level.

Another Method.

As it may be difficult to establish a horizontal line, so as to be in the same vertical plane with the summit of the mountain, it will perhaps be better to proceed in the following manner:

Trace out your base in the most convenient manner, so as to be horizontal: we shall here suppose that it is represented by $a\,b$ (Fig. 11.); let $s\,c$ be the perpendicular from the summit s to the horizontal plane passing through $a\,b$; and let c be the point where this plane is met by the perpendicular: if the lines $a\,c$ and $b\,c$ be drawn to that point, we shall have the triangles $s\,a\,c$ and $s\,b\,c$, right-angled at c; and the angles $s\,a\,c$ and $s\,b\,c$ may be found by measuring, from the points a and b, the apparent height of the mountain above the horizon: the angles $s\,a\,b$ and $s\,b\,a$, in the triangle $a\,s\,b$, must also be measured.

Fig. 11.



Now, since in the triangle $s\,a\,b$, the angles $s\,a\,b$ and $s\,b\,a$ are known,

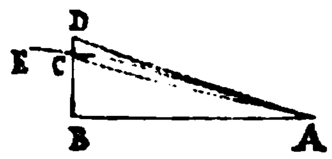
* For even in the case of this angle being a degree, they would not differ a ten thousandth part, which would suppose the distance of the stations from the mountain to be more than 100000 yards.

and also the side *a b*; any one of the other sides, such for example as *s a*, may be easily determined by plane trigonometry. In the triangle *a c s*, right-angled at *c*, as the angle *s a c* is known, the side *a c* and the perpendicular *s c* may be found in the same manner. When this is done, the method pointed out in the preceding operation must be employed: that is to say, find the depression of the level below the apparent level for the number of feet or yards comprehended in the line *a c*, and add it to the height *s c*: the sum will be the height of the point *s*, above the real level of the points *a* and *b*.

Example.—Let the horizontal length *a b* be 2000 yards, or 6000 feet: the angle *s a b* 80° 30′; and the angle *s b a* 85° 10′; consequently the angle *b s a* will be 14° 20′. By means of these data, the side *s a* of the triangle *a s b* will be found to be 8050 yards. On the other hand, if we suppose the angle *s a c* to have been measured, and to be 18°, the side *a c* will be found, by trigonometrical calculation, to be 7656 yards; and *s c*, perpendicular to the horizontal plane passing through *a b*, will be found equal to 2488. Now, as the depression of the real level below the apparent level at the distance of 7656 yards, is 12½ feet, or 4 yards 6 inches,* if this quantity be added to the height *s c*, we shall have 2492 yards 6 inches for the real height of the mountain.

Remark.—When either of these methods is employed, if the mountain to be measured is at a considerable distance, such as 20000 or 40000 yards, as its summit in that case will be very little elevated above the horizon, the apparent height must be corrected by making an allowance for refraction, otherwise there may be a very considerable error in the result. The necessity of this correction may be easily conceived by observing, that the summit C of the mountain B C (Fig. 12), is seen by a ray of light E C A, which is not rectilineal, but bent; so that the summit C is judged to be in D, according to the direction of the line A D, a tangent to the curve A C E, which in the small space A C may be considered as the arc of a circle. The angle D A B therefore, of the apparent height of the mountain, exceeds the height at which the summit would appear without refraction, by the quantity of the angle C A D; which must be determined. But it will be found that this angle C A D is nearly equal to half the refraction which would belong to the apparent height D A B. You must therefore find, in the tables, the refraction corresponding to the apparent height D A B of the mountain, and subtract the half of it from that height: the remainder will be that of the summit of the mountain, such as it would be seen without refraction.

Fig. 4.



Let us suppose, for example, that the summit of the mountain seen at the distance of 20000 yards appears to be elevated above the horizon 5 degrees: the refraction corresponding to 5 degrees is 9′ 54″, the half of which is 4′ 57″; if 4′ 57″ therefore be subtracted from 5°, the remainder will be 4° 55′ 3″ which must be employed as the real elevation.†

It may thence be seen, that, to proceed with certainty in such operations, it will be necessary to make choice of stations at a moderate distance from the mountain; so that its summit may appear to be elevated several degrees above the horizon; otherwise the difference of the refraction, which is very variable near the horizon, will occasion great uncertainty in the measurement.

We shall give hereafter another method for measuring the height of mountains, by means of the barometer; but in this case it is supposed that it is possible to ascend

* See the table in the additional remark.

† Montucla here employs the common tables of refraction used for nautical and astronomical purposes, such as that given in "Riddle's Navigation," and other works of the kind. In regard to terrestrial refraction, and the allowance made for it, see the additional remark at the end of this article.

to the summit of them. We shall also give a table of the heights of the principal mountains of the earth above the level of the sea. We shall here only observe that the highest mountains in the world, at least in that part of it which has hitherto been accessible to scientific men, are situated in the neighbourhood of the equator; and it is with justice that an historian of Peru says, that when compared with our Alps and our Pyrenees, they are like the towers and steeples of the churches in our cities, compared with common edifices. The highest yet known is one in the Himalayan range in India, which rises nearly 27000 feet in a perpendicular direction above the level of the sea.

As all the known mountains in Europe are scarcely two-thirds of the height of those enormous masses, the falsity of what the ancients, and some of the moderns, such as Kircher, have published respecting the height of mountains, will readily appear. According to these authors, Ætna is 4000 geometrical paces in height; the mountains of Norway 6000; Mount Hœmus and the Peak of Teneriff 10000; Mount Atlas and the Mountains of the Moon in Africa 15000; Mount Athos 20000; Mount Cassius 28000. It is asserted that these heights were found by means of their shadows; but nothing is more destitute of truth, and if ever any observer ascends to the summit of these mountains, or measures their height geometrically, they will be found very inferior to the mountains of Peru; as is the case with the Peak of Teneriff, which when measured geometrically was found to be only about 7000 feet.

Hence it appears that the elevation of the highest mountains is very little, when compared with the diameter of the earth, and that its regular form is not sensibly altered by them; for the mean diameter of the earth is about 7957¾ miles; therefore if we suppose the height of a mountain to be 3½ miles, it will be only the 2273d part of the diameter of the earth, which is less than the elevation of half a line on a globe six feet in diameter.

Additional Remark.—As Montucla has not here explained the method of finding the difference between the apparent and true level, we think it necessary to add a few observations on the subject. Two or more places are said to be on a true level, when they are equally distant from the centre of the earth. One place also is higher than another, or out of level with it, when it is farther from the centre of the earth; and a line equally distant from that centre in all its parts, is called the *line of true level.* Hence, because the earth is round, that line must be a curve, or at least parallel or concentric to it. But the line of sight, given by operations of levelling, which is a tangent, or a right line perpendicular to the semi-diameter of the earth at the point of contact, always rising higher above the true curve line of level the farther the distance, is called the *apparent line of level;* and the difference between the line of true level and the apparent, is always equal to the excess of the secant of the arch of distance above the radius of the earth. Hence it will be found that this difference is equal to the square of the distance between the places, divided by the diameter of the earth; and consequently it is always proportional to the square of the distance.

From these principles is obtained the following table, which shews the height of the apparent above the true level for every 100 yards of distance on the one hand, and for every mile on the other.

The common methods of levelling are sufficient for laying pavements of walks, or for conveying water to small distances, &c.; but in more extensive operations, as in levelling the bottoms of long canals, which are to convey water to the distance of many miles, and such like, the difference between the true and apparent level must be taken into account.

Dist.	Diff. of Level.	Dist.	Diff. of Level.
Yards.	Inches.	Yards	Inches.
100	0·026	1000	2·570
200	0·103	1100	3·110
300	0·231	1200	3·701
400	0·411	1300	4·344
500	0·643	1400	5·038
600	0·925	1500	5·784
700	1·260	1600	6·580
800	1·645	1700	7·425
900	2·081		

Dist.	Diff. of Level.		Dist.	Diff. of Level.	
Miles.	Ft.	In.	Miles.	Ft.	In.
$\frac{1}{4}$	0	$0\frac{1}{2}$	7	32	6
$\frac{1}{2}$	0	2	8	42	6
$\frac{3}{4}$	0	$4\frac{1}{2}$	9	53	9
1	0	8	10	66	4
2	2	8	11	80	3
3	6	0	12	95	7
4	10	7	13	112	2
5	16	7	14	130	1
6	23	11			

By means of these tables of reductions, the difference between the true and apparent level can be found by one operation; whereas the ancients were obliged to employ a great many; for being unacquainted with the correction answering to any distance, they levelled only from one 20 yards to another, as they had occasion to continue the work to some considerable extent.

These tables will answer several useful purposes: First, to find the height of the apparent level above the true, at any distance. If the given distance be contained in the table, the correction of the level will be found in the same line with it. For example, at the distance of 1000 yards the correction is 2·57, or nearly two inches and a half; and at the distance of ten miles, it is 66 feet 4 inches. But if the exact distance be not found in the table, multiply the square of the distance in yards by 2·57, and divide by 1000000, or cut off six places on the right for decimals, the rest will be inches; or multiply the square of the distance in miles by 66 feet 4 inches, and divide by 100.

2d. To find the extent of the visible horizon, or how far can be seen from any given height on a horizontal plane, as at sea, &c. Let us suppose the eye of an observer on the top of a ship's mast at sea, to be at the height of 130 feet above the water, it will then see about 14 miles all around; or from the top of a cliff by the sea side, the height of which is 66 feet, a person may see to the distance of nearly 10 miles on the surface of the sea. Also, when the top of a hill, or the light in a light-house, the height of which is 130 feet, first comes into the view of an eye on board a ship, the table shews that the distance of the ship from it is 14 miles, if the eye be at the surface of the water; but if the height of the eye in the ship be 80 feet, the distance will be increased by nearly 11 miles, making in all about 25 miles.

3d. Suppose a spring to be on the one side of a hill, and a house on an opposite hill, with a valley between them, and that the spring seen from the house appears, by a levelling instrument, to be on a level with the foundation of the house, which we shall suppose to be at the distance of a mile from it: this spring will be 8 inches above the true level of the house; and that difference would be barely sufficient for the water to be brought in pipes from the spring to the house, the pipes being laid all the way under ground.

4th. If the height or distance exceed the limits of this table: Then first, if the distance be given, divide it by 2, or by 3, or by 4, &c., till the quotient come within the distances in the table; then take out the height answering to the quotient, and multiply it by the square of the divisor, that is by 4, or by 9, or by 16, &c., which will give the height required. Thus, if the top of a hill be just seen at the distance of 40 miles; then 40 divided by 4, is 10, and opposite to 10 in the table will be found $66\frac{1}{3}$ feet, which multiplied by 16, the square of 4, gives $1061\frac{1}{3}$ for the height

of the hill. But when the height is given, divide it by one of these square numbers, 4, 9, 16, 25, &c., till the quotient come within the limits of the table, and multiply the quotient by the square root of the divisor, that is by 2, or 3, or 4, or 5, &c., for the distance sought. Thus, when the top of the peak of Teneriff, said to be about 3 miles or 15840 feet high, just comes into view at sea, divide 15840 by 225, or the square of 15, and the quotient is 70 nearly, to which in the table corresponds by proportion nearly $10\frac{3}{4}$ miles; which multiplied by 15, will give 154 miles and $\frac{3}{4}$ for the distance of the mountain.

In regard to the terrestrial refraction, which in measuring heights is to be taken into account also, as it makes objects to appear higher than they really are, it is estimated by Dr. Maskelyne at $\frac{1}{10}$ of the distance observed, expressed in degrees of a great circle. Thus if the distance be 10000 fathoms, its 10th part 1000 fathoms is the 60th part of a degree on the earth, or 1′, which is therefore the refraction in the altitude of the object at that distance.

Le Gendre, however, says he is induced by several experiments to allow only $\frac{1}{14}$th part of the distance for refraction in altitude. So that upon the distance of 10000 fathoms, the 14th part of which is 714 fathoms, he allows only 44″ of terrestrial refraction, so many being contained in the 714 fathoms.

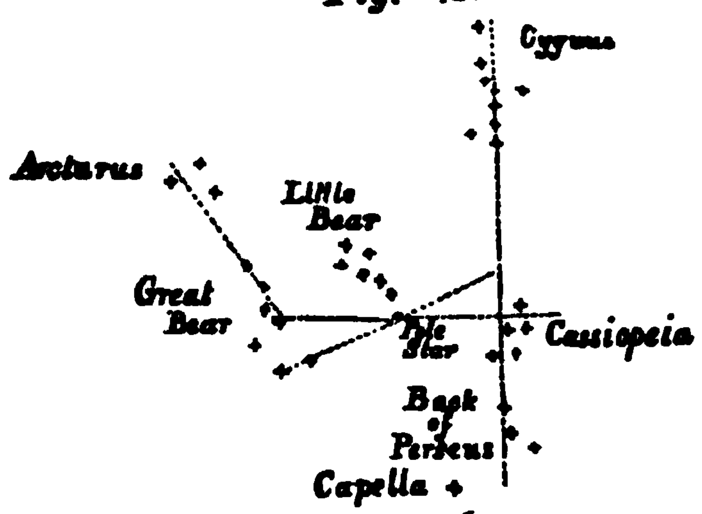
Delambre, an ingenious French astronomer, makes the quantity of terrestrial refraction to be the 11th part of the arch of distance. But the English measurers, Col. Ed. Williams, Capt. Mudge, and Mr. Dalby, from a multitude of exact observations made by them, determine the quantity of refraction to be the 12th part of the said distance. The quantity of this refraction however is found to vary, with the different states of the weather and atmosphere, from the 15th part of the distance to the 9th part; the medium of which is the 12th, as above mentioned.

PROBLEM XIX.

Method of knowing the Constellations.

To learn to know the heavens, you must first provide yourself with some good celestial charts, or a planisphere of such a size, that stars of the first and second magnitude can be easily distinguished. At the end of the present article we shall point out the best works on this subject.

Fig. 13.



Having placed before you one of these charts, that containing the north pole, turn your face towards the north, and first find out the great bear, commonly called Charles's wain (Fig. 13.) It may be easily known, as it forms one of the most remarkable groups in the heavens, consisting of seven stars of the second magnitude, four of which are arranged in such a manner as to represent an irregular square, and the other three a prolongation in the form of a very obtuse scalene triangle. Besides, by examining the figure of these seven stars, as exhibited in the chart, you will easily distinguish those in the heavens which correspond to them. When you have made yourself acquainted with these seven principal stars, examine on the chart the configuration of the neighbouring ones, which belong to the Great Bear; and you will thence learn to distinguish the other less considerable stars which compose that constellation.

After knowing the Great Bear, you may easily proceed to the Lesser Bear; for nothing will be necessary but to draw, as seen in the annexed figure, a straight line through two anterior stars of the square of the Great Bear, or the two farthest

distant from the tail: this line will pass very near the polar star, a star of the second magnitude, and the only one of that size in a pretty large space. At a little distance from it, there are two other stars of the second and third magnitude, which, with four more of a less size, form a figure, somewhat similar to that of the Great Bear; but smaller. This is what is called the Lesser Bear; and you may learn, in the same manner as before, to distinguish the stars which compose it.

Now, if a straight line be drawn through those stars of the Great Bear, nearest to the tail, and through the polar star, it will conduct you to a very remarkable group of five stars arranged nearly in this form M: these are the constellations of Cassiopeia, in which a very brilliant new star appeared in 1572; though soon after it became fainter, and at length disappeared.

If a line, perpendicular to the above line, be next drawn, through this constellation, it will conduct, on the one side, to a very beautiful star called Algenib, which is in the back of Perseus; and, on the other, to the constellation of the Swan, remarkable by a star of the first magnitude. Near Perseus is the brilliant star of the Goat, called Capella, which is of the first magnitude, and forms part of the constellation of Auriga.

After this, if a straight line be drawn through the two last stars of the tail of the Great Bear, you will come to the neighbourhood of Arcturus, one of the most brilliant stars in the heavens, which forms part of the constellation of Bootes.

In this manner you may successively employ the knowledge you have obtained of the stars of one constellation, to enable you to find out the neighbouring ones. We shall not enlarge farther on this method; for it may be easily conceived, that we cannot proceed in this manner through the whole heavens: but any person of ingenuity, in the course of a few nights, may learn by these means to know a great part of the heavens; or at any rate the principal stars.

The ancients were not acquainted with, or rather did not insert into their catalogue, more than 1022 fixed stars, which they divided into 48 constellations; but their number is much greater, even if we confine ourselves to those which can be distinguished by the naked eye. The Abbé de la Caille observed 1492 in the small space comprehended between the tropic of Capricorn and the south pole; a part of which he formed into new constellations. But this space is to the whole sphere, as 3 to 10 nearly; so that in our opinion, the whole number of the stars visible to the naked eye may be estimated at about 6500. It is a mere illusion that makes us conclude, on the first view, that they are innumerable; for if you take a space comprehended between four, five, or six stars of the second and third magnitude, and try to count those it contains, you will find that it can be done without much difficulty; and some idea may be thence formed of their total number, which will not much exceed that above stated.

The stars are divided into different classes, viz., stars of the first, second, third, &c. magnitude, as far as the sixth, which are the smallest perceptible to the naked eye. There are 20 of the first maguitude, 76 of the second, 223 of the third, 512 of the fourth, &c.

In regard to the constellatious, the number of those commonly admitted is 90; of which 33 belong to the northern hemisphere, 12 to the Zodiac, and the remaining 45 to the austral or southern hemisphere. We shall here give a catalogue of them, with the number of the principal stars of which each is composed, together with the names of some of the most remarkable stars: the constellations which have this mark * against them, are modern ones, the others ancient. The figures placed against the principal stars, denote their magnitudes.

I. PRINCIPAL CONSTELLATIONS NORTH OF THE ZODIAC.

No.	Constellations.	No. of Stars	Chief Stars.	No.	Constellations.	No. of Stars	Chief Stars.
1	Ursa Minor	24	Pole Star 2	19	Serpens	50	
2	Ursa Major	105	Dubhe 1	20	ScutumSobieski	76	
3	Perseus	72	Algenib 2	21	Hercules cum Ramo et Cerbero	117	Ras Algiatha 3
4	Auriga	66	Capella 1	22	*Serpentarius siveOphiucus	142	Ras Alhagus 3
5	*Bootes	54	Arcturus 1	23	*Taurus Poniatowski	7	
6	Draco	84	Rastaber 3	24	Lyra	24	Vega 1
7	*Cepheus	51	Alderamin 3	25	*Vulpecula et Anser	36	
8	*Canes Venatici scil. Asterian et Chara	63		26	Sagitta	12	
9	*Cor Caroli	3		27	Aquila	40	Altair 1
10	*Triangulum	10		28	Delphinus	13	
11	Triangulum minus	5		29	*Cygnus	82	Deneb Adige 1
12	*Musca	6		30	*Equuleus	10	
13	*Lynx	48		31	*Lacerta	16	
14	*Leo Minor	59		32	*Pegasus	88	Markab 2
15	*ComaBerenices	45		33	*Andromeda	71	Almaac 2
16	*Camelopardalus	78					
17	*Mons Menelaus	11					
18	CoronaBorealis	21					

II. CONSTELLATIONS IN THE ZODIAC.

No.	Constellations.	No. of Stars	Chief Stars.	No.	Constellations.	No. of Stars	Chief Stars.
1	Aries	67		7	Libra	55	ZubenichMali 2
2	Taurus	143	Aldebaran 1	8	Scorpio	37	Antares 1
3	Gemini	87	Castor and Pollux 1.2	9	Sagittarius	73	
4	Cancer	87		10	Capricornus	54	
5	Leo	101	Regulus 1	11	Aquarius	119	Scheat 3
6	Virgo	117	SpicaVirginis 1	12	Pisces	115	

III. PRINCIPAL CONSTELLATIONS SOUTH OF THE ZODIAC.

No.	Constellations.	No. of Stars	Chief Stars.	No.	Constellations.	No. of Stars	Chief Stars.
1	*Phœnix	13		15	Canis Major	31	Sirius 1
2	*Officina Sculptoria	12		16	*Equuleus Pictorius	8	
3	Eridanus	76	Achernar 1	17	*Monoceros	31	
4	*Hydrus	31		18	Canis Minor	18	Procyon 1
5	*Cetus	70	Menkar 2	19	*Chameleon	10	
6	*FornaxChemica	14		20	*Pyxis Nautica	4	
7	*Horologium	12		21	*Piscis Volans	8	
8	*Reticulus Rhomboidalis	10		22	Hydra	43	Cor Hydræ 1
9	*Xiphias	7		23	*Sextans	43	
10	*Celapraxitellis	16		24	*Robur Carolinum	12	
11	*Lepus	18		25	*Machina Pneumatica	3	
12	*Columba Noachi	10		26	*Crater	11	Alkes 3
13	Orion	70	Betelguese 1	27	*Corvus	9	Algorab 3
14	Argo Navis	43	Canopus 1	28	*Crosiers	6	

No.	Constellations.	No. of Stars	Chief Stars.	No.	Constellations.	No. of Stars	Chief Stars.
29	*Musca	4		38	*Corona Australis	12	
30	*Apis Indica	11		39	*Pavo	14	
31	*Circinus	4		40	*Indus	12	
32	Centaurus	36		41	*Microscopium	10	
33	*Lupus	24		42	*Octans Hadleianus	43	
34	*QuadraEuclidis	12		43	*Grus	14	
35	*Trangulum Australe	5		44	*Toucan	9	
36	Ara	9		45	Piscus Australis	20	Fomalhaut 1
37	*Telescopium	9					

IV. NUMBER OF STARS OF EACH MAGNITUDE.

Constellations.	Constellations.	Magnitudes. I.	II.	III.	IV.	V.	VI.	Total Number of Stars.
In the Zodiac	12	5	16	44	120	183	646	1014
In the N. Hemisphere	33	6	24	95	200	291	635	1251
In the S. Hemisphere	45	9	36	84	190	221	323	865
	90	20	76	223	512	695	1604	3130

We shall not here enter into any physical details respecting the stars; as we reserve these for another place, where we shall speak of their distances, magnitudes, motion, and various other things relating to this subject; such as new stars, changeable or periodical stars, &c.

The best celestial charts were for a long time those of Bayer's Uranometria, a work in folio, published in 1603, and which has gone through a great many editions. But these charts have given place to the magnificent Celestial Atlas of Flamsteed, published in folio at London, in 1729; a work indispensably necessary to every practical astronomer. Of the other charts or planispheres, those of Pardies, published in 1673, in six sheets, magnificently engraved by Duchange, are esteemed. We have also the two planispheres of De la Hire, in two sheets. Senex, an English engraver, published likewise two new planispheres, according to the observations of Flamsteed; one of them in two sheets, where the two hemispheres are projected on the plane of the equator; and the other where they are projected on the plane of the ecliptic. Those who have not the Celestial Atlas of Flamsteed must provide themselves with either of these planispheres. The modern astronomers, and particularly La Caille, having added a great number of new constellations to the old ones in the southern hemisphere, two new planispheres have on that account been formed. One of them, by M. Robert, consists of two sheets, where the ground of the heavens is coloured blue; so that the constellations are very distinctly seen. It is constructed according to the newest observations; and it is accompanied with useful instructions respecting the method of knowing the heavens.

As it is of the greatest importance to astronomers, to be acquainted with the constellations and stars of the Zodiac, because the planets move in that circular band, Senex, before mentioned, published, about half a century ago, The Starry Zodiac, from Flamsteed's Observations; and as it was difficult to be procured at Paris, the Sieur Dheuland, engraver, gave, in 1755, a new edition of it; with such corrections as the interval between that period and the time when Senex published his edition, had rendered necessary. He was directed in this undertaking by M. de Seligny, a

young officer in the service of the East-India Company. To the Zodiac of Dheuland is annexed a minute catalogue of the Zodiacal stars, with their longitudes and latitudes, reduced to the year 1755. This catalogue comprehends 924 stars; but the author, to render his work more useful in nautical observations, gives to his Zodiac ten degrees of latitude, on each side of the ecliptic. It may be readily seen, from what has been here said, that those who are not possessed of the Celestial Atlas of Flamsteed, must procure the Zodiac and Catalogue of Dheuland, or rather of Seligny, and that even possessing the former work does not supersede the necessity of the latter.

A new edition of Flamsteed's Atlas, reduced to a third of its original size, has since been published, with a planisphere of the austral stars observed by La Caille. M. Fortin, the author, reduced all the stars to the year 1780; and added a chart of the stars representing the different figures which they form, together with their relative positions.

To the above list we may add the large Celestial Atlas lately published by Professor Bode, of Berlin, consisting of twenty sheets.

Remark.—Since the period when mankind began to observe the stars, various astronomers, at different times, have undertaken to exhibit, in charts, their places, relative distances, and magnitudes. To the works of this kind before mentioned, we may add also the *Cœlum Stellatum* of Julius Schiller, 1627; the *Firmamentum Sobescianum* of Hevelius, 1690, in 54 sheets; and Doppelmayer's Celestial Atlas, Nuremberg 1742. In the year 1729 Flamsteed's Celestial Atlas was published in 28 sheets, containing 2919 stars, observed by that astronomer at Greenwich, and divided into 56 constellations. In the year 1776, an edition of it, reduced to the quarto form, was published at Paris by Fortin, in 30 sheets; in the year 1796 La Lande and Mechain published the same plates, considerably improved and enlarged with seven new constellations. In the year 1782 M. Bode published the same Atlas in 34 sheets, small folio; but he added, besides the old observations, a great many new ones, and above 2100 fixed stars and nebulæ. In the year 1748, a new *Uranographia*, of the same kind as that of Bayer, to consist of 50 sheets, was announced to be published by subscription in England. Dr. Bevis, a noted astronomer, was at the head of this undertaking, and some of the sheets were engraved; but the work was never completed.* The Atlas now published by professor Bode, in 20 sheets, is constructed according to an entirely new projection. Flamsteed's charts were each 21 inches in breadth and 28 inches in length; those of Bode's Atlas are 26 inches in breadth and 38 in length. Flamsteed's Atlas contains only 56 constellations on 28 sheets; that of Bode contains 106 on 18 sheets, together with the stars around the south pole, and two hemispheres. Of late years, by the continued assiduity of astronomers, the number of stars observed has been much increased. Dr. Herschel, with his excellent telescopes, has discovered above 2500 nebulæ, groups of stars, and double stars. Baron von Zach of Gotha constructed a new and complete catalogue of the fixed stars, from his own observations; but Professor Bode for the greatest number of his improvements was indebted to La Lande. This meritorious astronomer supplied him at different times with new stars, amounting altogether to about 6000, which were observed by himself and his nephew Le François, at the Military school, with a mural quadrant by Bird. But the first manuscripts transmitted by La Lande, contained the right ascensions only to minutes of time; and consequently were not accurately enough defined for the large scale on which these charts are constructed. Professor

* Another little known Celestial Atlas, which at least is not mentioned by La Lande, is that of Corbinianus Thomas, a Benedictine and professor of mathematics at Augsburg. It is entitled "Firmamentum Firmianum," in honour of the then bishop of the house of Firmian, and was published at Augsburg in small folio, in the year 1731. In this Atlas the northern crown is called "Corona Firmiana."

Bode therefore inserted only some of these stars into his charts, being obliged to leave out the greater part of them. La Lande sent afterwards more correct positions; and though the professor encountered many difficulties in reducing them, in consequence of errors in the transcribing or calculation, he was enabled to add to his charts some thousands of new stars, furnished by the above astronomer. The professor however found several vacuities, and being desirous that the improvement introduced into his work should be uniform, he resolved to supply these deficiencies from his own observations. He began therefore in the month of December 1796, at the royal observatory of Berlin, to search for and observe new stars, with a mural quadrant by Bird; and by these means was enabled to enrich his Atlas with some hundreds of stars, of the 6th and 7th magnitudes, not to be found in any of the catalogues.

Flamsteed, for his charts, made choice of a kind of projection by which, especially under great declinations, no proper idea is given of the real figure of the circles of the sphere. In these charts the parallels to the equator are straight lines, which intersect the meridians, where the cosines of their distance from the mean meridian falls. They appear therefore as crooked lines; the meridians or great circles appear also crooked, and the parallels or less circles straight lines, entirely contrary to the real form which these circles of the sphere exhibit. Professor Bode therefore made choice of another kind of projection, namely, that conical projection described by Kastner in his Geometrical Treatises, and in which the semi-diameter of the mean parallel is the cotangent of its declination. The mean meridian, on the other hand, is lengthened where these cotangents fall; and from this point as a centre are drawn the parallel circles at every 5 degrees. At this centre the value of the angle of right ascension, for example 10 degrees, is made = sin. decl. 10°; and the meridians are drawn as straight lines. By this construction the degrees of ascension are kept in the proper proportion to those of declination, in the mean zones lying between the parallels, as far as they extend east or west; and the principal stars which each sheet exhibits, fall in these mean zones. Each sheet generally contains about 75°, on the equator, of right ascension, and 54° in declination. When the equator falls in the middle of the chart, the parallels and meridians are straight lines, placed at equal distances, and intersecting each other at right angles. The polar regions are delineated according the stereographic projection. The scale of these charts, the two polar ones excepted, is 10° declination to 4 inches English.

The names of all the constellations are given in Latin, according to the general practice; the original constellations, when they form the principal figures in the chart, are completely shaded; but in such a manner that the smallest stars and the nebulous spots are apparent. The names are given in large Roman shaded characters. The constellations introduced in modern times are shaded in the punctured manner; and the names are added in large open Roman characters. Besides the Arabic and Latin names already known, the old Arabian names are also added to many of the stars. The epoch of the right ascension of these stars is fixed at the 1st of January, 1801.

The Society for the Diffusion of Useful Knowledge have published two sets of celestial maps on the *gnomonic projection*. The smaller is of a quarto size, and the larger of 25 inches square, and forms one of the most useful celestial Atlases hitherto published. The late Professor Harding, well known to astronomers as the discoverer of the planet Juno, published an Atlas of the heavens, which is considered exceedingly accurate—especially that part of it where planets may be expected to appear. At the death of the Professor many copies of this valuable Atlas were in possession of the family, and several copies were purchased on the occasion by English astronomers.

An Atlas has for some time been in progress of construction from actual observations made by several astronomers in Germany and one or two in England, each taking a separate part of the heavens and filling up from his observations skeleton forms with which he is furnished. This work when finished will doubtless be of standard character.

CHAPTER II.

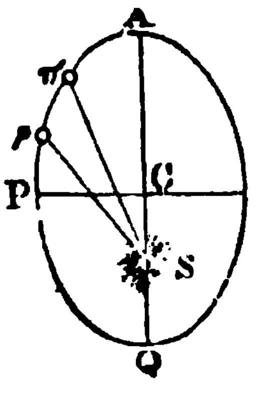
A SHORT VIEW OF THE PRINCIPAL FACTS IN REGARD TO PHYSICAL ASTRONOMY, OR THE SYSTEM OF THE UNIVERSE.

There is no difference of opinion at present among enlightened philosophers, in regard to the position of the planets and of the sun. All those capable of estimating the proofs deduced from astronomy and physics, admit that the sun occupies the centre of an immense space, in which the following planets revolve around him at different distances, viz., Mercury and Venus; the Earth, always accompanied by the moon; Mars; Pallas and Vesta, discovered by Dr. Olbers; Ceres, discovered by M. Piazzi; Juno, by Harding; and Jupiter, followed by his four moons or satellites; Saturn, surrounded by his ring, and accompanied by seven satellites; the Georgian planet, discovered by Dr. Herschel, together with its satellites; and lastly a great number of comets, which have been shewn to be nothing else but planets having orbits very much elongated.

The path in which each of the planets moves around the sun is not a circle, but an ellipsis more or less elongated, in one of the foci of which that luminary is placed; so that when the planet is at the extremity of the axis, beyond the centre, it is at its greatest distance from the sun; and when at the other extremity of that axis, it is at its nearest distance. This ellipsis however is not very much elongated: that described by Mercury is the most of all of the ancient planets; for the distance of its focus from the centre is equal to a fifth part of its semi-axis. That of Venus is nearly a circle. In the orbit of the earth, the distance from the focus to the centre is only about a 57th part of the semi-axis. The last discovered planet, Pallas, it is said has its orbit the most elongated of any, its eccentricity being about one third of its mean distance from the sun.

The motion of all these bodies around the sun is regulated by two celebrated laws, the discovery of which has rendered the name of Kepler immortal. The first of these laws, which relates to the motion of a planet in the different points of its orbit, is, that it always moves in such a manner, that the area described by the radius vector, or the straight line drawn from the planet to the sun, increases uniformly in equal times, or is always proportional to the time; so that if a planet, for example, employs 30 days in moving from A to π (Fig. 14.) and 20 in moving from π to p, the mixtilineal area A S π will be to the mixtilineal area π S p, as 30 to 20; or A S π is to A S p, as 30 to 50, or as 3 to 5. In double the time therefore this area is double, and so on; whence it follows, that when the planet is at its greatest distance, it moves with the least velocity in its orbit. The ancients laboured under a mistake, when they imagined that the retardation which they observed in the motion of any of the heavenly bodies, such as the sun for example, was a mere optical illusion: this retardation is partly real, and partly apparent.

Fig. 14.



The second law, discovered by Kepler, is that which regulates the distances of the planets from the sun, and their periodical times, or the times of their revolutions. According to this law, the cubes of the mean distances of two planets from the sun, around which they perform their revolutions, are always in proportion to each other as the squares of their periodical times; thus, if the mean distances of two planets from the sun, be the one double of the other, since the cubes of these distances will be as 1 to 8, the squares of the periodical times will be as 1 to 8; consequently the times themselves will be to each other as 1 to the square root of 8, which is 2⅚ nearly.

This rule holds good, not only in regard to the principal planets, those which revolve about the sun, but also in regard to the secondary planets, which revolve around a primary planet, as the four satellites of Jupiter and the seven satellites of Saturn. If the earth had two moons, they also would observe this law in regard to each other by a mechanical necessity.

These two laws, first discovered by Kepler, from his observations and those of Tycho Brahe, were afterwards confirmed and proved by Newton, from the principles and laws of motion; so that those who deny truths so well established, must be incapable of feeling the force of a demonstration.

We know of no secondary cause that could have any influence in regulating the distances of the planets from the sun, yet there appears a relation between the distances which is too remarkable to be considered accidental. This was first remarked by Bode of Berlin, who remarked that a planet was wanting to complete the relation; and that want has since been supplied by the discovery of the four new planets at almost the precise distances from the sun which Bode suggested that a planet ought to be. According to Bode (and it is nearly true), the distances of the planets may be expressed as follows:—that of the Earth being 10.

Mercury	2^2	$= 4$
Venus	$2^2 + 3\cdot2^0$	$= 7$
Earth	$2^2 + 3\cdot2^1$	$= 10$
Mars.................	$2^2 + 3\cdot2^2$	$= 16$
New planets	$2^2 + 3\cdot2^3$	$= 28$
Jupiter..............	$2^2 + 3\cdot2^4$	$= 52$
Saturn	$2^2 + 3\cdot2^5$	$= 100$
Georgium Sidus......	$2^2 + 3\cdot2^6$	$= 196$

We shall now lay before our reader every thing most remarkable in regard to those celestial bodies of which we have any knowledge, beginning with the sun. They who can behold this sublime picture without emotion, ought to be classed among those stupid beings whose minds are insensible to the most magnificent works of the Deity.

I.—*Of the Sun.*

The sun, as we have already said, is placed in the centre of our system, as a source of light and heat, to illuminate and vivify all the planets subordinate to it. Without his benign influence the earth would be a mere block, which in hardness would surpass marble and the most compact substances with which we are acquainted; no vegetation, no motion would be possible: in a word, it would be the abode of darkness, inactivity, and death. The first rank therefore among inanimate beings cannot be refused to the sun; and if the error of addressing to a created object that adoration which is due to the Creator alone, could admit of excuse, we might be tempted to excuse the homage paid to the sun by the ancient Persians, as is still the case among the Guebres, their successors, and some savage tribes in America.

The sun is, or seems to be, a globe of fire, the diameter of which is equal almost to 111 times that of the earth, being about 883217 English miles; its surface therefore is 12321 times greater than that of the earth; and its mass 1367631 times. Its distance from the earth, according to the latest observations, is about 95 millions of miles.

This enormous mass is not absolutely at rest: for modern astronomers have found that it revolves round its axis, in 25 days 12 hours. This motion takes place, on an axis inclined to the plane of the ecliptic about 7½°; so that the equator of the sun has the same inclination to the earth's orbit.

This phenomenon was discovered by means of the spots, with which the surface of the sun is covered at certain periods: with the assistance of a telescope, these

spots, which are dark, and generally of a very irregular form, and which often remain some months, may be observed on the disk of this luminary. They were first discovered by Galileo, who thus gave a mortal blow to the opinion of the philosophers of that time, some of whom, treading in the steps of Aristotle, considered the celestial bodies as unalterable. He repeatedly observed, at different periods, large spots on the sun's disk; saw them always approach in the same direction, and almost in a straight line to one of the edges; then disappear, and re-appear afterwards at the other edge; whence he concluded that the sun had a rotary motion about his axis. It is remarked that these spots employ 27 days 12 hours to return to the same point of the disk where they began to be observed; hence it follows that they require 25 days 12 hours, to perform a complete revolution;* and consequently the sun employs that time in revolving about his axis.

It thence follows, also, that a point in the sun's equator moves about four times and a third as fast as a point of the terrestrial equator, during its diurnal motion; for, the circumference of a solar great circle being 111 times as great, these points would move with the same velocity if the period of the sun's revolution were 111 days. But being only 25 days and some hours, it is about four times and a third as rapid.

Astronomers have also had the curiosity to measure the extent of some of these solar spots; and have found that they are sometimes much larger than the whole earth.

In regard to the nature of these spots; some philosophers have conjectured, that they can be nothing else than parts of the nucleus of the sun which remain uncovered, in consequence of the irregular movements of a fluid violently agitated. An English astronomer, Professor Wilson of Glasgow, revived this idea in the Philosophical Transactions for 1773, with this difference, that according to his theory the luminous matter of the sun is not fluid, but of such a consistence that, under particular circumstances, there may be sometimes formed in it considerable excavations, which discover a portion of the nucleus. The sloping sides of these excavations, according to his opinion, form the faculæ, or that border less luminous, without being black, with which these spots are generally surrounded. This theory he endeavours to establish, by examining the phenomena that ought to be exhibited by such excavations, according to the manner in which they might present themselves to an observer.

Other philosophers have supposed these spots to be only clouds of fuliginous vapours, which remain suspended over the surface of the sun, in the same manner as the smoke that rises from Vesuvius at the time of an eruption; and which to an eye placed in the atmosphere would appear to cover a large tract of country. Some also have imagined them to consist of a kind of scum produced by the combustion of heterogeneous matters, which have fallen on the sun's surface. But, in all probability, nothing certain will ever be known on the subject. For considerable periods no large spots are seen on the sun's disk, and sometimes a great many are observed. In 1637 it is said they were so numerous, that both the heat and splendour of that luminary were in some measure diminished by them.† If the opinion of Descartes, respecting the incrustation of the stars, and their conversion into opake planets, had been then known, some apprehensions might have been entertained of seeing the sun, to the great misfortune of the human species, undergo this strange metamorphosis.

We shall here remark that a certain figure of the sun, given on the authority of Kircher, and copied in various maps of the world, ought to be considered merely as

* The reason of this difference is, that while the sun performs a complete revolution on its axis, the earth, moving in its orbit, advances about 25 degrees towards the same side: on which account the spot must still pass over about 25 degrees, before it can be in the same point of view in regard to the earth.

† In September 1830, they were very numerous, and many of them large.

an imaginary production. No observations have ever been made by any astronomer, that can serve as the least foundation for it.

In 1683, Cassini discovered that the sun not only has a proper light of his own, but that he is accompanied by a kind of luminous atmosphere, which extends to an immense distance, since it sometimes reaches the earth. But this atmosphere is not of a form nearly spherical, like that of the earth : it is lenticular, and situated in such a manner that its greatest breadth coincides almost with the prolongation of the solar equator. We indeed often see, during very serene weather, and a little after sunset, a light somewhat inclined to the ecliptic, several degrees broad at the horizon, and decreasing to a point, which rises to the height of 45°. It is principally towards the equinoxes that this phenomenon is observed; and as it has been since seen, and in various places, by a great number of astronomers, these appearances cannot perhaps be accounted for, but by supposing around the sun an atmosphere such as that above mentioned.

It has been observed that this Zodiacal light is most distinct about the first of March, at about 7 o'clock in the evening; but it has been seen in January, and according to M. Toulquier it is always seen at Guadaloupe in fine weather.

Doctor Herschel has two ingenious papers in the Philosophical Transactions for 1795 and 1802, containing many new and curious speculations on the nature and constitution of the sun, his light, &c. Dissatisfied with the old terms used to denote certain appearances on the surface of the sun, Dr. Herschel rejects them ; and instead of the words, spots, nuclei, penumbræ, luculi, &c., he substitutes openings, shallows, ridges, nodules, corrugations, indentations, pores, &c. He imagines that the body of the sun is an opake habitable planet, surrounded and shining by a luminous atmosphere, which being at times intercepted and broken, gives us a view of the sun's body itself, which are the spots, &c. He conceives that the sun has a very extensive atmosphere, consisting of elastic fluids, that are more or less lucid and transparent, and of which the lucid ones furnish us with light. "This atmosphere, he thinks, is not less than 1843, nor more than 2765, miles in height: and he supposes that the density of the luminous solar clouds need not be much more than that of our aurora borealis, in order to produce the effects with which we are acquainted. The sun then, if this hypothesis be admitted, is similar to the other globes of the solar system, with regard to its solidity—its atmosphere—its surface diversified with mountains and valleys—the rotation on its axis—and the fall of heavy bodies on its surface; it therefore appears to be a very eminent, large, and lucid planet, the principal one in our system, disseminating its light and heat to all the bodies with which it is connected.

II.—*Of Mercury.*

Mercury is the smallest of all the ancient planets, and the nearest the sun : its distance from that luminary is about $\frac{37}{95}$ of that of the earth : Mercury therefore revolves about the sun at the distance of about 37 millions of miles. On account of this position, it is never more than 28° 20′ from the sun, and on this account it is very difficult to be seen. When at about its greatest elongation from the sun it appears through a good telescope as a crescent like the moon towards her quadratures.

It has not yet been ascertained from any observations whether Mercury has a motion round its axis, which however is very probably the case.

This planet completes its revolution round the sun in 87 days 23 hours 15 minutes; and its diameter is to that of the earth as 2 to 5; so that its bulk is to that of the earth as 8 to 125.

The distance of Mercury from the sun being no more than $\frac{37}{95}$ of that of the earth; and as heat increases in the inverse ratio of the squares of the distance; it thence

follows that, *cæteris paribus*, it is nearly seven times as hot in that planet as on our earth. This heat even far exceeds that of boiling water. If Mercury therefore has the same conformation as our earth, and is inhabited, the beings by which it is peopled must be of a nature very different from those of the latter. In this there is nothing repugnant to reason; for who will dare to confine the power of the Deity to beings almost similar to those with which we are acquainted on the earth? We shall shew hereafter that the conformation of the surface of Mercury, and the nature of the circumambient fluid, may be such as to make it not impossible for such beings as ourselves to exist in it.

III.—*Of Venus.*

Venus is the most brilliant of all the planets in the heavens. This planet, as is well known, sometimes precedes the sun; and on that account is called *Lucifer*, or the morning star: sometimes it follows him, appearing the first after he is set; and on that account is distinguished also by the name of *Vesper*, or the evening star.

This planet revolves about the sun at a distance from him, which is to that of the earth from the sun, as 68 to 95; consequently its distance from the sun is about 68 millions of miles: its greatest elongation from the sun, in regard to us, is about 48°, and it exhibits the same phases as the moon.

The revolution of Venus around the sun is performed in 224 days 16 hours 49 minutes: its diameter, according to the latest and most correct observations, is nearly the same as that of the earth, and consequently it is of equal bulk also. Changeable spots have been discovered on the surface of Venus, which serve to prove the revolution of that planet about its axis; but the period of this revolution is not yet fully ascertained. M. Bianchini makes it to be 24 days, and M. Cassini 23 hours 20 minutes. For our part we are inclined to adopt the latter opinion; but unfortunately these spots, seen by Maraldi and Cassini, are no longer visible, even with the help of the best telescopes, at least in Europe: at present not a single spot can be observed in this planet; and therefore the questiou must remain undetermined till new ones are seen.

Venus may sometimes pass between the earth and the sun, in such a manner as to be seen on the disk of the latter, where it appears as a black spot, of about a minute apparent diameter. It was seen for the first time passing over the sun's disk in Nov. 1631; it was again observed under the like circumstances on the 6th of June, 1761; and the same observation was made on the 3d of June, 1769. It will not be again seen passing over the sun's disk, till the 9th of December, 1874. In the observation of this phenomenon, all the states of Europe interested themselves, as the foundation of the best method of finding the sun's parallax, from which his distance from the earth may be computed, and thence his distance from any other planet whose time of revolution is known.

IV.—*Of the Earth.*

The Earth, which we inhabit, is the third in the order of the planets hitherto known. Its orbit, the semi-diameter of which is about 95 millions of miles, comprehends within it those of Venus and Mercury. It performs its revolution about the sun in 365 days 6 hours 11 minutes; for it is necessary that a distinction should be made between the real and complete revolution of the earth, and the tropical revolution or what is called the solar year. The latter cousists of 365 days 5 hours 49 minutes; because it represents only the time which the sun employs in returning to the same point of the equinoctial; but as the equinoctial points go back every year 50″, which makes the stars seem to advance the same quantity, in the same period; when the earth has returned to the point of the vernal equinox, it must still pass over 50″ before it can attain to the point of the fixed sphere, where the equinox was the

preceding year. But as it employs for this purpose about 20 minutes, these added to the tropical year will give, as the time of the complete revolution, from a point of the fixed sphere to the same point again, 365 days 6 hours 11 minutes, as mentioned above.

During a revolution of this kind, the earth, in consequence of the laws of motion, always maintains its axis parallel to itself; and it performs its revolution around this axis, with respect to the fixed stars, in 23 hours 56 minutes; for it is in regard to the fixed stars that this revolution ought to be measured, and not in regard to the sun, which has apparently advanced in the same direction about a degree per day. This parallelism of the earth's axis produces the variation of the seasons; as it exposes sometimes the northern and sometimes the southern part to the direct influence of the sun's rays.

This parallelism however is not absolutely invariable. In consequence of certain physical causes, it has a small motion, by which it deviates from it, at each revolution, about 50 seconds; as if it had a conical motion, exceedingly slow, around the moveable and supposed axis of the ecliptic. On account of this motion, the apparent pole of the world, among the fixed stars, is not fixed; but revolves about the pole of the ecliptic, and approaches certain stars, while it recedes from others. The polar star has not always been the nearest the arctic pole; nor is it yet at its greatest degree of proximity: it will attain to this situation about the year 2100 of our æra, and its distance from the pole at that period will be 28′ or 29; the arctic pole will then recede more and more from it, so that in the course of ages there will be another polar star, and even others after that in succession.

The axis of the earth is inclined to the plane of the ecliptic, at present, in an angle of nearly 23° 28, which causes the inclination of the ecliptic to the equator, and produces the different changes of the seasons. This inclination is also variable, and, according to modern observations, decreases about a minute every century: the ecliptic therefore slowly approaches towards the equator, or rather the equator towards the ecliptic; and if this motion takes place with the same velocity, and in the same direction, the equator will coincide with the ecliptic in about 140,000 years; and then a perpetual spring, as well as an equality of the days and nights, will prevail all over the earth.

But it has been shewn by Laplace, that all variable planetary phenomena are periodical, and restricted with respect to their amounts within certain and comparatively narrow limits; one of which being attained, they recede again towards the opposite one; but the times of periodical variation are many of them of great extent.

V.—*Of the Moon.*

Of all the celestial bodies which surround us, and by which we are illuminated, the most interesting, next to the sun, is the moon. Being the faithful companion of our globe during its immense revolution, she often supplies the place of the sun, and by her faint light consoles us for the loss we sustain when the rays of that luminary are withdrawn. It is the moon which raising, twice every day, the waters of the ocean, produces in them that reciprocal motion, known under the name of the flux and reflux; a motion which is perhaps necessary in the economy of the globe.

The mean distance of the moon from the earth is about $60\frac{1}{4}$ semi-diameters of the latter, or 240,000 miles. Her diameter is in proportion to that of the earth, as 20 to 73, or nearly as to 3 to 11; so that her mass, or rather bulk, is to that of the earth, nearly as 1 to $48\frac{3}{4}$.

The moon is an opake body; but we do not think it necessary to adduce here any proof of this assertion. She is not a polished body, like a mirror; for if that were the case, it would scarcely transmit to us any light, as a convex mirror disperses the rays in such a manner that an eye, at any considerable distance, sees only one point

on the surface illuminated; whereas the moon transmits to us from her whole disk a light sensibly uniform.

To this we may add, that observation shews in the body of the moon asperities still greater, considering her magnitude, than those with which the earth is covered. If the moon indeed be attentively viewed, some days after her conjunction, the boundary of the shaded part will be seen as it were indented; which can arise only from the effect of its inequalities. Besides, at a little distance from that boundary, in the part not yet illuminated, there are observed luminous points, which, increasing gradually as the luminous part approaches them, are at length confounded with it, and form the indentations above mentioned: in short, the shadows of those parts, when they are entirely illuminated, are seen to project themselves to a greater or less distance, and to change their position, according as they are illuminated on the one side or the other, and in a direction more or less oblique. It is in this manner that the summits of the mountains on our earth are illuminated, while the neighbouring valleys and plains are still in obscurity; and that their shadows are projected to a greater or less distance, on the right or the left, according to the elevation and position of the sun. Galileo, the author of this discovery, measured the height of one of these lunar mountains geometrically; and found it to be about 3 leagues, which is nearly double the height of the most elevated peaks of the highest mountains known on the earth. But later astronomers, by more accurate measurements, have found that few of the lunar mountains rise above a mile in height, and that the majority do not reach above half that height.

The best time to observe the shadows is about half moon, when the separation of light from the dark part is a straight line; the shadows are then seen of their full extent, not being foreshortened.

We have already spoken of the names given by astronomers to these spots, and of their use in astronomy. We shall therefore not repeat them here, but proceed to something more interesting. On the surface of the moon there are spots of different kinds, some luminous, and others in some measure obscure. It was long considered as fully established that the most luminous parts were land, and the obscure parts sea; for it was said, as water absorbs a part of the light, it must transmit a weaker splendour than the land, which reflects it very strongly. But this reasoning is not well founded; for if these spots, which are obscure in regard to the rest of the moon, consisted of water, when illuminated obliquely, as they are in respect to us during the first days after the conjunction, they ought to transmit to us a very lively light; as a mirror which seems black to those not placed in the point to which it reflects the solar rays, appears on the other hand exceedingly bright to an eye situated in that point.

Others have hence been induced to believe that these obscure parts are immense forests; and this indeed may be more probable. We have no doubt that if the vast forests still in Europe, and those of America, were seen at a great distance, they would appear darker than the rest of the earth's surface.*

But is this observation sufficient to make us conclude that these spots are really forests? We do not think it is; and the reasons are as follow:

It is in a manner proved, that the moon has no atmosphere; for, if she had, it would produce the same effects as ours. A star, on the moon approaching it, would change its colour: and its rays, broken by that atmosphere, would give it a very irregular motion, even at a considerable distance from the moon. But nothing of this kind is observed. A star covered by the dark edge of the moon suddenly disappears,

* About the eighth day of the moon's age, towards the upper part of the moon, and not far from the line which divides the light from the dark part, is seen a straight and deep excavation running through an extensive range of hills. If we were to meet with such an excavation on the globe we inhabit, we should certainly consider it a monument of ancient art.

without changing its colour, or experiencing any sensible refraction. Some astronomers indeed have imagined that they saw lightning in the moon, during total eclipses of the sun; but this no doubt was an illusion, owing to their eyes being fatigued by looking too attentively at the sun. Besides, if there were clouds and vapours in the moon, they would sometimes be seen to conceal certain known parts of her surface; as an observer placed in the moon would certainly see certain pretty large portions of the earth, such as whole provinces, concealed sometimes for days, and even weeks, by those clouds, which frequently cover them, during as long a period. M. de la Hire has shewn that an extent as large as Paris would be perceptible to an observer in the moon, if viewed through a telescope which magnified objects about 100 times.

But if there be no dense atmosphere, no elevation of vapours, on the surface of the moon, it is difficult to conceive how there can be any kind of vegetation in it; and if this be the case, it can produce neither plants, trees, nor forests, and consequently no animals. It is therefore probable that the moon is not inhabited; besides, if it were inhabited by animals nearly similar to man, or endowed with some kind of reason, it is hardly to be supposed that they would not make some changes in the surface of that globe. But since the invention of the telescope, to the present time, no alteration has been observed in its surface.

The moon always presents to the earth very nearly the same face; and therefore she must have a rotary motion about an axis, nearly perpendicular to the ecliptic, the duration of which forms the lunar month; or in one of its hemispheres there must be some cause, which makes it incline towards the earth. The latter conjecture is more probable; for why should this revolution of the moon around its axis be performed exactly in the period of its rotation about the earth. However, as the moon always presents the same face to the earth, it thence follows, that her whole surface is illuminated by the sun, in the course of a lunar month; the days therefore in the moon are equal to about 15 of ours, and the nights of the same duration.

But if we suppose, notwithstanding what has been said, that there are inhabitants in the moon, they will enjoy a very singular spectacle: an observer placed towards the middle of the lunar disk, for example, will always see the earth motionless towards his zenith, or having only a motion of nutation, in consequence of reasons which we shall explain hereafter. In a word, each inhabitant of that hemisphere will always see the earth in the same point of the horizon; while the sun will appear to perform his revolution in a month. On the contrary, the inhabitants of the other hemisphere will never see the earth; and if there are astronomers in it, some of them no doubt will undertake a voyage to the hemisphere which is turned towards us, for the purpose of observing this sort of motionless moon, suspended in the heavens like a lamp, and the more remarkable as it must appear to the lunar inhabitants of a diameter four times as large as that of the moon appears to us; with a great variety of spots performing their revolutions in the interval of 24 hours: for there can be no doubt that our earth, intersected by vast seas, large continents, and immense forests, such as those of America, must exhibit to the moon a disk variegated with a great many spots, more or less luminous.

We have said that the moon always presents the same disk to the earth; but strictly speaking this is not exactly the case; for it has been found that the moon has a certain motion, called libration, in consequence of which the parts nearest the edge alternately approach to or recede from that edge by a kind of vibration. Two kinds of libration are in particular distinguished; one called a libration in latitude, by which the parts near the austral or the boreal poles of the moon, seem to vibrate from north to south, and from south to north, through an arc which may comprehend about 5 degrees. This however is a mere optical effect, produced by the parallelism of the moon's axis of rotation, which is inclined $2\frac{1}{2}$ degrees to the ecliptic.

The other libration is that in longitude; which takes place around the above axis, at an angle of nearly 7¼ degrees; and as both are combined, it needs excite no wonder that this phenomenon should have long been an object of research to philosophers. However, it is evident that the inhabitants of the moon, if there really be any, who are situated near the edge of the disc turned towards the earth, must see our globe alternately rise and set, describing an arc of only a few degrees.

The moon is a little depressed at the poles in consequence of her rotation, but because she presents always the same face to the earth she must be elongated in the direction of that axis which points towards the earth. According to the theory this excess ought to be about the 3000th part of the least axis. It is remarkable that the satellites of Jupiter also present always the same face to the planet, and there can be no doubt that it is a general consequence of the combined laws of gravitation and revolution.

The planes of the earth's equator and the moon's orbit, and the plane drawn through the moon's centre parallel to the ecliptic, have always very nearly the same intersection.

VI.—*Of Mars.*

Mars, which may be easily distinguished by its reddish splendour, is the fourth in the order of the primary planets. Its orbit incloses that of Mercury, Venus, and the earth; consequently the motions of these planets must exhibit to the inhabitants of Mars the same phenomena, as are presented by Mercury and Venus to the inhabitants of our globe.

The revolution of Mars around the sun is performed in 686 days 23 hours 30 minutes, or nearly two years. Its mean distance from the sun is more than 1½ that of the earth, or about 144 millions of miles.

Spots are observed sometimes on the disc of Mars, by which it is proved that it revolves on an axis almost perpendicular to its orbit; and that this revolution is completed in 24 hours 39 minutes. The days therefore, to the inhabitants of Mars, if there are any, must be nearly equal to ours; and the days and nights in this planet must be of the same length, since its equator coincides with its orbit.

Mars has a very dense atmosphere, and when either of the poles emerges from darkness into the rays of the sun, it is found to be decidedly brighter than the other parts of the surface; and the extent of the bright spot gradually becomes less during the time that the pole remains in the light. This appearance has been conceived to be caused by snow deposited during the polar winter, and the gradual melting of it during the summer. The diameter of Mars is about 4100 miles.

VII.—*Of Jupiter.*

The next planet to Mars, of the ancient ones, is Jupiter. Its distance from the sun is above 5 times that of the earth, being 490 millions of miles. The period of its revolution around the sun is 11 years 317 days 12 hours 20 minutes. Its diameter, compared with that of the earth, is as 11 to 1; so that its bulk is 1331 times as great as that of our globe.

This bulk does not prevent Jupiter from revolving around his axis with much more rapidity than our earth. The spots observed on the disc of this planet have indeed shewn that this revolution is performed in 9h. 56m.; so that it is more than twice as quick, and as any point in the equator of Jupiter is eleven times as far distant from the axis as a point of the earth's equator is from the terrestrial axis, it thence follows that this point in Jupiter moves with a velocity about twenty-four times as great.

It has therefore been observed that the body of Jupiter is not perfectly spherical: it is an oblate spheroid, flattened at the poles, and the diameter of its equator is to

that passing from the one pole to the other, according to the latest observations made with the most perfect instruments, as 14 to 13.

On the four new Planets, commonly, from their smallness, called Astroids.

These planets CERES, PALLAS, JUNO, and VESTA, are situated between the orbits of Mars and Jupiter, and are nearly at the same distance from the sun.

Ceres was discovered by Piazzi, on January 1, 1801. It is of a ruddy though not very deep colour, and, being surrounded by an extensive and dense atmosphere, it exhibits a distinct disc when viewed with a magnifying power of 200. Like the atmosphere of the earth, that of Ceres is very dense near the planet, and becomes rarer at a greater distance. As this planet approaches the earth its apparent size increases much more rapidly than it ought to do from the diminution of the distance; an effect which, Schroeter observes, arises from the finer exterior strata becoming visible as it approaches.

It performs its revolution round the sun in about four years seven months and ten days. Its mean distance from the sun is about 2·669 times that of the earth; and its diameter has been variously estimated at from 163 to 1630 miles.

Pallas was discovered March 28th, 1802, by Dr. Olben, of Bremen. It is nearly of the same apparent magnitude as Ceres, but of a less ruddy colour. It is surrounded with a nebulosity, but of less extent than that of Ceres. But Pallas is distinguished from all other planets by the great inclination of its orbit to the plane of the ecliptic. While the other planets revolve in orbits which are nearly circular, and deviating only a few degrees from the plane of the ecliptic, that of Pallas is inclined to this plane about 35 degrees; and while the mean distance of this planet from the sun is nearly the same as that of Ceres, from the greater eccentricity of the orbit of Pallas, the orbits of these two planets intersect each other, a phenomenon quite anomalous in the solar system. The diameter of this planet, like that of Ceres, has not been satisfactorily determined: it has been estimated at from 80 to upwards of 2000 miles.

Juno was discovered by Mr. Harding, at Lilienthal, on September 1st, 1804. It is of a reddish colour, and is free from the nebulosity which surrounds Ceres and Pallas. It is probably the smallest of the new planets; but it is distinguished by the great eccentricity of its orbit, its greatest distance from the sun being double its least distance. Its time of revolution is about 4 years and 123 days; and its mean apparent diameter, as seen from the earth, is, according to Schroeter, 3·057."

Vesta.—From the regularity observed in the distances of the old planets from the sun, some astronomers supposed that a planet existed between the orbits of Mars and Jupiter. The discovery of Ceres seemed to confirm this conjecture, which, however, was overturned by the discovery of Pallas and Juno. Dr. Olben, however, imagined that these small celestial bodies might be the fragments of a large planet, which had been burst by some internal convulsion, and that several more might be discovered between the orbits of Mars and Jupiter. And he conceived that if they originated in this manner, though their orbits might be differently inclined to the ecliptic, yet as they must all have diverged from the same point, they must have two common points of reunion in opposite regions of the heavens. These nodes he found, from observation on the planets that had been discovered, to be in Virgo and the Whale. With the hope therefore of detecting other fragments of the supposed planet, Dr. Olben examined thrice every year all the little stars in these two opposite constellations, and he discovered Juno in the Whale; and on March 29th, 1807, he discovered Vesta in Virgo.

The appearance of this planet is similar to that of a star of the 5th or 6th magnitude, and it may be seen on a clear night by the naked eye. It is not surrounded by any nebulosity; and, with a power of 636, Dr. Herschel saw no appearance of a

planetary disc. Its orbit intersects that of Pallas, but not in the same place where it is cut by Ceres. Its time of revolution is about 3 years and 66 days.

Sir David Brewster has some ingenious speculations on the origin of these small planets, and he is decidedly of opinion that the phenomena which they exhibit lead to the conclusion that they are the dispersed fragments of a single planet. See his edition of Ferguson's Astronomy, and the article Astronomy in the Edinburgh Encyclopoedia.

The axis of Jupiter is almost perpendicular to the plane of its orbit; for its inclination is only 3 degrees: the days and nights therefore in this planet must be nearly equal at all seasons.

The surface of Jupiter is for the most part interspersed with spots, in the form of bands; some of them obscure, and others luminous; at certain periods they are scarcely visible; nor are uniformly marked throughout their whole extent; so that they are, as it were, interrupted: their number also varies, and they can be very well seen by the assistance of a telescope magnifying 50 times, but best when Jupiter is at his least distance from the earth. The year 1773 was exceedingly favourable for these observations; because Jupiter was then as near to the orbit of the earth as possible.

The distance of Jupiter from the sun being above 5 times that of the earth, it is evident that the sun's diameter must appear five times less, or about 6 minutes only; consequently the splendour of the sun at Jupiter will be 25 times less than it is to the earth. But a light 25 times less than that of the sun is still pretty strong, and more than sufficient to produce a very clear day: the inhabitants therefore of Jupiter, for it is probable that there are some in this planet, will have no great cause to complain.

But if they are treated less favourably in this respect than the inhabitants of the earth, they possess advantages in others; for while the earth has only one moon, to make up for the absence of the sun, Jupiter has four. These moons, or satellites, were first discovered by Galileo; and they enabled him to reply to those who objected, in opposition to the earth's motion, the impossibility of conceiving how the moon could accompany the earth during its revolution; Galileo's discovery reduced them to silence.

The satellites of Jupiter revolve around him in the periods, and at the distances indicated in the following table.

Order of the Satellites.	Dist. in semi-diameters of Jupiter.	Periodical Times.		
		D.	H.	M.
I.	6·048	1	18	28
II.	9·623	3	13	14
III.	15 350	7	3	43
IV.	27·998	16	16	32

The inhabitants of Jupiter then, in this respect, enjoy much greater advantages than those of the earth; for having four moons, some of them must be always above the horizon which is not illuminated by the sun; they will even sometimes see the whole four, one as a crescent, another full, and a third half full; they will see them eclipsed, as we see the moon deprived of her light from time to time, when she enters the shadow projected by the earth, but with this difference, that, being much nearer to Jupiter, considering his bulk, they cannot pass behind him, in regard to the sun, without suffering an eclipse.

When the brilliancy of the satellites of Jupiter is examined at different times, it is found to undergo considerable change. By comparing the mutual positions of the satellites with the times when they attain the maximums of light, Sir Wm. Herschel concluded, that, like our moon they all turned round their axes in the same time that they performed their revolution round Jupiter.

From the theory of reciprocal attraction, La Place discovered two remarkable theorems concerning the motions of these satellites.

First. The mean motion of the first satellite added to twice the mean motion of the third, is rigorously equal to thrice the mean motion of the second satellite.

Second. The mean longitude of the first satellite *minus* three times that of the second, *plus* twice that of the third, is exactly equal to a semi-circle or 180 degrees.

By following out these laws, we find, 1st. When the second and third satellites of Jupiter are simultaneously eclipsed, the first is always in conjunction with Jupiter. 2d. When the first and third satellites are simultaneously eclipsed, the difference between either of their longitudes and that of the second is 60°. 3d. When the first and second are simultaneously eclipsed, the difference between either of their longitudes and that of the third is 90°.

It is obvious from these results, that a wonderful provision is made in the system of Jupiter to secure to the planet the benefit of his satellites; for *it is impossible for the planet to be deprived of the light of all the satellites at the same time.*

Astronomers, however, not contented with establishing the existence of these moons attached to Jupiter, have done more; for they have calculated their eclipses with as much correctness, at least, as those of our moon. The Nautical Almanac, and other astronomical Ephemerides, exhibit for each day of the month, the aspects of the satellites of Jupiter, and announce the hour, minute, and second at which their eclipses will commence, and whether they will be visible or not on the horizon of the place; they give also the time when any of these satellites will be hid behind the disc of Jupiter, or disappear by passing before it. These predictions are not matters of mere curiosity, since they are of great utility in determining the longitude.

VIII.—*Of Saturn.*

Saturn, which is still farther from the sun than Jupiter, exhibits a most singular spectacle, on account of his seven moons, and the ring by which he is surrounded. He performs his revolution around the sun in 29 years 174 days 6 hours 36 minutes; and his mean distance from that luminary is about 9½ times as great as that of the earth, or 900 millions of miles.

At such an immense distance the apparent diameter of the sun, to a spectator in Saturn, is no more than $\frac{1}{10}$ of what is to us; and its light as well as heat must be 90 times less. An inhabitant of Saturn transported to Lapland, or even to the polar regions, covered with perpetual ice, would experience there an insupportable heat; and would no doubt perish sooner than a man immersed in boiling water; while an inhabitant of Mercury would freeze in the most scorching climates of our torrid zone.

By noticing carefully the changes of certain dark spots on the disc of Saturn, Sir Wm. Herschel found that he revolved on his axis in 10h. 16m., and that the axis of rotation is perpendicular to the plane of the ring.

Nature seems to have been desirous to indemnify Saturn for his great distance from the sun, by giving him seven moons, which are called his satellites. Their distances from the centre of Saturn, in semi-diameters of that planet, and the periods of their revolution, are as expressed in the following table.

Satellites.	Distances.	Revolutions.		
		D.	H.	M.
I.	5·284	1	21	18
II.	6·819	2	17	41
III.	9·524	4	12	25
IV.	22·081	15	22	41
V.	64·359	79	7	48
VI.	4·300	1	8	53
VII.	3·351	0	22	40

Of these satellites, five were discovered by Cassini and Huygens, before the year 1685; and it was imagined there were no more, till two were discovered by Dr. Herschel in 1787 and 1788. These are nearer to Saturn than any of the other five; but to prevent confusion in the numbers, with regard to former observations, they are called the 6th and 7th satellites.

The inclination of the first four satellites to the ecliptic, is from 30 to 31 degrees. The fifth describes an orbit inclined in an angle of from 17 to 18 degrees to the orbit of Saturn. Dr. Herschel observes that this satellite turns once round its axis exactly in the time in which it revolves about Saturn; and in this respect it resembles our moon.

We shall not here enlarge on the advantages which this planet must derive from so many moons; what we have said in regard to Jupiter is applicable in a greater degree to Saturn also.

But something still more singular than these seven moons, is the ring by which Saturn is surrounded. Let the reader conceive a globe placed in the middle of a flat thin circular body, with a concentric vacuity: and that the eye is placed at the extremity of a line oblique to the plane of this circular ring. Such is the aspect exhibited by Saturn, when viewed through an excellent telescope; and such is the position of a spectator on the earth. The diameter of Saturn is to that of the vacuity of the ring, as 3 to 5; and the breadth of the ring is nearly equal to the interval between the ring and Saturn. It is fully proved that this interval is a vacuity; for a fixed star has been seen between the ring and the body of the planet; this ring therefore maintains itself around Saturn as a bridge would do concentric to the earth, and having every where a uniform gravity.

What is called *the ring* of Saturn certainly consists of at least *two* rings, concentric with the planet and each other, both lying in one plane, and separated from each other by a very narrow interval. It has been surmised by some observers that the outer ring consists of several narrow ones. In the fourth volume of the Memoirs of the Royal Astronomical Society, there is a very interesting account by Captain Kater, of some observations which he and some other persons made on the ring; and the conclusion which all the observers came to was, that the outer ring was certainly composed of several separate ones.

Any tolerable telescope will shew the division of the ring which divides the outer from the inner part; and Professor Struve has given the following dimensions of the planet and the rings from micrometrical measurements with the magnificent Fraunhofer achromatic telescope, at the Dorpat observatory.

	Miles.
Exterior diameter of exterior ring	176418
Interior ditto ditto	155272
Exterior ditto interior ditto	151690
Interior ditto ditto	117339
Equatorial diameter of the planet	79160
Interval between the planet and interior ring	19090
Interval between the rings	1791
Thickness of the ring, not exceeding	100

(See Mem. Astron. Soc., vol. 3.)

That the ring is an opake substance is shewn by its casting a deep shadow on the body of the planet, on the side next the sun; and receiving the shadow of the planet on the other side. The time of revolution has been found to be 10h. 29m. 17s.

Recent micrometrical measurements seem to indicate that the rings, though nearly, are not *accurately*, concentric with the planet. And it has been stated that this want of perfect concentricity is essential to the stability of the system of which the rings

form so conspicuous a part. Sir J. Herschel, in his Astronomy, has the following striking remarks on the phenomena of Saturn:

"The rings of Saturn must present a magnificent spectacle from those regions of the planet which lie above their enlightened sides, as vast arches spanning the sky from horizon to horizon, and holding an invariable situation among the stars. On the other hand, in the regions beneath the dark side, a solar eclipse of 15 years' duration must afford (to our ideas) an inhospitable asylum to animated beings, ill compensated by the faint light of the satellites. But we shall do wrong to judge of the fitness or unfitness of their condition from what we see around us, when perhaps the very combinations which convey to our minds images of horror, may be in reality theatres of the most striking and glorious displays of beneficent contrivance."

This body, of a conformation so singular, is alternately illuminated on each side by the sun; for it makes, with the plane of Saturn's orbit, an invariable angle of about 31° 20′; always remaining parallel to itself; in consequence of which it presents to the sun, sometimes the one face, and sometimes the opposite one; the inhabitants, therefore, of the two hemispheres of Saturn enjoy the benefit of it alternately.

Saturn is seen sometimes from the earth without his ring; but this phenomenon may be easily explained.

Saturn's ring may disappear in consequence of three causes. 1st. It disappears when the continuation of its plane passes through the earth and the sun; for in that case its surface is in the shade, or too weakly illuminated by the sun to be visible at so great a distance; and its edge is too thin, even though illuminated, to be seen from the earth. This phenomenon is observed when Saturn's place is about 19° 45′ of Virgo and Pisces.

2d. The ring of Saturn must disappear also, when the continuation of its plane passes between the earth and the sun; for the flat part of the ring, which is then turned towards the earth, is not that illuminated by the sun. It cannot therefore be seen from the earth; but its shadow may be seen projected on the disk of Saturn.

The nature of this singular ring affords much matter for conjecture. Some have supposed that it may be a multitude of moons, all circulating so near each other, that the distance between them is not perceptible from the earth, which gives them the appearance of one continued body. But this is very improbable.

Others have imagined that it is the tail of a comet, which, passing very near Saturn, has been stopped by it. But such an arrangement of a circulating fluid would be something very extraordinary. In our opinion, while we admire this work of the sovereign Artist, the Creator of the universe, we must suspend our conjectures respecting the nature of it, till a farther improvement in telescopes shall enable us to obtain new facts to support them.

The distance of Saturn from the sun is so great, that, if it be inhabited by intelligent beings, it is very doubtful whether they have any knowledge of our existence, and much less of that of Mercury and Venus; for in regard to them, Mercury will never be farther from the sun than 2° 25′, Venus than 4° 15′, and the Earth than 6°; Mars will be distant from the sun only about 9°, and Jupiter 28° 40′: it will therefore be much more difficult for the Saturnians to see the first three or four of these planets, than it is for us to observe Mercury; which can scarcely ever be seen, as it is almost always concealed among the rays of the sun.

It is however true that the light of the sun is, on the other hand, very weak; and that the constitution of Saturn's atmosphere, if it has one, may be of such a nature, that these planets are visible as soon as the sun has set.

IX.—*Of the Georgian Planet.*

It was long supposed that Saturn was the remotest planet in our system; but from inequalities in the motions of Jupiter and Saturn, astronomers at length began to suspect that another more distant planet existed; and this conjecture was confirmed in 1781, when Dr. Herschel discovered a new planet, which he called the *Georgium Sidus*, in honour of the then reigning king of England, George III. The French call it Herschel, in honour of the discoverer; and Professor Bode, of Berlin, gave it the name of Uranus, who was the father of Saturn, as Saturn was of Jupiter. An interesting history of the discovery was presented to the Academy of Sciences at Brussels, in May 1785, by Baron von Zach of Gotha, and is inserted in the first volume of the Memoirs of that Academy.

The distance of this planet from the sun is immense; being about 1800 millions of miles, which is double that of Saturn. It performs its annual revolution in 83 years 150 days and 18 hours of our time; and its motion in its orbit must consequently be above 7000 miles an hour. To a good eye, unassisted by a telescope, it appears like a faint star of the fifth magnitude; and it cannot readily be distinguished from a fixed star with a less magnifying power than 200. Its apparent diameter, to an observer on the earth, subtends an angle of no more than 4 seconds; but its real diameter is about 35000 miles, and therefore it must be about 80 times as big as the earth. Hence we may infer, as the earth cannot be seen under an angle of quite a second by the inhabitants of the Georgian planet, that it has never yet been discovered by them, unless their eyes and instruments are considerably better than ours. The orbit of this planet is inclined to the ecliptic at an angle of 46 minutes 26 seconds; but as no spots have been discovered on its surface, the position of its axis, and the length of its day and night, are not known.

On account of the immense distance of the Georgian planet from the sun, it was highly probable that it was accompanied with several satellites or moons; and the high powers of Dr. Herschel's telescopes indeed enabled him to discover six; but there may be some others which he has not yet seen. The first and nearest the planet, revolves at the distance from it of 12¼ of its semi-diameters; and performs its revolution in 5 days 21 hours 25 minutes; the second revolves at the distance from the primary of 16½ of its semi-diameters, and completes its revolution in 18 days 17 hours 1 minute: the third, at the distance of 19 semi-diameters, in 10 days 23 hours 4 minutes; the fourth, at 22 semi-diameters, in 13 days 11 hours 5 minutes; the fifth, at 44 semi-diameters, in 38 days 1 hour 49 minutes; and the sixth, at 88 semi-diameters, in 107 days 16 hours 40 minutes. It is remarkable that the orbits of these satellites are almost all at right angles, to the plane of the ecliptic; and that the motion of every one of them, in their own orbits, is retrograde, or contrary to that of all the other known planets.

X.—*Of Comets.*

Comets are not now considered, as they were formerly, to be signs of celestial vengeance; the forerunners of war, famine, or pestilence. Mankind in those ages must have been exceedingly credulous to imagine that scourges confined to a very small portion of the globe, which itself is but a point in the universe, should be announced by a derangement of the natural and immutable order of the heavens. Neither are comets, as supposed by the greater part of the ancient philosophers, and those who trod in their footsteps, meteors accumulated in the middle of the air. Astronomical observations made at the same time, in different parts of the earth, have shewn that they are always at a distance much greater than that even of the moon; and consequently that they have nothing in common with the meteors formed in our atmosphere.

The opinions entertained by some ancient philosophers, such as Apollonius the Myndian, and particularly Seneca, have been since confirmed. According to these philosophers, comets are as old and as durable as the planets themselves; their revolutions are regulated in the same manner; and if they are seldom seen, it is because they perform their course in such a manner, that in a part of their orbits they are so far distant from the earth as to become invisible; so that they never appear but when in the lower part of them.

Newton and Halley, who pursued the same path, have proved by the observations of different comets, which appeared in their time, that they describe elliptical orbits around the sun, which is placed in one of the foci; and that the only difference between these orbits and those of the planets is, that the orbits of the latter are nearly circular, whereas those of comets are very much elongated; in consequence of which, during a part of their course, they approach near enough to our earth to become visible; but during the rest they recede so far from us, as to be lost in the immensity of space. These two philosophers have taught us also, by the help of a small number of observations, made in regard to the motion of a comet, how to determine the distance at which it has passed, or will pass, the sun; as well as the period when it is at its least distance, and its place in the heavens for any given time. Calculations made according to these principles agree in a surprising manner with observations.

The modern philosophers have even done more: they have determined the periods of the return of some of these comets. The celebrated Dr. Halley, considering that comets, if they move in ellipses, ought to have periodical revolutions, because these curves return into themselves, examined with great care the observations of three comets, which appeared in 1531 and 1532, 1607, and 1682; and having calculated the position and dimensions of their orbits, found them to be nearly the same, and consequently that these comets were only one, the revolution of which was completed in about 75 years: he therefore ventured to predict that this comet would re-appear in 1758 or 1759 at latest. It is well known that this prediction was verified at the time announced; hence it is certain that this comet has a periodical revolution around the sun, in 75 years and a half.* According to the dimensions of its orbit, determined by observations, its least distance from the sun is [illegible] of the semi-diameter of the earth's orbit; it afterwards recedes to a distance which is equal to $35\frac{1}{2}$ of these semi-diameters; so that its greatest elongation from the sun is about four times as great as that of Saturn. The inclination of its orbit to the ecliptic, is 17° 40′, in a line proceeding from 23° 45′ of Taurus to 23° 45′ of Scorpio.

There are still two comets, the return of which is expected with some sort of foundation; viz., that of 1556, expected in 1848; and that of 1680 and 1681, which it is supposed, though with less confidence, will re-appear about 2256. The latter, by the circumstances which attended its apparition, seems to be the same as that seen, according to history, 44 years before the Christian æra, also in 531, and in 1106; for between all these periods there is an interval of 575 years. There is reason therefore to suppose that this comet has an orbit exceedingly elongated, and that it recedes from the sun about 135 times the distance of the earth.

What is very remarkable also in this comet is, that in the lower part of its orbit it passed very near the sun; that is, at a distance from its surface which scarcely exceeded a sixth part of the solar diameter; hence Newton concludes, that at the time of its passage it was exposed to a heat 2000 times greater than that of red-hot

* Its re-appearance in 1835, in the very place in which observers had been instructed to look for it, is one of the greatest triumphs of modern science. A very interesting series of observations on this comet were made at the Cape of Good Hope, by Sir J. Herschel, and Mr. Maclear, in 1836. There is a very elaborate paper on this comet published as a supplement to the Nautical Almanac for 1839.

iron. This body therefore must be exceedingly compact, to be able to resist so prodigious a heat, which there is reason to think would volatilize all the terrestrial bodies with which we are acquainted.

Within these few years two small comets have been discovered, which revolve round the sun in comparatively short periods. The first, which is called Encke's comet (from the name of the astronomer who first predicted its return), completes its revolution in about 3 years and 4 months. The second, called Biela's comet, which was first observed by an Austrian officer whose name was Biela, revolves in about 6 years and 8 months. Both these comets have been repeatedly observed; they have no tails; and Biela's is so transparent that Sir J. Herschel saw some small stars through the centre of it. It appears indeed to consist of extremely attenuated vapour; and it is wonderful that a thing so light should continue its course through the regions of space, obeying, with the most exact regularity, the laws of gravitation.

At present there are more than 100 comets, the orbits of which have been calculated; so that their position, and the least distance at which they must pass the sun, are known. When a new comet therefore shall appear, and describe the same, or nearly the same path, we may be assured that it is a comet which has appeared before: we shall then know the period of it revolution, and the extent of its axis, which will determine the orbit entirely: in short we shall be enabled to calculate the times of its return, and other circumstances of its motion, in the same manner as those of the other planets.

Comets have this in particular, that they are often accompanied by a train or tail. These tails or trains are transparent, and of greater or less extent; some have been seen which were 45, 50, 60, and even 100 degrees in length, as was the case with those of the comets which appeared in 1618 and 1680. Sometimes however the tail consists merely of a sort of luminous nebula, of very little extent, which surrounds the comet in the form of a ring, as was observed in the comet of 1585: it frequently happens that these tails cannot be seen unless the heavens be exceedingly serene, and free from vapours. The celebrated comet, which returned about the end of the year 1758, seemed at Paris to have a tail scarcely 4 degrees in length; whereas some observers at Montpelier found it to be 25°; and it appeared still longer to others at the Isle of Bourbon.*

In regard to the cause which produces the tails of comets, there are only two opinions which seem to be founded on probability. According to Newton, they are vapours raised by the heat of the sun, when the comet descends into the inferior regions of our system. It is therefore observed that the tail of a comet is longest when it has passed its perihelion; and it always appears longer the nearer it approaches to the sun. But this opinion is attended with considerable difficulties. According to M. de Mairan, these tails are a train of the zodiacal light, with which comets become charged in passing between the earth and the sun. It is remarked that comets which do not reach the earth's orbit, have no sensible tail; or are at most surrounded by a ring. Of this kind was the comet of 1585, which passed the sun at a distance $\frac{1}{10}$ greater than that of the earth; the comet of 1718, which passed at a distance almost equal to that of 1729, that is, at a distance nearly quadruple; and that of 1747, which passed at a distance more than double. It is indeed true, that the comet of 1664, which passed at a greater distance from the sun than that of the earth, appeared with a tail, but it was of a moderate size; and as the distance of its perihelion was very little more than that of the earth from the sun, and as the solar

* The tail of Halley's comet underwent some remarkable changes during the appearance of the comet in 1835—6. A beautiful set of drawings, by Mr. P. Smith, of the appearance of the tail at the Cape of Good Hope, have been engraved and published in vol. 10, Mem. Royal Ast. Soc.

atmosphere extends sometimes beyond the earth's orbit, no objection of any great weight can thence be made, in opposition to the opinion of M. de Mairan.

We shall remark, in the last place, that while the other planets perform their revolutions in orbits very little inclined to the ecliptic, and proceed in the same direction, comets on the other hand move in orbits, the inclination of which to the ecliptic amounts even to a right angle. Besides, some move according to the order of the signs, and are called *direct;* others move in a contrary direction, and are called *retrograde.* These motions being combined with that of the earth, give them an appearance of irregularity, which may serve to excuse the ancients for having been in an error respecting the nature of these bodies.

It has been already said that there are some comets which pass very near the earth; and hence a catastrophe fatal to our globe might some day take place, had not the Deity, by particular circumstances, provided against any accident of the kind.

A comet, indeed, like that of 1744, which passed at a distance from the sun only greater by about a 50th than the radius of the earth's orbit, should it experience any derangement in its course, might fall against the earth or the moon, and perhaps carry away from us the latter. As a multitude of comets descend into the lower regions of our system, some of them, in their course towards the sun, might pass so near the orbit of our earth, as to threaten us with a similar misfortune. But the inclination of the orbits of comets to the ecliptic, which is exceedingly varied, seems to have been established by the Deity to prevent that effect. It would be a curious calculation to determine the least distances at which some of these comets pass the earth; we should by these means be enabled to know those from which we have any thing to apprehend: that is, if it could be of any utility to be acquainted with the period of such a catastrophe; for where is the advantage of foreknowing a danger which can neither be retarded nor prevented?

It would seem however, from the extreme tenuity of the matter of which comets are generally formed, that except from actual contact, the more weighty bodies of the system have little to apprehend from them. One comet passed near if not among the satellites of Jupiter; and the orbit of the comet was altogether deranged by the attraction of the planet, but the motions of the satellites were not found to be affected in the slightest degree.

An English astronomer, who possessed more imagination and learning than soundness of judgment, the clebrated Whiston, entertained an opinion that the deluge was occasioned by the earth's meeting with the tail of a comet, which fell down upon it in the form of vapours and rain: he advanced also a conjecture, that the general conflagration, which according to the sacred Scriptures is to precede the final judgment, will be occasioned by a comet like that of 1681; which returning from the sun, with a heat two or three thousand times greater than of red-hot iron, will approach so near the earth as to burn even its interior parts. Such assertions are bold; but they rest on a very weak foundation: and in regard to a general deluge, occasioned by the tail of a comet, we need be under very little apprehension on that head; for if we consider the extreme tenuity of the ether in which the comets float, it may be readily conceived that the whole tail of a comet, even if condensed, could not produce a quantity of water sufficient for the effect ascribed to it by Whiston.

Cassini thought he observed that comets pursue their course in a kind of Zodiac, which he even denoted by the following verses:—

Antinous, Pegasusque, Andromeda, Taurus, Orion,
Procyon, atque Hydrus, Centaurus, Scorpio, Arcus.

But the observations of a great number of comets have shewn that this supposed Zodiac of comets has no reality.

XI.—*Of the Fixed Stars.*

As it now remains for us to speak of the fixed stars, we shall here collect every thing most curious in the modern astronomy on this subject.

The fixed stars may be easily distinguished from the planets. The former, at least in our climates, and when they are of a certain magnitude, have a splendour accompanied with a twinkling called *scintillation*. But one thing by which they are particularly distinguished is, that they do not change their place in regard to each other, at least in a sensible manner: they are therefore a kind of fixed points in the heavens, to which astronomers have always referred the positions of the moving bodies, such as the moon, the planets, and the comets.

We have said that the fixed stars in our climates exhibit a sort of twinkling. This phenomenon seems to depend on the atmosphere; for we are assured that in certain parts of Asia, where the air is exceedingly pure and dry, as at Bender-Abassi, the stars have a light absolutely fixed; and that the scintillation is never observed, except when the air is charged with moisture, as is the case in winter. This observation of M. Garcin, which was published in the History of the Academy of Sciences for 1743, deserves to be farther examined.

The distance between the fixed stars and the earth, is so immense, that the diameter of the earth's orbit, which is 190 millions of miles, is in comparison of it only a point; for in whatever part of its orbit the earth may be, the observations of the same star shew no difference in its aspect; so that it has no sensible annual parallax. Some astronomers however assert that they discovered, in certain fixed stars, an annual parallax of a few seconds. Cassini, in a memoir on this parallax, says he observed in Arcturus an annual parallax of seven seconds, and in the star called Capella one of eight.* This would make the distance of the sun from the former of these stars equal to about 20250 times the radius of the earth's orbit, which, being 95 millions of miles, would give for that distance 1923750000000 miles. Between the fixed stars and the Georgian planet, which is the most distant of our system, there would therefore remain a space equal to more than 10000 times the distance of that planet from the sun.

Placed at such an immense distance from us, what can the fixed stars be but immense bodies, which shine by their own light; in short, suns, similar to that which affords us heat, and around which our earth performs its revolutions? It is very probable also that these suns, accumulated as we may say on each other, have the same destination as ours; and are the centres of so many planetary systems, which they vivify and illumine. It would however be ridiculous to form conjectures respecting the nature of the beings by which these distant bodies are peopled; but of whatever kind they may be, who can believe that our earth, or our system, is the only one inhabited by beings capable of enjoying the pleasure which arises from the contemplation of such noble works? Who can believe that an immense whole, a creation almost without bounds, should have been formed for an imperceptible point, a quantity infinitely small?

The apparent diameter of the fixed stars is in no manner magnified by the best telescopes; on the contrary, these instruments, while they increase their splendour, seem to diminish their magnitude so much, that they appear only as luminous points; but they shew in the heavens a multitude of other stars, which cannot be observed without their assistance. Galileo, by means of his telescope, which was far inferior to those now employed, counted in the Pleiades 36 stars, invisible to the naked eye; in the sword and belt of Orion 80; in the nebula of Orion's head 21, and in

* It may be safely affirmed that this estimate (for it can have been nothing more) of Cassini's is greatly in excess. It is doubtful whether the parallax of any fixed star amounts to 1″.

that of Cancer 36. Father de Rheita says he counted 2000 in Orion, and 188 in the Pleiades. In that part of the Austral hemisphere, comprehended between the pole and the tropic, the Abbé de la Caille observed more than 6000 of the 7th magnitude, that is to say perceptible with a good telescope, of a foot in length; a longer telescope shews others apparently more distant, so in progression perhaps without end. What immensity in the works of the Creator! And how much reason to exclaim with the Psalmist: "The heavens declare the glory of God, and the firmament sheweth his handy work!"

The fixed stars seem to have a common and general motion, by which they revolve around the pole of the ecliptic, at the rate of a degree in 72 years. It is in consequence of this motion that the constellations of the zodiac have all changed their positions. Aries occupies the place of Taurus, the latter that of Gemini, and so of the rest; so that the constellations or signs have advanced about 30 degrees beyond the divisions of the zodiac to which they gave names. But this motion is only apparent, and not real; and arises from the equinoctial points going back every year about 51 seconds on the ecliptic. The explanation of this phenomenon however is of such a nature as not to come within the object of this work.

It has always been believed that the fixed stars have no real motion, or at least no other than that by which they change their longitude. But it has been discovered, by the very accurate observations of modern astronomers, that some of them have a small motion peculiar to themselves, by which they slowly change their places. Thus Arcturus, for example, has a motion by which it approaches the ecliptic about 4 minutes every 100 years. The distance between this star and another very small one, in its neighbourhood, has been sensibly changed in the course of the last century. Sirius also seems to have a motion in latitude, of more than 2 minutes per century, by which it recedes from the ecliptic. A similar motion has been observed in Aldebaran or the Bull's Eye, in Rigel, in the eastern shoulder of Orion, in the Goat, the Eagle, &c. Some others seem to have a peculiar motion in a direction parallel to the equator, as is the case with the brilliant star in the Eagle; for in the course of 48 years it has approached one star in its neighbourhod 73", and receded from another 48". All the stars perhaps are subject to a similar motion; so that in a series of ages the heavens will afford a spectacle very different from what they do at present. So true it is that nothing in the universe is permanent! In regard to the cause of this motion, however astonishing it may at first seem, it will appear less so, if it be recollected that it has been demonstrated by Newton, that a whole planetary system may have a progressive and uniform motion in space, without the particular motion of the different parts being thereby disturbed. It needs therefore excite no surprise that suns, as the fixed stars are, should have a motion of their own. The state of rest being of one kind only, and that of motion in any direction being infinitely varied, we ought rather to be astonished to see them absolutely at rest, than to discover in them any movement.

But these are not the only phenomena exhibited to us by the fixed stars; for some have appeared suddenly, and afterwards disappeared. The year 1572 is celebrated for a phenomenon of this kind. In the month of November of that year, an exceedingly bright star suddenly appeared in the constellation of Cassiopeia: its splendour at first was equal to that of Venus when in its perigeum, and then to that of Jupiter when he exhibits the greatest brightness; three months after its appearance it was only like a fixed star of the first magnitude: its splendour gradually decreased till the month of March 1574, at which time it entirely disappeared.

There are other stars which appear and disappear regularly at certain periods; of this kind is that in the neck of the whale. When in its state of greatest brightness it is nearly equal to a star of the second magnitude; it retains this splendour for about

fifteen days, after which it becomes fainter, and at length disappears; it then reappears, and attains to its greatest splendour, after a period of about 330 days.

The constellation of the Swan exhibits two phenomena of the same kind; for in the breast of the Swan there is a star which has a period of 15 years, during 10 of which it is invisible; it then appears for 5 years, varying in its magnitude and splendour. Another, which is situated in the neck near the bill, has a period of about 13 months. In the same constellation a star was observed in 1670 and 1671, which disappeared in 1672, and has never since been seen.

Hydra also has a star of the same kind, which is attended with this remarkable circumstance, that it appears only 4 months; after which it remains invisible for 20, so that its period is about two years.

Some stars seem to have become extinct since the time of Ptolemy; for he enumerates some in his catalogue which are not now to be seen: others have changed their magnitude; this diminution of size is proved in regard to several of the fixed stars; among this number may be classed the star B in the Eagle, which at the beginning of the last century was the second in splendour, but which at present is scarcely of the third magnitude. Of this kind also is a star in the left leg of Serpentarius or Ophiuchus.

It now remains that we should say a few words respecting those stars called *nebulæ*. They are distinguished by this name, because, when seen by the naked sight, they appear only like a small luminous cloud. There are three kinds of them. Some consist of an accumulation of a great number of stars crowded together, and as it were heaped upon each other; but when viewed through a telescope, they are seen distinct, and without any nebulous appearance. Among these is the famous nebula of Cancer, or the *præsepe Cancri*, forming a collection of 25 or 30 stars, which may be counted by means of a telescope. Similar groups may be seen in various parts of the heavens.

Other nebulæ consist of one or more distinct stars, but accompanied or surrounded by a whitish spot, through which they seem to shine. There are two of this kind in Andromeda; one in the girdle, and another smaller about a degree farther south than the former. Of this kind also is that in the head of Sagittarius; that between Sirius and Procyon; that in the tail of the Swan; and three in Cassiopeia. It is probable that our sun appears under this form, when seen from the neighbourhood of those fixed stars which are situated towards the prolongation of his axis; for he has around him a lenticular and luminous atmosphere, which extends nearly to the earth. The Abbé de la Caille counted, in the Austral hemisphere, fourteen stars surrounded in this manner with nebulosities; but the most remarkable appearance of this kind, is that of the nebula in the sword of Orion; for, when viewed through a telescope, it is found to be formed of a whitish spot, nearly triangular, and containing seven stars, one of which is itself surrounded by a small cloud, brighter than the rest of the spot. One is almost inclined to believe that this spot has experienced some alteration since the time of Huygens, by whom it was discovered.

The third kind of nebulæ is composed of a white spot, in which no stars are seen when viewed with the telescope. Fourteen of this kind are found in the Austral hemisphere, among which the celebrated spots, near the South pole, called by sailors the *Magellanic clouds*, hold the first rank. They are like small detached portions of the Milky Way. But it may be thought an error to ascribe the splendour of that part of the heavens to small stars accumulated there in a greater multitude than any where else; for it does not contain a number, visible by common telescopes, sufficient to produce that effect; and there are portions of the Milky Way no less brilliant than the rest, though no stars are observed in them, unless with the very highest improved instruments.

Respecting the milky way, nothing certain is known; but we may conjecture, not without probability, that it consists of some matter similar to that of the solar atmosphere, and which is diffused throughout that celestial space.* If our whole system indeed were filled with a similar matter, it would exhibit to the neighbouring fixed stars the same appearance as the milky way. But why are all these systems, with which that part of the heavens is interspersed, filled with this luminous matter? To this question no answer certainly can be given.

We shall here remark, that the famous new star in Cassiopeia had its origin in the milky way, and was perhaps formed by a prodigious quantity of this luminous matter being precipitated on some centre. But it is more difficult to explain why, and in what manner, the star disappeared. This origin of the new star may acquire some probability, if it be true that in the part of the milky way where it was seen, there is a vacuity similar to the other parts of the heavens.

Many of the fixed stars, when examined with telescopes, are found to be double, or to consist of two stars. Sir Wm. Herschel has enumerated more than 500 of such stars within 30″ of each other, and Struve and other modern observers have increased the list to at least five times the number. Some of the stars which form these double ones are within less than one second of each other. In catalogues they are divided into classes, the closest forming the first class.

On observing the direction of the line joining the centre of these stars with respect to the meridian, Sir Wm. Herschel found that some of them formed dynamic systems, actually revolving round each other, or round a common centre, in circular or elliptical orbits of different degrees of eccentricity; and by continuing to watch their motions, the times of revolution of several are now known with very considerable accuracy.

The stars forming γ *Virginis* revolve in about 629 years, *Castor* in about 253 years, 70 Ophiuchi in about 80 years, and ξ *Ursæ* in about 58 years.

The following list of a few of the larger class of double stars may be useful for the trial of telescopes:—

Object.	R. A. 1830.	N.P.D. 1830.	Dist. of Stars.
γ Arietis	1H 44M	71° 33′	9″
Castor	7 23	57 45	5
ι Piscium	1 53	88 5	5
ζ Aquarii	22 20	90 55	4
μ Draconis	17 2	35 18	4
Andron 37	23 51	57 13	4
Boötis 44	14 58	41 38	3
ξ Ursæ	11 9	57 30	2
ζ Boötis	14 33	75 31	1
γ Virginis	12 33	90 29	1¼

Besides these double stars, others are found to present more complicated combinations, consisting of three, of four, or sometimes of a greater number of separate stars; and it not unfrequently happens that the separate stars are of different colours.

XII.—*Recapitulation of what has been said respecting the System of the Universe.*

We shall terminate this chapter with a familiar comparison, calculated to shew, by known and common measures, the small space which our planetary system occupies in the immensity of the universe; and the poor figure, if we may be allowed the expression, which our earth makes in it. This consideration will no doubt serve to

* Unless, with Dr. Herschel, we suppose it is a far extended stratum of stars, by us seen edgeways.

humble those proud beings, who, though they occupy but an infinitely small portion of this atom, have the vanity to think that the universe was created for them.

To form an idea of our system as compared with the universe, let us suppose the sun to be in Hyde Park, as a globe of 9 feet 3 inches diameter: the planet Mercury will be represented by a globule of about $\frac{1}{4}$ of a line in diameter, placed at the distance of 37 feet. Venus will be a globe of a little more than a line in diameter, circulating at the distance of 68 feet from the same centre: if another globule, a line in diameter, be placed at the distance of 95 feet, it will represent the earth, that theatre of so many passions, and so much agitation; on the surface of which the greatest potentate scarcely possesses a point, and where a space often imperceptible excites, among the animalcula that cover it, so many disputes, and occasions so much bloodshed. Mars, which in magnitude is somewhat inferior to the earth, will be represented by a globule of a little less than a line in diameter, and placed at the distance of 144 feet; Jupiter, by globe 10 lines in diameter, 490 feet from the central globe; Saturn, about 7 lines in diameter, at the distance of about 900 feet; and the Georgian planet, 4 lines in diameter, at the distance of 1800 feet.

But the distance from the Georgian planet to the nearest fixed stars, is immense. The reader may perhaps imagine that, according to the supposition here made, the first star ought to be placed at the distance of two or three leagues. This is the idea which one might form before calculation has been employed; but it is very erroneous, for the first, that is to say the nearest star, ought to be placed at the same distance as that between London and Edinburgh, which is more than 300 miles. Such then is the idea which we ought to have of the distance between the sun and the nearest of the fixed stars; and there is reason even to think that it is much greater, for we have supposed, in this calculation, that the parallax of the earth's orbit is the same as the horizontal parallax of the sun, that is to say 8·5″. But certainly this parallax is much less; for it can hardly be believed that it could have escaped astronomers had it been so great.

Our solar system then, that is the system of our primary and secondary planets, which circulate around the sun, is to the distance of the nearest fixed stars almost as a circle of 1800 feet radius would be to a concentric one of 300 miles radius; and in the first circle our earth would occupy a space a line in diameter, appearing like a grain of mustard seed.

Another comparison, proper to convey some idea of the immense distance between the sun, which is the centre of our system, and the nearest of the neighbouring bodies of the same nature, is as follows: It is well known that the velocity of light is so great, that it passes over the distance between the sun and the earth in about half a quarter of an hour: in a second and a half it would go to the moon and return, or rather it would go fifteen times round the earth in a second. What time would light then employ in coming to us from the nearest of the stars?—Not less than 108 days; or if the annual parallax be only two or three seconds, it would require a year and more.

What immense distance then between this inhabited point and the nearest of its neighbours! Is it not probable that in this vast interval there are planets which will remain for ever unknown to the human species?

Modern astronomy indeed has discovered that this space is not entirely desert: it is now known that about a hundred comets move in it, at greater or less distances, but do not penetrate to a very great depth. Those of 1531, 1607, 1682, and 1759, the only ones the periods and orbits of which are known, do not immerge farther than about $37\frac{1}{2}$ times the radius of the earth's orbit, or four times the distance of Saturn from the sun. If that of 1681 has a revolution of 575 years, as supposed, it must recede from us about 130 times the distance of the earth from the sun, or about 14 times that of Saturn from the same body; which is only a point when compared with the nearest of the fixed stars. But there are comets perhaps which perform their revo-

lution only in 10000 years, and which scarcely approach so near the sun as Saturn: in that case these would penetrate into the immense space, which separates us from the first of the fixed stars, as far as a fifteenth part of its depth.

Those desirous of seeing a great many curious conjectures respecting the system of the universe, the habitation of the planets, the number of the comets, &c., may consult a work by M. Lambert, member of the Royal Academy of Berlin, entitled "Système du Monde," Bouillon 1770, 8vo. Every one almost is acquainted with the "Pluralité des Mondes," of Fontenelle; the "Cosmotheoros," of Huygens, the "Somnium," of Kepler, and the "Iter extaticum," of Kircher. The first of these, the "Pluralité des Mondes," is an ingenious and pleasing work, but a little too affected. The second is learned and profound, and, like Kepler's "Somnium," will please none but astronomers. In regard to the last, however much we may esteem the memory of Kircher, it can be considered in no other light than as a production altogether pedantic and ridiculous.

CHAPTER III.

OF CHRONOLOGY, AND VARIOUS QUESTIONS RELATING TO THAT SUBJECT.

All polished nations keep an account of the time which has elapsed, and of that which is to come, by means of periods that depend on the motions of the heavenly bodies; and this is even one of those things which distinguish man in a state of civilization, from man in the animal and savage state: for, while the former is enabled at every moment to count that part of the duration of his existence which has elapsed; to foresee, at an assigned period, the recurrence of certain events, labours or duties; the latter, though in some measure happier, since he enjoys the present without recollecting the past, or anticipating the future, cannot tell his age, nor foresee the period of the renovation of his most common occupations: the most striking events of which he has been a witness, or in which he has had a share, exist in his mind only as past; while the civilized man connects them with precise periods and dates, by which they are arranged in their proper order. Without this invention, every thing hitherto done by mankind would have been lost to us; there would be no historical records; and men, whose existence in the social state requires the united efforts of its different members in certain circumstances, could not employ that concurrence of action, which is necessary. No real civilized society therefore can exist without an agreement to count time in a regular manner; and hence the origin of chronology, and the various computations of time employed by different nations.

But, before we proceed farther, it will be proper to present the reader with some definitions, and a few historical facts, necessary for comprehending the questions which will be proposed in the course of this article.

There are two kinds of year employed by different nations; one of which is regulated by the course of the sun, and the other by that of the moon. The first is called the solar, and the second the lunar year. The solar year is measured by a revolution of the sun through the ecliptic, from one point of the equinoctial, that of the vernal equinox for example, to the same point again; and, as already said, consists of 365 days 5 hours 49 minutes.

The lunar year consists of twelve lunations; and its duration is 354 days 8 hours 44 minutes 3 seconds. Hence it follows that the lunar year is about 11 days shorter than the solar; consequently, if a lunar and a solar year commence on the same day, at the end of three years the commencement of the former will have advanced 33 days before that of the latter. The commencement therefore of the lunar year passes successively through all the months of the solar year, in a retrograde direction. The Arabians and Mussulmans in general, count only by lunar years; and the Hebrews and Jews never employed any other.

But the most polished and enlightened nations have always endeavoured to combine these two kinds of year together. This the Athenians accomplished by means of the famous golden cycle, invented by Meto, the celebrated mathematician whom Aristophanes made the object of his satirical wit; and the same thing is done at present by the Europeans, or the Christians in general, who have borrowed from the Romans the solar year for civil uses; and from the Hebrews their lunar year for their ecclesiastical purposes.

Before Julius Cæsar, the Roman calendar was in the utmost confusion; but it is here needless to enter into any details on the subject: it will be sufficient to observe, that Julius Cæsar, being desirous to reform it, supposed, according to the suggestion of Sosigines his astronomer, that the duration of the year was exactly 365 days 6 hours. He therefore ordered that, in future, there should be three successive years of 365 days, and a fourth of 366. This last year was afterwards distinguished by the name of *bissextile*, because the day added every fourth year followed the sixth of the calends which was counted twice; and because, to avoid any derangement in the denomination of the following days, it was thence called *bissexto calendas*. Among us it is added to the end of February, which has then 29 days instead of 28, which is the number it contains in common years. This form of year is called the *Julian year*, and the calendar in which it is employed is called the *Julian calendar*.

But Julius Cæsar was mistaken, when he considered the year as consisting exactly of 365 days 6 hours; as it contains only 365 days 5 hours 49 minutes; and hence it follows that the equinox always retrogrades in the Julian year 11 minutes annually; which gives precisely three days in 400 years. Hence it happened that the equinox, which at the time of the council of Nice corresponded to the 21st of March, after the lapse of about 1200 years, that is to say, in the year 1500, fell about the 11th. Pope Gregory XIII. being desirous to reform this error, suppressed, in 1582, ten consecutive days; counting, after the 11th of October, the 21st: and by these means brought back the vernal equinox following to the 21st of March; and, in order that it might never deviate any more, he proposed that three bissextiles should be suppressed in the course of 400 years. For this reason the years 1700 and 1800 were not bissextile, though they ought to have been so according to the Julian calendar; the case will be the same with the year 1900, but the year 2000 will be bissextile; in like manner the years 2100, 2200, and 2300 will not be bissextile; but 2400 will; and so of the rest.

All this is sufficient, and more than sufficient, for the solar year. But the great difficulty of our calendar arose from the lunar year, which it was necessary to combine with it; for as the Christians had their origin among the Jews, they were desirous of connecting their most solemn festival, that of Easter, with the lunar year; because the Jews celebrated their Passover at a certain lunation, viz. on the day of the full moon which immediately followed the vernal equinox. But the council of Nice, that the Easter of the Christians might not concur with the Passover of the Jews, ordained, that the former should celebrate their festival on the Sunday after the full moon which should take place on the day of the vernal equinox, or which should immediately follow it. Hence has arisen the necessity of forming periods of lunations, that the day of the new or full moon may be found with more facility, in order to determine the paschal moon.

The council of Nice supposed the cycle of Meto, or the golden number, according to which 235 lunations are precisely equal to 19 solar years, to be perfectly exact. After the period of 19 years, therefore, the new and full moons ought to take place on the same days of the month. It was thence easy to determine, in each of these years, the place of the lunations; and this was what was actually done by means of the epacts, as shall be hereafter explained.

But in reality 235 lunations are less, by an hour and a half, than 19 solar Ju-

lian years; whence it happens, that in 304 years the new moons retrograde a day towards the commencement of the year: and consequently four days in 1216 years. On this account, about the middle of the 16th century, the new and full moons had anticipated, by four days, their ancient places: so that Easter was frequently celebrated contrary to the disposition of the council of Nice.

Gregory XIII. undertook to remedy this irregularity by an invariable rule, and proposed the problem to all the mathematicians of Europe: but it was an Italian physician and mathematician who succeeded best in solving it, by a new disposition of the epacts, and which the church adopted. This new arrangement is called the *Gregorian calendar*. It began to be used in Italy, France, Spain, and other Catholic countries, in 1582. It was soon adopted, at least in what concerns the solar year, even by the Protestant states of Germany: but they rejected it in regard to the lunar, and preferred finding the day of the paschal full moon by astronomical calculation: the Roman Catholics therefore do not always celebrate Easter at the same time as the Protestants in Germany. The English were the most obstinate in rejecting the Gregorian year, and almost for the same reason which made them long exclude Peruvian bark from their pharmacopeia: that is to say, because they were indebted for it to the Jesuits: but they at length became sensible that whatever is good in itself, and useful, ought to be received were it even from enemies: and they conformed to the method of computing time employed in the rest of Europe. This change did not take place till the year 1752. Before that period, when the French counted the 21st of the month, the English counted only the 10th. In the course of ages they would therefore have had the vernal equinox at Christmas, and the winter at Midsummer. The Russians are the only people of Europe who still adhere to the Julian calendar.

After this short historical sketch, we shall now proceed to the principal problems of chronology.

PROBLEM I.

To find whether a given year be Bissextile or not; that is to say, whether it consists of 366 days.

Divide the number which indicates the given year by 4, and if nothing remains the year is bissextile: if there be a remainder, it shews the number of the year current after bissextile. We shall here propose, as an example, the year 1774. As 1774 divided by 4 leaves 2 for remainder, we may conclude that the year 1774 was the second after bissextile.

To this rule however there are some limitations. 1st. If the year is one of the centenaries posterior to the reformation of the calendar by Gregory XIII., that is to say 1582, it will not be bissextile unless the number of the centuries which it denotes be divisible by 4; thus 1600, 2000, 2400, 2800 have been or will be bissextiles; but the years 1700, 1800, 1900, 2100, 2200, 2300, 2500, 2600, 2700, were not, or will not be, bissextiles, for the reason already mentioned.

2d. If the year be centenary, and anterior to 1582, but without being below 474, it has been bissextile.

3d. Between 459 and 474 there was no bissextile.

4th. There was none among the first six years of the Christian æra.

5th. As the first bissextile after the Christian æra was the seventh year, and as the bissextiles regularly followed each other every four years till 459; when the given year is between the 7th and the 459th, first subtract 7 from it, and then divide it by 4; if nothing remains, the year has been bissextile; but if there be any remainder, it will shew what year after bissextile the proposed year was. Let the proposed year, for example, be 148: if 7 be subtracted, the remainder is 141, which divided by 4,

leaves 1 for remainder; consequently the year 148 of the Christian æra was the first after bissextile.

Of the Golden Number and Lunar Cycle.

The golden number, or lunar cycle, is a revolution of 19 solar years, at the end of which the sun and moon return very nearly to the same position. The origin of it is as follows.

Since the solar Julian year, as already said, consists of 365 days 6 hours; and as the duration of one lunation is 29 days 12 hours 49 minutes; it has been found, by combining these two periods, that 235 lunations make nearly 19 solar years; the difference being only 1h. 31m. It is therefore plain that after 19 solar years the new moons ought to take place on the same days of the month, and almost at the same hour. In the first of these solar years, if the new moon happen on the 4th of January, the 2d of February, &c., at the end of 19 years the new moons will take place also on the 4th of January, the 2d of February, &c.; and this will be the case eternally, if we suppose that 235 lunations are exactly equal to 19 solar revolutions. Hence is sufficient to have once determined, during 19 solar years, the days of the month on which the new moons happen; and when it is known what rank a given year holds in this period, we can immediately tell on what days of each month the new moons fall.

The invention of this cycle appeared to the Athenians to be so ingenious, that, when proposed by the astronomer Meto, it was received with acclamations, and inscribed in the public square in golden letters: hence the name of the golden number. It is distinguished also by the less pompous denomination of the lunar cycle, or cycle of Meto, from the name of its inventor.

PROBLEM II.

To find the Golden Number of any given year; or the rank which it holds in the Lunar Cycle.

To the given year add 1, and divide the sum by 19: if nothing remains, the golden number of the given year will be 19; but if there be a remainder, which must necessarily be less than 19, it will be the golden number required.

Let the given year, for example, be 1802. If 1 be added to 1802, and if the sum 1803 be divided by 19, the remainder will be 17; which indicates that 17 is the golden number of 1802, or that this year is the 17th of the lunar cycle of 19 years.

If the year 1728 be proposed, it will be found, by a similar operation, that the remainder is nothing: which shews that the golden number of that year was 19.

The reason of adding 1 to the given year, is because the first year of the Christian æra was the second of the lunar cycle, or had 2 for its golden number.

If any year before the Christian æra be proposed, such as the 25th for example, substract 2 from that number, and divide 23 the remainder by 19; if 4 the remainder be then taken from 19, the result will be the golden number of the year 25 before Jesus Christ; which in this case is 15.

Remark.—It may be readily seen, that when the golden number of any year has been found, the golden number of the following year may be obtained by adding 1 to the former. The golden number of the preceding year may be obtained also by subtracting 1 from the golden number already found. Thus, having found the golden number of the year 1802, which is 17, by adding 1 to it, we shall have 18 for that of the year 1803; and 1 subtracted from it will give 16 for the golden number of 1801.

Of the Epact.

The epact is nothing else than the number of days denoting the moon's age at the

end of a given year. The formation of it may be easily conceived by considering that the lunar year, which consists of 12 lunations, is less than a Julian year by about 11 days; therefore if we suppose that a lunar and a solar year begin together on the 1st of January, the moon at the end of the year will be 11 days old; for 12 complete lunations, and 11 days of a thirteenth, will have elapsed; and therefore the moon, at the end of the second year, will be 22 days old, and at the end of the third 33. But as 33 days exceed a lunation, one of 30 days is intercalated, by which means that year has 13 lunations; and consequently the moon is only 3 days old at the end of the third year.

Such then is the progress of the epacts. That of the first year of the lunar cycle is 11; this number is afterwards continually added, and when the sum exceeds 30, if 30 be subtracted, the remainder will be the epact; except in the last year of the cycle, where the product of the addition being only 29, the same number is deducted to have 0 for epact: this announces that the new moon happens at the end of that year, which is also the beginning of the next one. The order of the epacts therefore is 11, 22, 3, 14, 25, 6, 17, 28, 9, 20, 1, 12, 23, 4, 15, 26, 7, 18, 29.

This arrangement would have been perfect and perpetual, if 19 solar years, of 365 days 6 hours, had been exactly equal to 235 lunations, as supposed by the ancient astronomers; but unfortunately this is not the case. On the one hand, the solar year consists only of 365 days 5 hours 49 minutes; and besides, 235 lunations are less than 19 Julian years by one hour and a half; so that in 304 years the real new moons are anterior, by one day, to the new moons calculated in this manner. Hence it happened that in the middle of the 16th century, they preceded by four days those found by calculation; as four revolutions of 304 years had elapsed between that period and the council of Nice, at which the use of the lunar cycle had been adopted for computing the time of Easter, it was therefore found necessary to correct the calendar, that this festival might not be celebrated, as was often the case, contrary to the intention of that council; and with this view some changes were made in the calculation of the epacts, which form two cases. One of them is that when the proposed year is prior to the reformation of the calendar, or to 1582: the second is when the years are posterior to that epoch. We shall illustrate both cases in the following problem.

PROBLEM III.

Any year being given, to find its Epact.

I. If the proposed year be anterior to 1582, though posterior to the Christian era, which forms the first case; find by the preceding problem the golden number for the given year, and having multiplied it by 11, subtract 30 from the product as many times as possible: the remainder will be the epact required.

Let the given year, for example, be 1489. Its golden number, by the preceding problem, is 8, which multiplied by 11 gives 88; and this product divided by 30 leaves for remainder 28: the epact of the above year therefore was 28.

In like manner, if 1796 be considered as a Julian year, that is to say, if those who have not adopted the new style of reformation in the calendar wished to know the epact of that year, it would be necessary first to find the golden number, which is 11; this multiplied by 11 gives 121; and the latter divided by 30, leaves 1 for remainder. Hence it appears that the epact of 1796, considered as a Julian year, was 1.

II. We shall now suppose that the given year is posterior to the reformation of the calendar, or to the year 1582; which forms the second case. In this case, multiply the golden number by 11, and from the product subtract the number of days cut off by the reformation of Gregory XIII., that is to say 10, if the year is between

1582 and 1700; 11 between 1700 and 1900; 12 between 1900 and 2200, &c.; divide what remains after this deduction by 30, and the remainder will be the epact required.*

Let it be proposed, for example, to find the epact of the Gregorian year 1693, the golden number of which was 3: multiply 3 by 11, and from 33, the product, subtract 10: as the number 23 cannot be divided by 30, that number was the epact of the year 1693.

If the epact of the year 1796 were required, the golden number of which was 11; multiply 11 by 11, and from the product 121 subtract 11, which will leave 110: this number divided by 30, gives for remainder 20, which was the epact of the year 1796.

If the epact of the year 1802 were required, the golden number of which is 17; multiply 17 by 11, and from the product 187 subtract 11; the remainder, 176, divided by 30, leaves for remainder 26, which therefore is the epact for the year 1802.

Remarks.—The epact, according to the Julian calendar, may be found without a division, in the following manner: Assign to the upper extremity of the thumb of the left hand the value of 10; to the middle joint 20, and to the last or root 30, or rather 0. Count the golden number of the proposed year on the same thumb, beginning to count 1 at the extremity, 2 on the middle joint, 3 on the root; then 4 at the extremity, 5 on the joint, 6 on the root; and so on, till you come to the golden number found; to which, if it falls on the root, nothing is to be added, because the value assigned to it was 0: but if it falls on the extremity add 10 to it; and if on the middle joint 20; because these were the values assigned to them. The sum, if less than 30, will be the epact required; if greater than 30, subtract 30 from it and the remainder will be the epact.

Thus, if the epact of 1489 were required: as the golden number of that year was 8, count 8 on the thumb, as above mentioned, beginning to count 1 on the extremity, 2 on the middle joint, 3 on the root; then 4 on the extremity, and so on. Because 8, in this case, falls on the middle joint, add to it 20, and the sum 28 will be the epact of the above year 1489. In like manner, if the epact of 1726 be required, the golden number of which is 17; count 1 on the extremity of the thumb, 2 on the middle joint, &c., till you complete 17, which will fall on the joint; and if 20, the value assigned to that joint, be then added to the golden number, the sum will be 37; from which if 30 be subtracted, there will remain 7 for the epact of 1726, according to the Julian calendar.

By the same artifice the epact for any year of the 17th century might be found; provided 20 be assigned to the extremity of the thumb, 10 to the joint, and 0 to the root; and that you begin to count 1 on the root, 2 on the joint, and so on.

PROBLEM IV.

To find the day of the New Moon in any proposed Month of a given Year.

First find the epact of the given year, as taught in the two preceding problems; and add to it the number of months, reckoning from March inclusively: subtract the sum from 30 if less, or from 60 if greater; and the remainder will give the day of the new moon.

Let it be required, for example, to find on what day the new moon happened in the month of May 1802. The golden number of 1802 was 17, which multiplied by 11 gives 187; and if 11 be subtracted, according to the rule, we shall have for remainder 176: this divided by 30 leaves 26 = the epact of that year, as before found.

* When the golden number is 1, if the year be posterior to 1900, add 30 to it before you multiply by 11, and then proceed as above directed.

Now the number of months from March, including May, is 2; and 2 added to the epact makes 28, which subtracted from 30 leaves 2: new moon therefore took place on the 2d of May 1802. Accordingly the Almanacs shew it was new moon on the 2d at 1h. 43m. in the morning.

Remark.—In calculations of this nature, great exactness must not be expected. The irregular arrangement of the months which have 31 days, the mean numbers necessary to be assumed in the formation of the periods from which these calculations are deduced, and the inequality of the lunar revolution, may occasion an error of nearly 48 hours.

More correctness may perhaps be obtained by employing the following table; which indicates what ought to be added to the epact for each commencing month.

January	0	May	3	September....	8
February......	2	June	4	October	8
March	1	July...............	5	November....	10
April	2	August	6	December	10

PROBLEM V.

To find the Moon's Age on any given day.

To the epact of the year add, according to the above table, the number belonging to the month in which the proposed day is; and to this sum add the number which indicates the day: if the result be less than 30, it will be the moon's age on the given day; if it be 30, it shews that new moon took place on that day; but if it exceeds 30, subtract 30 from it, and the remainder will be the age of the moon.

Let it be required, for example, to find what was the age of the moon on the 20th of March 1802. The epact of 1802 was 26, and the number to be added for the month of March, according to the preceding table, is 1: this added to 26 makes 27, and 20, the number of the proposed day, added to 27, makes 47; from which if 30 be subtracted, the remainder is 17 = the moon's age on the 20th of March; and this indeed is agreeable to what is indicated by the Almanacs.

The moon's age, during the present century, may be found with sufficient exactness by the following rule.

From the given year subtract 1800, multiply the remainder by 109, and divide the product by 295. To the tenth part of the remainder add the day of the month, and the number from the subjoined table; and the sum, if it does not exceed 30, is the moon's age,—if it exceed 30, the excess is the moon's age.

TABLE.

	Jan.	Feb.	Mar.	Ap.	May.	June.	July.	Aug.	Sep.	Oct.	Nov.	Dec.
Common Year	5	6	5	6	7	8	9	10	12	12	14	14
Leap Year ..	5	7	6	7	8	9	10	11	13	13	15	15

Thus taking the example proposed above, 1800 deducted from 1802 leaves 2; and 2 multiplied by 109 gives 218 for a product, which divided by 295 gives 0 for a quotient, and 218 for a remainder; the tenth parts of which is 21·8 or 22 nearly.

Remainder	22
Day of month	20
Tab. No.................	5
	47
Deduct	30
Moon's age	17 days.

Of the Solar Cycle and Dominical Letter.

The solar cycle is a perpetual revolution of 28 years, the origin of which is as follows:

1st. The seven first letters of the alphabet A B C D E F G are arranged in the calendar in such a manner, that A corresponds to the 1st of January, B to the 2d, C to the 3d, D to the 4th, E to the 5th, F to the 6th, G to the 7th, A to the 8th, B to the 9th, and so on through several revolutions of seven. The seven days of the week, called also feriæ, are represented by these seven letters.

2d. Because a year of 365 days contains 52 weeks and 1 day, and as that remaining day is the first of a 53d revolution, a common year of 365 days ought to begin and end with the same day of the week.

3d. According to this disposition, the same letter of the alphabet corresponds to the same day of the week, throughout the course of a common year of 365 days.

4th. As these letters all serve alternately to indicate Sunday, during a series of several years, they have on that account been called *dominical letters*.

5th. It hence follows that if a common year begins by a Sunday, it will end by a Sunday; the 1st of January therefore of the following year will be a Monday, which will correspond to the letter A; and the 7th will be a Sunday, which will correspond to the letter G, which will be the dominical letter of that year. For the same reason the dominical letter of the following year will be F, that of the next one E, and so on, circulating in an order retrograde to that of the alphabet. From this circulation of the letters has arisen the name of *solar cycle;* because Sunday among the pagans was called *dies solis*, the day of the sun.

6th. If there were no days to be added for bissextile years, all the different changes of the dominical letters would take place in the course of seven years. But this order being interrupted by the bissextile years, in which the 24th of February corresponds to two different feriæ of the week: the letter F, for example, which would have indicated a Saturday in a common year, will indicate a Sunday in a bissextile year: or if it indicated a Sunday in a common year, it will indicate a Sunday and a Monday in a bissextile, &c. Hence it follows that in a bissextile year, the dominical letter changes, and that the letter which marked a Sunday in the commencement of the year, will mark a Monday after the addition of the bissextile. This is the reason why two dominical letters are assigned to each bissextile year; one which serves from the 1st of January to the 24th of February, and the other from the 24th of February to the end of the year; so that the second dominical letter would naturally be that of the following year, if a day had not been added for the bissextile.

7th. All the possible varieties to which the dominical letters are subject, both in common and in bissextile years, take place in the course of 4 times 7, or 28 years; for after 7 bissextiles the dominical letters return and circulate as before. This revolution of 28 years has been called the *solar cycle*, or the *cycle of the dominical letter*.

PROBLEM VI.

To find the Dominical Letter of any proposed year.

I. To find the dominical letter of any given year, according to the Gregorian Calendar, add to the number of the year its fourth part, or, if it cannot be exactly divided by 4, the least nearest to it; from the sum subtract 5 for 1600, 6 for the following century 1700, 7 for 1800, and 8 for 1900 and 2000; because the years 1700, 1800, and 1900, are not bissextiles; 9 for 2100, 10 for 2200, and 11 for 2300 and 2400, because the three years 2100, 2200, and 2300 will not be bissextiles; divide what remains by 7, and the remainder will be the dominical letter required, counting from

the last letter G towards A the first; so that, if nothing remains, the dominical letter will be A; if 1 remains, the dominical letter will be G; if 2 remains, it will be F; and so of the rest.

Thus, to find the dominical letter of the year 1802: add its fourth part 450, which makes 2252, and from this sum subtract 7; if the remainder 2245 be divided by 7, the remainder 5 will shew that the dominical letter is C, since it is the fifth, counting in a retrograde order, from the last letter G.

We must here observe, that to find with more certainty, by this operation, the dominical letter of a bissextile year, it will be necessary to find first the dominical letter of the preceding year, which will serve till the 24th of February of the bissextile year; after which the next letter in the retrograde order must be used for the remaining part of the year. Thus, if it be required to find the dominical letter of the year 1724; first find that of 1723, by adding to it its nearest less fourth part, 430; subtracting 6 from the sum 2153, and dividing the difference 2147 by 7: the remainder 5 shews that the dominical letter of the year 1723 was C; which is the fifth of the first seven letters of the alphabet, counting in the retrograde order. Since it is known that C was the dominical letter of 1723, it may be readily seen that B was the dominical letter of the following year 1724. But as 1724 was bissextile, B could be used only till the 24th of February, after which A, the letter preceding B, was employed to the end of the year; hence it is seen that A and B were the two dominical letters of the year 1724. In like manner the dominical letters of any future bissextile year may be found.

2d. To find the solar cycle, or rather the current year of the solar cycle, corresponding to a given year; add 9 to the proposed year, and divide the sum by 28: if nothing remains, the solar cycle of that year is 28; but if there be any remainder it indicates the number of the solar cycle required.

Thus, if the solar cycle of 1802 be required; add 9, which makes 1811, and divide this sum by 28; the remainder, being 19, shews that 19 is the solar cycle of 1802.

The reason of this rule is, that the first year of the Christian æra was the 10th of the solar cycle; or in other words, that at the commencement of this æra 9 years of the solar cycle were elapsed.

Remarks.—The solar cycle of any year whatever may be found with great ease, and without division, by means of the subjoined table.

Years.	Solar Cycle.	Years.	Solar Cycle.	Centuries.	Solar Cycle.	Centuries.	Solar Cycle.
1	1	10	10	100	16	1000	20
2	2	20	20	200	25	2000	12
3	3	30	2	300	20	3000	4
4	4	40	12	400	8	4000	24
5	5	50	22	500	24	5000	16
6	6	60	4	600	12	6000	8
7	7	70	14	700	0	7000	0
8	8	80	24	800	16	8000	20
9	9	90	6	900	4	9000	12

The method of constructing this table is as follows:—

Having placed opposite to the first ten years the same numbers as the solar cycles of these years, and 20 for the solar cycle of the 20th; instead of putting down 30, for the 30th year, set down only 2, which is the excess of 30 above 28, or above the period of the solar cycle. For the 40th year inscribe the numbers which correspond to 30 and to 10, that is 2 and 10; and so of the rest, always subtracting 28 from the

sum when it is greater. Having thus shewn the method of constructing this table, we shall now explain the use of it.

In the first place, if the proposed year, the solar cycle of which is required, be in the above table, look for the number opposite to it in the column on the right, marked solar cycle at the top, and add 9 to it; the sum will be the solar cycle required: thus if 9 be added to 12, which stands opposite to the year 2000, we shall have 21 for the solar cycle of that year.

But, if the given year cannot be found exactly in the above table, it must be divided into such parts as are contained in it. If the numbers corresponding to these parts be then added, their sum increased by 9 will give the solar cycle of the required year; provided this sum is less than 28; if greater, 28 must be subtracted from it as many times as possible.

Let it be required, for example, to find by the above table the solar cycle of the year 1802. Divide 1802 into the three following parts 1000, 800, 2, and find the numbers corresponding to them in the right hand columns, which are 20, 16, 2; the sum of these is 38, and 9 added makes 47; from which if 28 be subtracted we shall have for remainder 19, the solar cycle of 1802.

II. The reason of adding 9 to the sum of all these numbers, is because the solar cycle, before the first year of the Christian æra, was 9; consequently this cycle had begun 10 years before the birth of Christ, which may be ascertained in this manner:—

Knowing the solar cycle of any year, either by tradition or in any other manner, that of the year 1693 for example, which was 22; subtract 22 from 1693, and divide the remainder 1671 by 28; then subtract 19, which remains, from 28, and the remainder 9 will be the solar cycle before the first year of the Christian æra.

III. A table to shew the golden number of any proposed year might be constructed in the same manner; with this difference, that instead of subtracting 28 it would be necessary to subtract 19, because the period of that cycle is 19; and that instead of adding 9, it would be necessary to add only 1; because the golden number, before the first year of the Christian æra, was 1: consequently this cycle began two years before the birth of Christ; that is to say, the golden number for the first year of the Christian æra, was 2, &c.

IV. The dominical letter of any proposed year may be found by another method; and when this letter is known, it will serve to shew the letter which corresponds to every day throughout the whole of the same year.*

Divide by 7 the number of days which have elapsed between the first of January and the proposed day inclusively; and if nothing remains the required letter will be G; if there be any remainder, it will indicate the number of the required letter, reckoning according to the order of the alphabet, A 1, B 2, &c.

Thus, to find the dominical letter of the year 1802; take any Sunday, the 28th of February for example, and find how many days have elapsed between it inclusively, and the 1st of January: as the number is 59, divide this number by 7, and the remainder 3 will shew that C, the third letter of the alphabet, is the dominical letter required.

The days which have elapsed between the first of January and any given period of the year, may be readily found by means of the following table; but it is to be observed that after February, in bissextiles, the number of days must be increased by unity.

* It is here to be observed, that when you wish to find the dominical letter, the proposed day must be a Sunday; otherwise you will find only the letter which belongs to some other day.

	Days.		Days.
From Jan. to Feb.	31	From Jan. to August	212
Jan. to March	59	Jan. to Sept.	243
Jan. to April	90	Jan. to Oct.	273
Jan. to May	120	Jan. to Nov.	304
Jan. to June	151	Jan. to Dec.	334
Jan. to July	181	Jan. to Jan.	365

PROBLEM VII.

To find what day of the Week corresponds to any given day of the Year.

To the given year add its fourth part, or, when it cannot be found exactly, its nearest least fourth part; and to the sum add the number of days elapsed since the first of January, the proposed day included: from the last sum subtract 14, for the present century, and divide what remains by 7: the remainder will indicate the day of the week, counting Sunday 1, Monday 2, Tuesday 3, and so on: if nothing remains, the required day is a Saturday.

Thus, if it be required to know what day of the week corresponded to the 27th of April 1802; add to 1802 its nearest least fourth part 450, and to the sum 2252, add 117, the number of days elapsed between that day inclusive and the 1st of January. If 14 be subtracted from the last sum, which is 2369, and if 2355 which remains be divided by 7, the remainder will be 3: consequently the 27th of April 1802 was a Tuesday.

Remark.—If the proposed year be between 1582 and 1700, it will be necessary to deduct only 12 from the sum formed as above.

If the year be anterior to 1582, it will be necessary to deduct only 2; because in 1582 ten days were suppressed from the calendar. As a bissextile was suppressed in 1700, which makes an eleventh day suppressed, 13 must be subtracted if the given year be in the last century.

For the same reason 14 must be subtracted in the present century, 15 in the twentieth and twenty-first, and so on.

PROBLEM VIII.

To find Easter-Day and the other Moveable Feasts.

By the reformation of the calendar, the 14th day of the paschal moon was brought back to the same season in which it was found at the time of the council of Nice, and from which it had removed more than four days. According to the decree of that council, Easter ought to be celebrated on the first Sunday after the 14th day of the moon, if this 14th day should happen on or after the 21st of March. Hence it is obvious that Easter cannot happen sooner than the 22nd of that month, nor later than the 25th of April: which on that account have been called the paschal limits. The following is a table of these limits from the year 1700 to 1900.

Lunar Cycle.	Paschal Limits.	Lunar Cycle.	Paschal Limits.	Lunar Cycle.	Paschal Limits.
1	April 13	8	March 27	14	March 21
2	April 2	9	April 15	15	April 9
3	March 22	10	April 4	16	March 29
4	April 10	11	March 24	17	April 17
5	March 30	12	April 12	18	April 6
6	April 18	13	April 1	19	March 26
7	April 7				

By means of this table Easter may be found in the following manner. First find the golden number or lunar cycle of the year, and opposite to it, in the above table, will be found the day of the month on which the paschal full moon happens in that year. The Sunday immediately following is Easter-day, according to the Gregorian calendar. If the full moon happens on a Sunday, Easter-day will be the Sunday following.

Thus, if Easter-day 1802 were required, as the golden number of that is 17, opposite to it will be found April 17th; and as the following day, or the 18th, is a Sunday, Easter-day happens on the 18th of April.

Second Method.

Easter may be found also by means of the following table, which consists of nine columns, each divided into seven parts. The first column contains the dominical letters, the seven following the epacts, and the ninth the day on which Easter falls.

TABLE FOR FINDING EASTER.

	23	22	21	20	19			26 March
	18	17	16	15	14	13	12	2 April
A	11	10	9	8	7	6	5	9 April
	4	3	2	1	*	29	28	16 April
	27	26	25	24				23 April
	23	22	21	20	19	18		7 March
	17	16	15	14	13	12	11	3 April
B	10	9	8	7	6	5	4	10 April
	3	2	1	*	29	28	27	17 April
	26	25	24					24 April
	23	22	21	20	19	18	17	28 March
	16	15	14	13	12	11	10	4 April
C	9	8	7	6	5	4	3	11 April
	2	1	*	29	28	27	26	18 April
	25	24						25 April
	23							22 March
	22	21	20	19	18	17	16	29 March
D	15	14	13	12	11	10	9	5 April
	8	7	6	5	4	3	2	12 April
	1*	29	28	27	26	25	24	19 April
	23	22						23 March
	21	20	19	18	17	16	15	30 March
E	14	13	12	11	10	9	8	6 April
	7	6	5	4	3	2	1	13 April
	*	29	28	27	26	25	24	20 April
	23	22	21					24 March
	20	19	18	17	16	15	14	31 March
F	13	12	11	10	9	8	7	7 April
	6	5	4	3	2	1	*	14 April
	29	28	27	26	25	24		21 April
	23	22	21	20				25 March
	19	18	17	16	15	14	13	1 April
G	12	11	10	9	8	7	6	8 April
	5	4	3	2	1	*	29	15 April
	28	27	26	25	24			22 April

To use this table, the epact and dominical letter for the given year must be found. Thus if 1802 were proposed, the dominical letter of which is C, and the epact 26; look in one of the cells, opposite to that inscribed C, for the epact 26, and opposite to it will be found, in the last column on the right, the 18th of April, which is Easter-day.

Third Method.

If the epact of the proposed year does not exceed 23, subtract it from 44; and the remainder, if less than 31, will give the paschal limits in March; if greater than 31, the surplus will be the paschal limits in April.

But if the epact is greater than 23, subtract it from 43, or from 42, when it is 24 or 25; the remainder will be the day of the paschal limits in April, and the Sunday following will be Easter.

Remark.—Since all the other moveable feasts are regulated by Easter, when the day on which it falls is known, it will be easy to find the rest. Septuagesima Sunday is nine weeks or 64 days before it, both the Sundays included. Ash-Wednesday is the 47th day preceding Easter, and the Sunday following Ash-Wednesday is the first Sunday in Lent. Ascension-day is 40 days, Pentecost or Whit-Sunday is 50 days, and Trinity Sunday is 57 days, after Easter.

PROBLEM IX.

To find on what day of the week each month of the year begins.

As it has been usual in the calendars to mark the seven days of the week with the first seven letters of the alphabet, always calling the 1st of January A, the 2d B, the 3d C, the 4th D, the 5th E, the 6th F, the 7th G, and so on throughout the year; the letters answering to the first day of every month in the year, according to this disposition, may be known by the following Latin verses:

Astra Dabit Dominus, Gratisque Beabit Egenos,
Gratia Christicolæ Feret Aurea Dona Fideli.

Or by these French verses:—

Au Dieu De Gloire Bien Espere;
Grand Cœur, Faveur Aime De Faire.

Or by the well known English ones:—

At Dover Dwells George Brown Esquire,
Good Caleb Finch, And David Frier.

Where the first letter of each word is that belonging to the first day of each month, in the order from January to December.

Now, as these letters, when the dominical letter is A, indicate the day of the week by the rank which they hold in the alphabet, it is evident in that case that January begins on a Sunday, February on a Wednesday, March on a Wednesday, April on a Saturday, and so on. But when the dominical letter is not A, count either backwards or forwards from the letter of the proposed month, till you come to the dominical letter of the year, and see how many days are between them; for, as the dominical letter indicates Sunday, it will be easy, by reckoning back, to find the day of the week corresponding to the letter of the proposed month.

Thus, if it were required to find on what day of the week February 1802 began; as the dominical letter of 1802 is C, and as the letter corresponding to February is D, which is the one immediately following C, in the order of the alphabet, it is evident that February began on a Monday. In like manner if April 1802 were proposed, as the letter G which belongs to that month is the third from C, the dominical letter, it may be readily seen that April 1802 began on a Thursday.

The day of the week on which any proposed month begins, may be found also by means of the following table:—

MONTHS.	A	B	C	D	E	F	G
January	Sunday	Satur.	Friday	Thurs.	Wedn.	Tues.	Mond.
February	Wedn.	Tues.	Mond.	Sunday	Satur.	Friday	Thurs.
March	Wedn.	Tues.	Mond.	Sunday	Satur.	Friday	Thurs.
April	Satur.	Friday	Thurs.	Wedn.	Tues.	Mond.	Sunday
May	Mond.	Sunday	Satur.	Friday	Thurs.	Wedn.	Tues.
June	Thurs.	Wedn.	Tues.	Mond.	Sunday	Satur.	Friday
July	Satur.	Friday	Thurs.	Wedn.	Tues.	Mond.	Sunday
August	Tues.	Mond.	Sunday	Satur.	Friday	Thurs.	Wedn.
September	Friday	Thurs.	Wedn.	Tues.	Mond.	Sunday	Satur.
October	Sund.	Satur.	Friday	Thurs.	Wedn.	Tues.	Mond.
November	Wedn	Tues.	Mond.	Sunday	Satur.	Friday	Thurs.
December	Friday	Thurs.	Wedn.	Tues.	Mond.	Sunday	Satur.

To use this table, look for the dominical letter of the given year at the top, and in the column below it, and opposite to each month will be found the day on which it begins. Thus, as the dominical letter for 1802 is C, it will be seen, by inspecting the table, that January began on a Friday, February on a Monday, March on a Monday, April on a Thursday, and so of the rest.

PROBLEM X.

To find what Months of the year have 31 *days, and those which have only* 30.

Fig. 15.

Raise up the thumb A (Fig. 15.) the middle finger C, and the little finger E, of the left hand; and keep down the other two, viz. the fore finger B, which is next to the thumb, and the ring-finger D, which is between the middle finger and the little finger. Then begin to count March on the thumb A, April on the fore finger B, May on the middle finger C, June on the ring-finger D, July on the little finger E, and continue to count August on the thumb, September on the fore-finger, October on the middle finger, November on the ring-finger, and December on the little finger; then beginning again continue to count January on the thumb and February on the fore-finger: all those months which fall on the fingers raised up A, C, E, will have 31 days; and those which fall on the fingers kept down, viz. B and D, will have only 30, except February, which in common years has 28 days, and in bissextiles 29.

The number of the days in each month may be known also by the following lines:—

Thirty days hath September,
April, June and November;
All the rest have thirty-one,
Except February alone.

PROBLEM XI.

To find to what Month of the Year any lunation belongs.

In the Roman calendar, each lunation is considered as belonging to that month in which it terminates, according to this ancient maxim of the computists,

In quo completur, mensi lunatio detur.

Hence, to determine whether a lunation belongs to a certain month of any given year, as the month of May 1693 for example; having found, by Prob. 5, that the moon's age on the last day of May was 27; this age 27 shews that the lunation ends in the next month, that is to say in June, and consequently that it belongs to that month. It indicates also that the preceding lunation ended in the month of May, and therefore belonged to that month.

PROBLEM XII.

To determine the Lunar Years which are common, and those which are embolismic.

This problem may be readily solved by means of the preceding, from which we easily know that the same solar month may have two lunations. For two moons may end in the same month, which has 30 or 31 days, as November, which has 30; or one moon may end the first of that month, and the following moon on the last or 30th of the same month: this year then will have had 13 lunations; and consequently will be embolismic. We shall here give an example.

In the year 1712, the first moon having ended on the 8th of January, the second on the 6th of February, the third on the 8th of March, the fourth on the 6th of April, the fifth on the 6th of May, the 6th on the 4th of June, the seventh on the 4th of July, the eighth on the 2d of August, the ninth on the 1st of September, the tenth on the 1st of October, the eleventh also on the 30th of the same month, the twelfth on the 29th of November, and the thirteenth on the 28th of December; we know that this year, as it had thirteen moons, was embolismic.

We know that all the civil lunar years of the new calendar, which begin on the first of January, are embolismic, when they have for epact 29, 28, 27, 26, 25, 24, 23, 22, 21, 19; and also 18, when the golden number is 19.

Thus we know, that in the year 1693, the epact of which was 3, the lunar civil year was embolismic; that is to say, had thirteen moons: this happened because the month of August had two lunations, one of which ended on the first, and the following one on the thirtieth of the same.

PROBLEM XIII.

An easy method of finding the Calends, Nones, and Ides, of any month in the year.

The denomination of Calends, Nones, and Ides, was a singularity in the Roman Calendar; and as these terms frequently occur in classical authors, it may be useful to know how to reduce them to our method of computation. This may be easily done by means of the three following Latin verses.

Principium mensis cujusque vocato calendas:
Sex Maius nonas, October, Julius et Mars;
Quator at reliqui: dabit idus quidlibet octo.

Which have been thus translated into French:

A Mars, Juillet, Octobre et Mai
Six Nones les gens ont donné;
Aux autres mois quatre gardé;
Huit Ides à tous accordé.

The meaning of these verses is, that the first day of each month is always called *the calends;*

That in the months of March, May, July and October the *nones* are on the seventh day, and in all the other months on the fifth;

Lastly, that the *ides* are eight days after the nones, viz. on the fifteenth of March. May, July and October; and on the thirteenth of the other months.

It must now be observed that the Romans counted the other days backwards; always decreasing, and that they gave the name of nones to those days of the month which were between the calends and nones of that month; that of ides to those days which were between the nones and ides of that month; and the name of calends to those days which remained between the ides and the end of the preceding month.

Thus in the four months of March, May, July and October, where the nones had six days, the second day of the month was called *sexto nonas;* that is to say the sixth day before the nones, the preposition *ante* being here understood. In like manner the third day was called *quinto nonas;* that is to say the fifth day of the nones, or before the nones; and so of the rest. But instead of calling the sixth day of the month *secundo nonas*, they said *pridie nonas;* that is, the day preceding the nones. They said also *postridie calendas*, the day after the calends; *postridie nonas*, the day after the nones; *postridie idus*, the day after the ides.

PROBLEM XIV.

To find what day of the Calends, Nones, or Ides, corresponds to a certain day of any given month.

To solve this problem, attention must be paid to the remark already made, that all the days between the calends and the nones belong to the nones; that those between the nones and the ides bear the name of ides; and that those between the ides and calends of the following month, have the name of the calends of that month. This being premised, the following method must be pursued.

1st. If the day of the month belongs to the calends, add 2 to the number of the days in the month, and from the sum subtract the given number; the remainder will be the day of the calends.

Thus, for example, to find to what day of the Roman calendar the 25th of May corresponds, it is first to be observed that it belongs to the calends, since it is between the ides of May and the calends of June. As the month of May has 31 days, add 2 to this number, which will make 33; and if 25 be subtracted from the sum, the remainder 8 will shew that the 25th of May corresponds to the 8th of the calends of June; that is to say, the 25th of May among the Romans was called *octavo calendas Junii.*

2d. If the day of the month belongs to the ides or the nones, add 1 to the number of days elapsed between the first of the month and the ides or nones inclusively; from this sum subtract the given number, which is the day of the month, and the remainder will be exactly the day of the nones or ides.

We shall suppose, for example, that the given day is the 9th of May, which belongs to the ides; as it is between the seventh day of the nones and the fifteenth day of the ides. If 1 be added to 15, and 9 be subtracted from the sum 16, the remainder 7 will shew that the 9th of May corresponds to the 7th of the ides of that month; that is, the 9th of May among the Romans was called *septimo idus Maii.*

In like manner, if the proposed day be the 5th of May, which belongs to the nones, because it is between the 1st and 7th; add 1 to 7, and from the sum 8, subtract 5, or the given day of the month; the remainder 3, shews that the 5th of May corresponds to the 3d of the nones; or that the Romans called the 5th of May, *tertio nonas Maii.*

PROBLEM XV.

The day of the Calends, Ides, or Nones, being given; to find the corresponding day of the month.

This problem may be solved by a method similar to that employed in the preceding, but with this difference, that instead of subtracting the day of the month, to obtain that of the calends, &c., the latter is subtracted to obtain the day of the month.

Let it be required, for example, to find what day of the month corresponds to the 6th of the calends of June, which the Romans expressed by *sexto calendas Junii.* As the calends are counted in a retrograde order from the 1st of June towards the ides of May, it is evident that the 6th of the calends of June corresponds to some day in the month of May; and as that month has 31 days add 2 to 31, and from the sum 33 subtract 6, or the given day of the calends, the remainder 27 shews that the 9th of the calends of June corresponds to the 27th of May.

The same operation must be employed, in regard to the nones and the ides.

Remark.—The above two questions may be easily solved also by means of a table of the Calends, Nones, and Ides, which will be found with other tables at the end of this part.

Of the Cycle of Indiction.

The cycle of indiction is a period of fifteen years, distinguished by that name, according to some authors, because it served to indicate the year in which a certain tribute was paid to the Roman republic; and hence it is called the *Roman Indiction.*

It is called also the *Pontifical Indiction*, because employed by the court of Rome in its bulls, and in all its decrees. The following, it is said, is the origin of this custom. In the year 312, Constantine issued an edict, by which he authorised the exercise of the Christian religion throughout the whole empire. Some years after, the council of Nice was assembled, which in 328 condemned the heresy of Arius: in the space therefore of fifteen years, Christianity triumphed over persecution and heresy; and on that account it was considered as a memorable period. To preserve the remembrance of it, the cycle of indiction was established; the commencement of which was fixed at the 1st of January 313, to make it begin with the solar year; though the epoch of this cycle, according to the institution of Constantine, had been fixed at the month of September 312, the date of his edict in favour of the Christians. It was the emperor Justinian however who first ordered, that the method of computing by the indiction should be introduced into the public acts.

But, whatever may have been its origin, which Petau considers as very doubtful, it is certain that the first year of the indiction was the year 313 of the Christian æra. The year 312 therefore must have corresponded to 15 of the indiction, had this method of computation been then in use; and if 312 be divided by 15, the remainder will be 12; which shews that the 12th year of the Christian æra was the 15th of the indiction. consequently this cycle must have begun three years before the birth of Christ: or, in other words, the first year of the Christian æra corresponded to the fourth of the indiction, and hence we have a solution of the following problem.

PROBLEM XVI.

To find the number of the Roman Indiction which corresponds to any given year.

Add 3 to the given year, and divide the sum by 15: the remainder will indicate the current year of the indiction.

Let it be required, for example, to find the indiction of the year 1802. If 3 be added to 1802, we shall have 1805, and if this sum be divided by 15, the remainder will be 5. Hence it appears that the indiction for 1802 is 5.

Of the Julian Period; and some other periods of the like kind.

The Julian period is formed by combining together the lunar cycle of 19 years, the solar of 28, and the cycle of indiction of 15. The first year of this period is supposed to be that which corresponded to 1 of the lunar cycle, 1 of the solar cycle, and 1 of the cycle of indiction.

If the numbers 19, 28, and 15, be multiplied together, the product, 7980, will be the number of years comprehended in the Julian period; and we are assured by the laws of combination, that there cannot be in one revolution two of these years which have at the same time the same numbers.

This period is merely an artificial one, invented by Julius Scaliger; but it is convenient on account of its extent, as we can refer to it the commencement of all known æras, and even the creation of the world, were that epoch certain; for, according to the common chronology, it was only 3950 years before the Christian æra. But the commencement of the Julian period goes 4714 years beyond that æra; and hence it follows that the creation of the world corresponds to the year 764 of the Julian period.

The method by which it is found that the year of the birth of Jesus Christ was the 4714th of the Julian period, is as follows. It is shewn, by a retrograde calculation, that if the three cycles, viz., the solar, lunar, and that of indiction, had been in use at the birth of Christ, the year in which he was born would have been the 2d of the lunar cycle, the 10th of the solar, and the fourth of the cycle of indiction. But these characters belong to the year 4714 of the above period, as will be seen in the following problem. That year therefore must be adapted to the year of the birth of Christ; from which if we proceed backwards, calculating the intervals of anterior events, from the profane historians and sacred Scriptures, it will be found that there were 3950 years between that period and the creation of Adam. If 3950 then be subtracted from 4714, the remainder will be 764; so that the Julian period is anterior to the creation of the world by 764 years.

PROBLEM XVII.

Any year of the Julian period being given; to find the corresponding year of the lunar cycle, the solar cycle, and the cycle of indiction.

Let the given year of the Julian period be 6522. Divide this number by 19, and the remainder 5, neglecting the quotient, will be the golden number; divide the same number by 28, and the remainder 26 will be the year of the solar cycle; if 6522 be then divided by 15, the remainder 12 will indicate the indiction. If nothing remains, when the given year has been divided by the number belonging to one of these cycles, that number itself is the number of the cycle. Thus, if the year 6525 were proposed; when divided by 15, nothing remains, and therefore the indiction is 15.

But if it were required to find what year of the Christian æra corresponds to any given year of the Julian period; such for example as 6522, nothing is necessary but to subtract from it 4714; the remainder 1808 will be the number of years elapsed since the commencement of the Christian æra.

All this is so plain that it requires no farther illustration.

PROBLEM XVIII.

The Lunar and Solar Cycles and the Cycle of Indiction corresponding to any year being given; to find its place in the Julian period.

Multiply the number of the lunar cycle by 4200, that of the solar cycle by 4845, and that of the indiction by 6916.

Add together all these products, and divide the sum by 7980; the number which remains will indicate the year of the Julian period.*

Let the lunar cycle be 2, the solar 10, and the indiction 4; which is the character of the first year of the Christian æra. In this case $4200 \times 2 = 8400$; $4845 \times 10 = 48450$; and $6916 \times 4 = 27664$; the sum of these products is 84514, which divided by 7980, leaves for remainder 4714. The year therefore in the Julian period, to which the above characters correspond, is the 4714th, or the origin of the Julian period is 4713 years anterior to the Christian æra.

Remarks.—I. There is another period, called the Dionysian, which is the product of the lunar cycle 19, and the solar cycle 28; consequently it comprehends 532 years. It was invented by Dionysius Exiguus, about the time of the council of Nice, to include all the varieties of the new moons and of the dominical letters; so that, after 532 years, they were to recur in the same order, which would have been very convenient for finding Easter and the moveable feasts; but as it supposed the lunar cycle to be perfectly correct, which is not the case, this period is no longer used.

II. As among the cycles of the Julian period there is one, viz. that of indiction, which is merely a political institution—that is to say, which has no relation to the motions of the heavenly bodies—it would have been of more utility perhaps, to substitute in its place that of the epacts, which is astronomical, and contains 30 years: the number of years of the Julian period would, in this case, have been 15960. This period of 15960 years was called by the inventor of it, Father John Louis d'Amiens, a Capuchin friar, *the period of Louis the Great*. But it does not appear that it met with that reception from chronologists, which the author expected.

Of some Epochs or Periods celebrated in History.

The first of these epochs is that of the Olympiads. It takes its name from the Olympic games, which, as is well known, were celebrated with great solemnity every four years, about the winter solstice, throughout all Greece. These games were instituted by Hercules; but having fallen into disuse, they were revived by Iphitus, one of the Heraclidæ, or descendants of that hero, in the year 776 before Jesus Christ; and after that time they continued to be celebrated with great regularity; till the conquest of Greece by the Romans put an end to them. The æra or epoch of the Olympiads, begins therefore at the summer solstice of the year 776 before Christ.

* The year of the Julian period may be found also by the following general rule: Multiply the golden number by 3780, and the indiction by 1064; subtract the sum of these products from the product of 4845 multiplied by the solar cycle; divide the difference, if it can be done, by 7980, and the remainder will be the year of the Julian period.

The reason of this rule may be found in the solution of the following algebraic problem: To find a number which, divided by 28, shall leave for remainder a; divide by 19, shall leave b; and by 15, shall leave c.

Call the three quotients, arising from the division of the required number according to the terms of the problem, x, y, z. Then the number will be $= 28x + a = 19y + b = 15z + c$. From the first equation $28x + a = 19y + b$, we have $y = x + \frac{9x+a-b}{19}$. Now since $\frac{9x+a-b}{19}$ is an integer number, let us suppose it $= m$, then $m = \frac{9x+a-b}{19}$, and $x = 2m + \frac{m-a+b}{19}$, or making $\frac{m-a+b}{19} = n$, or $m = 19n + a - b$, we have, by substitution, $x = 19n + 2a - 2b$. Therefore $28x + a = 532n + 57a - 56b = 15z + c$ by the third quotient; and by resolving this equation in the same manner, putting p and q to denote the successive fractions, we shall find the number sought to be $15z + c = 7980q + 4845a - 37806b - 1064c$.

PROBLEM XIX.

To convert years of the Olympiads into years of the Christian æra, and vice versâ.

1st. To solve this problem, subtract unity from the number of the olympiads, and multiply the remainder by 4; then add to the product the number of years of the olympiad which have been completed, and from the last sum subtract 775; or, if the sum be less, subtract it from 776: in the first case, the result will be the current year of the Christian æra, and in the second, the year before that æra.

Let the proposed year, for example, be the third of the seventy-sixth olympiad. Unity subtracted from 76 leaves 75, which multiplied by 4 gives for product 300. The complete years of an olympiad, while the third is current, are 2: if 2 therefore be added to 300, we shall have 302. But as 302 is less than 775, we must subtract the former from 776, and the remainder 474, will be the current year before Jesus Christ.

As a second example we shall take the 2d year of the 201st olympiad. If 1 be subtracted from 201, the remainder is 200; which multiplied by 4 gives 800, and 1 complete year being added makes 801. But 775 subtracted from 801 leaves 26; which is the year of the Christian æra, corresponding to the 2d year of the 201st olympiad.

2d. To convert years of the Christian æra into years of the olympiads; the number of years, if anterior to the birth of Christ, must be subtracted from 776; or, if posterior to that period, 775 must be added to them: if the result be divided by 4, the quotient increased by unity will be the number of the olympiad; and the remainder, also increased by unity, will be the current year of that olympiad.

Let the proposed year, for example, be 1715. By adding 775, the sum is 2490; and this number divided by 4, gives for quotient 622, with a remainder of 2. The year 1715 therefore was the 3d year of the 623d olympiad; or more correctly, the last six months of the year 1715, with the first six months of 1716, corresponded to the 3d year of the 623d olympiad.

II. The æra of the Hegira is that used by the greater part of the followers of Mahomet: it is employed by the Arabs, the Turks, the various nations in Africa, &c.; consequently, it is necessary that those who study their history, should be able to convert the years of the hegira into those of the Christian æra, and *vice versâ.*

For this purpose, it must be first observed, that the years of the hegira are nearly lunar; and as the lunar year, or twelve complete lunations, forms 354 days 8 hours 48 minutes; if the year were always made to consist of 354 or 355 days, the new moon would soon sensibly deviate from the commencement of the year. To prevent this inconvenience, a period of 30 years has been invented, in which there are ten common years, that is to say of 354 days; and 11 embolismic, or of 355 days. The latter are the 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21st, 24th, 26th, and 29th.

It is to be observed also, that the first year of the hegira began on the 15th of July 622, of the Christian æra.

PROBLEM XX.

To find the year of Hegira which corresponds to a given Julian year.

To resolve this problem, it must first be observed that 288 Julian years form nearly 235 years of the Hegira.

This being supposed, let us take, as example, the year 1770 of the Christian æra. Now as 621 years complete of our æra had elapsed when the hegira began, we must first subtract these from 1770, and the remainder will be 1149. We must then employ this proportion: if 228 Julian years give 235 years of the hegira, how many

will 1149 give, the answer will be 1184, with a remainder of 99 days. The year 1770 therefore, of the Christian æra, corresponded, at least in part, to the year 1184 of the hegira.

On the other hand, if it be required to find the year of the Christian æra which corresponds to a given year of the hegira, the reverse of this operation must be employed: the number thence resulting will be that of the Julian years elapsed since the commencement of the hegira; and by adding 621, we shall have the current year after the birth of Christ.

We shall say nothing further on this subject, but terminate the present article with a few useful tables. The first contains the dates of the principal events recorded in history, and of the commencement of the most celebrated æras; the second is a table of the golden numbers for every year from the birth of Christ to 5600; the third a table of the dominical letters from 1700 to 5600; the fourth a table of the index letters for the same period; the fifth a table of the epacts; and the sixth a table of the calends, nones, and ides.

A TABLE

OF THE YEARS OF THE MOST REMARKABLE EPOCHS OR ÆRAS AND EVENTS.

Remarkable Events.	Julian Period.	Years of the world.	Years before Christ.
The creation of the world	706	0	4007
The Deluge, or Noah's flood	2362	1656	2351
Assyrian monarchy founded by Nimrod	2537	1831	2176
The birth of Abraham	2714	2008	1999
Kingdom of Athens founded by Cecrops	3157	2451	1556
Entrance of the Israelites into Canaan	3262	2556	1451
The destruction of Troy	3529	2823	1184
Solomon's temple founded	3701	2995	1012
The Argonautic expedition	3776	3070	937
Lycurgus formed his laws	3829	3103	884
Arbaces 1st king of the Medes	3838	3132	875
Olympiads of the Greeks began	3938	3232	775
Rome built, or Roman æra	3961	3255	752
Æra of Nabonassar	3967	3261	746
First Babylonish captivity by Nebuchadnezzar	4107	3401	606
The 2d ditto and birth of Cyrus	4114	3408	599
Solomon's temple destroyed	4125	3419	588
Cyrus began to reign in Babylon	4177	3471	536
Peloponnesian war began	4282	3576	431
Alexander the Great died	4390	3684	323
Captivity of 100,000 Jews by Ptolemy	4393	3687	320
Archimedes killed at Syracuse	4506	3800	207
Julias Cæsar invaded Britain	4659	3953	54
He corrected the calendar	4667	3961	46
The true year of Christ's birth	4709	4003	4

Christian Æra begins here.

Remarkable Events.	Julian Period.	Years of the world.	Years since Christ.
Dionysian or vulgar æra of Christ's birth	4713	4007	0
Christ crucified, Friday April 3d	4746	4040	33
Jerusalem destroyed	4783	4077	70
Adrian's wall built in Britain	4833	4127	120
Dioclesian epoch, or that of Martyrs	4997	4291	284
The council of Nice	5038	4332	325
Constantine the Great died	5050	4344	337

Remarkable Events.	Julian Period.	Years of the world.	Years since Christ.
The Saxons invited into Britain	5158	4452	445
Hegira or flight of Mohammed	5335	4629	662
Death of Mohammed	5343	4637	630
The Persian yesdegird	5344	4638	631
Sun, moon, and planets seen from the earth	5899	5193	1186
Art of printing discovered	6153	5447	1440
Constantinople taken by the Turks	6166	5460	1453
Reformation begun by Martin Luther	6230	5524	1517
The calendar corrected by Pope Gregory	6295	5589	1582
Sir Isaac Newton born	6355	5649	1642
Made president of the Royal Society	6416	5710	1703
Died, March 20th	6440	5734	1727

TABLE OF SOME OTHER REMARKABLE EVENTS, RELATING CHIEFLY TO THE ARTS AND SCIENCES.

	A. D.
Use of bells introduced into churches	605
Alexandrian library destroyed, and Egypt conquered by the Saracens	641
Organs first used in churches	660
Glass invented by a bishop, and brought to England by a Benedictine monk	663
Arabic ciphers introduced into Europe by the Saracens	991
Astronomy and Geography brought to Europe by the Moors	1120
Silk manufacture introduced at Venice from Greece	1209
Spectacles invented by a monk of Pisa	1299
The mariner's compass invented or improved by Flavio	1302
Gunpowder invented by a monk of Cologne	1330
The art of weaving cloth brought from Flanders to England	1331
Cannon first used in the English service by the Governor of Calais	1383
First company of linen-weavers settled in England	1386
Cards invented for the amusement of the French king	1391
Algebra brought to Europe from Arabia	1400
Great guns first used in England at the siege of Berwick	1405
Paper made of linen rags invented	1417
Printing invented in Germany	1441
Engraving and etching invented	1459
Cape of Good Hope discovered	1488
Geographical maps and sea charts brought to England	1489
America discovered by Columbus	1492
Algebra taught by a Friar at Venice	1495
First voyage round the world by Magellan	1522
Variation of the compass discovered by Cabot	1540
Iron cannon and mortars made in England	1543
Glass first manufactured in England	1557
First proposal of settling a colony in America	1583
Bomb-shells invented at Venloo	1588
Telescopes invented by Jansen, a spectacle-maker of Holland	1590
Art of weaving stockings invented by Lee in Cambridge	1590
Watches brought to England from Germany	1597
Thermometers invented by Drebbel, a Dutchman	1610
Galileo first observed three of Jupiter's satellites, January 7th	1610
Logarithms invented by Lord Napier of Scotland	1614
Circulation of the blood discovered by Hervey	1619
Gazettes first published at Venice	1630

Transit of Mercury over the sun's disc first observed by Gassendi, Nov. 17th.	1631
Galileo condemned by the inquisition	1633
French academy established, January	1635
Transit of Mercury observed by Cassini, Nov. 11th	1636
Polemoscope invented by Hevelius	1637
Transit of Venus observed by Horrox, November 24th	1639
Barometers invented by Torricelli	1643
Royal academy of painting founded by Louis XIV.	1643
Galileo first applied the pendulum to clocks	1649
Air-pump invented by Otto Gueric of Magdeburg	1654
Huygens first discovered a satellite of Saturn, March 25th	1655
Royal Society of London established, July 15th	1663
Royal Academy of inscriptions and belles-lettres founded	1663
Academy for sculpture established in France	1664
The observatory of Paris founded	1664
Magic lantern invented by Kircher	1665
Academy of sciences established in France	1666
Cassini discovered 4 of Saturn's satellites in the course of a few years	1671
The royal observatory at Greenwich built	1676
The anatomy of plants made known by Grew	1680
The Newtonian philosophy was published	1686
The academy of sciences founded at Berlin	1701
Academy of sciences established at Petersburgh	1724
Aberration of the fixed stars discovered and accounted for by Bradley	1727
Transit of Mercury observed by Cassini, Nov. 11th	1736
Academy of sciences founded at Stockholm	1750
New style introduced into Great Britain, Sept. 3d, being reckoned Sept. 14th	1752
British Museum established at Montague-House, by an act of parliament	1753
Transit of Venus over the sun, June 6th	1760
Royal academy of arts established at London	1768
Transit of Venus over the sun's disk, June 3d, 1769	1769

EMINENT BRITISH PHILOSOPHERS AND MATHEMATICIANS.

	Died.
Arbuthnot, John, M.D.	1705
Bacon, Roger, philosopher	1294
Bacon, Lord, ditto	1626
Barrow, Isaac, mathematician	1677
Boyle, Robert, phil.	1691
Brerewood, Edward, phil. and math.	1613
Briggs, Henry, math.	1630
Cheyne, George, phys. and phil.	1748
Clark, Samuel, phil. and math.	1729
Cook, James, navigator	1779
Derham, William, philosopher	1735
Dudley, Sir Robert, phil. and math.	1639
Evelyn, John, phil.	1706
Ferguson, James, phil. and mech.	1776
Graham, George, math. and mech.	1751
Gregory, James, prof. St. Andrew's	1675
Gregory, David, prof. Oxford, astronomy	1708
Gunter, Edmund, astron.	1626
Hales, Stephen, phil.	1761
Halley, Edmund, astron.	1742

	Died.
Harriot, Thomas, math.	1621
Harrison, John, inventor of the time-keeper	1776
Harvey, William, phys. discoverer of the circulation of the blood	1657
Horrox, Jeremiah, astron.	1641
Keil, John, math. and astron.	1721
Locke, John, phil.	1704
Long, Robert, astron.	1770
Lyons, Israel, math.	1775
Maclaurin, Colin, math.	1746
Newton, Sir Isaac, math. and phil.	1727
Pell, John, math.	1685
Pemberton, Henry, phil.	1771
Ray, John, phil.	1705
Simpson, Thomas, math.	1761
Watts, Isaac, phil. and math.	1748
Whiston, William, astron.	1752
Wilkins, John, phil.	1672
Wren, Sir Christopher, math.	1723

TABLE OF THE GOLDEN NUMBERS,

FOR EVERY YEAR SINCE THE BIRTH OF CHRIST TO THE YEAR 5600.

The centenary years; that is, the last years of each century.						0	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800
						1900	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000	3100	3200	3300	3400	3500	3600	3700
						3800	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800	4900	5000	5100	5200	5300	5400	5500	5600
Intermediate years.						GOLDEN NUMBERS.																		
						1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15
1	20	39	58	77	96	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16
2	21	40	59	78	97	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17
3	22	41	60	79	98	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18
4	23	42	61	80	99	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19
5	24	43	62	81		6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1
6	25	44	63	82		7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2
7	26	45	64	83		8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3
8	27	46	65	84		9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4
9	28	47	66	85		10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5
10	29	48	67	86		11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6
11	30	49	68	87		12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7
12	31	50	69	88		13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8
13	32	51	70	89		14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9
14	33	52	71	90		15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10
15	34	53	72	91		16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11
16	35	54	73	92		17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12
17	36	55	74	93		18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13
18	37	56	75	94		19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14
19	38	57	76	95		1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15

TABLE OF THE DOMINICAL LETTERS,

FROM 1700 TO 5600.

Centenary years; that is, the last years of each century.	1700 2100 2500 2900 3300 3700 4100 4500 4900 5300	1800 2200 2600 3000 3400 3800 4200 4600 5000 5400	1900 2300 2700 3100 3500 3900 4300 4700 5100 5500	2000 2400 2800 3200 3600 4000 4400 4800 5200 5600
Intermediate years.	C	E	G	BA
1 29 57 85	B	D	F	G
2 30 58 86	A	C	E	F
3 31 59 87	G	B	D	E
4 32 60 88	FE	AG	CB	DC
5 33 61 89	D	F	A	B
6 34 62 90	C	E	G	A
7 35 63 91	B	D	F	G
8 36 64 92	AG	CB	ED	FE
9 37 65 93	F	A	C	D
10 38 66 94	E	G	B	C
11 39 67 95	D	F	A	B
12 40 68 96	CB	ED	GF	AG
13 41 69 97	A	C	E	F
14 42 70 98	G	B	D	E
15 43 71 99	F	A	C	D
16 44 72	ED	GF	BA	CB
17 45 73	C	E	G	A
18 46 74	B	D	F	G
19 47 75	A	C	E	F
20 48 76	GF	BA	DC	ED
21 49 77	E	G	B	C
22 50 78	D	F	A	B
23 51 79	C	E	G	A
24 52 80	BA	DC	FE	GF
25 53 81	G	B	D	E
26 54 82	F	A	C	D
27 55 83	E	G	B	C
28 56 84	DC	FE	AG	BA

TABLE OF THE INDEX LETTERS,

FROM 1700 TO 5600.

C	1700	Metemptosis.*	p	3700	Met.
C	1800	Met. & proemptosis.†	n	3800	Met.
B	1900	Met.	n	3900	Met. and proem.
B	2000	Bissextile.	n	4000	Bissextile.
B	2100	Met. and proem.	m	4100	Met.
A	2200	Met.	l	4200	Met.
u	2300	Met.	l	4300	Met. and proem.
A	2400	Bissext. and proem.	l	4400	Bissextile.
u	2500	Met.	k	4500	Met.
t	2600	Met.	k	4600	Met. and proem.
t	2700	Met. and proem.	i	4700	Met.
t	2800	Bissextile.	i	4800	Bissextile.
s	2900	Met.	i	4900	Met. and proem.
s	3000	Met. and proem.	h	5000	Met.
r	3100	Met.	g	5100	Met.
r	3200	Bissextile.	h	5200	Bissext. and proem.
r	3300	Met. and proem.	g	5300	Met.
q	3400	Met.	f	5400	Met.
p	3500	Met.	f	5500	Met. and proem.
q	3600	Bissext. and proem.	f	5600	Bissextile.

* Metemptosis, or the solar equation, is the suppression of a day. There was a metemptosis in the year 1800, because that year, which ought naturally to have been bissextile, was not so. Since the reformation of the calendar it takes place three times in 400 years.

† Proemptosis, or the lunar equation, is the anticipation of the new moon. There is a proemptosis in about every 300 years, because the new moon takes place then a day sooner than it ought to do.

TABLE OF THE EPACTS, FROM THE YEAR 1700 TO 5600.

Index Letters.	GOLDEN NUMBERS.																		
	i	ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii	xiii	xiv	xv	xvi	xvii	xviii	xix
	EPACTS.																		
C	•	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii
B	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	•	xi	xxii	iii	xiv	xxv		xvii
A	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi
u	xxvii	viii	xix	•	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv
t	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	•	xi	xxii	iii	xiv
s	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii
r	xxiv	v	xvi	xxvii	viii	xix	•	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii
p	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	•	xi
q	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x
n	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	•	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix
m	xx		xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii
l	xix	•	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii
k	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	•	xi	xxii	iii	xiv	xxv	vi
i	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v
h	xvi	xxvii	viii	xix	•	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv
g	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	•	xi	xxii	iii
f	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii

A TABLE OF THE CALENDS, NONES, AND IDES.

Days of the Month.	April, June, Sept., November.	January, August, December.	March, May, July, October.	February.
1	Calendæ.	Calendæ.	Calendæ.	Calendæ.
2	IV	IV	VI	IV
3	III	III	V	III
4	Prid. Non.	Prid. Non.	IV	Prid. Non.
5	Nonæ.	Nonæ.	III	Nonæ.
6	VIII	VIII	Prid. Non.	VIII
7	VII	VII	Nonæ.	VII
8	VI	VI	VIII	VI
9	V	V	VII	V
10	IV	IV	VI	IV
11	III	III	V	III
12	Prid. Id.	Prid. Id.	IV	Prid. Id.
13	Idus.	Idus.	III	Idus.
14	XVIII	XIX	Prid. Id.	XVI
15	XVII	XVIII	Idus.	XV
16	XVI	XVII	XVII	XIV
17	XV	XVI	XVI	XIII
18	XIV	XV	XV	XII
19	XIII	XIV	XIV	XI
20	XII	XIII	XIII	X
21	XI	XII	XII	IX
22	X	XI	XI	VIII
23	IX	X	X	VII
24	VIII	IX	IX	VI
25	VII	VIII	VIII	V
26	VI	VII	VII	IV
27	V	VI	VI	III
28	IV	V	V	Prid. Cal.
29	III	IV	IV	Martii.
30	Prid. Cal.	III	III	
31	Mensis sequentis.	Prid. Cal. Mens. seq.	Prid. Cal. Mens. seq.	

USE OF THE FOREGOING TABLES.

1st. *Table of the Golden Numbers.*

This table contains the centenary years, that is to say, the last years of each century, arranged in cells at the top, and the intermediate years in the ten cells on the left hand. The centenary years which have the same golden number, are placed in different cells, but below each other in a line, as 1800, 3700, 5600. The golden numbers belong, some to the centenary years, and others to the intermediate years. The former are placed in a row by themselves below the centenary years, and are as follow: 1, 6, 11, 16, 2, 7, 12, 17, 3, 8, 13, 18, 4, 9, 14, 19, 5, 10, and 15. The latter will be found in a line with the intermediate years distributed in 30 different cells.

I. Now to find the golden number of a centenary year, for example 1800; first look for the centenary year in the cell to which it belongs, and immediately below it, in the row at the bottom standing by itself, will be found 15, which was the golden number of that year.

II. To find the golden number of an intermediate year, 1802 for example. Find the centenary year 1800 in its proper cell, and the intermediate year 2 in the cells on the left hand; then on a line with 2, and exactly below 1800, will be found 17, the golden number of 1802.

2d. *Table of the Dominical Letters.*

The centenary years are arranged in this table, as in the preceding, in the four cells at the top, and the intermediate years in the seven cells on the left. All the centenary years which have the same dominical letter are arranged together in one cell. Those which have C for dominical letter in the first, those which have E in the second, those which have G in the third, and those which have B A in the fourth. As in 40 centenary years, the number comprehended in this table, there are 10 bissextiles, these 10 years have been placed in the fourth cell, and the other 30 in the first three. The intermediate years placed horizontally in the same cell, differ by 28 years, because the solar cycle contains only that number. Thus the difference between 1 and 29 in the first cell, is 28, and the case is the same with 29 and 57, &c. Each collateral cell contains four perpendicular rows, consisting each of four numbers, because a bissextile recurs every four years. The four first dominical letters, in the four upper cells, viz., B, D, F, G, correspond to the numbers 1, 29, 57, 85, in the first cell of intermediate years; the case is the same with the dominical letters in the next row, A, C, E, F, in regard to the numbers 2, 30, 58, 86; and so on throughout the table.

I. To find the dominical letter of a centenary year, 1800 for example. Look for 1800, which stands in the second cell at the top, and immediately below it will be found the letter E.

II. To find the dominical letter of an intermediate year, as 1802. First find the centenary year 1800 in its proper cell; then look for 2 among the intermediate years, on a line with which, and below the cell containing 1800, will be found the letter C.

3. *Table of the Index Letters, and Table of the Epacts.*

The use of the first of these tables will readily appear, when we have explained the nature of the second. The table of the epacts contains the golden numbers in the horizontal column at the top; the index letters are arranged in the first perpendicular column, and the epacts in columns parallel to it. Now if the epact of any year be required; first find the golden number of the proposed year, and in the table of index letters, the letter corresponding to the century; then look for the same letter in the table of the epacts, and also for the golden number at the top; and on a line with the index letter, and directly below the golden number, will be found the epact required.

Let it be proposed, for example, to find the epact of 1802, the golden number of which is 17. Look in the table of the index numbers, and it will be found that the letter corresponding to 1800 is C; then find C in the first column on the left of the table of epacts; and on a line with it, and directly below xvii among the golden numbers, will be found xxvi, the epact of the year 1802. The epact of any other year, till the year 5600, may be found in like manner.

4th. *Table of the Calends, Nones, and Ides.*

This table requires little explanation: look for the given month at the top, and in the column below it, and opposite to the proposed day, will be found the corresponding day of the Roman calendar. The day of our calendar, corresponding to any given day of the Roman calendar, may be found with the same ease.

PART SEVENTH.

CONTAINING THE MOST USEFUL AND INTERESTING PROBLEMS IN GNOMONICS OR DIALLING.

GNOMONICS, or Dialling, is the art of tracing out on a plane, or even on any surface whatever, a sun-dial; that is, a figure, the different lines of which, when the sun shines, indicate, by the shadow of a style, the different hours of the day. This science depends therefore on Geometry and Astronomy, or at least on a knowledge of the sphere.

As many people construct sun-dials without having a clear idea of the principle which serves as a basis to this part of the Mathematics, it may not be improper to begin with an explanation of it.

The General Principle of Sun-dials.

Conceive a sphere, with its twelve horary circles or meridians, which divide the equator, and consequently all its parallels, into 24 equal parts. Let this sphere be placed in a situation suited to the position of the dial; that is, let its axis be directed to the pole of the place for which the dial is constructed, or elevated at an angle equal to the latitude. Now if we suppose a horizontal plane cutting the sphere through its centre, the axis of the sphere will represent the style, and the different intersections of the horary circles with that plane will be the hour-lines; for it is evident, that if the planes of these circles were infinitely produced, they would form in the celestial sphere the horary circles, which divide the solar revolution into twenty-four equal parts. When the sun therefore has arrived at one of these circles, that of three in the afternoon for example, he will be in the plane of the similar circle of the sphere above mentioned; and the shadow of the axis or style will fall upon the line of intersection which that circle forms with the horizontal plane: this line then will be the line of 3 o'clock; and so of the rest.

Fig. 1.

B

All this is illustrated in Fig. 1, which represents a part of the sphere, with six of the horary circles. P *p* is the axis, in which all these circles intersect each other; A H B *h* the horizontal plane, or horizon of the sphere, indefinitely continued; A B the meridian; D E the diameter of the equator, which is in the meridian; D H E *h* the circumference of the equator, of which D H E is a half, and D H a quarter. This quarter of the equator is divided into six

equal parts, D 1, 1 2, 2 3, 3 4, 4 5, 5 6, and through these pass the horary circles, the planes of which evidently cut the horizon in the lines C 1, C 2, C 3, C 4, C 5, C 6: these are the hour-lines, and if we suppose them continued to A F, which is perpendicular to the meridian C A, they will give the hour-lines C I, C II, C III, C IV, C V, C VI. The style will be a portion C S of the axis of the sphere; which consequently ought to form with the meridian, and in its plane, an angle S C A, equal to the height of the pole or P C A.

Should this reasoning appear too dry and tedious, another method may be employed to acquire a clear idea of the principles of dialling. Construct a solid sphere, divided by its twelve horary circles, and cut it in such a manner that one of its poles shall form with the plane of the section an angle equal to the height of the pole of the given place.

If the sphere, cut in this manner, be then made to rest on a horizontal plane, with its pole directed towards the pole of the world, the points where the horary circles intersect the horizontal plane, will be readily seen; and the common section of all the circles, which is the axis, will shew the position of the style.

For the sake of illustration, we have here supposed the section of the sphere to be formed by a horizontal plane; but if the plane were vertical, the case would be similar, and the lines of intersection would be the hour-lines of a vertical dial. If the plane be declining or inclining, we shall have a declining or inclining dial: it may even be easily seen that this holds good in regard to every surface, whatever be its form, convex, concave or irregular, and whatever may be its position.

The style is an iron rod, generally placed in an inclined direction, the shadow of which serves to point out the hours: as before said, it is a portion C S of the axis of the sphere; and in that case it shews the hour by the shadow of its whole length.

An upright style, however, such as S Q, is sometimes given to dials; but in that case it is only the shadow of the summit S that indicates the hour, because this summit is a point of the axis of the sphere.

The centre of the dial is the point C, where all the hour-lines meet. It sometimes happens however that these do not meet. This is the case in dials which have their plane parallel to the axis of the sphere; for it is evident that in such dials the intersections of the horary circles must be parallel lines. These dials are called *dials without a centre*. Vertical east and west dials, and dials turned directly towards the south, and inclined to the horizon at an angle equal to the latitude, or which if produced would pass through the pole, are of this number.

The meridian line, as is well known, is the intersection of the plane of the meridian with the plane of the dial; when the plane of the dial is vertical, it is always perpendicular to the horizon.

The substylar line is that marked out by the plane perpendicular to the plane of the dial, passing through the style. As this line is of great importance in declining dials, it is necessary to have a very distinct idea of it. For this purpose, conceive a perpendicular let fall on the plane of the dial, from any point in the style; and that a plane is made to pass through the style and the perpendicular: this plane, which will necessarily be perpendicular to that of the dial, will cut it in a line passing through the centre, and through the bottom of the perpendicular, and this line will be the substylar line

This line is the meridian of the plane; that is, it shews the moment at which the elevation of the sun above that plane is greatest. Care however must be taken not to confound this meridian with the meridian of the place, or the south line of the dial; for the latter is the intersection of the plane of the dial with the meridian of the place, which is the plane passing through the zenith of the place and the pole; whereas the meridian of the plane of the dial, is the intersection of that plane

with the meridian, or the horary circle passing through the pole and the zenith of the plane.

In the horizontal plane, or any other which has no declination, the substyle and the meridian of the place coincide; but in every plane not turned directly towards the south or the north, these lines form greater or less angles.

Lastly, the equinoctial is the intersection of the plane of the equator with the dial: it may easily be seen that this line is always perpendicular to the substyle.

PROBLEM I.

To find the Meridian Line on a horizontal plane.

To find the meridian line is the basis of the whole art of constructing sun-dials; but as it is at the same time the basis of all astronomical operations, and as we have already treated of it in that part of this work which relates to astronomy, it would be needless to repeat here what has been already said on the subject. We shall therefore confine ourselves to one ingenious and little-known operation.

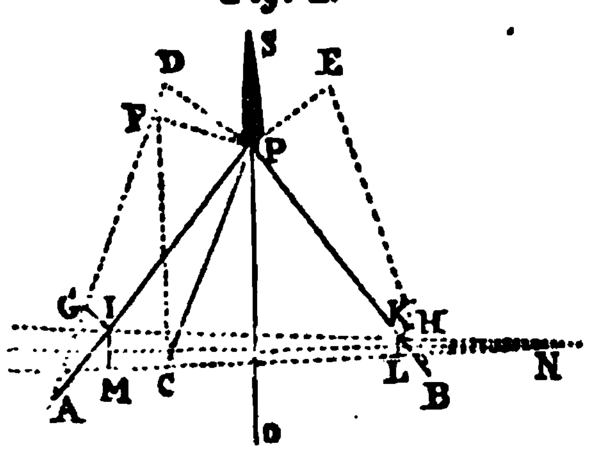
We shall give also hereafter a method of determining the position of the meridian line at all times, and in all places, provided the latitude be known.

PROBLEM II.

To find the Meridian by the observation of three unequal shadows.

The meridian line on a horizontal plane is found generally by means of two equal shadows of a perpendicular style; the one observed in the forenoon and the other in the afternoon. For this purpose, several concentric circles are described from the bottom of the style; but notwithstanding this precaution, it may happen that it will be impossible to have two shadows equal to each other. This inconvenience however may be remedied by three observations instead of two. For this ingenious method, we are indebted to a very old author on Gnomonics, named *Muzio oddi da Urbino*, who published it in a treatise entitled "Gli Orologi solari nelle superficie piane." This author was exceedingly devout; for he piously thanks our Lady of Loretto for having communicated to him, by inspiration, the precepts he has taught in his work. The operation is as follows.

Fig. 2.



Let P (Fig. 2.) be the bottom of the style, and PS its height; and let three shadows projected by it be PA, PB, and PC; which suppose to be unequal, and let PC be the shortest of them. From the point P draw PD, PE, and PF, perpendicular to PA, PB, and PC, and all equal to each other, as well as to PS. Draw also the lines DA, EB, and FC, on the two largest of which, viz. DA and EB, assume DG and EH equal to FC; then from G and H draw GI and HK, perpendiculars to PA and PB, and join the points I and K by an indefinite line: make IM and KL perpendicular to IK, and equal to GI and KH; and draw ML, which will meet IK in the point N: if through N and C the line CN be drawn, it will be perpendicular to the meridian; consequently by drawing, from P, the line PO perpendicular to CN, it will be the meridian required.

As the demonstration of this problem would be too long, we must refer the reader to the fifth book of a work by Schootten, entitled "Exercitationes Mathematicæ."

PROBLEM III.

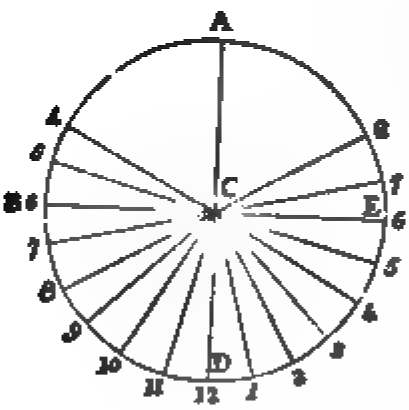
To find the Meridian on a plane, or the substylar line.

After what has been already said, in regard to the substylar line, this operation will be easy; for since this line is the meridian of the plane, nothing is necessary but to consider it as if it were horizontal, and to trace out on it the meridian by the same method: the line resulting will be the substyle, the determination of which is very necessary for constructing inclined or declining dials, and those which are both at the same time.

PROBLEM IV.

To describe an Equinoctial Dial.

Fig. 4.



From any point C (Fig. 3.) as a centre, describe a circle A E D B; and having drawn the two diameters intersecting each other at right angles in the centre C, divide each quadrant into six equal parts; and draw the radii C 1, C 2, C 3, and so on, as seen in the figure. These radii will shew the hours by means of a style perpendicular to the plane of the dial, which must be placed in the plane of the equator; that is, in such a manner as to form with the horizon an angle equal to the complement of the latitude. The line A D must coincide with the plane of the meridian, and in north latitude the point A must be directed towards the south.

Remarks.—I. When this equinoctial dial is erected, if the hour-lines look towards the heavens, it is called a *superior* dial, but if they are turned towards the earth, an *inferior*.

II. A superior equinoctial dial shews the hours of the day only in the spring and summer; and an inferior one only during the autumn and winter; but at the equinoxes, when the sun is in the equator, or very near it, equinoctial dials are of no use, as at those periods they are never illuminated by the sun.

III. At London the elevation of the plane of the equator is 38° 29′, which is the complement of the elevation of the pole: the angle therefore which the plane of an equinoctial dial at London should form with the horizon, ought to be 38° 29′.

IV. It hence appears that it is easy to construct an universal equinoctial dial, which may be adjusted to any elevation of the pole whatever. For this purpose, join together two pieces of ivory, or copper, or any other matter, A B C D and C D E F, (*Fig.* 4), by means of a hinge at C D: then describe on the two surfaces of the piece A B C D, two equinoctial dials; and in the centre I place a style extending both ways in a direction perpendicular to A B C D. At G, in the middle of the piece C D E F, fix a magnetic needle, covered with a plate of glass, and towards the edge of the same piece apply a quadrant H L divided into degrees, and passing through an aperture H, made to receive it in the upper piece A B C D. The degrees and minutes must begin to be counted from the point L.

Fig. 4.

E

F C

When this dial is to be used, place the needle in the meridian, making a proper allowance for the variation; and cause the two pieces A B C D and C D E F to form an angle B C F, equal to the elevation of the equator at the given place; that is, equal

to the complement of the latitude. If care be then taken to turn the quadrant towards the south, either of these equinoctial dials will shew the hours at that place, except on the day of the equinox.

PROBLEM V.

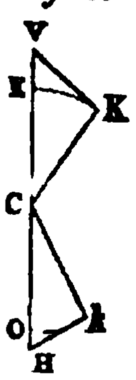
Construction of the most important of the other regular Dials.

Regular dials are those which have the hour lines, forming equal angles on each side of the meridian: these dials therefore are, the equinoctial, the horizontal, the north and south vertical, and the polar. Having already spoken of the equinoctial and horizontal, we shall now proceed to the north and south vertical dials.

Of the South Vertical Dial.

If the vertical dial be turned directly towards the south; then make the angle E C K or the arc E K (Fig. 5.) equal to the height of the pole; if C K V be then made a right angle, the point V will be the centre of the dial; and the angle C V K, which will then be equal to the complement of the latitude or of the elevation of the pole, will denote the angle which the style, in the plane of the meridian, ought to form with the plane of the dial.

Fig. 5.



Of the North Vertical Dial.

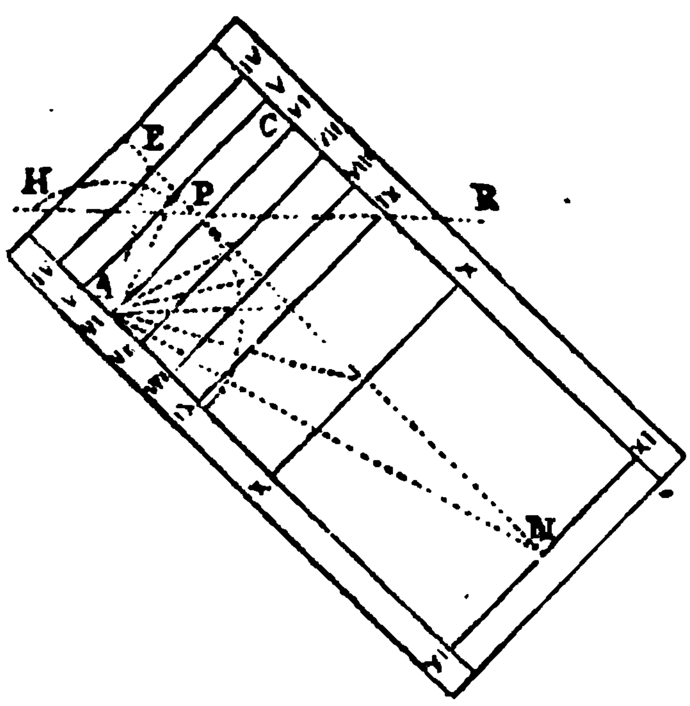
If the vertical dial be north, make, as before, the angle O C *k* (Fig. 5.) equal to the height of the pole, and the angle C *k* H a right angle: the point H will be the centre of the dial; and the angle C H *k* will be that which the style forms with the meridian. The style, instead of being inclined downwards, must be turned in a contrary direction, as may be readily conceived when we consider the position of the pole in regard to a vertical plane turned directly towards the north.

PROBLEM VI.

Of Vertical East and West Dials.

Next to the dials already described, the simplest are those which directly front the east or the west. The method of constructing them is as follows:—

Fig. 6.

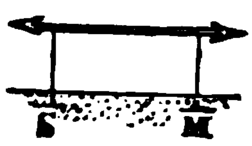


Draw the horizontal line H R, (Fig. 6.) and assume it in any point P, for the bottom of the style, the upper extremity of which is intended to shew the hours. At the point P, make, towards the left for an east dial, and towards the right for a west one, the angle H P E, equal to the complement of the latitude of the pole above the horizon; and continue E P to N. The line E N will be the equinoctial. Then through the point P draw the line C A, in such a manner as to form with the line H R the angle A P H, equal to the elevation of the pole; then A C, which will intersect the equinoctial E N at right angles, will be the hour-line of VI in the morning, and also the substylar line.

When these lines have been traced out, the hour lines may be drawn in the following manner. In the substylar line A C, assume a point A, at any distance from the point P according to the intended size of the dial; and from A, as a centre, describe a semicircle of any radius at pleasure. Divide this semicircle into twelve equal parts,

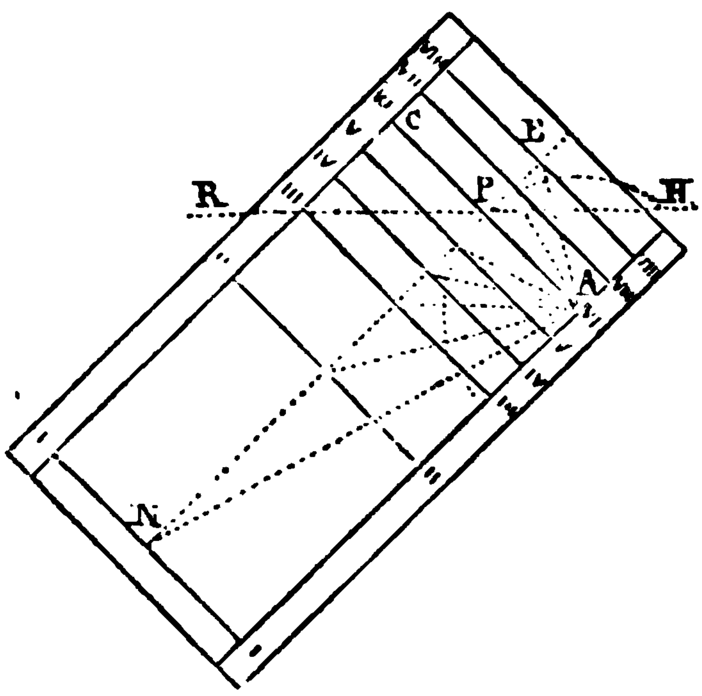
beginning at the point P, and then from the centre A draw dotted lines through each of the points of division in the semicircle, till they meet the equinoctial E N: if lines parallel to the substylar line be then drawn through the points where these dotted lines cut the equinoctial, they will be the hour lines required, the substylar line being that of VI in the morning. The parallels above the substylar line, in the east dial, will correspond to IV and V in the morning; those below it to VII, VIII, &c. in the afternoon.

Fig. 7.



The style, the figure of which is seen in Fig. 7. is placed parallel to the line of VI, on two supports raised perpendicular to the plane of the dial, and at a distance above it equal to that of VI. hours from III or from IX. It is here evident that a west is exactly the same as an east dial, only in a contrary situation (Fig. 8.); but instead of marking on it the morning hours, as IV, V, VI, &c., you must inscribe on it those of the afternoon, as I, II, III, IV, &c. If an east dial be traced out on a piece of oiled paper, and if the paper be then inverted, but not turned upside down, on holding it between you and the light, you will see a west dial.

Fig. 8.

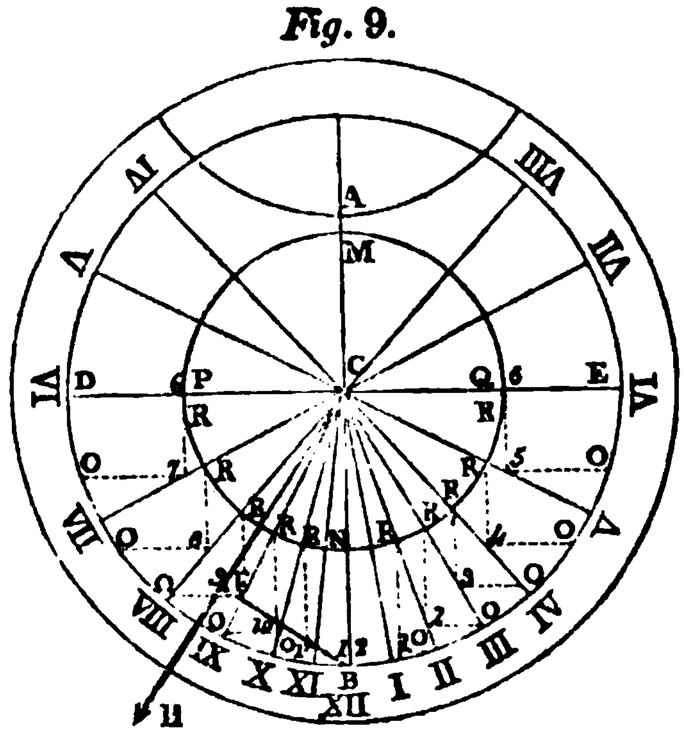


It may be easily seen that these dials cannot shew the hour of noon: for the sun does not begin to illuminate the latter till that hour, and the former ceases to be illuminated at the same period.

PROBLEM VII.

To describe a horizontal or a vertical South Dial, without having occasion to find the horary points on the equinoctial.

Fig. 9.



Let the line A B (Fig. 9.) be the meridian of the dial, which we suppose a horizontal one; and let C be its centre: make the angle H C B equal to the elevation of the pole, in order to find the position of the style, and from the point B, assumed at pleasure; but in such a manner that C B shall be of a proper length, draw B F perpendicular to C H. If we conceive the triangle B F C raised vertically above the plane of the dial, it will represent the style.

From the point C, with the radius C B, describe a circle B D A E; and from the same centre, with the radius B F, describe another circle M Q N P.

Divide the whole circumference of the first circle into 24 equal parts, B O, O O, O O, &c., and then divide the second circle into the same number of equal parts, N R, R R, &c.: from the points of division O, of the great circle, draw lines perpendicular to the meridian; and from the corresponding points R of the less circle, draw lines parallel to that meridian. These parallels and perpendiculars will meet in certain points, which will serve to determine the hour-lines. For example, the lines O 3, R 3, which proceed from the third of the corresponding points of divison, will meet

in the point 3; through which if C 3 be drawn, it will be the position of the line of 3 o'clock; and so of the rest.

It is evident that the larger the circles, the more distinct will be the intersections formed by the lines drawn through the points of division O and R.

It is remarkable that all these points of intersection are found in the circumference of an ellipse, the greater axis of which is equal to twice C R; and the less P Q to twice C N, or twice B F.

The reason of this construction will be easily discovered by geometricians.

PROBLEM VIII.

To trace out a Dial on any plane whatever, either vertical or inclined, declining or not, on any surface whatever, and even without the sun shining.

This problem, as may be seen, comprehends the whole of Gnomonics; and the operation may be practised by any person who knows how to find the meridian, and to construct an equinoctial dial. The solution of it is as follows.

Fig. 10.

Having made the necessary preparation (Fig. 10.), trace out a meridian line on a table, according to the method taught in the first problem; and by means of this meridian, place an equinoctial dial in such a situation, that the plane of it shall be raised at the proper angle; that is, at an angle equal to the elevation of the equator, or complement of the latitude, and that its south line shall coincide with the above meridian. Adjust along the axis a piece of packthread, which being stretched shall meet the plane on which the dial is to be described: the point where it meets this plane is that where the style or axis ought to be placed, so as to form one straight line with the packthread and the style of the equinoctial dial.

When this is done, and when the axis of the dial has been fixed, hold a candle or taper before the equinoctial dial, in such a manner, that the style shall shew noon; the shadow projected, at the same time, by the packthread, or the axis of the dial about to be constructed, will be the south line. You must therefore assume a point which, together with the centre, will determine that line. If you then change the position of the taper, so that the equinoctial dial shall shew one o'clock, the shadow projected by the packthread, or the axis of the proposed dial, will be the hour-line of 1; and so of the rest.

Remarks.—I. If the plane, on which the dial is to be described, be situated in such a manner that it cannot be met by the axis continued, according to the preceding method, two supporters must be affixed to the plane, for the purpose of receiving a rod of iron, so as to make one line with the packthread; and the operation may then be performed as above described.

II. Instead of an equinoctial dial, a horizontal one may be employed; provided it be placed in such a manner, that the south line corresponds with the meridian which has been traced out.

III. This operation may be performed in the day time when the sun shines. In

this case you must employ a mirror, the reflection of which will produce the same effect as the taper or candle.

PROBLEM IX.

To describe a Vertical Dial on a pane of glass, which will shew the hours without a style by means of the solar rays.

Ozanam relates that he once constructed a vertical declining dial on a pane of glass in a window, which had no style; and by which the hours could be known when the sun shone.

I detached, says he, from the window frame on the outside a pane of glass, and described upon it a vertical dial, according to the declination of the window and the height of the pole above the horizon; taking as the height of the style the thickness of the window frame. I then fixed the pane of glass against the frame in the inside; having given to the meridian line a situation perpendicular to the horizon, as it ought to have in vertical dials. I then cemented to the window frame on the outside, opposite to the dial, a piece of strong paper, not oiled, in order that the surface of the dial might be more obscure. And that I might be able to know the hours without the shadow of a style, I made a small hole in the paper with a pin, opposite to the bottom of the style, which I had marked out. As this hole represented the extremity of the style, the rays of the sun passing through it formed on the glass a luminous point; which, while the rest of the dial was obscure, indicated the hours in an agreeable manner.

PROBLEM X.

In any latitude, to find the Meridian by one observation of the sun, and at any hour of the day.

Provide an exact cube, each side of which is about 8 inches; and describe on the upper face a horizontal dial, adapted to the latitude of the place. On the vertical face, which stands at right angles to the meridian of this dial, describe a vertical one; on the adjacent face to the left an east dial, and on the opposite one a west dial, each of which must be furnished with the proper style.

When you are desirous of finding the meridian on a horizontal plane, place this quadruple dial on it, so that the vertical one shall nearly face the south; and gradually turn it till three of these dials all shew the same hour: when this takes place, you may be assured that the three dials are in their proper position. If a line be then drawn with a pencil, or other instrument, along one of the lateral sides of the cube, it will be in the true direction of the meridian.

It is indeed evident that these three dials cannot shew the same hour, unless they are all placed in a proper position in regard to the meridian; their concurrence therefore will shew that they are properly placed; and that their common meridian is the meridian of the place.

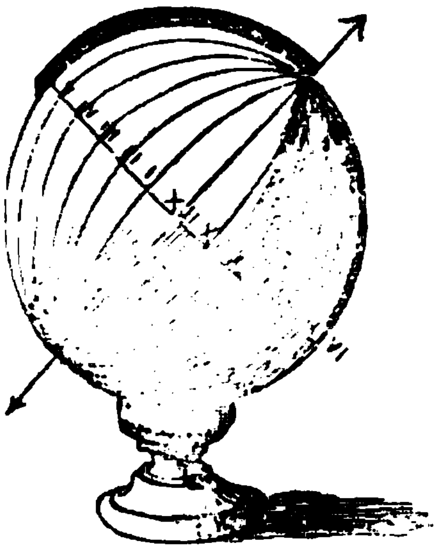
PROBLEM XI.

To construct a Dial on the convex surface of a globe.

This dial, which is the simplest and most natural of all, is formed by dividing the equatorial circle into 24 parts. If a globe be placed on a pedestal, in such a manner that its axis shall be in the plane of the meridian, and exactly elevated according to the height of the pole of the place, nothing then will be necessary to complete the dial, but to divide its equator into 24 equal parts.

The globe (Fig. 11.) in this state, may be used without any farther apparatus; for one half of it being enlightened by the sun, the boundary of the illumination will exactly follow on the equator, the motion of the sun from east to west. At noon,

Fig. 11.



it will fall on those points of the equator turned directly to the east and west. At one o'clock, it will have advanced 15°; and so on. To render this globe then fit for being employed as a dial; VI must be inscribed at the division which corresponds with the meridian; VII at the following one, and so of the rest; so that the twelfth will be exactly in the point turned towards the west; then I, II, III, &c. will be under the horizon. Nothing then will be necessary but to observe what division corresponds with the boundary of the light and shadow; for the number belonging to that division will be the hour.

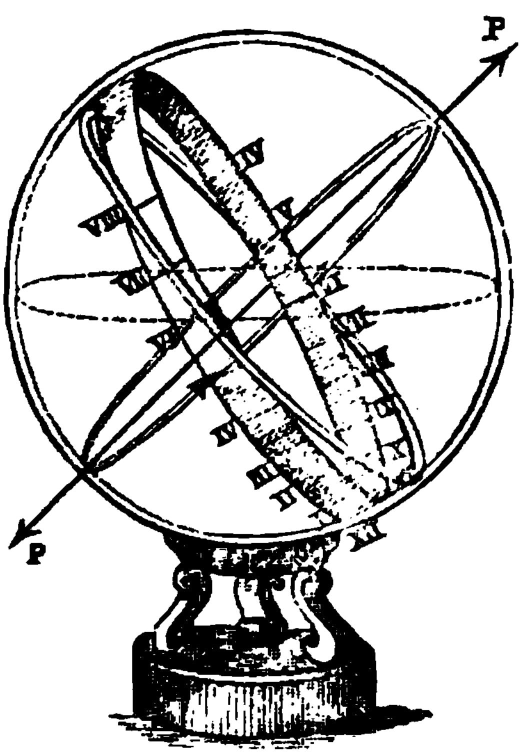
This dial however is attended with a very great inconvenience: as the boundary between the light and shadow is always badly defined, it cannot be precisely known where it terminates; it will therefore be better to employ this dial in the following manner.

Adapt to this globe a half meridian, made of a piece of flat wire, 7 or 8 lines in breadth, and half a line in thickness, and moveable at pleasure around its axis, which must be the same as that of the globe. Then, when you wish to know the hour, move the half meridian in such a manner, that it shall project the least shadow possible, and this shadow will shew the hour on the equator. In this case however it is evident that the numbers naturally belonging to the points of division in the meridian, should be inscribed on them; that is, XII at the meridian, I at the following division, towards the west, and so on.

PROBLEM XII.

Another kind of Dial, in an Armillary Sphere.

Fig. 12.



This dial is equally simple as the preceding, and is attended with this advantage, that it may serve by way of ornament in a garden.

Conceive an armillary sphere (Fig. 12.) consisting only of its two colures, its equator, and zodiac, and furnished with an axis passing through it. If we suppose this sphere to be placed on a pedestal, in such a manner that one of its colures shall supply the place of a meridian, and that its axis shall be directed towards the pole of the place, it is evident that the shadow of this axis, by its uniform motion, will shew the hours on the equator. If the equator therefore be divided into 24 equal parts, and if the numbers belonging to the hours be inscribed at these divisions, the dial will be constructed.

But as the equator, in general, is not of sufficient thickness, the hours must be marked on the inside of the zone which represents the zodiac, and which, on that account, should be painted white. But in this case, care must be taken not to divide each quarter of the zodiac into equal parts; for the shadow of the axis, which passes over equal arcs on the equator, will pass over unequal ones on the zodiac: these divisions will be narrower towards the points of the greatest declination of that circle; so that the division in the zodiac nearest to the solstitial colures, instead of 15°, which are equal to the interval of an hour on the equator, ought to comprehend only 13° 45′; the second 14° 15′; the third 15° 20′; the fourth 15° 25′; the fifth 15° 55′; and the sixth, or nearest the equinoxes, 16° 20′. It is in this manner that

the zodiacal band, on which the hours are marked, must be divided; otherwise there will be several minutes of error; but each interval may be divided into four equal parts for quarters, without any sensible error. Transversal lines may then be drawn through the breadth of the zodiac, taking care to make them concur in the pole. We have seen dials of this kind constructed by ignorant artists, who paid no attention to the above remark, and which therefore were very incorrect.

PROBLEM XIII.

To construct a Solar Dial, by means of which a blind person may know the hours.

This may appear a paradox; but we shall shew that a sun-dial might be erected near an hospital for the blind, by which its inhabitants could tell the hours of the day.

If a glass globe, 18 inches in diameter, be filled with water, it will have its focus at the distance of 9 inches from its surface; and the heat produced in this focus will be so considerable, as to be sensible to the hand placed in it. This focus also will follow the course of the sun, since it will always be diametrically opposite to it; and therefore to construct the proposed dial, we may proceed as follows.

Let the globe be surrounded by a portion of a concentric sphere, 9 inches distant from its surface, and comprehending only the two tropics, with the equator, and the two meridians or colures; and let the whole be exposed to the sun in a proper position; that is, with the axis of the globe parallel to that of the earth.

Let each of the tropics and the equator be divided into 24 equal parts; and let the corresponding parts be connected by a small bar, representing a portion of the hour circle comprehended between the two tropics. By these means all the horary circles will be represented in such a manner, that a blind person can count them, beginning at that which corresponds to noon, and which may be easily distinguished by some particular form.

When a blind person then wishes to know the hour by this dial, he will first put his hand on the meridian, and count the hour circles on the bars which represent them; when he comes to the bar on which the focus of the solar rays fall, he will readily perceive it by the heat, and consequently will know how many hours have elapsed since noon; or how many must elapse before it be noon.

Each interval between the principal bars, that indicate the hours, may be easily divided by smaller ones, in order to have the half-hours and quarters.

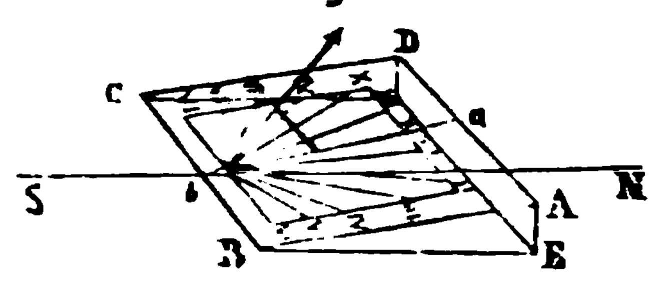
PROBLEM XIV.

Method of arranging a Horizontal Dial, constructed for any particular latitude, in such a manner as to make it shew the hours in any place of the earth.

Every dial, for whatever latitude constructed, may be disposed in such a manner as to shew the hour exactly in any given place; but we shall here confine ourselves to a horizontal dial, and shew how it may be employed in any place whatever.

1st. If the latitude of the place be less or greater than that of the place for which the dial has been constructed, after exposing it in a proper manner, that is, with its meridian in the meridian of the place, and its axis turned towards the north, nothing will be necessary but to incline it till its axis forms with the horizon an angle equal to the latitude of the place in which it is to be used. Thus, for example, if it has been constructed for the latitude of Paris, which is 49° 50′, and you wish to employ it at London, in latitude 51° 31′; as the difference of these two places is 1°41′, the plane of the dial must make with the horizon an angle of 1° 41′, as seen in the figure (Fig. 13.), where S N is the meridian,

Fig. 13.



A B C D the plane of the dial, and A B E, or $a\,b\,e$, the angle of the inclination of that plane to the horizon. If the latitude of the primitive place of the dial be less than that of the place for which it is used, it must be inclined in a contrary direction.

2d. When the second method of rendering a horizóntal dial universal is employed, the hour-lines must not be described on it, but only the points of division in the equinoctial line. In regard to the style, it must be moveable, in the following manner. Let A B C (Fig. 14.) represent the triangle in the plane of the meridian, where N B C is the axis or oblique style, and A B the radius of the equator. The style must be moveable, though it always remain in the plane of the meridian, so that the radius A B of the equator, having a joint in the point A, may form the angle B A C equal to a given angle; that is, equal to the complement of the latitude. For this reason a groove must be formed in the meridian, so as to admit this triangle to be raised up or lowered, always remaining in the plane of the meridian.

Fig. 14.

When every thing has been thus arranged, to adapt the dial to any given latitude, such as that of 51° 31′, for example, take the complement of 51° 31′, which is 38° 29′, and make the angle B A C = 38° 29′. The style then will be in the proper position, and the dial being exposed to the sun, with its meridian corresponding to the meridian of the place, the shadow of the style, which ought to be pretty long, will shew the hour at the place where it intersects the equinoctial.

PROBLEM XV.

Method of constructing some Tables necessary in the following problems.

There are three tables frequently employed in Gnomonics, and which we shall have occasion to make use of hereafter. These are:

1st. A table of the angles which the hour-lines form with the meridian on an horizontal dial, according to the different latitudes.

2d. A table of the angles which the azimuth circles, passing through the sun at different hours of the day, form with the meridian, according to the different latitudes, and the sun's place in the ecliptic.

3d. A table of the sun's altitude at different hours, on a given day, and in a place the latitude of which is given.

From the latter is deduced the sun's zenith distance, at different hours of the day, in a given place, and on a given day; for the sun's zenith distance is always the complement of his altitude.

The first of these tables may be easily calculated by means of the following proportion:

As radius is to the sine of the latitude of the given place, so is the tangent of the angle which measures the sun's distance from the meridian, at a given hour, to the tangent of the angle which the hour-line forms with the meridian.

By means of this analogy, we have calculated the following table, which we conceive will be sufficient: as it comprehends the whole extent of Great Britain, and particularly the latitude of London.

A TABLE

OF THE ANGLES WHICH THE HOUR-LINES FORM WITH THE MERIDIAN ON A HORIZONTAL DIAL, FOR EVERY HALF DEGREE OF LATITUDE, FROM 50° TO 59° 30′.

Latitude.	A. M. I. XI.	A. M. II. X.	A. M. III. IX.	A. M. IV. VIII.	A. M. V. VII.	A. M. VI. VI.
50° ′	11° 38′	23° 51′	37° 27′	53° 0′	70° 43′	90° 0′
50 30	11 41	24 1	37 40	53 11	70 51	90 0
51	11 46	24 10	37 51	53 24	70 58	90 0
51 31	11 51	24 19	38 4	53 36	71 6	90 0
52	11 55	24 27	38 14	53 46	71 13	90 0
52 30	12 0	24 36	38 25	53 58	71 20	90 0
53	12 5	24 45	38 37	54 8	71 27	90 0
53 30	12 9	24 54	38 48	54 19	71 34	90 0
54	12 14	25 2	38 58	54 29	71 40	90 0
54 30	12 18	25 10	39 8	54 39	71 47	90 0
55	12 23	25 19	39 19	54 49	71 53	90 0
55 30	12 28	25 27	39 29	54 59	71 59	90 0
56	12 32	25 35	39 40	55 8	72 5	90 0
56 30	12 36	25 43	39 50	55 18	72 12	90 0
57	12 40	25 51	39 59	55 27	72 17	90 0
57 30	12 44	25 58	40 9	55 37	72 22	90 0
58	12 48	26 5	40 18	55 45	72 27	90 0
58 30	12 52	26 13	40 27	55 54	72 33	90 0
59	12 56	26 20	40 36	56 2	72 39	90 0
59 30	13 0	26 27	40 45	56 10	72 44	90 0

We have not marked, in this table, the angles formed by the lines V hours in the morning and VII hours in the evening, IV hours in the morning and VIII in the evening, because these lines are only a continuation of others; for example, that of IV hours in the morning, is the continuation of IV in the evening; that of VIII hours in the evening is the continuation of VIII in the morning; and so of the rest.

The use of this table may be easily comprehended. If the place for which a horizontal dial is required, corresponds with any latitude of the table, such as 52 for example, it may be seen at one view, that the hour-lines of XI and I must form, with the meridian, an angle of 11° 55′, at the centre of the dial; that of X and II an angle of 24° 27′; and so of the rest.

If the latitude be not contained in the table, the proportional parts may be taken without any sensible error. Thus, if it were required to find the angle which the hour-line of I or XI forms with the meridian, on a dial for the latitude of 54° 15′; as the difference of the horary angles, for 54° and 54° 30′, is 4′, take the half of 4, and add it to 12° 14′, which will give 12° 16′ for the horary angle between the hours of I or XI and the meridian, on a dial for the latitude of 54° 15′. The same operation may be employed for the other horary angles.

It is necessary to observe that this table, though constructed for horizontal dials, may be used also for vertical south or north dials; for it is evident that a south vertical dial, for any particular place, is the same as a horizontal dial for another, the latitude of which is the complement of the former. Thus a south vertical dial for the latitude of London 51° 31′, is the same as a horizontal dial for the latitude of 38° 29′, and *vice versa*.

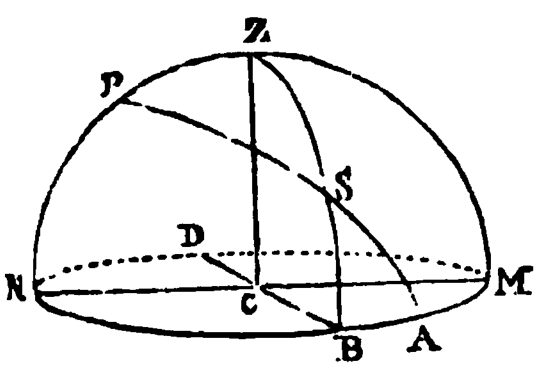
It is in the construction of these vertical dials that the utility of such tables will be most apparent; for as these dials are in general very large, the common rules of Gnomonics cannot easily be applied to them. To remedy this inconvenience, when

the centre and equinoctial of the dial have been fixed, assume, as radius, that part of the meridian comprehended between the equinoctial and the centre, and divide it into 1000 parts; then find in some table, or by calculation as above shewn, for the given latitude—that is, for its complement if a vertical dial is to be constructed, the tangents of the angles which the hour-lines form with the meridian, at I, II, III, IV, &c., and lay them off on both sides on the equinoctial: the points where they terminate will be the horary points of I and XI hours, II and X hours, &c.

Let us suppose, for example, that a south vertical dial is to be constructed for the latitude of 51° 31′, the complement of which is 38° 29′. A vertical south dial for lat. 51° 31′, may be considered as a horizontal dial for the latitude of 38° 29′. But the angles which the hour-lines form with the meridian on a horizontal dial, for that latitude, are 9° 28′; 19° 46′; 31° 53′; 47° 9′; 66° 42′; 90° 0′; the tangents of which, radius being divided into 1000 parts, are 166, 359, 622, 1078, 2321, *infinite*. If the portion of the meridian therefore, comprehended between the centre and the equinoctial, be divided into 1000 parts, and if 166 of these parts be set off on each side of the meridian, we shall have the points of XI and I hours; if 359 parts be then laid off in the same manner, we shall have the points of X and II hours; and so of the rest. Straight lines drawn from the centre, to each of these points, will be the hour-lines.

The last tangent, which corresponds to VI hours, being infinite, indicates that the hour-line corresponding to it must be parallel to the equinoctial.

Fig. 15.



In order to give an idea of the construction of the second table, let the circle M B N D (Fig. 15.), represent the horizon of the place; Z its zenith, P the pole, Z B the azimuth circle passing through the sun, and P S A the horary circle in which the sun is at any proposed time of the day; it is here evident, that if the hour be given, the angle Z P S is known; that the day of the year being given, the sun's distance from the equator is known, and consequently the arc P S, which in our hemisphere is the fourth part of a great circle, minus the sun's declination, if it be north, or plus that declination, if it be south; and lastly, that if the elevation of the pole be given, the arc P Z which is its complement, is also known. In the spherical triangle Z P S, we have therefore given the arcs Z P and P S, with the included angle Z P S; and hence we may find the angle P Z S, which subtracted from 180 degrees, will leave the angle M Z B or M C B, the sun's azimuth from the south.

In the same triangle, we can find the side Z S, the complement of the sun's altitude at the same time; and consequently the altitude itself.

By these means, the following tables have been constructed, for the latitude of London 51° 31′. Those who are tolerably versed in spherical trigonometry, may easily construct similar tables for any other latitude.

A TABLE OF THE SUN'S AZIMUTH FROM THE SOUTH, AT HIS ENTRANCE INTO EACH OF THE TWELVE SIGNS, AND AT EACH HOUR OF THE DAY, FOR THE LATITUDE OF LONDON, 51° 31′.

Hours.	♋	♊ ♌	♉ ♍	♈ ♎	♓ ♏	♒ ♐	♑
XI. I	28° 2′	26° 9′	22° 18′	19° 13′	16° 19′	14° 46′	14° 9′
X. II	50 50	48 7	42 9	36 25	31 49	28 53	27 49
IX. III	68 11	65 22	58 48	51 57	46 3	42 7	40 39
VIII. IV	82 2	79 27	72 55	65 41	59 0	54 24	
VII. V	93 54	91 25	85 28	78 10	71 8		
VI. VI	105 7	102 54	97 8	90 0			
V. VII	116 5	114 5					
IV. VIII	127 23						

A TABLE OF THE SUN'S ALTITUDE AT HIS ENTRANCE INTO EACH OF THE TWELVE SIGNS, AND AT EACH HOUR OF THE DAY, FOR THE LATITUDE OF LONDON 51° 31′.

Hours.	♋	♊ ♌	♉ ♍	♈ ♎	♓ ♏	♒ ♐	♑
XII.	61° 57′	58° 41′	49° 51′	38° 29′	26° 43′	18° 17′	15° 2′
XI. I	59 40	56 34	48 2	36 57	25 30	17 32	13 54
X. II	53 44	50 56	43 4	32 37	21 42	13 40	10 32
IX. III	45 41	43 7	35 52	26 7	15 50	8 15	5 17
VIII. IV	36 40	34 14	27 21	18 8	8 18	1 16	
VII. V	27 22	24 56	18 12	9 17	1 17		
VI. VI	18 10	15 41	8 53				
V. VII	9 26	6 50					
IV. VIII	1 31						

PROBLEM XVI.

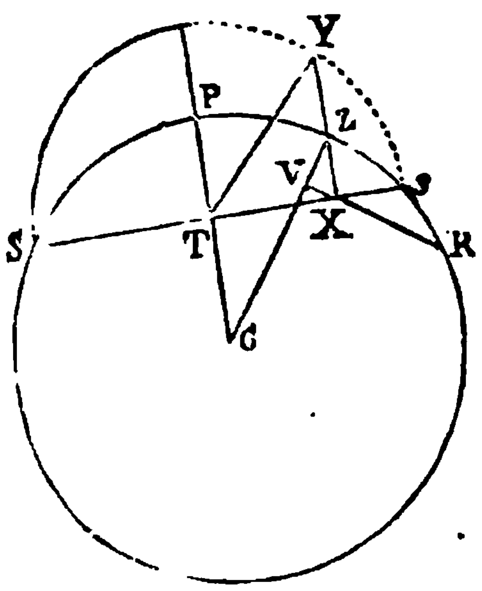
The sun's altitude, the day of the month, and the elevation of the pole, being given; to find the hour by a geometrical construction.

We give this construction merely as a geometrical curiosity; for it is certain that the same thing can be performed with much greater accuracy by calculation. However, as the solution of this problem forms a very ingenious example of the graphic solution of one of the most complex cases of spherical trigonometry, we have no doubt that it will afford gratification to our readers; or at least to such of them as are sufficiently versed in geometry to comprehend it.

Let us return then to Fig. 15, in which P Z represents the complement of the latitude or elevation of the pole; Z S the complement of the sun's altitude, which is known, being given by the supposition; and P S the sun's distance from the pole, which is also given, since the declination of the sun, or his distance from the equator each day, is known. In the triangle Z P S therefore, there are given the three sides, to find the angle Z P S, the hour angle, or angle which the horary circle, passing through the sun, forms with the meridian. This case then is one of those in spherical trigonometry, where the three sides of an oblique triangle being given, it is required to find the angles; and which may be solved geometrically in the following manner.

In the circumference of a circle, which must be sufficiently large to give quarters of degrees, (Fig. 15 and 16), assume an arc equal to P Z, and draw the two radii C P and C Z. On the one side of this arc make P s equal to the arc P S, and on the other

Fig. 16.



ZR equal to the arc ZS: from the points R and s let fall, on the radii PC, CZ, two perpendiculars ST and RV, which will intersect each other in some point X: then, if ST be radius, we shall have TX for the cosine of the required angle, which may be constructed in the following manner:

From the centre T, with the radius TS or TS, which is equal to it, describe a quadrant, comprehended between TP and TX continued; if XY be then drawn parallel to TP, the arc YS will be the one required, or the measure of the hour angle SPZ; therefore YTX will be equal to that angle.

By a similar construction we might find the angle Z, the complement of which is the sun's azimuth; but this is sufficient in regard to an operation which is rather curious than useful.

This construction is much simpler and far more elegant, than that given by Ozanam, for the solution of the same problem.

GNOMONICAL PARADOX.

Every sun-dial, however accurately constructed, is false, and even sensibly so, in regard to the hours near sun-set.

The truth of what is here asserted, will be readily perceived by astronomers, who are acquainted with the effects of refraction. The following observations will make it sensible to our readers.

It is a fact now well known to all philosophers, that the heavenly bodies always appear more elevated than they really are, except when they are in the zenith. This phenomenon is produced by the refraction, which the rays of light, proceeding from them, experience in the atmosphere; and the effect of it is very considerable in the neighbourhood of the horizon; for when the centre of the sun is really on the horizon, he still appears to be elevated more than half a degree, or 33 minutes, which in our latitudes is the quantity of the horizontal refraction. The centre of the sun then is really on the horizon, and astronomically set, when his lower limb does not touch the horizon, but is still distant from it an apparent semi-diameter of the sun.

Let us suppose then, that on the day of the equinox, for example, the hour indicated by a vertical west-dial, near the time of sun-setting, has been observed at the moment when a well regulated clock strikes six: the shadow of the style ought to be on the hour of six, and it would indeed be so if the sun were on the horizon; but being elevated 33 minutes above the horizon, the shadow of the style will be within six hours, for it is by the apparent image of the sun that this shadow is formed: it will even not reach that line till the sun has still descended 33′, for which he will employ, in the latitude of London, about 3m. 28s. of time. But, in a sun-dial, an error of 3m. 28s. is more than sensible.

If the sun be at the summer solstice; as he employs in the latitude of London more than 4′ to descend vertically 33 minutes on the horizon, on account of the obliquity with which the tropic cuts that circle, the difference will be more sensible as the space passed over by the shadow between the hours of seven and eight, is sufficiently great to suffer an error of a twelfth or a fifteenth to be very perceptible. We have seen, on a dial of this kind, the point of the shadow, which ought to have fallen on the line of seven o'clock, more than an inch distant from it; though at all the other hours of the day the dial was very exact, and corresponded with an excellent watch which was compared with it.

PROBLEM XVII.

To describe a Portable Dial on a Quadrant.

As the construction of this dial depends also on the sun's titude at each hour of the day, in a determinate latitude, according to his place in te zodiac, the tables before mentioned must be employed here also.

A

Fig. 17.

Let A B C then, (Fig. 17.), be a quadrant, the centre of which is A. From the centre A describe, at pleasure, seven quadrants equally distant from each other, to represent the commencement of the signs of the zodiac: the first and last being assumed as the tropics, and that in the middle as the equator. Mark on each of these parallels of the signs, the points of the hours, according to the altitude which the sun ought to have at these hours, which may be found in the table above mentioned. To determine, for example, the point of II in the afternoon, or X in the morning, for the latitude of London, when the sun enters Leo; as the table shews that the sun's altitude is at that time 50° 56', make in the proposed quadrant the angle B A O equal to 50° 56', and the place where the parallel of the commencement of Leo is intersected by the line A O, will be the required point of II in the afternoon and X in the morning.

Having made a similar construction for all the other hours, on the day of the sun's entrance into each sign, nothing will be necessary but to join, by curved lines, all the points belonging to the same hour, and the dial will be completed. Then fix a small perpendicular style in the centre A, or place on the radius A C, or any other line parallel to it, two sights, the holes of which exactly correspond; and from the centre A suspend a small plummet by means of a silk thread.

When you use this instrument, place the plane of it in such a manner as to be in the shade; and give such a direction to the radius that the shadow of the small style shall fall on the line A C, or that the sun's rays shall pass through the two holes of the sights: the thread from which the plummet is suspended will then shew the hour, by the point where it intersects the sun's parallel.

To find the hour with more convenience, a small bead is put on the thread, but in such a manner as not to move too freely. If this bead be shifted to the degree and sign of the sun's place, marked on the line A C, and if the instrument be then directed towards the sun, as above mentioned, the bead will indicate the hour on the hour-line which it touches.

Remark.—To render this dial more commodious, it will be better, instead of the signs, to mark the days of the month on which the sun enters them. For example, instead of marking the small circle with the sign ♑, mark December 21; close to the second place on one side January 21, instead of ♒, the sign of Aquarius; and on the other November 21, instead of ♐, the sign of Sagittarius, &c.; for if we suppose the equinoxes invariably fixed at the 21st of March and the 21st of September, the days on which the sun enters the different signs of the zodiac will be nearly the 21st of each month: to use the dial, nothing will then be necessary but to know the day of the month.

PROBLEM XVIII.

To describe a Portable Dial on a card.

This dial is generally called the Capuchin, because it resembles the head of a Capuchin friar with the cowl inverted. It may be described on a small piece of pasteboard, or even a card, in the following manner.

Fig. 18.

Having described a circle, Fig. 18, at pleasure, the centre of which is A, and the diameter B 12, divide the circumference into 24 equal parts, or at every 15 degrees, beginning at the diameter B 12. If each two points of division, equally distant from the diameter B 12, be then joined by parallel lines, these parallels will be the hour-lines; and that passing through the centre A, will be the line of six o'clock.

Then at the point 12, make the angle B 12 ♈ equal to the elevation of the pole, and having drawn through the point ♈, where the line 12 ♈ intersects the line of 6 o'clock, the indefinite line ♋ ♑, perpendicular to the line 12 ♈, draw from the extremities of the line ♋ ♑, the lines 12 ♋, and 12 ♑, which will each make with the line 12 ♈, an angle of 23½ degrees, which is the sun's greatest declination.

The points of the other signs may be found on this perpendicular ♋ ♑, by describing from the point ♈, as a centre, through the points ♋, ♑, the circumference of a circle, and dividing it into 12 equal parts, or at every 30 degrees, to mark the commencement of the 12 signs. Join every two opposite points of division, equally distant from the points ♋, ♑, by lines parallel to each other, and perpendicular to the diameter ♋, ♑: these lines will determine, on this diameter, the commencement of the signs; from which, as centres, if circular arcs be described through the point 12, they will represent the parallels of the signs; and therefore must be marked with the appropriate characters as seen in the figure.

A slit must be made along the line ♋ ♑, to admit a thread furnished with a small weight, sufficient to stretch it; and in which it must glide, but not too freely; so that its point of suspension can be shifted to any point of the line ♋ ♑ at pleasure.

These arcs of the signs will serve to indicate the hours when the sun shines, in the following manner: Having drawn at pleasure the line C ♑, parallel to the diameter B 12, fix at its extremity C a small style in a perpendicular direction, and turn the plane of the dial to the sun, so that the shadow of the style shall cover the line C ♑: the thread and plummet being then freely suspended from the sun's place, marked on the line ♋ ♑, will indicate the hour on the arc of the same sign at the bottom.

The thread may be furnished with a small bead to be used as in the preceding problem.

Remark.—This dial originated from an universal rectilineal dial constructed by Father de Saint-Rigaud, a jesuit, and professor of mathematics in the college of Lyons, under the name of *Analemma Novum*. But though Ozanam has given a con-

spicuous place to it in his Recreations, as well as to another universal rectilineal analemma, it appeared to us that his description of them was too complex to be admitted into a work of this kind.

PROBLEM XIX.

Method of constructing a Ring-dial.

Portable ring-dials are sold by the common instrument-makers; but they are very defective. The hours are marked in the inside on one line, and a small moveable band, with a hole in it, is shifted till the hole correspond with the degree and sign of the sun's place marked on the outside. Such dials however, as already said, are defective; for as the hole is made common to all the signs of the zodiac, marked on the circumference of the ring, it indicates justly none of the hours but noon: all the rest will be false. Instead of this arrangement, therefore, it will be necessary to describe, on the concave surface of the ring, seven distinct circles, to represent as many parallels of the sun's entrance into the signs; and on each of these must be marked the sun's altitude on his entrance into the sign belonging to the parallel to which the circle corresponds. When these points are marked, they must be joined by curved lines, which will be the real hour-lines, as has been remarked by Deschales.

Fig. 19.

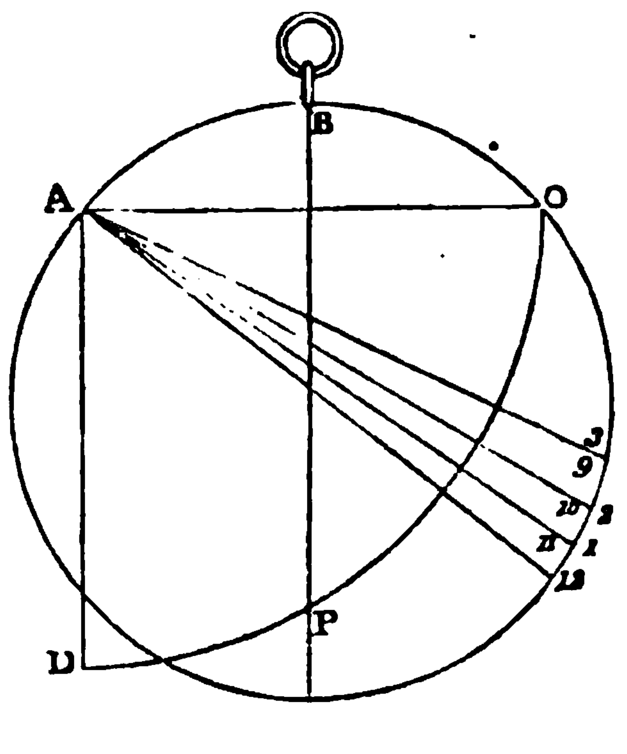
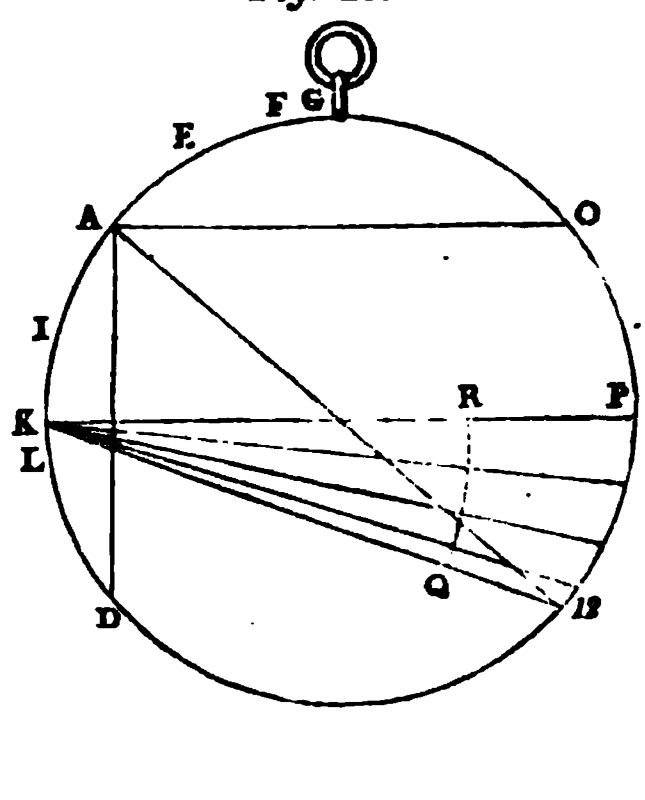


Fig. 20.



Having provided a ring, Fig. 19, or rather described a circle of the size of the ring which is to be divided; and having fixed on B as the point of suspension, make B A and B O, on each side of B, equal to 51° 31′, for the latitude of the place, suppose London, that is, equal to the distance of the zenith from the equator: then through the points A and O draw the chord A O, and A D perpendicular to it: if the line A 12 be then draw through A and the centre of the circle, the point 12 will be the hour of noon on the day of the equinox.

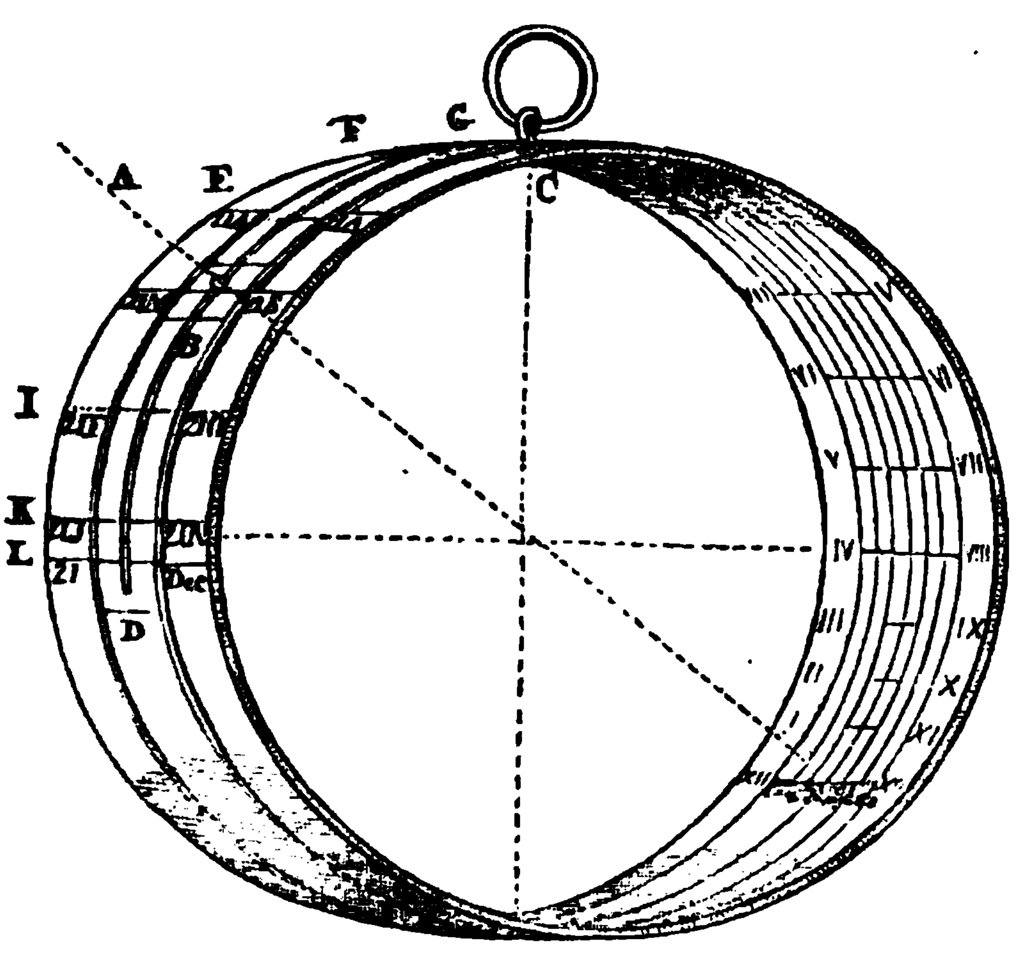
To find the other hour-points for the same day, at the commencement of Aries Libra; from the centre A describe the quadrant O D; and from the point O, set off toward P the sun's altitude at the different hours of the day, as at I and II, 2 and 10. &c.; the lines drawn from the centre A through these points of division, if continued to the circumference of the circle B 12 A, will give the hour-points for the day of the equinox.

To obtain the hour-divisions on the circles corresponding to the other signs, first set off, on both sides of the point A Fig. 20, the sun's declination when he enters each of the signs, viz. the arcs A E and A I of 23 degrees, for the commencement of Taurus or Virgo; of Scorpio or Pisces; A F of 40° 26′ for the commencement of Gemini and Leo; A K equal to it for the

commencement of Sagittaris and Aquarius; and A G and A L of 47° for the commencement of Cancer and Capricorn.

Now to find the hour-points on the circle, (that corresponding to the commencement of Aquarius, for example,) through the point K, which corresponds to the sun's entrance into that sign, draw K P parallel to A O, and also the line K 12 : from the same point K describe, between K 12 and the horizontal line K P, the arc Q R; on which set off, from R towards Q, the sun's altitude at the different hours of the day, when he enters Sagittarius and Aquarius, as seen in the figure; and if lines be then drawn from K to these points of division, you will have the hour-points of the two circles corresponding to the commencement of Sagittarius and Aquarius. By proceeding in the same manner for the sun's entrance into the other signs, you will have the hour-points of the circles which correspond to them.

Fig. 21.



Then trace out, on the concave surface of the circle, seven parallel circles (Fig. 21.), that in the middle for the equinoxes; the two next on each side for the commencement of the signs Taurus and Virgo, Scorpio and Pisces; the following two on the right and left for Gemini and Leo, Sagittarius and Aquarius; and the last two for Cancer and Capricorn: if the similar hour-points be then joined by a curved line, the ring-dial will be completed.

The next thing to be done, is to adjust properly the hole which admits the solar rays; for it ought to be moveable, so that on the day of the equinox it may be at the point A; on the day of the summer solstice at G; on the other days of the year in the intermediate positions. For this purpose the exterior part of the ring C B D must have in the middle of it a groove, to receive a small moveable ring or hoop, with a hole in it. The divisions L, K, I, A, E, F, G, must be marked on the outside of this part of the ring by parallel lines, inscribing on one side the ascending signs, and on the other the descending: when this construction has been made, it will be easy to place the hole of the moveable part A on the proper division, or at some intermediate point; for if the ring be pretty large, each sign may be divided into two or three parts.

To know the hour, move the hole A to the proper division, according to the sign and degree of the sun's place; then turn the instrument in such a manner that the sun's rays, passing through the hole, may fall on the circle corresponding to the sign in which the sun is: the division on which it falls will shew the hour.

Remark.—I. To render the use of this instrument easier, instead of the divisions of the signs, the days corresponding to the commencement of the signs might be

marked out on it: for example, June 21 instead of ♋; April 20, August 20, instead ♉ and ♍, and so on.

II. The hole A might be fixed, and the most proper position for it would be that which we originally assigned to the day of the equinox; but in this case, the hour of noon, instead of being found on a horizontal line, for all the circles of the signs, according to the preceding method, would be a curved line; and all the other hour-lines would be curved lines also. As this would be attended with a considerable degree of embarrassment and difficulty, it will be better, in our opinion, that the hole A should be moveable.

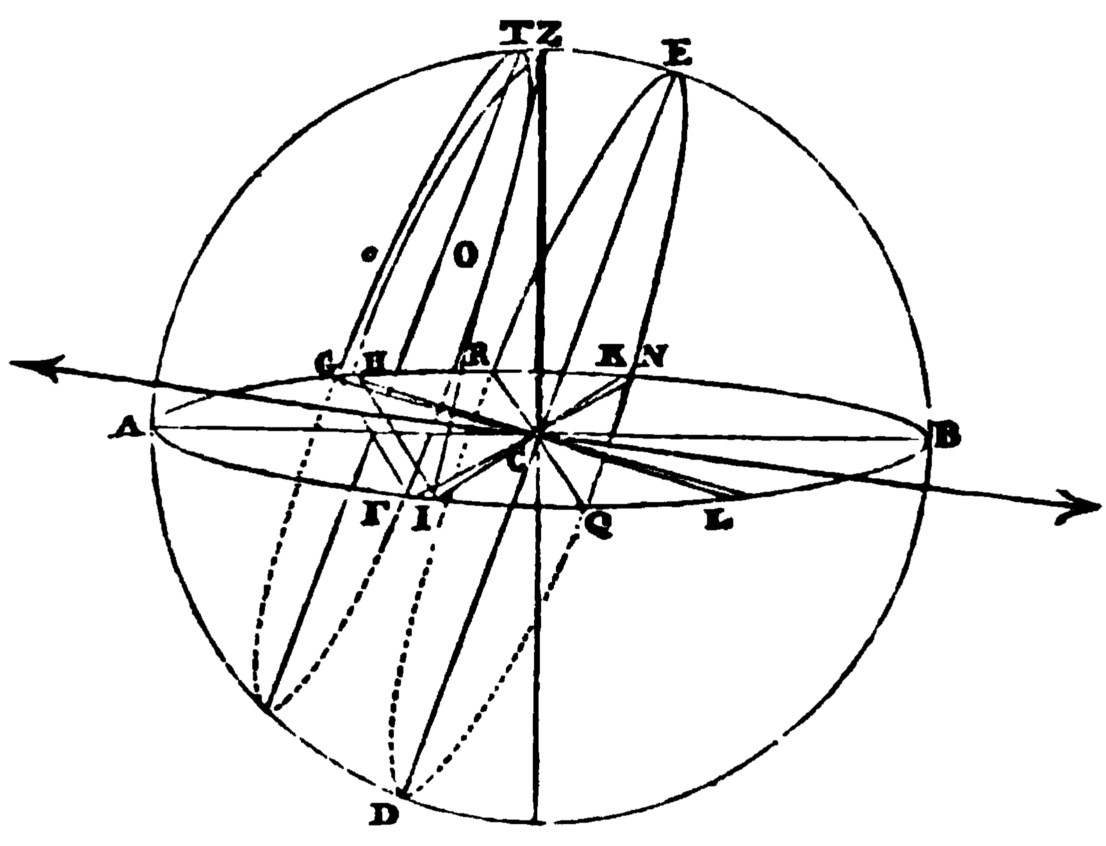
PROBLEM XX.

How the shadow of a style, on a Sun-dial, might go backwards, without a miracle.

This phenomenon, which on the first view may appear physically impossible, is however very natural, as we shall here shew. It was first remarked by Nonius or Nugnez, a Portuguese mathematician, who lived about the end of the sixteenth century. It is founded on the following theorem.

In all countries, the zenith of which is situated between the equator and the tropic, as long as the sun passes beyond the zenith, towards the apparent or elevated pole, he arrives twice before noon at the same azimuth, and the same thing takes place in the afternoon.

Fig. 22.



Let Z (Fig. 22) be the zenith of any place situated between E the equator, and T the point through which the sun passes on the day of the summer solstice; let the circle H A Q B K H represent the horizon; E B Q one half of the equator; T F the eastern part of the tropic above the horizon, and G T the western part. It is here evident, that from the zenith Z there may be drawn an azimuth circle, such as Z I, which shall touch the tropic in a point O, for example: and which shall fall on the horizon in a point I, situated between the points Q and F, which are those where the horizon is intersected by the equator and the tropic; and, for the same reason, there may be drawn another azimuth, as Z H, which shall touch in *o* the other part of the tropic.

Let us now suppose that the sun is in the tropic, and consequently rising in the point F; and let a vertical style, of an indefinite length, be erected in C. Draw also the lines I C K, and F C N; it is evident that at the moment of sun-rise the shadow of the style will be projected in C N; and that when the sun has arrived at the point of contact O, the shadow will be projected in C K. While the sun is passing over F O, it will move from C N to C K, but when the sun has reached the meridian, the shadow will be in the line C B; it will therefore have gone back from C K to C B:

from sunrising to noon then it will have gone from C N to C K, and from C K to C S: consequently it will have moved in a contrary or retrograde direction, since it first moved from the south towards the west, and then from the west towards the south.

Let us next suppose that the sun rises between the points F and I. In this case the parallel he describes before noon will evidently cut the azimuth Z I in two points; and therefore, in the course of a day, the shadow will first fall within the angle K C L; it will then proceed towards C K, and even pass beyond it, going out of the angle; but it will again enter it, and, advancing towards the meridian, will proceed thence towards the east, even beyond the line C L, from which it will return to disappear with the setting of the sun within the angle L C R.

It is found by calculation, that in the latitude of 12 degrees, when the sun is in the tropic on the same side, the two lines C N and C K form an angle of 9° 48′; to pass over which the shadow requires 2 hours 7 minutes.

PROBLEM XXI.

To construct a Dial, for any latitude, on which the shadow shall retrograde, or move backwards.

For this purpose incline a plane, turned directly south, in such a manner, that its zenith shall fall between the tropic and the equator, and nearly about the middle of the distance between these two circles: in the latitude of London, for example, which is 51° 31′, the plane must make an angle of about 38°. In the middle of the plane, fix an upright style of such a length, that its shadow shall go beyond the plane; and if several angular lines be then drawn from the bottom of the style towards the south, about the time of the solstice, the shadow will retrograde twice in the course of the day, as above-mentioned.

This is evident, since the plane is parallel to the horizontal plane, having its zenith under the same meridian, at the distance of 12 degrees from the equator towards the north: the shadows of the two styles must consequently move in the same manner in both.

Remark.—Some may here say, that this is a natural explanation of the miracle, which, as we are told in the Sacred Scriptures, was performed in favour of Hezekiah, king of Jerusalem; but God forbid that we should entertain any idea of lessening the credibility of this miracle. Besides, it is very improbable, if the retrogradation which took place on the dial of that prince had been a natural effect, that it should not have been observed till the prophet announced it to him, as a sign of his cure; for in that case it must have always occurred when the sun was between the tropic and the zenith: the miracle therefore, recorded in the Scriptures, remains unimpeached.

PROBLEM XXII.

To determine the Line traced out, on the plane of a Dial, by the summit of the style

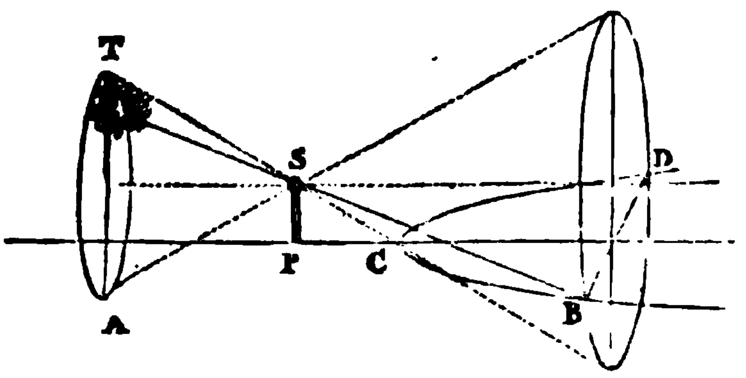
We here suppose that the sun, in the course of a diurnal revolution, does not sensibly change his declination; for if he did, the curve in question would be of two complex a nature, and very difficult to determine.

Let the sun then be in any parallel whatever. It may be easily seen that the central solar ray, drawn to the point of the style, describes a conical surface, unless the sun be in the equator; consequently the shadow projected by that point, which is always directly opposite to it, passes over, in its revolution, the surface of the opposite cone, which is united to it by its summit. Nothing then is necessary but to know the position of the plane which cuts the two cones; for its intersection with the conical surface, described by the shadow, will be the curve required.

Those therefore who have the least knowledge of conic sections will be able to

solve the problem. For, 1st, If the proposed place be under the equator, and the plane horizontal, it is evident that this plane intersects the two opposite cones at the summit: consequently the track of the shadow will be an hyperbola B C D (Fig. 23.), having its summit turned towards the bottom of the style.

Fig. 23.



But it may be easily seen, that as the sun approaches the equator, this hyperbolic line becomes flatter and flatter; and at length, on the day of the equinox, is changed into a straight line; that it afterwards passes to the other side, and always becomes more and more curved, till the sun reaches the tropic, &c.

We shall here add, that the sun rises every day in one of the asymptotes of an hyperbola, and sets in the other.

2nd. In all places situated between the equator and the polar circles, the track of the shadow, on a horizontal plane, is still an hyperbola; for it may be easily seen that this plane cuts the two opposite cones, united at their summits, which are described by the solar ray that passes over the point of the style; since in all these latitudes the two tropics are intersected by the horizon.

3d. In all places situated under the polar circle, the line described by the shadow on a horizontal plane, when the sun is in the tropic, is a parabolic line: but that described on other days is hyperbolic.

4th. In places situated between the polar circle and the pole, as long as the sun rises and sets, the tract described by the shadow of the summit of the style is an hyperbola: when the sun has attained to such a high latitude that he only touches the horizon, instead of setting, the track is a parabola; and when the sun remains the whole day above the horizon, it is an ellipsis, more or less elongated.

5th. Lastly, it may be easily seen that under the pole the track of the shadow of the summit of the style is always a circle; since the sun, during the whole day, remains at the same altitude.

Corollary.—As the arcs of the signs are nothing else than the track of the shadow of the summit of the style, when the sun in his diurnal motion passes over the parallel belonging to the commencement of each sign, it follows that these arcs are all conic sections, having their axis in the meridian or substylar line. In horizontal dials, constructed for places between the equator and the polar circles, and in all vertical dials, whether south, north, east, or west, constructed for places in the temperate zone, they are hyperbolas. This may be easily perceived, on the first view, in most of the dials in our latitude.

These observations, which perhaps may be considered by common gnomonists as of little importance, appeared to us worthy the consideration of those more versed in geometry; especially as some of them may not have attended to them. For this reason we resolved to give them a place in this work.

PROBLEM XXIII.

To describe the Arcs of the Signs on a Sun-dial.

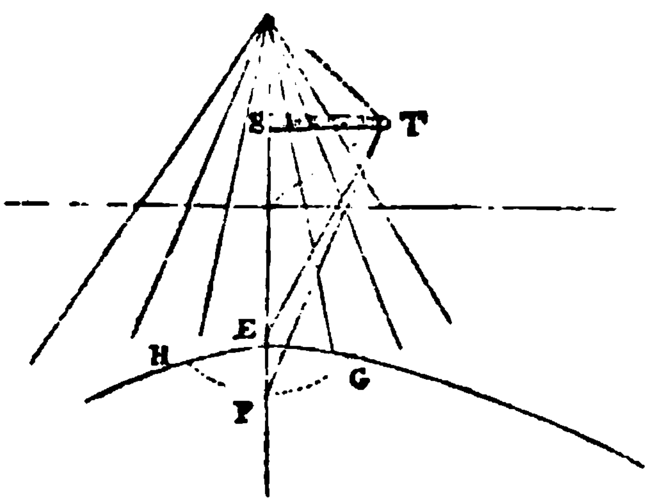
Of the appendages added to sun-dials, the arcs of the signs may be classed among the most agreeable; for by their means we can know the sun's place in the different signs, and as we may say can follow his progress through the zodiac. We therefore thought it our duty not to omit, in this work, the method of describing them.

For the sake of brevity, we shall suppose that the plane is horizontal. First de-

scribe a dial such as the position of the plane requires, (that is, a horizontal one,) and fix in it an upright style, terminated by a spherical button, or by a circular plate, having in its centre a hole, of a line or two in diameter, according to the size of the dial. Then proceed as follows:—

Let it be required, for example, to trace out the arc corresponding to the commencement of Scorpio or Pisces. First find, by the table of the sun's altitude, at each hour of the day in the latitude of London, for which we suppose the dial to be constructed, the altitude when he enters these two signs. As this altitude is 26° 43′, make the triangle S T E, Fig. 24, in which S T is the height of the style, and such that the angle S E T shall be equal to 26° 43′: the point E will be the first point of the arc of these two signs.

Fig. 24.



Then find, in the same table, the sun's altitude at one in the afternoon of the same day, which will be found equal to 25° 30′; and construct the triangle S T F, in such a manner that the angle F shall be 25° 30′; then from the bottom of the style S, as a centre, with the radius S F, describe an arc of a circle, intersecting the lines of I and XI hours in the two points G and H; these will be the points of the arc of those signs on the lines of I and XI.

If the same operation be repeated for all the other hours, you will have as many points, through which if a curved line be drawn, by means of a very flexible ruler, you will obtain the arc of the signs Scorpio and Pisces.

By employing the like construction, the arcs belonging to the other signs may be obtained.

Of the different kinds of Hours.

Every thing hitherto said has related only to the equinoctial and equal hours; such as those by which time is reckoned in England, the day being supposed to begin at midnight, and the hours being counted to the following midnight, to the number of 24, or twice twelve. This is the most common method of computing the hours in Europe. The astronomical hours are almost the same; the only difference is, that the latter are counted, to the number of 24, from the noon of one day to the noon of the day following.

But there are some other kinds of hours, which it is proper we should here explain; because they are sometimes traced out on sun-dials: such are the natural or Jewish hours, the Babylonian, the modern Italian, and those of Nuremburg.

The natural or Jewish hours begin at sun-rise; and there are reckoned to be 12 between that period and sun-set: hence it is evident that they are not of equal length, except on the day of the equinox: at every other time of the year they are unequal. Those of the day, in our hemisphere, are longer from the vernal to the autumnal equinox: those of the night are, on the other hand, longer while the sun is passing through the other half of the zodiac.

The Babylonian hours were of equal length, and began at sun-rise; they were counted, to the number of 24, to sun-rise of the day following.

The modern Italian hours, for the ancient Romans counted nearly as we do from midnight to midnight, are reckoned to the number of 24, from sun-set to sun-set of the day following; so that on the days of the equinox noon takes place at the 18th hour, and then, as the days lengthen, the astronomical noon happens at 17½ hours,

then at 17 hours, &c.; and *vice versâ*. This singular and inconvenient method has had its defenders, and that even among the French; who have found that with a pencil, and a little astronomical calculation, one may fix the hour of dinner with very little embarrassment.

However, as these hours are still used throughout almost the whole of Italy, we think it our duty to shew here the method of describing them, by way of a Gnomonical curiosity.

PROBLEM XXIV.

To trace out, on a Dial, the Italian Hours.

Describe first on the proposed plane, which we here suppose to be a horizontal one, a common horizontal dial, with the astronomical or European hours: delineate on it also the arcs of the solsticial signs, Cancer and Capricorn; as well as the equinoctial line, which is the arc of the equinoctial signs.

Then observe that, on the days of the equinox, noon, for a dial constructed at London, takes place at the end of the 18th Italian hour; and on the day of the summer solstice at 17 minutes after the 16th hour. Noon, therefore, or 12 hours, counted according to the astronomical hours, corresponds, on the day of the equinox, to the 18th Italian hour; and on the day of the solstice to 17 minutes after the 16th; consequently the 18th Italian hour, on the day of the summer solstice, will correspond to 17 minutes past 2, counted astronomically. Join therefore, (Fig. 25), by a straight

Fig. 25.

line, the point of noon marked on the equinoctial line, and that of 2 hours 17 minutes on the tropic or arc of the sign Cancer, and inscribe there 18 hours. Join also by transversal lines 1 hour on the equinoctial and 3h. 17m. on the arc of Cancer; then 2h. and 4h. 17m., &c.; and before noon 11h. and 1h. 17m.; 10h. and 12h. 17m.; 9h. and 11h. 17m. &c.; efface then the astronomical hours, which we suppose ought not to appear, and continue the above transversal lines till they meet the parallel of Capricorn, inscribing at their extremities the proper numbers; by which means you will have your dial traced out as seen.

Remark.—It may be easily seen, by the above example, what calculation will be necessary for a latitude different from that of London, where the length of the day, at the summer solstice, is 16 hours 34 minutes, and at the winter solstice only 7 hours 44 minutes. In another latitude, where the longest day is only 14 hours and the shortest 10, noon at the summer solstice will take place at the end of the 17th Italian hour. Noon therefore, or 12 hours, counted astronomically, will on the day of the solstice correspond to the 17th Italian hour; and consequently the 18th Italian hour,

at the same period, will correspond to 1 in the afternoon counted astronomically. To have the hour-line of the 17th Italian hour, therefore, nothing will be necessary, but to join the point of 1 in the afternoon, on the arc of Cancer, and the point of noon on the equinoctial. And the case will be the same with the other hours.

PROBLEM XXV.

To trace out on a Dial the lines of the natural or Jewish hours.

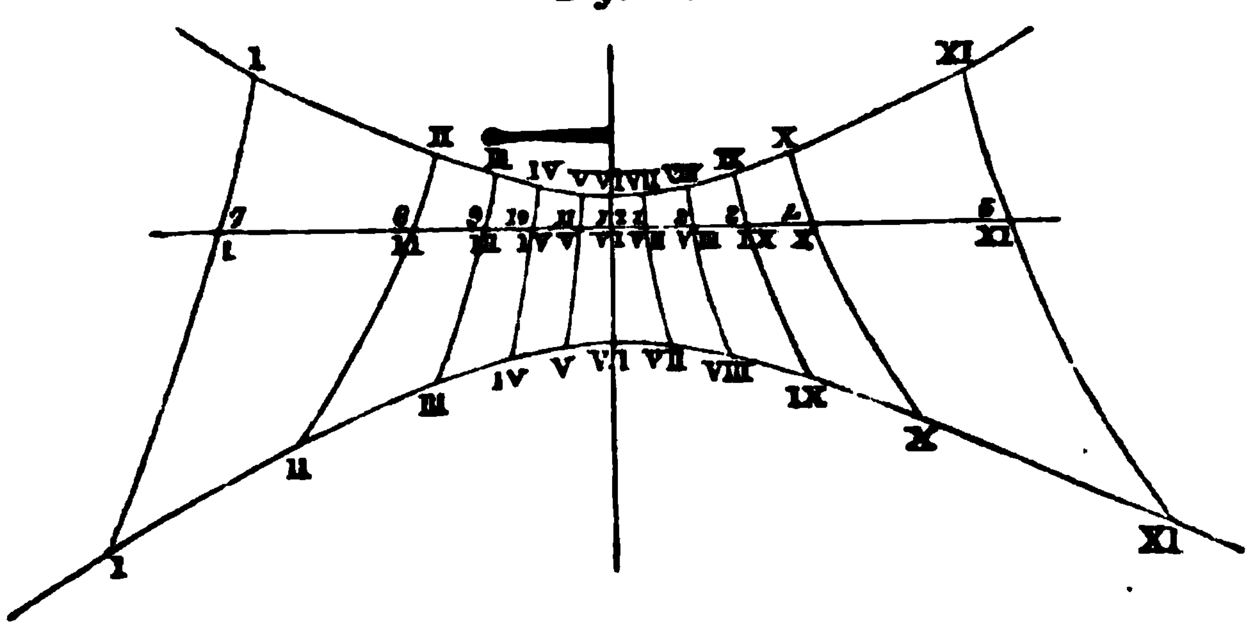
We have already said, that the equal hours which can be counted from sun-rise to sun-set, to the number of twelve, are called the natural hours; for it is this interval of time which really forms the day.

This kind of hours may be easily traced out on a dial, which we shall here suppose to be horizontal. For this purpose, it will be first necessary to draw the equinoctial, and the two tropics, by the preceding methods.

Now it must be observed, that as, in the latitude of London, the sun, on the day of the summer solstice, rises at 3h. 43m., and sets at 8h. 17m., the interval between these periods is equal to 17h. 34m.; consequently, if we divide this duration into 12 parts, each of these will be about 1½ hour: for this reason, draw lines from the centre of the dial to the points of division on the equinoctial, corresponding to 5½ hours, to 7 hours, to 8½ hours, to 10 hours, to 11½ hours, to 1 hour, and so on; but marking only, on the tropic of Cancer, the points of intersection which these hours form with it.

In like manner, as the sun at the winter solstice, in the latitude of London, rises at 8h. 8m., and sets at 3h. 52m., the duration of the day is only 7 hours 44 minutes; which being divided into 12 parts, gives for each about 40 minutes, or ⅔ of an astronomical hour. Draw therefore the hour-lines corresponding to 8⅔ hours, to 9⅓ hours, to 10 hours, and so on; marking only the points where they intersect the tropic of Capricorn; then, if the corresponding points of division, on the two tropics and the equinoctial, be joined by a curved line, the dial will be described, as seen Fig. 26.

Fig. 26.



If more exactness be required, it will be necessary to trace out two more parallels of the signs, viz. those of Taurus and Scorpio, and to find on each, by a similar process, the points corresponding to the natural hours: the natural hour-lines may then be made to pass through five points, by which means they will be obtained with much more exactness.

APPENDIX.

We shall conclude this subject by giving a general method of describing sun-dials, whatever be the declination or inclination of the plane.

This method is founded on the consideration that any plane whatever is always a horizontal plane to some plane on the earth; for a plane being given, it is evident that there is some point of the earth the tangent or horizontal plane of which is parallel to it. It is evident, also, that two such parallel planes will shew the same hours at the same time. Thus, for example, if we suppose at London a plane inclining and declining in such a manner, as to be parallel to the horizontal plane of Ispahan; then a dial traced out on that plane, as if it were horizontal, will give the hours of Ispahan; so that when the shadow falls on the substyle, we may say that it is noon at Ispahan, &c.

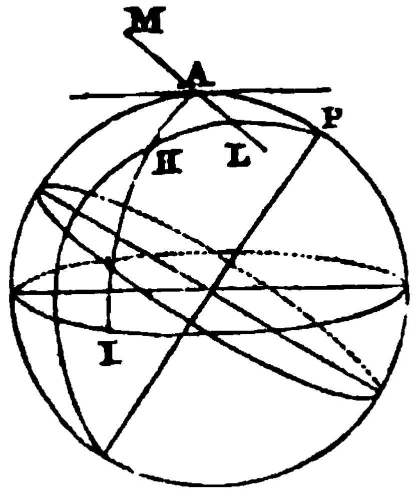
But as the hours of Ispahan are not those wanted at London, it is necessary that we should find out the means of delineating those of London, which will not be attended with much difficulty, when the difference of longitude between these two cities is known. Let us suppose then that it is exactly 45 degrees, or 3 hours: when it is noon at London then, it will be 3 in the afternoon at Ispahan; and when it is 11 in the forenoon at the former, it will be 2 in the afternoon at the latter, &c. Consequently, on this dial, which we suppose to be horizontal, if we assume the line of 3 o'clock as that of noon, and mark it 12; and if we assume the other hour-lines in the same proportion, we shall have at London the horizontal dial of Ispahan, which will indicate, not the hours of Ispahan, but those of London, as required.

We flatter ourselves that we have here explained the principle of this method in a manner sufficiently clear, to make it plain to such of our readers as have a slight knowledge of geometry or astronomy; but to render the application of it more familiar, we shall illustrate it by an example.

Let us suppose then, at London, a plane forming with the horizon an angle of 12 degrees, and declining towards the west 22½ degrees.

The first operation here is, to find the longitude and latitude of that place of the earth where the horizontal plane is parallel to the given plane.

Fig. 27.



For this purpose, let us conceive an azimuth A I perpendicular to the given plane (Fig. 27.), and in this azimuth, which we suppose to be traced out on the surface of the earth, let us assume, on that side which is towards the upper part of the plane, an arc A H, equal to the inclination of that plane to the horizon: the extremity of this arc, that is the point H, will be that point of the earth where the horizon is parallel to the given plane. This is so easy to be comprehended that it requires no demonstration. Let us next conceive a meridian P H, drawn from the pole P to the point H: it is evident that this will be the meridian of the given plane; and that the angle A P H, formed by this meridian and that of London, will give the difference of longitude of the two places. We must therefore determine this triangle, and to find it we have three things given, viz., 1st,

A P the complement of the latitude of London, which is 38° 29′; 2d, A H the distance of London from the place, the horizontal plane of which is parallel to the given plane, and which is 12°; 3d, the angle P A H, comprehended between these two sides, which is equal to the right angle H A L plus P A L, or that which the plane forms with the meridian.

By resolving this spherical triangle, it will be found, that the angle at the pole A P H, or that formed by the two meridians, is 5° 59′; which is the difference of longitude between the two places A and H.

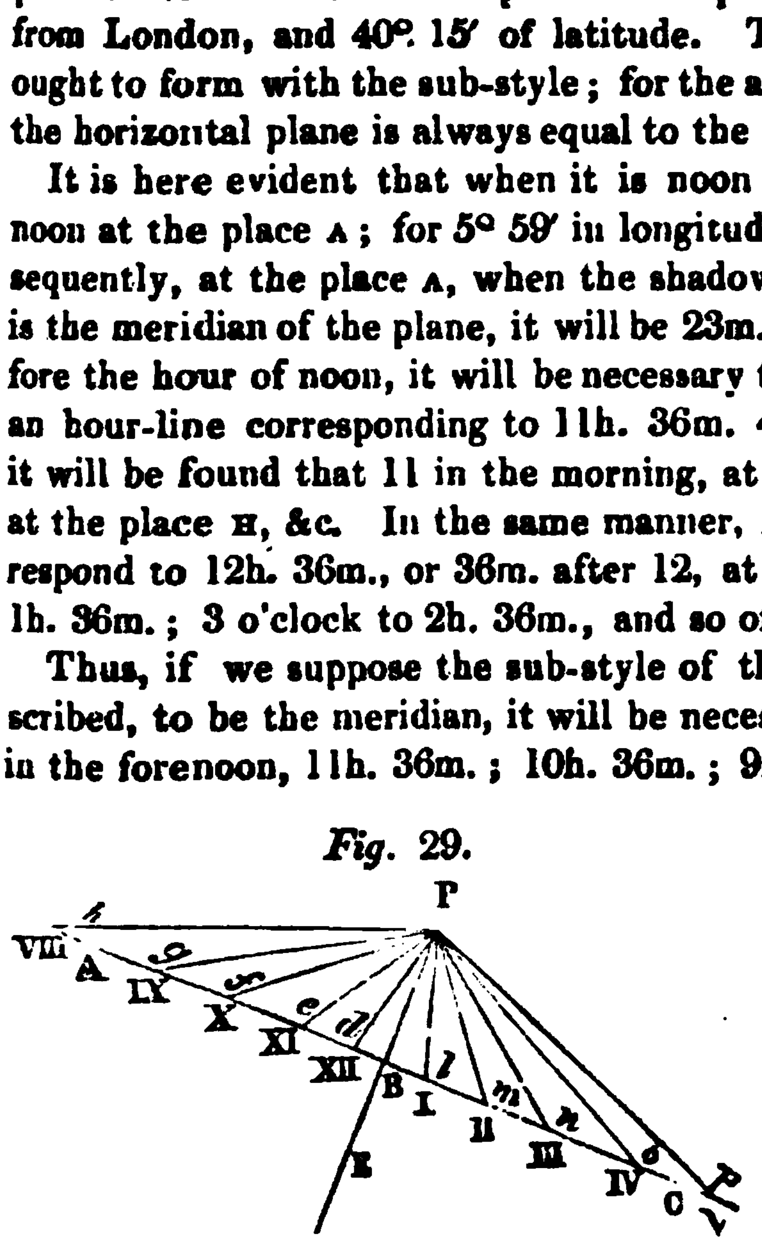
The latitude of the place H will be found also by the solution of the same triangle; for it is measured by the complement of the arc P H, of the triangle P A H: according to calculation it is 40° 15′.*

Thus, a plane inclining 12° at London, and declining to the west 22½ degrees, is parallel to the horizontal plane of a place which has 5° 59′ of longitude west from London, and 40° 15′ of latitude. The latter also is the angle which the style ought to form with the sub-style; for the angle which the axis of the earth forms with the horizontal plane is always equal to the latitude.

It is here evident that when it is noon at the place H, it will be 23m. 56s. after noon at the place A; for 5° 59′ in longitude correspond to 23m. 56s. in time. Consequently, at the place A, when the shadow of the style falls on the sub-style, which is the meridian of the plane, it will be 23m. 56s. after twelve at noon. To find therefore the hour of noon, it will be necessary to draw, on the west side of the sub-style, an hour-line corresponding to 11h. 36m. 4s., or 11h. 36m. By the like reasoning, it will be found that 11 in the morning, at the place A, will correspond to 10h. 36m. at the place H, &c. In the same manner, 1 in the afternoon, at the place A, will correspond to 12h. 36m., or 36m. after 12, at the place H: 2 o'clock will correspond to 1h. 36m.; 3 o'clock to 2h. 36m., and so of the rest.

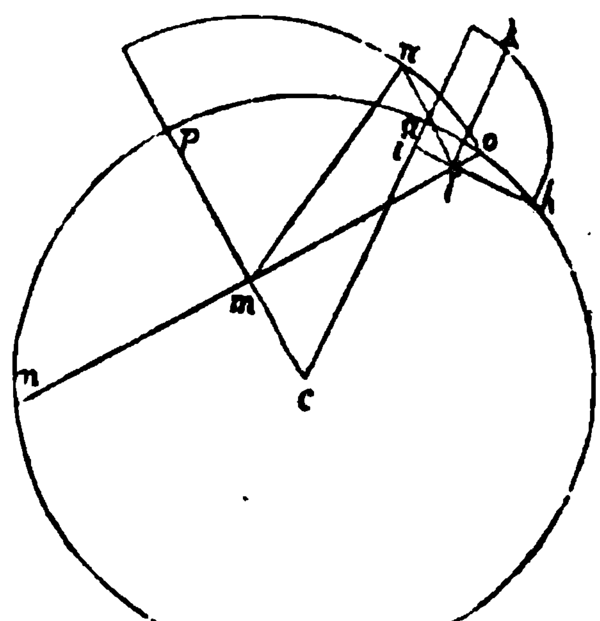
Thus, if we suppose the sub-style of the plane, on which the dial ought to be described, to be the meridian, it will be necessary to describe a dial which shall indicate in the forenoon, 11h. 36m.; 10h. 36m.; 9h. 36m.; 8h. 36m., &c.; and in the afternoon 12h. 36m.; 1h. 36m.; 2h. 36m.; 3h. 36m.; 4h. 36m., &c.

Fig. 29.



When these calculations have been made, the dial may be easily constructed. For this purpose, first find, by Prob. 3, the sub-style, which is the meridian of the plane. We shall suppose that it is P E (Fig. 29.), and that P is the centre of the dial. Having assumed P B of a

Fig. 28.



* Trigonometrical calculation may be avoided by means of a graphic operation exceedingly simple. In a circle of a convenient size (Fig. 28.), assume an arc *p a* equal to P A (Fig. 27.); make *a h* equal to A H; and from the point *h* let fall a perpendicular *h i*, on the radius C *a*. On *h i* describe a quadrant, or make *h k* equal to the arc which measures the declination of the plane, or equal to the supplement of the angle P A H: draw *k l* perpendicular to *h i*, and from the point *l*, draw *l m* perpendicular to the radius *c p*, and let *l m* be continued till it meet the circle in *n*: the arc *p n* will be equal to P H, and if an arc of a circle be described on *m o*, and if *l π* be drawn perpendicular from the point *l*, so as to meet this arc in *π*, the angle *π m l* will be equal to the required angle P of the triangle A P H.

convenient length, draw, through the point B, the line A B C, perpendicular to P B: if A be the western side, the line P d which corresponds to 11 hours 36 minutes, or which is distant from the meridian 24 minutes in time, may be found by making use of the following analogy:

As radius is to the cosine of the latitude, which is 40° 15′; so is the tangent of the hour-angle corresponding to 24m. in time, or the tangent of 6°, to a fourth term, which will be the tangent of the angle B P d.

By this analogy, it will be found equal to 80 parts, of which P D contains 1000: if 80 of these parts therefore, taken from a scale, be set off from B towards d, and if P d be then drawn, we shall have the hour-line of 11 hours 36 minutes for the plane of the dial, or of the place H.

The line P e, of 10 hours 36 minutes, will be found in like manner, by this analogy:

As radius is to the cosine of 40° 15′; so is the tangent of the hour-angle corresponding to 10h. 36m., or the tangent of 21° to the tangent of the angle B P e.

This tangent will be found equal to 293 of the above parts: if this number of parts therefore, taken from the same scale, be laid off from B to e, we shall have the hour-line P e corresponding to 10 hours 36 minutes.

The lines of the other hours before noon may be found in the like manner: the two first terms of the analogy are the same, and the third is always the tangent of an angle successively increased by 15°: these tangents therefore will be those of 6°, 21°, 36°, 51°, 66°, the logarithms of which must be added to the cosine 40° 15′; and if the logarithm of radius be subtracted, the remainders will be the logarithms of the tangents of the hour-lines: these tangents themselves will be for B d, B e, &c. 80, 293, 554, 942, 1732, 4814, &c. in parts of which the radius or P D contains 1000.

A similar operation must be performed for the hours in the afternoon. As 36m. in time correspond to 9°, the first hour-angle will be 9°; the second, by adding 15°, will be 24°; the third 39°; the fourth 54°, &c. The following proportions then must be employed: As radius is to the cosine of 40° 15′; so is the tangent of 9°, or 24°, or 39°, &c. to a fourth term, which will be the tangent of the angle B P l, or B P m, or B P n, &c.

Hence, if the logarithm of the sine of 49° 45′ be successively added to the logarithmic tangent, of 9°, 24°, 39°, 54°, &c., and if radius be subtracted from the different sums, we shall have the logarithms of the tangents of the angles which the hour-lines P l, P m, P n, &c. form with the sub-style; and these tangents themselves will respectively be 121, 339, 618, 1050, 1968, 7268, parts of which P B contains 1000. If these numbers therefore, taken from the same scale as before, by means of a pair of compasses, be set off from B to l, from B to m, from B to n, &c., and if the lines P l, P m, P n, P o, &c. be then drawn, the dial will be nearly completed; as nothing will be necessary but to mark the point d with XII, because P d is the meridian of the place A; and to mark the other hour-points with the numbers which belong to them, as seen in the figure.

To avoid the trouble of tracing out more hour-lines than are necessary, it will be proper first to determine at what hour the sun rises and sets on the given plane, at the time of the longest day; which may be easily done by means of the following consideration.

It may be readily seen that if we suppose two parallel planes, in two different places of the earth, the sun will begin to illuminate both of them at the same moment; and that he will also set to both at the same time. The plane of the dial in question, being parallel to the horizontal plane of a place which has 40° 15′ of north latitude, nothing is necessary but to know at what hour the sun will rise

in regard to that plane on the longest day. But it will be found that in the latitude of 40° 15′ the longest day is 15 hours 24 minutes; or that the sun rises on that day 7 hours 42 minutes before noon, and sets at 42 minutes past 7 in the evening. It will be sufficient then, on the dial in question, to make the first hour-line in the morning that of 4 hours 15 minutes, and the last in the evening 7 hours 30 minutes.

PART EIGHTH.

CONTAINING SOME OF THE MOST CURIOUS PROBLEMS IN NAVIGATION.

Navigation may be classed among those arts which do the greatest honour to the human invention; for in no department of science is the ingenuity of man displayed to more advantage than in this art, by which he conducts himself through the wide expanse of the ocean, without any other guide than the heavenly bodies and a compass; by which he subdues the winds, and even employs them to enable him to brave the fury of the ocean, which they excite against him: in short, an art which connects in social intercourse the two worlds; forms the principal source of the industry, commerce, and opulence of nations. Hence one of our poets very justly says,

Le trident de Neptune est le sceptre du monde.

But this is not a proper place for entering into a dissertation on the utility of navigation. We shall only observe, that navigation may be considered under two points of view. According to the first, it is a science which depends on astronomy and geography: considered in this manner it is called Piloting, which is the art of determining the course that ought to be pursued in order to go from one place to another, and of knowing at all times that point of the earth at which a ship has arrived. According to the other, it is an art founded on mechanics and the moving powers of the vessel: considered under this point of view, it is called manœuvring, and teaches how to give to that ponderous mass, which cleaves the billows, the necessary direction by means of the sails and the rudder.

We shall here present the reader with every thing most curious in both these parts of navigation.

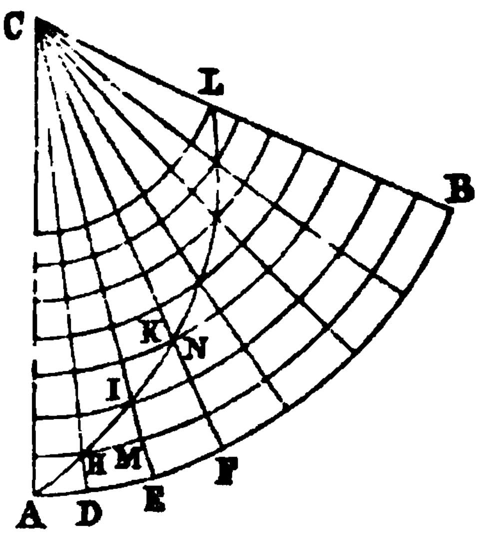
PROBLEM I.

Of the curve which a vessel describes on the surface of the sea, when she sails on the same point of the compass.

When a ship is about to set sail, it is necessary to find out the proper course: that is, to determine the direction in which she ought to proceed, in order to arrive, in the shortest time and with the greatest safety, at the place of her destination. When this direction, or the angle it forms with the meridian, has been determined, it is always pursued, unless particular circumstances prevent it. A vessel, by thus steering for several days, on the same point of the compass, describes a line which always forms the same angle with the meridians: this is what is called the *loxodromic* line, or oblique course; and there hence results, on the surface of the earth, a peculiar curve, the nature and properties of which have excited the attention of mathematicians. On these properties the practical rules of navigation have been founded; and, as they are very remarkable, they deserve to be explained.

We presume that the reader is acquainted with the nature of the compass, the different points, &c.; and with the elements of navigation; for it is impossible that we should here enter into details merely elementary.

Fig. 1.



Let us suppose that the sector A C B (Fig. 1.) represents a portion of the spherical surface of the earth, of which C is the pole, and A B the equator; or only the arc of a parallel comprehended between two meridians, as A C and B C; and that C D, C E, and C F, represent so many meridional arcs, very near to each other.

Let a vessel depart from the point A of the arc A B, the meridian of which is A C; and proceed on a course forming with that meridian the angle C A H, less than a right angle, for example an angle of 60 degrees; the vessel will describe the line A H, by which means she will always change her meridian. When she arrives at H, under the meridian C D, let her continue in the same course, making with the meridian the angle C H I, equal to the former; and so on, describing the lines A H, H I, I K, &c., always making the same angle (60°) with the meridians C A, C H, C I, C K, &c. As her course is continually inclined to the meridian at an angle of 60 degrees, it may be readily seen that the line A H I K will not be the arc of a great circle on the surface of the sphere; for it is demonstrated in spherics, that if A H K were a circle of this kind, the angle C H I would be greater than C A H, and C I K greater than C H I, and so on. The case would be the same if the curve A H I K were an arc of a lesser circle of the sphere; hence there is reason to conclude, that the curve described by a ship, when she always proceeds on the same course, is a peculiar curve, which constantly approaches the pole.

Remarks.—I. It is here evident, that when the loxodromic angle vanishes; that is, when the vessel steers directly north or south, the loxodromic line is an arc of the meridian.

But, if the angle be a right angle, and if the vessel be under the equator, she will describe an arc of the equator. In the last place, if out of the equator, she will describe a parallel.

II. If the loxodromic line A K L, be divided into several parts, so small that they may be considered as straight lines, and if as many parallels or circles of latitude be made to pass through the points of division H, I, K, &c., all these circles will be equal and equally distant from each other; so that, by making meridional arcs to pass through the same points of division, the portions of these meridians, such as D H, M I, N K, &c., will be equal, as well as the corresponding arcs A D, H M, I N, &c. This equality however will not be in degrees, but in miles, as may be easily demonstrated; for the triangles A D H, H M I, I N K, &c., are evidently similar, because the hypothenuses, A H, H I, I K, &c., being equal in length, the other sides will be respectively equal also. On the other hand, it is evident that if A D, which is part of a great circle, be equal in length, or in miles, to H M, which is part of a lesser circle, the latter must contain a greater number of minutes or degrees than the former.

III. When a very small portion of the loxodromic line, such as A H, has been passed over, always pursuing the same course, on the vessel's arrival at H, if the difference of latitude, or the arc D H, be determined by observation, it will be easy to find the distance sailed A H; since D H is to A H, as the sine of the angle H A D, which is known, is to radius. If the angle C A H, for example, be 60 degrees, and consequently H A D 30 degrees; and if D H be equal to half a degree, or 30 nautical miles, the distance A H will be 60 nautical miles; for the sine of 30 degrees is exactly equal to half the radius.

IV. If the course and distance sailed be known, the difference of latitude may be found in like manner.

V. The loxodromic angle C A H, or H A D, being known, as well as the difference of latitude D H, the value of the arc A D may be found; for D H is to A D as the sine of the angle H A D is to its cosine. But when the length of the arc of a parallel, or the number of miles it contains, is known, the degrees and minutes it contains may be determined also. In this manner, the difference of longitude produced by the vessel's change of position, while passing over the small loxodromic arc A H, is obtained; and if the same operation be performed in regard to all the other small arcs H M, I N, &c., we shall have the whole difference of longitude, produced by the vessel's passing over any loxodromic arc A K. The difficulty of this operation arises from these arcs being dissimilar, though equal in length. But geometricians have found means to avoid these calculations, by ingenious tables or other operations, the explanation of which does not fall within the plan of this work.

VI. This curved line has one property which is very singular, that it always approaches the pole without ever reaching it. This evidently follows from the nature of it; for if we suppose it to arrive at the pole, it will intersect all the meridians in that point; consequently, since it cuts each meridian under the same angle, it will cut them all at the pole under the same inclination, which is absurd; since they are all inclined in that point to each other. It will therefore approach the pole more and more, making an infinite number of circumvolutions around it, but without ever reaching it. Hence, according to mathematical rigour, a ship which continually pursues the same course, the cardinal points excepted, will always approach the pole, without ever arriving at it.

VII. Though the loxodromic line, when it forms an acute angle with the meridians, must make an infinite number of circumvolutions around the pole before it reaches it, its length is however finite; for it can be demonstrated, that the length of a loxodromic line, such as A K L, is to the length of the arc of the meridian that indicates the difference of latitude, as radius to the cosine, or sine complement, of the angle which the loxodromic line forms with the meridian; consequently the difference of latitude is to the loxodromic distance sailed, as the cosine of the above angle is to radius.

The above remark is principally intended for geometricians; and exhibits a kind of paradox which must astonish those to whom truths of this kind are not familiar: those, however, who comprehend the preceding demonstrations, can entertain no doubt of it. But, for the sake of farther illustration, let us suppose a loxodromic line inclined to the meridian at an angle of 60 degrees, with its infinite circumvolutions around the pole; if we employ the following proportion, As the cosine of 60 degrees, or the sine of 30°, is to radius, so is 90 degrees difference of latitude to a fourth term, this fourth term will be the absolute length of the loxodromic line. But the sine of 30 degrees is equal to half the radius; and hence it follows that the fourth part of the circle is the half of the above loxodromic line; or this line, notwithstanding the infinite number of its circumvolutions, is exactly equal to a semicircle of the sphere.

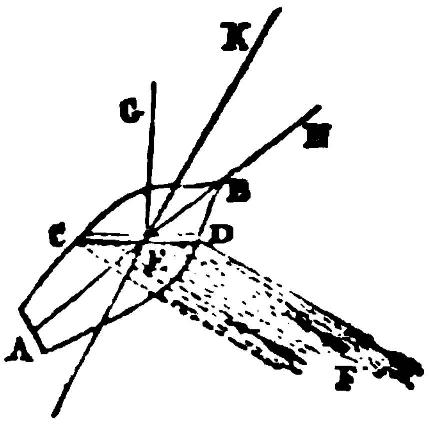
PROBLEM II.

How a Vessel may sail against the Wind.

What is here proposed will no doubt seem a paradox to those unacquainted with the principles of mechanics. Nothing however is more common in navigation, as this is always done when a vessel, according to the nautical term, is beating up on different tacks, or keeping as near to the wind as possible. But when we say a vessel can sail against the wind, we do not mean that she can proceed on a course directly opposite to the point from which the wind blows; it is only by making an acute angle

with the rhumb line passing through that point, which is sufficient; for by several tacks she can then advance in a direction contrary to that of the wind.

Fig. 2.

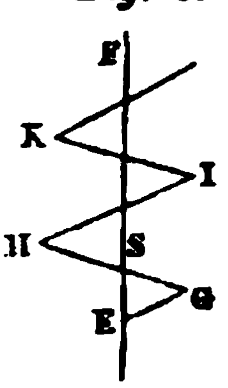


Let us suppose a vessel, (Fig. 2.), the keel of which is A B, and let one of the sails C D be set in such a manner, as to form with the keel an angle B E D of 40 degrees: if the direction of the wind be E F, making with the same keel an angle of 60 degrees, for example, it is evident that the angle D E F will be 20 degrees; consequently the sail will be impelled by a wind falling on it at an angle of 20 degrees. But according to the principles of mechanics, the action of a power falling obliquely on any surface, is exercised in a direction perpendicular to that surface, and therefore if E G be drawn perpendicular to C D, the line E G will be the direction according to which the effort of the wind is exercised on the sail C D, but with a diminished force on account of the obliquity of the stroke.

If the vessel were round, it would proceed in that direction; but as, in consequence of its length, it can move with much greater facility in the direction of its keel E H, than according to any other, it will assume a direction E K, somewhere between E G and E H, but much nearer to the latter than to the former, almost in the ratio of its facility to move according to E H and E G. The angle K E F therefore, which the ship's course forms with the direction of the wind, may be an acute angle. If the angle K E H, for example, be 10 degrees, the angle K E F will be 70½ degrees, consequently the vessel will lie almost two points nearer to the wind. But it is shewn by experience, that a vessel may be made to go on a course still nearer to the direction of the wind, or to lie closer to it by about one point more; for if the vessel be well constructed, there are 22 of the 32 points comprehended in the compass, which may serve to make her proceed to the same place.

It is indeed true, that the nearer a ship lies to the wind, or to speak in common terms, the sharper the angle of the wind's incidence on the sail, the less will be its force to push the vessel forwards; but this is compensated by the quantity of sail that may be set, for in this case none of the sails hurt each other, and a vessel can absolutely carry all her sails. What therefore is lost in consequence of the weakness of the force exerted on each, is gained by the quantity of surface exposed to the wind.

Fig. 3.



It may be easily conceived how advantageous this property of of vessels is to navigation; for whatever be the wind, it may be employed to convey a ship to any determinate place, even if it should blow directly from that quarter. For let us suppose (Fig. 3.) that the direct course is from E to F, and that the wind blows in the direction F S; the vessel must be kept as near the wind as possible to describe the line E G, making with E F the acute angle F E G; having proceeded some time in the direction E G, the vessel must then tack about to run down G H; then H I; then I K; and so on; by which means she will always approach nearer to the place of her destination.

PROBLEM III.

Of the force of the Rudder, and the manner in which it acts.

The force by which the rudder of a ship makes her move in any direction at pleasure, excites no small degree of astonishment; especially when we consider the weak action of the enormous rudders with which some of the barges that navigate our rivers and canals are furnished. The cause of this phenomenon we shall here endeavour to explain and illustrate.

The rudder of a barge or vessel has no action unless impelled by the water. It is the force resulting from this impulse, which being applied in a direction transversal to the poop, tends to make the vessel turn around a point of its mass, called the spontaneous centre of rotation. The prow of the vessel describes around this point an arc of a circle, in a direction opposite to that described by the poop; hence it follows that the prow of the vessel turns towards that side to which the rudder is turned, consequently opposite to that side towards which the tiller or lever of the rudder is moved. Hence, when the tiller is moved to the starboard side, the vessel turns towards the larboard, and *vice versa.*

A force, and even a certain degree of intensity, must therefore be applied to the rudder to make the vessel turn; and on this account the construction of the vessel is so contrived, as to increase this force as much as possible; for while the barges which navigate our rivers are in general very broad behind, and screen as we may say the rudder, so that the water flowing along their sides can scarcely touch it, the stern of vessels intended for sea are made narrow and slender, so that the water flowing along their sides must necessarily strike against the rudder, if in the least moved from the direction of the keel. Let us therefore endeavour to estimate nearly the force which results from this impulse.

A vessel of 900 tons, when fully laden, draws 13 or 14 feet of water, and its rudder is about 2 feet in breadth. Let us now suppose that the vessel moves with the velocity of 2 leagues per hour, which makes 176 yards per minute, or about 9 feet per second; if the rudder be turned in such a manner as to make with the keel continued an angle of 30 degrees, the water flowing along the sides of the vessel will impel the rudder under the same angle, that is, 30 degrees. The part of the rudder under water being 14 feet in length and 2 in breadth, presents a surface of 28 square feet, impelled at an angle of 30 degrees, by a body of water flowing with the velocity of 9 feet per second. But the action of such a current, if it impelled a similar surface in a perpendicular direction, would be 2205 pounds, which must be reduced in the ratio of the square of the sine of incidence to that of radius, or in the ratio $\frac{1}{4}$ to 1, since the sine of 30 degrees is $\frac{1}{2}$, radius being 1. The effort therefore of the water will be 551 pounds. Such is the force exercised perpendicularly on the rudder; and to find the quantity of this force that acts in a direction perpendicular to the keel, and which makes the vessel turn, nothing is necessary but to multiply the preceding effort by the cosine of the angle of inclination of the rudder to the keel, which in this case is $\sqrt{\frac{3}{4}}$ or 0·866, which will give 477 pounds.

The above computation is made on the old supposition, that the force of the water is diminished in proportion as the square of the sine of the incident angle is less than the square of the radius. But, by more accurate experiments it is found (Dr. Hutton's Math. and Philos. Dictionary, Tab. 3, Resistance), that at an angle of 30 degrees the absolute force is diminished only in the ratio of 840 to 278; hence then, the whole force 2205 pounds, reduced in this ratio, comes out 730 pounds, for the effective or perpendicular force on the rudder, to turn it or indeed the ship about, supposing the rudder held or fixed firm in that position.

But there is one cause which renders this effort more considerable: the water which flows along the sides of the vessel does not move in a direction parallel to the keel, but nearly parallel to the sides themselves which terminate in a sort of angle at the stern-post, or piece of timber which supports the hinges of the rudder; so that this water bears more directly on the rudder by an angle of about 30 degrees: hence, in the above case, the angle under which the water impels the rudder will be nearly 60 degrees; we must, therefore, make this proportion: As the square of the radius is to the square of the sine of 60 degrees, or as 1 is to $\frac{3}{4}$; so is 2205 to 1653. The force therefore which acts in a direction perpendicular to the keel, is 1653 pounds. Or,

by the table in the dictionary above quoted, as 840 is to 729 (for 60°), so is 2205 to 1913 pounds, the perpendicular force.

This effort will no doubt appear very inconsiderable when compared with the effect it produces, which is to turn a mass of 900 tons; but it must be observed that this effort is applied at a very great distance from the point of rotation and from the vessel's centre of gravity; for this centre is a little beyond the middle of the vessel towards the prow, as the anterior part swells out, while the posterior tapers towards the lower works, in order that the action of the rudder may not be interrupted. On the other hand, it can be shewn that what is called the spontaneous centre of rotation, the point round which the vessel turns, is also a little beyond the middle and towards the prow; hence it follows, that the effort applied at the extremity of the keel, towards the stern, acts to move the vessel's centre of gravity, by an arm of a lever 12 or 15 times as long as that by which this centre of gravity, where the weight of the vessel is supposed to be united, exerts its action. And lastly, there is no comparison between the action exercised by this weight when floating in water, and that which it would exert if it were required to raise it only one line. It needs therefore excite no surprise, that the weight of one ton, applied with this advantage, should make the vessel's centre of gravity revolve around its centre of rotation.

If the ship, instead of going at the rate of two leagues per hour, sails at the rate of three, the force applied to the rudder will be to that applied in the former case, in the ratio of 9 to 4; consequently, if the position of the rudder be as above supposed, the actual force will be 3719 pounds, or rather 4304 pounds: if the velocity of the vessel were 4 leagues per hour, this force in the same position of the rudder would be 4 times as much as at first, or 6612 pounds, or rather 7652 pounds.

Hence it is evident why a vessel, when moving with rapidity, is more sensible to the action of the helm; for when the velocity is double, the action is quadrupled: this action then follows the square or duplicate ratio of the velocity.

PROBLEM IV.

What angle ought the rudder to make, in order to turn the vessel with the greatest force?

If the water moves in a direction parallel to the keel when it impels the rudder, it will be found that this angle ought to be 54 degrees 44 minutes; but, as already observed, the water is carried along in an angular manner towards the direction of the keel continued; which renders the problem more difficult. If we suppose this angle to be 15 degrees, which Bouguer considers as near the truth, it will be found that the angle in question ought to be 46 degrees 40 minutes.

Ships do not receive the whole benefit of this force; for the length of the tiller does not permit the helm to form with the keel an angle of more than 30 degrees.

PROBLEM V.

Can a vessel acquire a velocity equal to or greater than that of the wind?

This can never take place in a direct course, or when the ship sails before the wind; for besides that in this case a part of the sails hurt or intercept the rest, it is evident that if the vessel should by any means acquire a velocity equal to that of the wind, it would no longer receive from it any impulse; its velocity then would begin to slacken in consequence of the resistance of the water, until the wind should make an impression on the sails equal to that resistance, and then the vessel would continue to move in an uniform manner, without any acceleration, with a velocity less than that of the wind.

But, when the course of the vessel is in a direction oblique to that of the wind,

this is not the case. Whatever may be its velocity, the sail is then continually receiving an impulse from the wind, which still approaches more to equality, as the course approaches a direction perpendicular to that of the wind: therefore, however fast the vessel advances, it may continually receive from the wind a new impulse to motion, capable of increasing its velocity to a degree superior to that even of the wind itself.

But for this purpose it is necessary that the construction of the vessel should be of such a nature, that, with the same quantity of sail, it can assume a velocity equal to $\frac{4}{11}$ or $\frac{3}{4}$ that of the wind. This is not impossible, if all the canvass which a vessel can spread to the wind, in an oblique course, were exposed in one sail, in a direct course. This then being supposed, Bouguer shews, that if the sails be set in such a manner as to make with the keel an angle of about 15 degrees, and if they receive the wind in a perpendicular direction, the vessel will continually acquire a new acceleration, in the direction of the keel, until her velocity be superior to that of the wind, and that in the ratio of about 4 to 3.

It is indeed true, that, as the masts of vessels are placed at present, it is not possible that the yards can form with the keel an angle less than 40 degrees; but some navigators assert, that by means of a small change this angle might be reduced to 30 degrees. In this case, and supposing that the vessel could acquire in the direct line a velocity equal to $\frac{3}{4}$ that of the wind, the velocity which it would acquire by receiving the wind on the sails at right angles, might extend to 1·034 that of the wind, which is a little more than unity, and therefore somewhat more than the velocity of the wind.

If we suppose the same velocity possible in the direct course, and that the sail forms with the keel an angle of 40 degrees, it will be found that the velocity acquired by the vessel in an oblique course, will be nearly $\frac{13}{16}$ the velocity of the wind.

This at least will be the case, if in this position of the sails, in regard to the wind, they do not hurt or obstruct each other. If all these circumstances therefore be combined, it appears that though it is possible, speaking mathematically, that a vessel can move with the same velocity as the wind, or even with a greater, it will be very difficult to produce this effect in practice.

PROBLEM VI.

Given the direction of the wind, and the course which a vessel must pursue in order to reach a proposed place; what position of the sails will be most advantageous for that purpose?

Let us suppose that the wind blows from the north, and that the ship's course is due east. If the ship, when her head is directed to that point, has her yards parallel to the keel, her progress will be $= 0$; as she will receive no impulse but in a direction perpendicular to the keel. On the other hand, if the yards be perpendicular to the keel, as the sails will not catch the wind, the vessel in this case again will not move. Thus, from the first position to the latter, the impulse in the direction of the keel, and consequently the velocity, goes on first increasing, and then decreasing. There is some position therefore at which this impulse is strongest, or what is called a *maximum*, and which will make the vessel move with the greatest velocity. The question is to determine it.

Geometricians have solved the problem, and have found, that to determine this angle, that between the wind and the proposed course must be divided in such a manner, that the tangent of the apparent angle, which the wind forms with the yard, shall be double to that which the yard forms with the course, or with the keel. In this case, therefore, the sail at first must be placed in such a situation, as to make

with the keel an angle of 35 degrees 16 minutes, and consequently with the wind an angle of 54 degrees 44 minutes.

We say the sail at first must be set in this manner; for as soon as the vessel has acquired a greater velocity, this angle will cease to be the most favourable, and will become less so, the more the velocity is accelerated, as must be the case, till the impulse of the wind be in equilibrio with the resistance which the vessel suffers from the water; but in proportion as the velocity is accelerated, the wind strikes the sail more obliquely, and loses its force: for this reason the sail must be disposed in such manner, as to form with the keel an angle always more acute, and this angle may be reduced to 30 degrees and less; so that the wind shall make with the sail an angle of 60 degrees and more.

We have here considered the question independently of lee-way; but if this be taken into account, supposing it for example in the present case to be one point, it will be necessary to make the vessel's head lie a point nearer to the wind: the angle then which the wind forms with the course will be from 78 to 79 degrees; and it will be found that on the outset, the angle formed by the wind and the sail ought to be 48° 45′; and that of the yard with the keel 29° 45′, which must gradually be reduced to 24 or 25 degrees. By then steering WNW ½ W, the vessel will really proceed east with the greatest velocity possible, or nearly so; and as in the neighbourhood of those points which give a *maximum* the progressive increase is insensible, this greatest velocity will always be nearly obtained, even when the above angles are not very exact.

PROBLEM VII.

In what manner must a vessel at sea be directed, so as to proceed from any given place to another by the shortest course possible?

As the loxodromic line, which navigators generally follow at sea, is not the shortest way from one place to another, it is natural to ask whether there be not some means by which the shortest course can be pursued; for it is evident, *cæteris paribus*, that the way being shorter, the voyage would be sooner ended.

As this is no doubt possible, we shall first shew how it may be done, and then examine with what advantage it is attended.

Every one knows that the shortest way from one place to another, on the surface of the earth, is the arc of a great circle drawn from the one to the other. Nothing then is necessary but to keep the vessel continually on the arc of a great circle, or at least to deviate very little from it.

Let us suppose, then, that a vessel is bound from London to the island of Trinidad. It will be found by trigonometrical calculation, that the arc of a great circle drawn from London to Trinidad, makes at London with the meridian an angle of 69° 44′, and at Trinidad of 37° 30′; while that of the loxodromic line with the meridian is at London 50° 40′. The angle formed by the course with the meridian, at the time of departure, ought therefore to be 69° 44′.

But to keep the vessel in this great circle, it will be necessary to change the angle every day; and strictly speaking every hour and every moment; otherwise the vessel will describe small loxodromic lines, and not the arc of a great circle. The following method, which if not perfectly exact, approaches very near the truth, may be employed to effect this change.

As the angle at Trinidad is 37° 30′, it may be easily seen, that from the time of the vessel's departure, till that of her arrival at the place of destination, the angle of the course must be gradually diminished, from 69° 44′ to 37° 30′. Let us divide the difference, which is 32° 14′, into 10 equal portions, which will each be 3° 13′. Every time then that the difference of longitude is one tenth of the whole, or about 5° 37′, that is, when the vessel has made about 111 leagues of departure towards the

west, it will be necessary to keep 3° 13′ more to the south. By these means the vessel will be kept nearly on the arc of a great circle, passing through London and Trinidad.

These angles might be more exactly determined by means of trigonometry; that is, by drawing a meridian at about every four degrees of longitude, and successively solving the spherical triangles thence resulting; but if we examine what advantage would arise from this operation, it will be found of very little importance. The distance from Plymouth to Trinidad, measured on a great circle drawn from the one to the other, is about 1212 leagues; and if the loxodromic line drawn from the one to the other be measured, it will be found to be about 1254. It is therefore not worth while to seek for the shortest course to save about 40 leagues; especially as in sea voyages the principal object is not to pursue the shortest route, but to take advantage of the wind whatever it may be, in order to complete the voyage.

PROBLEM VIII.

What is the most advantageous form of construction for the prow of the vessel, in order that she may sail better, or be easier steered?

If one only of these objects were to be attained, that for example of cleaving the water with the greatest facility, the problem might be easily solved. The sharper a vessel is at the prow, the easier she can cut the water, and consequently will be better calculated for moving with rapidity.

But an object still more important than velocity, is that of being easily worked: without this property a vessel, like a refractory horse, would render useless the whole art of the navigator. But it is shewn, both by experience and reason, that a vessel, to be manageable, must be narrow towards the prow, in the part immersed, in order that the water which runs along her sides may strike the rudder with more facility. She will also be managed with more ease, the farther the centre of gravity is from the stern; and for this reason the most obtuse and the widest part of the vessel must be towards the head. This is actually the case in regard to all vessels destined for voyages.

Nature, in regard to this point, seems to have provided man with a model in the form of fishes; for it may be readily seen that the thickest part of the fish is towards the head, which in general is even pretty obtuse. Like our ships, they have much more need of being able to turn and direct themselves with ease, than to move with rapidity. The best vessel perhaps would be that constructed according to the exact dimensions of a migrating fish, such as the salmon; which seems to enjoy, in a greater degree than any other, the two properties of moving quick and directing itself with ease.

M. Camus, a gentleman of Lorraine, gives an account, in his Mechanics, of several experiments, from which he endeavours to shew, that the model of a vessel will move faster with the thick end foremost, than when cleaving the waves with the other, which is sharper: he even assigns reasons for this idea, but they are certainly ill founded. These experiments are in absolute contradiction to sound theory; and if ships have that form, it is not that they may move faster, but in consequence of the necessity which has been found, of sacrificing the advantage of velocity to that of being easily manœuvred.

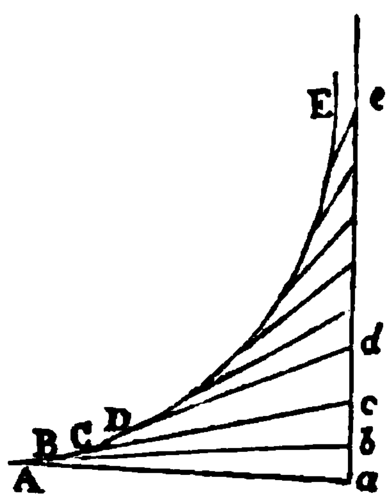
M. Montucla here, rather injudiciously, opposes theory to experiment, and censures Camus improperly, whose experiments and reasonings have been confirmed by the more accurate and extensive ones made, in the years 1793—1798, by the English Society for the improvement of Naval Architecture, as may be seen at large in the Report of their Committee, printed in the year 1800.

PROBLEM IX.

What is the most expeditious method of coming up with a vessel which is chased, and which is to the leeward?

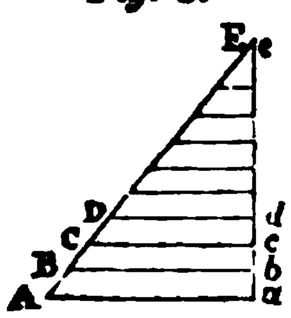
When a vessel is descried at sea, and you are desirous of coming up with her, you would be much mistaken if you directed the head of your own vessels towards the one you are pursuing; for unless the chase were proceeding on the same course exactly, you would either be obliged to change your direction every moment, or you would lose the advantage of the wind by falling to the leeward.

Fig. 4.



If a body *a* (Fig. 4.) moves in the line *a b c d*, and if it be proposed that another body A should come up with it, the body A ought not to be impelled in the direction A *a*; for in a few moments *a* will have advanced on the line in which it moves, and will have reached the point *b*, for example. Hence if we suppose that the body A always changes its course, directing itself towards the one it pursues, it will describe a curve such as A B C D E, and will at length reach the body *a* by going faster, but not by the shortest way. If it does not change its direction every moment, it will arrive at a point in the line *a d*, which the body *a* has already left, and will pass it, unless it set out to pursue it along the line *a d*, which would still make it lose time.

Fig. 5.



To cause the body A therefore to come up with *a*, in the least time possible, A must be directed to a point in the line *a e* (Fig. 5.), so situated, that A E and *a e* shall be to each other in the ratio of their respective velocities. But these lines will be in this ratio, if the body A, at every moment in its course, has that which it pursues similarly situated, in a direction parallel to the direction A *a*; that is, A *a* being directed to the south, if the body *a*, when it reaches *b*, is to the south of the body A when it arrives at B; for it is evident that the lines A E, *a e*, will then be proportional to the velocities of the two bodies, and they will arrive at the same time at E or *e*.

Navigators are sensible of this, both from practice and reason; for if a vessel at A espies another at *a*, the course of the latter *a e*, may be ascertained nearly without much difficulty, and the ship in chase, instead of directing her head towards *a*, will follow a course such as A B, inclined from *a*, and at the same time the bearing of the vessel in the direction A *a* will be taken by means of a compass; when A has proceeded some time, and reached B, for example, while *a* has reached *b*, the bearing of the vessel *a* in the direction B *b* will be again taken: if it be still the same, it is a sign that A is gaining ground, for A *a* and B *b* are parallel. If the chace falls a little behind, it shews that she may be pursued in a line making with the direction of her course, a less acute angle; but if she has got a-head, a line more inclined must be pursued to reach her; and if the line be as much inclined as possible, and approaches to parallelism, there is reason to conclude that the chase is a better sailer, and that all hope of reaching her must be given up.

It is here supposed that the chasing vessel has the advantage, or is to windward; for if she be to leeward, the manœuvring must be different, unless she has a great advantage in being able to lie near the wind. But this is not the proper place for enlarging on these manœuvres of the most ingenious of all arts.

PROBLEM X.

On determining the Longitude at sea.

To find the longitude at sea is a problem which has long engaged the attention of mathematicians; and they have been stimulated in their efforts by munificent rewards which have been offered by some of the maritime states of Europe. The British parliament offered £20000. for the discovery of a practical method which could be relied on within certain specified limits; and the reward was paid to Mr. Harrison for the improvements which he made in chronometers; one which he sent from England to Jamaica being found less than two minutes in error on its return.

Since the days of Harrison chronometers have been greatly improved; they are made of a more portable size, and are sold at a comparatively cheap rate. They form an essential part of the outfit of every respectable navigator, and have conferred great benefits on maritime science.

Formerly the mariner had no guide to his longitude but the dubious one depending on the compass and the log; and at the conclusion of a long voyage errors amounting to several degrees were not unfrequently found in the reckoning.

To determine the longitude by celestial observations it is necessary to find by independent processes the time at the place where you are; and also at the same instant the corresponding time at some assigned or known meridian, as that of the Royal Observatory at Greenwich, which is the meridian from which English seamen compute their longitude.

Now, with respect to the time at the place of observation, we may observe, that for every individual instant, every celestial object has a specific place in the heavens—changing its place as the time changes. At every instant its altitude, or its distance from the horizon, varies; the change of altitude being most rapid when the bearing of the object is *east* or *west*, and slowest when the bearing is north or south, that is, when the object is on the meridian.

If then the altitude of a celestial object be observed with a sextant or other like instrument, when the altitude is varying quickly, the true *mean time* at the place of observation may be found at the instant of taking the altitude. And if, when such an altitude is observed, the time by a chronometer be noted, whose error for Greenwich *mean time* on a given day, and daily gain or loss, are known, the true mean time at Greenwich may readily be inferred, and the difference between these local times (one for the meridian of the ship, and the other for that of Greenwich) is the longitude of the ship in time; and it may be converted into degrees by allowing 15 degrees for every hour of time.

This method of finding the local time at sea, which is that depending on observed altitudes in a known latitude, may be considered as the only one generally practicable at sea; on land many other methods may be used with advantage. We have adverted to one method of finding the *Greenwich time*, viz. that by chronometers whose errors and rates are settled before going to sea. We have now to give an account of another method of determining this important element, which within the last 80 years has been brought to a high degree of perfection—we mean that by lunar *observations*. By the successive efforts of men of science, building on the foundation laid by NEWTON, it is now found practicable to predict the moon's place in the heavens to a very minute degree of accuracy; and in fact, in the Nautical Almanac, her distance from the sun, nine fixed stars, and the four brightest planets, are given for every third hour of mean Greenwich time throughout the year, except when the moon is too near the sun to be visible. It must not however be understood that her distance is given from *all* of these objects for every day; a few conveniently situated on each side of her are selected for each day, and they are changed for others as the moon changes her place.

Now if with a sextant we observe the distance of the moon from one of the objects whose distances from her on the day of observation are given in the Nautical Almanac, we must first find by computation what the observed distance would have been if the observer had been at the centre of the earth. The methods which have been devised for making this computation are numberless, and many are of nearly equal merit. But for the details of the methods we must refer to works on practical navigation.

Having found the distance of the moon from the sun, star, or planet, as seen from the centre of the earth, we have only to turn to the Nautical Almanac for the given day, and find to what Greenwich mean time the distance corresponds; and comparing that time with the local mean time at the ship, the longitude is deduced as before.

In deducing the distance seen at the centre from that observed on the surface of the earth, the altitudes of both objects are necessary elements; and if either of the objects be at a suitable distance from the meridian, the mean time at the place may be deduced from its altitude, and the longitude found accordingly.

The longitude may also be found by comparing the mean time at the ship, of the immersion of one of Jupiter's satellites in his shadow, or the emersion of the satellite from the shadow, with the Greenwich time of the immersion or emersion as given for the day in the Nautical Almanac. Much difficulty has however been experienced in holding a telescope, of sufficient magnifying power for observing these eclipses, steady at sea; but we are assured that some persons have succeeded in doing so.

Eclipses of the moon have also been used for finding the Greenwich time; but the times of beginning and end, or the contact of the border of the earth's shadow with the spots on the moon, can seldom be noted with requisite precision.

Eclipses of the sun and the kindred phenomena of occultations of stars by the moon have occasionally been made available for the same object. They admit of great precision in the results, but the necessary calculations are long and intricate, and the phenomena occur too seldom to be of much use in the practice of navigation.

It is to the moon, however, that astronomers have in general looked for the solution of this problem; the comparative quickness of her angular motion rendering her decidedly the most eligible of all celestial objects for determining by her motions small intervals of time.

Some persons have recently proposed her *meridian altitudes* as observed in a latitude determined by other means, for finding the longitude; and there is no doubt that the longitude is involved as an element in the observed altitude. But no latitude can be determined at sea with the precision requisite to render this method of any value, and it is much to be wished that seamen were put on their guard against trusting to any such method of finding their longitude.

Before closing this article, we shall give an account of a method which has been extensively used within the last few years, for finding differences of longitude on land: the observations being the differences between the intervals of transit over the meridian of the moon's bright limb, and a star, as observed by transit instuments at two different places; availing ourselves of the form under which Mr. Riddle has put the problem in his "Treatise on Navigation and Nautical Astronomy."

It is evident (Mr. Riddle observes), that if the moon had no motion, and her semi-diameter did not change, the interval between the times of transit would be the same at both places, and that the difference of the intervals arises from and is equal to the increase of the moon's right ascension in the time of the limb's passing from one meridian to the other; or the westerly meridian must revolve in that time through an angle equal to the sum of the difference of longitude, and the increase of the moon's right ascension.

Therefore, if D = the different longitude I = the increase, in time of the moon's

right ascension in a sidereal hour, and I' the observed increase of her right ascension, (or *the difference of the observed intervals*) between the passing the two meridians, then $I : 1^h :: I' : D + I'$, whence $D = \frac{1^h - I}{I} I'$.

Now from the Nautical Almanac, the increase of the moon's right ascension in an hour may be taken out at once, and hence its increase in a sidereal hour may readily be found, whence the longitude may readily be computed.

Mr. Riddle gives a table of three pages, which contains the name of $\frac{1^h - I}{I}$ for every possible value of I, the argument of the table being I for an hour of mean time, but the number from the table corresponds to I for an hour of sidereal time; so that there is no necessity for reducing the mean time interval into a sidereal one. With the aid of the table the longitude may be found by adding two logarithms together.

PROBLEM XI.

If a vessel should be able to reach either of the poles, what method ought the commander to pursue, in order to steer in the direction of a determinate Meridian?

The difficulty which this problem seems, on the first view, to present, arises from this circumstance, that if a vessel were at either of the poles, to whichever side she might turn, her head would be directed towards the south or north. Every line drawn from that point to any point whatever in the horizon, is a meridian; and consequently at the pole there is neither east nor west. But if there is neither east nor west, how would she steer, or how would it be possible, all the meridians being similar, to find that in the direction of which it would be necessary to proceed, in order to reach the proposed place?

This however is not all: if a vessel should reach one of the poles, it is probable that the compass would become useless, or as the sailors say *run entirely mad;** and there are only two ways of navigating a vessel, either by the magnetic needle, or by observing the stars, or rather by both these methods combined.

Such is the problem which the astronomer who accompanied the Hon. Capt. Phipps, afterwards Lord Mulgrave, sent out to attempt a passage through the northern ocean, would have had to solve, had the expedition succeeded. If the progress of the vessel had not been stopped by the ice, he would have proceeded to the 90th degree of latitude in order to arrive by the shortest passage at the strait which separates Asia from America—a strait, the existence of which is now confirmed by the expeditions of the Russians, and by the researches of Captain Cook, and which lies in about the 176th degree of longitude. I proposed this problem to myself, in consequence of a new attempt which was about to be undertaken in France, by M. de Bougainville. I have heard that it was proposed to a celebrated astronomer, a member of the Royal Academy of Sciences: I do not know what answer he returned; but my solution is as follows:—

Had I been the navigator entrusted with the expedition, that I might not be taken by surprise, I should have provided myself with two or three good time-keepers, all exactly set to the time at the port of departure, which I suppose to be Brest.

Let us now suppose that the sea was found open, and that I had arrived at the north pole. I shall suppose also that my compass had become entirely useless; but that I had the sun on the horizon, which is the case in summer, and therefore such an expedition ought never to be undertaken but at that period, during which the sun is visible in those regions for several months. It is evident that by consulting my time-keepers, the moment when they indicated noon would be that when the sun was on the meridian of Brest; consequently had I been desirous of returning thither, nothing would

* There is no reason to believe that the compass would have no directive power at the terrestrial pole, as the magnetic pole and that of the earth are quite distinct.

have been necessary but to turn the ship's head towards the sun, and to steer on that course in such a manner, as to have the sun at the end of an hour 15 degrees to the starboard; at the end of two hours 30 degrees, &c. It may be readily conceived that by these means, though destitute of a compass, I should have kept my vessel pretty exactly on the line of the determinate meridian.

Now, if the meridian, on which it was necessary I should steer, had been distant from that of the place of departure 176°, as seems to be the case with that of the strait which separates Asia from America, it may be easily seen that I should have had nothing to do but to direct the ship's head within about 4 degrees of the point diametrically opposite to the sun, when the time-keepers indicated noon; or towards the sun itself when they indicated 16 minutes after midnight, and then to keep on this course by the method above described; changing every hour the angle formed by the ship's course with the azimuth passing through the sun. If we suppose the mouth of the strait in question to be, in regard to Brest, in the longitude already mentioned, it is evident that I should not have failed to enter it.

But, it is to be observed, that this expedient would be necessary only when very near to the pole: at the distance of ten degrees from it, other means of directing the ship's course might be employed. We shall not however enlarge farther on this subject; for it would be of very little use to point out these means, since the latest voyages seem to prove that the arctic pole, at the most favourable seasons, that is to say during the summer of our hemisphere, is surrounded by a covering of ice ten degrees at least in diameter, and which even extends farther towards Asia and America; or, in all probability, adheres to these two continents, except perhaps during some excessively hot summers. In a word, I am fully persuaded that the idea of traversing the frozen ocean, in order to proceed to the seas of China and Japan, is a mere chimera; and that if a vessel should even be able to get thither, by steering close along the shores of Asia or America, to the strait above mentioned, the voyage would be attended with so many dangers, and require circumstances so favourable, that it would be madness to attempt it. What indeed would become of a ship if, retarded by any of the accidents so common in those seas, she should be obliged to winter, nearly a whole year, in any port of the almost uninhabited northern coast of Asia? What assistance could she expect from the Samoeides, or any other of these nations still more barbarous? If the crew remained there, how could they secure themselves from the intense cold of these climates? If they quitted their vessel, to take up their lodging in a close hut, after carrying thither their provisions, would not the vessel be exposed to the danger of being plundered or burnt? Such an enterprise would require, that the commercial nation which undertook it, should have a port belonging to it in some advantageous situation, that ships obliged to winter in those cold regions might have a convenient place of shelter. But what appearance is there that Russia, the sole mistress of these countries, will ever consent to such a measure; especially as the Russian government so long concealed the information it had obtained in regard to the strait above mentioned?

PART NINTH.

CONTAINING SOME CURIOUS PARTICULARS IN REGARD TO ARCHITECTURE.

ARCHITECTURE may be considered under two points of view. According to the first, it is an art, the object of which is to unite utility and grace; to give to an edifice that form fittest for the purpose to which it is destined, and at the same time the most agreeable by its proportions; to strike the beholder by magnitude or extent, and to please by the harmony of the different parts and their relation to each other: the more an architect succeeds in uniting all these requisites, the more he will be entitled to rank among the eminent men who have distinguished themselves in this art.

But it is not under this point of view that we here consider it; we shall confine ourselves to the geometrical and mechanical part of Architecture, as it presents us with several curious and useful questions, which we shall lay before the reader.

PROBLEM I.

To cut a Tree into a Beam capable of the greatest possible resistance.

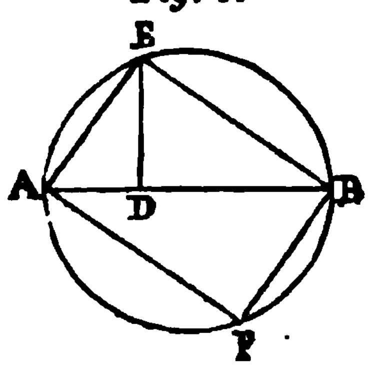
This problem belongs properly to Mechanics; but on account of its use in Architecture, we thought it might be proper to give it a place here, and to discuss it both geometrically and philosophically. We shall first examine it under the former point of view.

Galileo, who first undertook to apply geometry to the resistance of solids, has determined on a very ingenious train of reasoning, that when a body is placed horizontally, and fixed by one of its extremities, as is the case with a quadrangular beam projecting from a wall, if a weight be suspended from the other extremity, in order to break it, the resistance which it opposes is in the compound ratio of the horizontal dimension and the square of the vertical dimension. But this would be more correctly true, if the matter of the body were of a homogeneous and inflexible texture.

It has been shewn also, that if a beam is supported at both extremities, and if a weight, tending to break it, be suspended from the middle, the resistance it opposes is in the ratio of the product of the breadth and square of the depth, divided by half the length.

To solve therefore the proposed problem, we must cut from the trunk of the tree a beam of such dimensions, that the product of the square of the one by the other shall be the greatest possible.

Fig. 1.



Let A B then (Fig. 1.), be the diameter of the circle, which is the section of the trunk; the question is, to inscribe in this circle a rectangle, as A E B F, of such a nature, that the square of one of its sides A F, multiplied by the other side A E, shall give the greatest product. But it can be proved that, for this purpose, we must first take, in the diameter A B, the part A D equal to a third of it, and raise the perpendicular

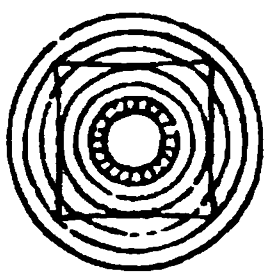
D E, till it meet the circumference in E: if B E and E A be then drawn, and also A F and F B parallel to them, we shall have the rectangle A E B F, of such a nature, that the product of the square of A F by B F, will be greater than that given by any other rectangle inscribed in the same circle. If a beam of these dimensions, cut from the proposed trunk, be placed in such a manner, that its greatest breadth A F shall be perpendicular to the horizon, it will present more resistance than any other that could be cut from the same trunk; and even than a square beam cut from it, though the latter would contain more matter.

Remark.—Such would be the solution of the problem, if the suppositions from which Galileo deduced his principles, in regard to the resistance of solids, were altogether correct. He indeed supposes that the matter of the body to be broken is perfectly homogèneous, or composed of parallel fibres, equally distributed around the axis, and presenting an equal resistance to rupture; but this is not entirely the case with a beam cut from the trunk of a tree which has been squared.

By examining the manner in which vegetation takes place, it has been found, that the ligneous coats of a tree, formed by its annual growth, are almost concentric; and that they are like so many hollow cylinders, thrust into each other, and united by a kind of medullary substance, which presents little resistance: it is therefore these ligneous cylinders chiefly, and almost wholly, which oppose resistance to the force that tends to break them.

But, what takes place when the trunk of a tree is squared, in order that it may be converted into a beam? It is evident—and it will be rendered more sensible by inspecting Fig. 2—that all the ligneous cylinders, greater than the circle inscribed in the square, which is the section of the beam, are cut off on the sides; and therefore the whole resistance almost arises from the cylindric trunk inscribed in the solid part of the beam. The portions of the cylindric coats which are towards the angles, add indeed a little strength to that cylinder, for they cannot fail of opposing some resistance to the breaking force; but it is much less than if the ligneous cylinder were entire. In the state in which they are they oppose only a moderate effort to flection, and even to rupture. For this reason, there is no comparison between the strength of a joist made of a small tree, and that of another which has been sawn, or cut with several others from the same beam or block. The latter is generally weak, and so liable to break, that joists, and other timber of this kind, ought to be carefully rejected from all wooden work which has to support any considerable weight.

Fig. 2.



We shall here add, that these ligneous and concentric cylinders are not all of equal strength. The coats nearest the centre, being the oldest, are also the hardest; while, according to theory, the absolute resistance is supposed to be uniform throughout.

It needs therefore excite no surprise, that experience should not entirely confirm, and even that it should sometimes oppose, the result of theory. Hence we are under considerable obligations to Duhamel and Buffon, for having subjected the resistance of timber to experiments; as it is of great importance in architecture to know the strength of the beams employed, in order that larger and more timber than is necessary may not be used.

But notwithstanding what has been said, it is very probable that the beam capable of the greatest resistance, which can be cut from the trunk of a tree, is not the square beam; for the following experiments made by Duhamel seem to prove, the size being the same, that the beam which has more depth in proportion than breadth, when the depth is placed vertically, presents so much more resistance; and even

without deviating very much from the law proposed by Galileo, viz., the compound ratio of the square of the vertical dimension and that of the breadth.

Duhamel indeed caused to be broken twenty square bars of the same volume, to determine what form of dressing would render them capable of the greatest resistance. They all had 100 square lines of base, and four of each sort were employed of the different dimensions, to compose the same area.

The first four, which were 10 lines in every direction, sustained a weight of 131 pounds.

Four others, which were 12 lines in one direction and 8⅓ in another, sustained each 154 pounds. The above law would give 157 pounds.

The next four, which were 14 lines in height and 7⅛ in breadth, supported each 164 pounds. Calculation would give 183 pounds.

Four more, which were 16 lines in height and 6¼ in breadth, sustained each 180 pounds. According to calculation they ought to have supported 209 pounds.

The last four, which were 18 lines in height and 5⅓ in breadth, sustained each 243 pounds. Calculation would have given only 233 pounds. It is very singular that in this case calculation should give less than experience; while in the other cases the result was contrary.

Buffon began experiments on a larger scale, in regard to the resistance of timber, an account of which may be seen in the Memoirs of the Academy of Sciences for the year 1741. It is to be regretted that he did not pursue this subject, on which no one could have thrown more light. It appears to result from these experiments, that the resistance increases less than in the square of the vertical dimension, and decreases in a ratio somewhat greater than the inverse of the length.

In short, the result of the whole is, that to solve the proposed problem, it would be necessary to have physical data of which we are not yet in possession; that the beam capable of the greatest resistance, that can be cut from the trunk of a tree, is not a square beam; and that in general many researches are to be made respecting the lightening of carpenters' work, which often contains forests of timber in a great part useless.

Our readers are referred to Barlow on the Strength of Timber, for a series of valuable experiments and important deductions on this subject.

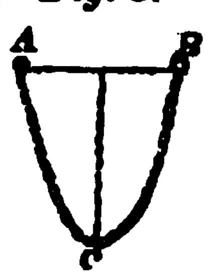
PROBLEM II.

Of the most perfect form of an Arch. Properties of the Catenarian Curve, and their application to the solution of this problem.

The most perfect arch, no doubt, would be that, the voussoirs of which being exceedingly thin, and even smooth on the sides in contact, should maintain themselves in complete equilibrium. It may easily be perceived that, in consequence of this form, very light materials might be employed; and we shall shew also that its push or thrust on the piers would be much less than that of any other arch of the same height, constructed on the same piers.

This property and this advantage are found in a curve well known to geometricians under the name of the Catenarian, and called by the French *la Chainette*. This name has been given to it because it represents the curve assumed by a chain A C B, Fig. 3, composed of an indefinite number of infinitely small and perfectly equal links, or by a rope perfectly uniform and exceedingly flexible, when suspended freely by its two extremities.

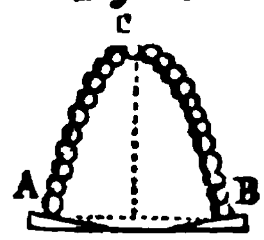
Fig. 3.



The determination of this curve was one of those problems which Leibnitz and Bernoulli proposed towards the end of the 17th century, in order to shew the superiority of their calcu-

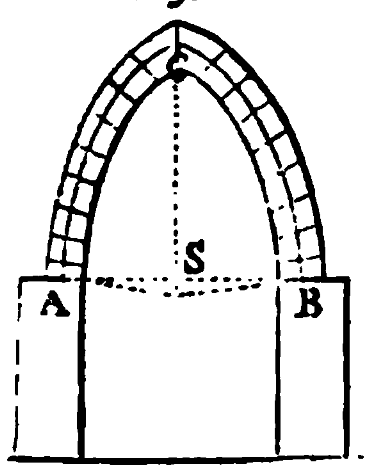
lation over the common analysis; which indeed is hardly sufficient to solve a problem of this nature. But we must here confine ourselves to a few of the properties of the curve in question.

Fig. A.



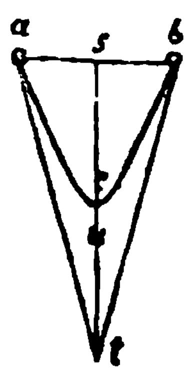
If the curve A B C, (Figs. 3., 4.), be disposed in such a manner, that its summit shall be uppermost; and if a multitude of globes be so arranged, that their centre shall be in the circumference of this curve, they will all remain motionless and in equilibrium: much more will this equilibrium subsist, if, instead of balls, we substitute thin voussoirs, the joints of which will pass through the points in contact, as they will touch each other in a surface far more extensive than the points in which we suppose the balls to touch each other.

Fig. 5.



Now to describe a curve of this kind is attended with no difficulty; for let us suppose that the space A B, comprehended between the two piers A and B of Fig. 5., is to be covered with an arch, and that the elevation of this arch is to be S C. Trace out on a wall a horizontal line *a b*, Fig. 6, equal to A B; then from the middle of *a b* draw *c* perpendicular to it, and equal to S C; and having fixed to the points, *a* and *b*, the two ends of a very flexible rope or chain, formed of small links perfectly equal and very moveable, so that when suspended freely it shall pass through the point *c*, mark out on the wall a sufficient number of the points or eyes of these links, without deranging them: the curve described through these points will be the one required; and nothing will be easier than to trace out the plan of it on the wall as represented by A C B, Fig. 5.

Fig. 6.



Then trace out at an equal distance, both without and within A C B, two curves, which will represent the extrados and intrados of the arch to be constructed. Divide the curve A C into any number of equal parts at pleasure; and through these points of division draw lines perpendicular to the curve, which may be done mechanically with sufficient exactness for practice: these perpendiculars will divide the arch into voussoirs; and you will thus have a plan of the arch described on the wall. From this plan it will be easy to construct the pannel or model boards for cutting the stones according to the proper form. If these operations are accurately performed, were the line A B a hundred feet, and the height S C still more, the voussoirs of this arch would maintain themselves in equilibrium, however small the part in contact might be: for, mathematically speaking, they ought to maintain themselves in equilibrium even if the surfaces in contact were highly polished and slippery: consequently the equilibrium will subsist much more when cut in the usual manner.

Now to find the force with which an arch of this kind pushes against its piers, or tends to overturn them, draw a tangent to the point *a* the commencement of the curve, Fig. 6., which may be done mechanically by assuming two points very near the curve, and drawing through these points a line which will meet in *t*, the axis *s c* continued.* This tangent being given, it can be demonstrated in mechanics that the whole weight of the semi-arch *a c*, is to the weight or force with which it pushes the

* This tangent may be drawn geometrically in the following manner: make use of this proportion: as 2 *s c* is to *a c* + *s c*, so is *a c* — *s c* to a fourth term, which we shall call *c u*, if you then say: as *c u* is to *a c*, so is *a s* to *s t*, the point *t* will be that where the tangent to the point *a* will meet the axis.

pier in a horizontal direction, as $s\,t$ is to $s\,a$. On the other hand, we must add to the weight of the pier the force with which the semi-arch presses upon it perpendicularly; that is to say, the absolute weight of the semi-arch: in this manner the thickness of the pier may be found, by the following arithmetical operation, which we shall here substitute for a geometrical construction, as the latter might appear too complex to the generality of our readers.

We shall suppose the span A B to be 60 feet, Fig. 5. and 6., and consequently A S will be 30 feet; we suppose S C to be 30 feet also, in order that we may compare the push or thrust of this arch with that of a semi-circular one. Let the length A C be 45 feet 1 inch 8 lines,* and the breadth of the arch 1 foot; for, on account of the reasons above mentioned, it may be constructed with safety in this light manner. If the height of the pier then be 40 feet, required the thickness it ought to have in order to overcome the thrust of the arch.

It will be found, on this supposition, that the tangent of the point a, the commencement of the catenarian curve or arch, will meet its axis $s\,c$ produced, in a point t so situated, that $s\,t$ will be $71\frac{7}{10}$ feet. If $s\,a$ be then divided by $s\,t$, we shall have the number $\frac{300}{717}$, which must be reserved, and which we shall call N.

Now take a fourth proportional to the height of the pier, the length A C of the semi-arch, and to its thickness, and let the half of this fourth proportional, which in this is $\frac{9}{16}$, be called D.

Then multiply A C by the thickness 1, and the product by double the reserved number N, which will give $37\frac{2}{7}$; to this number add the square of D, and extract the square root of the sum, which will be $6\frac{1}{6}$: if the above number D be taken from this root, we shall have 5 feet 7 inches, for the breadth of the pier.† The pier being constructed of materials homogeneous to the arch, it is certain that it will resist the force with which the latter tends to throw it down; for, to simplify the calculation, we have made a supposition which is not altogether exact, but which increases in some measure the breadth of the pier. This observation we think necessary, that we may not be accused of committing a wilful error.

If this breadth be compared with that necessary to support a circular arch forming a complete semi-circle, the latter will be found to be much greater, for it ought to be near 8 feet.

The push of an arch constructed on a circular foundation, such as the arch of a dome, being only about one half that exerted on its piers by a vault arch of the same thickness, it thence follows that, on the above supposition, the side of such a dome would require only $33\frac{1}{4}$ inches in thickness. But it can be demonstrated, even by the figure of the catenarian curve, that the arch may be but about a foot in thickness. Hence we may see how ill founded was the objection made to the architect of the church of Saint Genevieve, of its being impossible to construct, on the base he employed, the dome which he projected; for he could have done it even if we suppose his construction to be such as the author of the objection traced out to him, according to the precepts of Fontana, or rather according to the mode which that architect followed in the construction of his domes. What then would have been the case, had the architect alluded to, instead of first constructing a cylinder of 36 feet, which it however appears was never his design, made his arch rise immediately in a catenarian curve, above the circular cornice, which crowned his pendentives, or above a socle of small height? It is evident that the push of this arch would have been much less; and it would not be surprising if it should be found by calculation that his piers would have been capable of sustaining the arch raised above them, even suppos-

* It is found by calculation that this ought to be the length.

† This determination of the breadth or thickness of the pier, if not mathematically correct, may at least be considered as sufficiently near for practical purposes.

ing them insulated, and not allowing them any support from the re-entering angles of the church, which might have been made to rest against them.

We shall conclude with observing, that if it were required to find, by principles similar to those which gave rise to the discovery of the catenarian curve, the most advantageous form for a dome, the problem would be exceedingly difficult; for if we suppose this arch divided into small sectors, it will be evident that the weight of the voussoirs is not equal, and that their relation depends even on the form given to the arch. What has been here said, ought therefore to be considered only as an approximation of the most advantageous figure which the arch in that case ought to have..

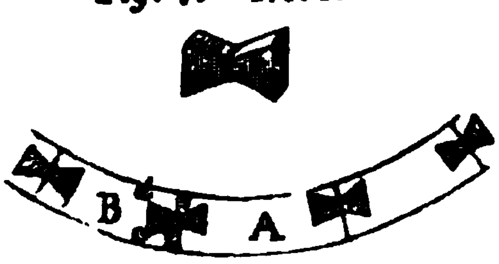
PROBLEM III.

How to construct a hemispherical arch, or what the French Architects call an arch en cul-de-four, which shall have no thrust on the piers.

The dispute carried on, some years ago, with a considerable degree of warmth, respecting the possibility of executing the cupola of the new church of Saint Genevieve, gave me an opportunity of examining whether, even on the supposition that the supporters would be necessarily too weak to resist the thrust of an arch of 63 feet in diameter, there might not be found some resources to render the construction of the cupola possible. I soon found that it was possible, by means of a very simple artifice, to construct a hemispherical arch, or an arch in the form of a semi-spheroid, which should have no sort of thrust on its piers, or on the cylindric tower by which it is supported. This will be readily conceived from the following reasoning and illustration.

It is evident that a hemispherical arch would exert no thrust on its support, if the first row were of one piece. But though this is impossible, the deficiency may be supplied, and such an arrangement may be made, that not only the first row, but that several of those above it, shall be disposed in such a manner, that their voussoirs can have no movement capable of disjoining them, as we shall here shew. The hemispherical arch will then exert no kind of thrust on its supporters; so that it may not only be sustained by the lightest cylindric pier, but even by simple columns, which would furnish the means of rearing a work very remarkable on account of its construction. Let us see, then, how the voussoirs of any row can be connected in such a manner as to have no motion tending to make them recede from the centre. There are several methods of accomplishing this object.

Fig. 7. *No.* 1.



No. 2.

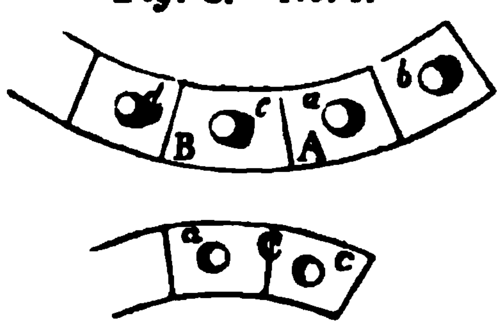
1st. Let A and B (Fig. 7. No. 2.) be two contiguous voussoirs, which we shall suppose to be three feet in length, and a foot and a half in breadth. Cut out on the contiguous sides two cavities in the form of a dove-tail, four inches in depth, with an aperture of the same extent at *a b;* five or six inches in length, and as much in breadth at *c d.* This cavity will serve to receive a double key of cast iron, as appears in the same Fig. No. 1; or even of common forged iron, which will be still more secure, as the latter is not so brittle as the former. These two voussoirs will thus be connected together in such a manner, that they cannot be separated without breaking the dove-tail at its re-entering angle; but as each of its dimensions in this place will be four inches, it may be easily seen that an immense force would be required to produce that effect; for we are taught, by well known experiments on the strength of iron, that it requires a force of 4500 pounds to break a bar of forged iron an inch square by the arm of a lever of 6 inches: consequently 288000 would be necessary to break a bar of 16 square inches, like that in question. Hence there is reason to conclude that these voussoirs

will be connected together by a force of 288000 pounds, and as they will never experience an effort to disjoin them nearly so great, as might easily be proved by calculation, it follows that they may be considered as one piece.

They might even be still farther strengthened, in a very considerable degree: for the height of these dove-tails might be made double, and a cavity might be cut in the middle of the bed of the upper voussoir, fit to receive it entirely: the dove-tail could not then be broken without breaking the upper voussoir also. But it may easily seen that, to produce this effect, an immense force would be required.

2d. But as some persons may condemn the use of iron in works of this kind, we shall propose another method, not attended with the same inconvenience, if it really be one;* and in which nothing is employed but stone combined with stone.

Fig. 8. *No.* 1.



No. 2.

Let A and B (Fig. 8.) be two contiguous voussoirs of the first row, and C the inverted voussoir of the upper next row, which ought to cover the joining. Each of the two former voussoirs being divided into two, cut out in the middle of each half a hemispherical cavity, half a foot in diameter; then take with great exactness the distance of the centres of the cavities *a* and *c*, which are in two contiguous voussoirs, and by these means cut out two similar cavities in the lower bed of the voussoir, which is to be placed in connection on the preceding. Then fill the cavities *a* and *c* with two globes of very hard marble, and place the upper voussoir in such a manner that these two globes shall fit exactly into the cavities of its lower bed. If this operation be dexterously performed throughout the whole range of the first, second, and third rows, it may be easily perceived, that all these voussoirs will form together one solid body, the parts of which cannot be separated; for the two voussoirs A and B cannot be disunited without breaking either the balls of marble which connect them with the upper voussoir, or breaking the upper voussoir through the middle. But even if we suppose this effect, which could not be produced without a force almost inconceivable, or at least far superior to the action of the arch, the two halves of the broken voussoir being themselves sustained in a similar manner by the superior voussoirs, no tendency to separate from each other could thence result: the three rows therefore of the arch would form only one piece, and there would be no thrust. It will be sufficient if the base of this arch have such a thickness as to prevent it from being crushed by its absolute weight; and a very moderate thickness, if the materials be good, will answer this purpose.

We think we have proved therefore, by these two methods, that a hemispherical arch might be constructed without any thrust on its supporters; consequently if we even suppose that the architect of Saint Genevieve had adopted the form of Fontana's domes, and had begun by raising on his pendentives a tower of about 36 feet in height, to be crowned by a hemispherical dome, it would not have been impossible to give it a solid construction.

PROBLEM IV.

In what manner the thrust of arches may be considerably diminished.

Architects, in our opinion, have not considered with sufficient attention the re-

* All architects, indeed, are not so nice in their choice of materials; but it appears to us that the frequent use of iron for strengthening buildings is subject to much inconvenience and danger. We at least wish that public monuments were constructed without it; for if they can support themselves without iron, it is needless: if iron is essential to strength, it will certainly be consumed in the course of time by rust; and the edifice will then tumble to pieces, or be greatly injured. The use of iron then in this case is attended with bad consequences.

sources afforded by mechanics, for diminishing, on many occasions, the thrust of arches. We shall therefore present the reader with some observations on that subject.

When the manner in which an arch tends to overturn its piers is analyzed, it appears that the arch necessarily divides itself somewhere in its flanks, and that the upper part acts in the form of a wedge or a lever on the remainder of the arch, and on the pier, which are supposed to form one body. This consideration then suggests, that to diminish the thrust of the arch, or increase the stability of the pier, the commencement of the flanks ought to be loaded; and that the thickness of the voussoirs near the key ought to be considerably lessened: in short, to make the arch, instead of having a uniform thickness throughout its whole extent, to be very thick at its origin, and at the key to be no thicker than what is necessary to resist the pressure of the flanks. It may be easily perceived, that as by this method a part of the force which acts to overturn it, is thrown upon that which resists being overturned, the latter will gain a great advantage over the former.

It is to arches in the form of a dome, in particular, that this consideration is applicable; and not only might this method be employed, but also heterogeneity of materials. For this purpose, let us suppose ourselves in the place of the architect of Saint Genevieve, and that it is necessary to construct his dome by first raising a round tower 36 feet in height, to be afterwards crowned by an arch, which we shall suppose to be hemispherical, though he was allowed to make it a little more elevated than that form, in order that it might appear hemispherical when seen at a moderate distance. It is found that giving to this arch the uniform thickness of a foot and a half, the tower ought to be 4½ feet in thickness at the utmost, which added to some necessary enlargement at the foundation, for the sake of solidity, exceeds the breadth of the basis which might be given to it in a part of its circumference. But, according to the above considerations, what would prevent this tower, and the first rows, even as far as towards the middle of the flanks of the arch, from being constructed of materials much more ponderous than the rest of the arch? For we are acquainted with some stones, such as hard and coarse marble, which weigh 230 pounds the cubic foot; while the Saint Leu, in the neighbourhood of Paris, weighs only 132, and brick much less. Instead of giving to the arch the uniform thickness of a foot and a half, why might it not be made three feet at the spring, and only eight inches towards the summit? But by making the following suppositions, namely that the tower and the first rows of the arch, as far as the middle of the flanks, are of the hard stone in the neighbourhood of Paris, which weighs 170 pounds the cubic foot, and the rest of brick which weighs only 130; and that the arch at its spring, as far as the middle, is 2½ feet in thickness, and only 8 inches towards the summit; we have found that the tower in question ought to be only 1 foot 8½ inches in thickness, to be in equilibrium with the thrust of the arch. If this tower therefore were made 3 feet in thickness, it is evident to the most timid architect, that it would be more than sufficient to counteract every effect of lateral pressure; and it would be still more so were it made 3½ feet in thickness to a certain height, such as that of 9 feet, for example, and then 3 feet or 2 feet 9 inches to the commencement of the arch; as a pier is strengthened by throwing to its lower part a portion of its thickness, instead of making it equally thick throughout; since the point on which it ought to turn, in order to be thrown down, is removed farther back.

But this is enough on a subject which we have introduced here occasionally.

PROBLEM V.

Two persons, who are neighbours, have each a small piece of ground, on which they intend to build; but, in order to gain as much room as possible, they agree to construct a stair common to both houses, and of such a nature, that the inhabitants shall

have nothing in common except the entrance and the vestibule. What method must the architect pursue to carry this plan into execution?

The stairs here proposed may be constructed in the following manner, of which there are some examples.

Fig. 9. No. 1.

Let Fig. 9. No. 1. be a plan of the stairs, the form of which is of such a nature as to ascend, without being too steep, from the lower to the first story in one revolution, or somewhat less. In a common vestibule A, the entrance to which is through a common door P, construct on the right at B the commencement of the ramp intended for the house on the right; and make it circulate from right to left, as far as a landing place, which must be constructed above the landing place B: the stairs may then be continued in the same manner to a second or third story.

Fig. 9. No. 2.

The commencement of the other stair-case must be on the side diametrically opposite at C; and must circulate in the same direction, in order to arrive, after one revolution, at a landing place forming the entrance to the first story of the house on the left; so that if the inside railing of these stair-cases be open, as it may be easily made, those who ascend or descend, by one of them, can see those who are on the other, without having any communication but by the common vestibule A, and the door of entrance. A section of this double stair-case is seen Fig. 9. No. 2.

At the castle of Chambord there is a stair-case nearly of this form, which serves for the whole building. For, as this edifice consists of four grand vestibules or immense saloons, placed opposite to each other, in the form of a Greek cross, and into which all the apartments open, Serlio, the architect, constructed the stair-case in the centre of this cross; and, by means of a double ramp, those who enter from the south vestibule on the ground floor, and who front the stair-case before them, arrive after one revolution at the southern vestibule or saloon of the first story, and vice versâ.

But though the form of this stair-case is very ingenious, it has some very great defects, which might have been easily avoided. 1st. The entrance of the stair-case, instead of being directly opposite to the middle of each saloon, is a little on one side. 2d. There is no landing place before the door which forms the entrance into this story. 3d. The interior railing, which might have been light, and almost entirely open, has only a very small number of apertures.

If the ground would admit, the same artifice might be employed to construct a stair-case with four ramps, all separate from each other, in order to ascend to four different apartments. The plan of a stair of this kind, which is said to have been constructed at Chambord, may be seen in Palladio. That of Serlio, on account of the four galleries to be entered, would no doubt have been much more beautiful, had it been built on the same plan; but we can assert that the stair of Chambord has only two ramps as above described.

Remark.—Some stairs are distinguished by another peculiarity, namely, the boldness of their construction. Such are those stairs in the form of a screw, the helix of which forms a spiral entirely suspended; so that there remains in the middle a vacuity

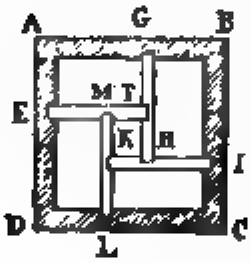
of greater or less extent. This bold construction depends on the manner in which the steps are cut, and their being fixed by one end in the wall which on one side supplies the place of a rail. A full account of the mechanism of them may be seen in most works on architecture.

PROBLEM VI.

To construct a floor with joists, the length of which is little more than the half of that necessary to reach from the one wall to the other.

Let the square A B C D (Fig. 10.), be the frame of the floor which is to be covered with joists a little more in length than the half of one of the sides A B. On the sides of this square assume the parts A G, B I, C L, and D E, equal to the given length of the joists, which must be arranged as seen in the figure; that is, first place E F, and introduce below it G H, with its end H resting on I K; and let K, the end of I K, rest upon L M, the end of which M must be made to rest upon the first joist E F. It may be easily proved, that in this position these joists will mutually support each other, without falling.

Fig. 10.

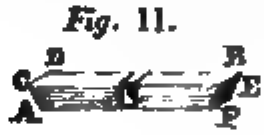


It is almost needless to observe that the end of each joist must be cut in such a manner as to enter a notch made for it in the joist on which it rests, and into which it ought to be well fitted. However, as a notch cut into a joist must lessen its strength, it would perhaps be better to make the end of each joist rest on an iron stirrup of a sufficient size, and affixed to them in a secure manner.

But it is not necessary that the joists should be a little longer than half the breadth of the frame to be covered: a floor may be constructed with pieces of wood much shorter, if they be cut and arranged in a proper manner.

Let us suppose, for example, that an area of 12 feet square is to be covered, and that the pieces of wood, intended to support the floor, are only 2 feet in length. Cut the extremities of one of these pieces of wood in an oblique form, or into a bevel, as represented by the section A C D or B E F (Fig. 11.); and in the middle of the same piece, form on each side a notch, for receiving the end of another piece cut in like manner. If the same operation be performed on all the rest, they may then be arranged as seen in the figure; a bare view of which will give a better idea of the artifice here employed, than a long description. The oblong spaces, which remain along the walls, may be filled up with pieces of wood half the length of the former. The scaffolding may then be removed with great safety, for these pieces of wood will form a solid floor, and will mutually support each other, provided none of them is destroyed; for it is to be observed that the breaking of one would make the whole fall to pieces.

Fig. 11.



Dr. Wallis, at the end of the third volume of his works, gives a great variety of these combinations, and he says that this invention was employed in some parts of England. But on account of the reasons already mentioned, it is to be considered rather as ingenious than useful, and fit only to be adopted when there is a great scarcity of timber, and for floors which have very little weight to support.

Remark.—Instead of pieces of wood, if we suppose stones to be cut in the same manner, it is evident that they would form a flat arch; but in this case, to avoid the danger of breaking, it would be necessary that they should be at most 2 feet in length, and of a suitable width and thickness. An arch of this kind is generally called the flat arch of M. Abeille; because it was proposed by that engineer to the Academy of

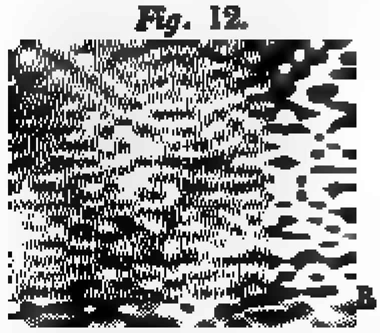
Sciences, in 1699. It is attended with this advantage, that its whole thrust is exerted on the four walls, which serve to support it; whereas a flat arch, constructed according to the usual method, exerts its thrust or push only against two. But this advantage is more than overbalanced by the danger of the whole tumbling to pieces, if one stone only should be deficient. Frezier has treated on this subject at some length, in his work on cutting stones; and has shewn how to vary the compartments of the intrados, or lower part, as well as of the extrados, or upper part, which might be formed with these arches. But we must here repeat that these things are more curious than useful, or rather that this construction is very dangerous.

PROBLEM VII.

Of suspended Arches, called by the French "Trompes dans l'Angle."

One of the boldest works in masonry, is that kind of arch called, by the French, *Trompe dans l'Angle.** Let us suppose a conical arch, as S A F B S (Fig. 12.) raised on the plane of a triangle A S B; if from the middle of the base there be drawn two lines E C and E D, which in general are parallel to the respective sides S D and S C; and if upon these be raised two planes D E F and C E F, perpendicular to the base; these two planes will cut off towards the summits, a part of the arch, as F D S C F, the half of which C F D C will be suspended, or project beyond the foundation. This truncated part of the conical arch F D S C F, is what is called a *trompe dans l'Angle*; because in general it is constructed in a re-entering angle to support some projecting part of an edifice. For this purpose, on the curvilinear planes C F and D F, there are raised walls which, though suspended, have sufficient strength, provided the voussoirs be exactly cut; are long enough to be inserted in the half which is not suspended; and provided also that this part is properly loaded.

Fig. 12.



Works of this kind are common; but the most singular is one at Lyons, which supports a considerable part of a house situated on the stone-bridge. One cannot see, without some uneasiness, the corner of this edifice, which is three or four stories in height, project several yards above the river. It is said to be the work of Desargues, a gentleman of the Lyonnese, and an able geometrician, who lived in the time of Descartes. In that case, this work must have stood about 150 years; which seems to prove that this kind of construction has a real and greater solidity than is commonly supposed.

Remark.—If the suspended arch be a right arch, that is to say a portion of a right cone A S B F; and if the section planes F E D and F E C be respectively parallel to S C and S D, the curves F C and F D, as is well known, will be parabolas, having their summit in D, and C E or D E for their axis. We must here take notice of a geometrical curiosity, which is, that the conical surface F C S D F, though a curve, and terminated in part by curved lines, is equal to a rectilineal figure; for if D G be drawn parallel to the axis S E, it can be demonstrated that the conical surface in question, is equal to one and a third of the rectangle of S B, or S F by E G.

PROBLEM VIII.

A gentleman has a quadrangular irregular piece of ground, as A B C D, *in which he is desirous of planting a quincunx, in such a manner, that all the rows of trees, whether transversal or diagonal, shall be right lines. How must he proceed to carry this plan into execution?*

* These arches are called *trompes*, because they have a resemblance to the mouth of a trumpet.

We shall suppose this quadrilateral to be so irregular, that the opposite sides A B and D C, (Fig. 13.), meet in a point F, and the sides A D and C B in another point E. Continue these sides two and two, to the points of meeting, E and F, which must be joined by a straight line E F; then through the point D draw a line parallel to E F; continue B C and B A till they meet that parallel in H and G, and divide G D and D H into the same number of equal parts, which we shall suppose to be four: if through the points of division in G D, as many straight lines be drawn to the point F; and if straight lines be drawn, in like manner, through the points of division in D H to the point E, these lines will intersect the sides of the quadrilateral, and each other, in points, which will be those where the trees must be planted, in order so solve the problem.

Fig. 13.

E I F

For the demonstration we might refer to Prob. 24 of Optics, where we have shewn how a quadrilateral, such as A B C D, may be the perspective representation of a parallelogram. We shall however here repeat it.

Through the points D and H draw the lines D *a* and H *b*, inclined to G H at an angle of 45 degrees from right to left; and through the points G and D two other lines, G *b* and D *c*, inclined also 45 degrees to G H, but in a contrary direction: these four lines will necessarily cut each other at right angles, and form a rectangle *a b c* D, of which, according to the rules of perspective, the quadrilateral A B C D would be the representation, to an eye situated in the point I, which divides E F into two equal parts, and is at a distance from the plane of the picture equal to I E or I F.

Let us suppose then that the oblong *a b c* D is divided into similar oblongs, by four lines parallel to its sides: these lines, if continued till they meet G D and D H, will divide them into the same number of equal parts; and as D C and G A B are the perspective representations of D *c* and G *a b*, the lines proceeding from the equal divisions of G D, and ending at the point F, will be the perspective representations of lines parallel to *a b* or D *c*. The case will be the same with the lines parallel to the two sides D *a* and *c b*. The small quadrilaterals then formed by these lines cutting each other in the quadrilateral A B C D, will be the perspective pictures of the oblongs into which *a b c* D is divided. But all the points which are in a straight line in the object, will be in a straight line in the picture; consequently, as the rows of trees planted at the angles of the divisions of the oblong *a b c* D necessarily form straight lines, both transversely and diagonally, their places in the quadrilateral A B C D, which are the pictures of these angles in the oblong, will also form straight lines in the same direction; for, in perspective representations, the pictures of straight lines are always straight lines.

If the opposite sides *a b* and *c* D, of the given quadrilateral be very unequal, they must not be divided into the same number of parts; for in that case they would be too unequal, since in a plantation of this kind the quadrilaterals ought to be nearly perfect squares. For example, if one side *a b* be 100 yards, and the other 40, by dividing each of them into 20, the divisions on one side would be 5, and on the other 2 yards, which would form figures too oblong. On this supposition, it would be much better to divide the first into 16, and the second into 6, which would give divisions almost square, namely of 6¼ yards in one direction, and 6⅔ in the other, but in this case there would be no diagonal row of trees, either in the oblong *a b c* D, or in the proposed quadrilateral A B C D. In short, by dividing one of the lines G D or D H into 16 parts, and the other into 6, all the rows of trees in the irregular figure will be straight lines.

To have a real quincunx,* it will be sufficient, after this operation, to draw, in each small quadrilateral of the plantation, two diagonals, and to plant a tree in the point where they intersect each other: all these new trees will form straight lines also.

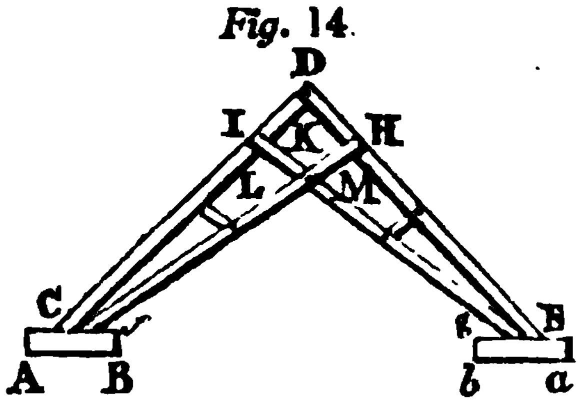
PROBLEM IX.

To construct the frame of a roof, which, without tie-beams, shall have no lateral thrust on the walls on which it rests.

We have seen at Paris, in a garden of the *faubourg Saint-Honoré*, a small building, in the form of a tent, the walls of which were only a few inches in thickness, and which were covered by a roof without tie-beams; the whole being lined in the inside, it had the real appearance of a tent. It was used as a summer apartment in the day-time, and formed a retreat truly delightful.

What surprised those who had any knowledge of architecture, was, how the roof of this small edifice would be constructed without tie-beams: for however light it might be, the walls were so thin, that any common roof must have overturned them. The artifice, said to have been the invention of M. Arnoult, superintendant of the theatres *des Menus-Plaisirs*, was as follows.

Fig. 14.



Two rafters, C D and E D (Fig. 14.), resting on the two beams A B and *a b*, were strongly joined together at the summit D. From the angles, which these two rafters formed at C and E, proceeded two other pieces of timber, which were well united to the beams at F and G, to the rafters at H and I, and also to each other at K, by means of a double notch. For the sake of greater security, the pieces C D and F H, and E D and G I, were bound together by two cross pieces at L and M. It is evident that these four inclined pieces can have no tendency to separate, or to exert any lateral thrust on the walls upon which the beams A B and *a b* are placed: for they cannot separate without rendering the angle D more obtuse. For this purpose, it would be necessary that the angle K should become more obtuse also; but the junctions at I and H oppose any movement of this kind: consequently this frame work will rest on the beams A B and *a b*, without separating in any manner, and will exert no lateral pressure against the walls.

It is hence evident that this artifice might be of great use in architecture, especially when it is required to cover an extensive building, the walls of which are thin, and to avoid the disagreeable effect produced by tie-beams, when not concealed from the sight.

PROBLEM X.

On measuring arches en cul-de-four, surhaussé, and surbaissé.

The apellation of *cul-de-four* is applied to vaults on a plan commonly circular, a section of which through the axis is an ellipsis, or as the French architects call it an *anse de panier*. They differ from hemispherical arches in this, that in the latter the height of the summit above the plane of the base is equal to the radius of that base; while in the former this height is greater or less: if greater, the arch is called *cul-de-four sur-*

Fig. 15.

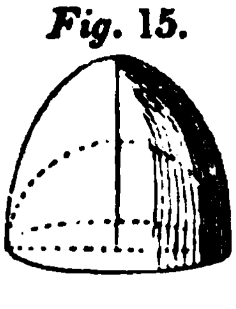
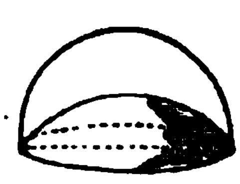


Fig. 16.



* A real quincunx is that where there is a tree in the middle of each square; for the word *quincunx* means five trees in a square, which cannot be arranged otherwise.

baissé. Both these arches are represented Figs. 15. and 16. The first is an arch *en cul-de-four surhaussé;* the second arch *en cul-de-four surbaissé.* In the language of geometry, the one is an elongated semispheroid, or an arch formed by the circumvolution of a semi-ellipse around its greater semi-axis; the other a semi-spheroid formed by the circumvolution of the same semi-ellipsis about its less semi-axis.

Books of architecture contain, in general, rules so false for measuring the superficial content of these arches, that we think it necessary to give here methods more correct. Bullet and Savot, for example, say that nothing is necessary but to multiply the circumference of the base by the height; as if the arch to be measured were hemispherical. This is an egregious error, and it is surprising those authors did not observe that if this rule were correct, the superficial content of some arches *en cul-de-four surbaissé,* would be less than the circle covered by them, which is absurd.

For let us suppose, by way of example, an arch of a foot in height, on a circle of seven feet diameter: the area of this circle, according to the approximation of Archimedes, will be equal to $38\frac{1}{2}$ square feet: but if the circumference, 22 feet, be multiplied by one foot in height, we should have only 22 square feet; which is not two-thirds of the base. In this case, the builder would be cheated of more than two-thirds of what he ought to receive. We shall therefore give rules for measuring such arches, sufficiently correct to be employed in the common purposes of architecture.

I.—*For arches en cul-de-four surhaussé, or the Oblong Spheroid.*

The radius of the base and the height of the arch being given, first make this proportion: As the height is to the radius of the base, so is the latter to a fourth term, the third of which must be added to two-thirds of the radius of the base.

Then find the circumference corresponding to a radius equal to that sum, and multiply this circumference by the height: the product will be the superficial content or curve surface nearly.

Example.—Let the height be 10 feet, and the radius of the base 8. Then say, as 10 is to 8, so is 8 to $6\frac{2}{5}$, the third of which is $2\frac{2}{15}$: two-thirds of 8 are $5\frac{1}{3}$, which added to $2\frac{2}{15}$, make $7\frac{7}{15}$ feet, or 7 feet 5 inches 7 lines.

But the circumference corresponding to $7\frac{7}{15}$ feet radius, or $14\frac{14}{15}$ feet diameter, is $46\frac{14}{15}$ feet, which multiplied by 10 feet, the height of the arch, gives for product $469\frac{1}{3}$ square feet, or 52 yards $1\frac{1}{3}$ foot.

By Bullet's rule, the superficial content would have been 55 yards 7 feet; the difference of which in excess is 3 yards 6 feet, or about a 14th of the whole; and this in an arch which does not deviate much from a hemisphere: if it deviated more, the error would be considerably greater.

II.—*For arches en cul-de-four surbaissé, or the Oblate Spheroid.*

The rule for these arches is nearly the same as the preceding. Find a third proportional to the height and the radius of the base; and add two-thirds of it to the radius of the base; then find the circumference corresponding to the sum as a radius, and multiply it by the height: the product will be the superficial content nearly of the given arch.

Let the radius of the base of an arch *en cul-de-four surbaissé* be 10 feet, and the height be 8. As 8 is to 10, so is 10 to $12\frac{1}{2}$, two-thirds of which are $8\frac{1}{3}$; on the other hand, the third of 10 is $3\frac{1}{3}$, which added to the former, gives $11\frac{2}{3}$ feet.

But the circumference corresponding to $11\frac{2}{3}$ feet radius, or $23\frac{1}{3}$ diameter is $73\frac{1}{3}$, which multiplied by the height, that is 8 feet, gives for product $586\frac{2}{3}$ feet, or 65 yards $1\frac{2}{3}$ foot = the superficial content of the arch.

According to Bullet's rule the superficial content would have been 55 yards 7 feet; which makes an error in defect of 9 yards 3⅜ feet, or above a 6th part of the whole surface.

Remark.—It would be easy to give, for those who are geometricians, rules still more exact; as it is well known that the dimensions of prolate spheroids depend on the measurement of a truncated elliptical or circular segment; and that of the surface of an oblate spheroid, on the measurement of an hyperbolical space; consequently the former may be determined by means of a table of sines and circular arcs, and the other by employing a table of logarithms.

In regard to the method above given, it is deduced from the same principles; but by considering a segment of a circle or hyperbola of a moderate extent, as a parabolic area, which when this segment forms but a small part of the space to be measured, is liable only to a very small error: in many cases this consideration supplies practical rules exceedingly convenient.

Some architects may perhaps ask: Of what advantage is it to be able to ascertain with precision the superficial content of these domes, as a few yards more or less can be of little importance? But it may be said in reply, that for the same reason, accurate measurement in general is of little utility. To such persons it is of no consequence that Archimedes has demonstrated that the surface of a hemisphere is equal to that of a cylinder of the same base and height; or, to speak according to their own terms, that the surface of a hemispherical arch is equal to the product of the circumference of the base by the height. If they employ, in regard to the arches in question, rules so erroneous, it is because they consider them as exact, and because they have been taught them by people so ignorant of geometry, as not to be able to give them better ones.

PROBLEM XI.

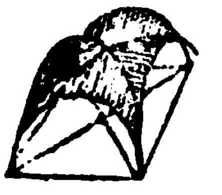
To measure Gothic or Cloister Arches, and Arches d'arête, or Groin Arches.

It frequently happens that on a square, an oblong, or polygonal space or edifice, an arch vault is raised, consisting of several *berceaux* or vaults, which commencing at the sides of the base, unite in a common point as a summit, and form in the inside as many re-entering angles or groins, as there are angles in the figure which serves as a base. These arches are called *arcs de cloître*, cloister arches. A representation of them is seen Fig. 17.

Fig. 17.

But if the space or edifice, a square for example, be covered with two *berceaux* or vaults (Fig. 18.) which seem to penetrate each other, and which form two ridges or re-entering angles, intersecting each other at the summit of the vault, such an arch is called an arch *d'arête*, or a groin arch. The most remarkable properties of these arches are as follow.

Fig. 18.



1st. The superficial content of every circular cloister arch, on any base, whether square or polygonal, is exactly double that of the base, in the same manner as the superficial content of a hemispherical arch, or arch *en cul-de-four* or *en plein ceintre*, is double that of the circular base.

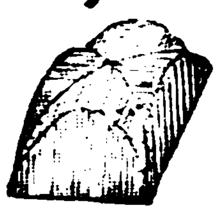
It may indeed be said that a hemispherical arch is only a cloister arch on a polygon of an infinite number of sides.

When the superficial content therefore of such an arch is to be measured, nothing will be necessary but to double the surface of the base, provided the *berceaux* be *en plein ceintre*, or a complete semi-circle; for if they are greater or less, they will have to the base, the same ratio that an arch *en cul-de-four surhaussé*, or *surbaissé*, has to the circle of its base.

2d. A cloister arch, and a groin arch on a square, form together the two complete *berceaux* or vaults, raised upon that square.

This may be readily seen in Fig. 19. Therefore if from two *berceaux* or vaults, the cloister arch be deducted, there will remain the groin arch, which in this case gives a simple method for measuring groin arches; for if the superficial content of the cloister arch be subtracted from the superficial content of the two vaults, the remainder will be that of the groin.

Fig. 19.



If the base, for example, be 14 feet in both directions, the circumference of the semi-circle of each will be 22 feet, and the superficial content will be 22 by 14, or 308 square feet; consequently the superficial content of both the *berceaux* will be 616 square feet. But the interior surface of the Gothic arch is twice the base, or twice 196, that is 392; and if this number be subtracted from 616, we shall have 224 square feet, for the superficial content of the groin arch.

3d. If the solid content of the interior of such an arch be required; multiply the base by two thirds of the height.

This is evident from the reason already given in regard to the superficial content; for arches of this kind, both in regard to their solidity and superficial content, are to a prism of the same base and height, in the same ratio as the hemisphere to the circumscribed cylinder.

4th. The solidity of the space contained by a groin arch on a square or oblong plane is $\frac{19}{21}$ of the solid having the same base and height, supposing the approximate ratio of the diameter of the circle to the circumference to be as 7 to 22.

This may be easily demonstrated also by observing, that the interior solid of such an arch is equal to the sum of the two vaults or demi-cylinders, minus once the solidity of the cloister arch, which is twice comprehended in this double, and consequently ought to be deducted.

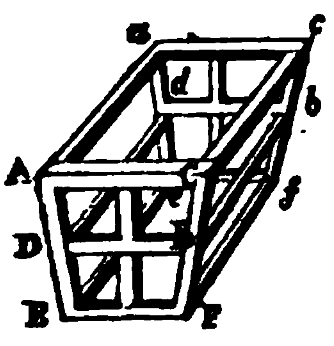
PROBLEM XII.

How to construct a wooden bridge of 100 *feet and more in length, and of one arch, with pieces of timber, none of which shall be more than a few feet in length.*

We shall here suppose that the pieces of timber intended for a bridge of this kind are 12 or 14 inches square, and only about 12 feet in length; or that particular circumstances have prevented rows of piles from being sunk in the bed of the river, to support the beams employed in constructing the work. In what manner must the architect proceed to build the bridge, notwithstanding these difficulties?

The execution of this plan is not impossible: for it might be accomplished in the following manner. First trace out, on a large wall, a plan of the projected bridge, by describing two concentric arches at such a distance from each other, as the length of the pieces of timber to be employed will admit; which we shall suppose, for example, to be 10 feet, giving them the form of an arc of 90 degrees from one pier to another: then divide this arc into a certain number of equal parts, in such a manner that the arc of each shall not exceed 5 or 6 feet.

Fig. 20.



On the supposition here made, of the distance of 100 feet between the two piers, an arc of 90 degrees which covers it would be 110 feet in length, and its radius would be 70 feet. Divide then this arc into 22 equal parts, of 5 feet each, and with the above pieces of timber, joined together, form a kind of voussoirs, 8 or 10 feet in height, 5 feet in breadth at the intrados, and 5 feet 8 inches 6 lines at the extrados; for such are the proportions of these arcs, according to the above dimensions. Fig. 20 represents one of these voussoirs, which, as it

is evident, consists of four principal pieces of strong timber, at least 10 inches square, which meet two and two at the centre of their respective arcs; of three principal cross bands at each face, as A C, B D, E F, *a c*, *b d*, *e f*, which must be exceedingly strong, and therefore ought to be 12 or 14 inches in height, and 10 inches in breadth; and lastly, of several lateral bands, between the two faces, to bind them together in different directions, and to prevent them from giving way. A voussoir of this kind may be about 6 feet in length, that is between the two faces A E F B and *a e f b*.

An arch must then be formed of these voussoirs, exactly in the same manner as if they were stone, and when they are all arranged in their proper places, the different pieces may be bound together according to the rules of art, either with pins or braces. Several arches or ribs of this kind must be formed, close to each other, according to the intended breadth of the bridge; and the pieces may be bound together in the same manner as the first, so as to render the whole firm and secure. By these means we shall have a wooden bridge of one arch, which it would be very difficult to construct in any other manner.

It now remains to be examined whether these voussoirs will have sufficient strength to resist the pressure which they will exert on each other. The following calculation will shew that there can be no doubt of it.

It appears, from the experiments of Muschenbroeck,* and the theory of the resistance of bodies, that a piece of oak 12 inches square, and 5 feet in length, can sustain in an upright position, without breaking, 264 thousand pounds; hence it follows that a cross band, as A C or E F, 5 feet in length, and 12 inches by 10, can support 220000; but for the greater certainty we shall reduce this weight to 150000: therefore, as we have six bands of this length, a few inches more or less, in each of these voussoirs, it is evident that the effort which one of these voussoirs is capable of sustaining, will be at least 900000 pounds. Let us now examine what is the real effort to be resisted.

We have found, by calculating, the absolute weight of such a voussoir, and even supposing it to be considerably increased, that it will weigh at most between 7 and 8 thousand pounds, or 7500. The weight then resting on one of the piers, most loaded, having 10 voussoirs to support, will be charged only with the weight of 75000 pounds: a weight however which, on account of the position of the voussoirs, will exert a pressure of 115000 pounds; but we shall suppose it to be even 120000. There is reason therefore to conclude from this calculation, that such a bridge would not only have strength to support itself, but also to bear, without any danger of breaking, the most ponderous burthens: it even appears that it would not be necessary to make the pieces of timber so strong.

If the expense of such a bridge be compared with that attending the common method, it will perhaps be found to be much less; for one of these voussoirs would contain no more than 140 or 150 square feet of timber, which at the rate of 2*s.* per foot, would be only £15; so that the 22 voussoirs, of one course or rib, would cost £330: consequently, if we suppose the breadth of the bridge to consist of four courses or ribs, the whole would amount only to £1320. It must indeed be allowed, that to complete such a bridge, other expenses would be required; but the object here proposed, was to shew the possibility of constructing it, and not to calculate the expense.

The idea of such a bridge first occurred to me in consequence of a dangerous passage I met with in the province of Cusco, in Peru; where I was obliged to cross a torrent, that flows between two rocks, about 125 feet distant from each other, and more than 150 feet in height. The inhabitants of the country have constructed

* Essais de Physique, vol. i. chap. 11.

there a *Travita*,* where I was in danger of perishing. When I arrived at the next village, I began to reflect on the best means of constructing in this place a wooden bridge, and I contrived the above expedient. I proposed my plan to the Corregidor, Don Jayme Alonzo y Cuniga, a very intelligent man, who, being fond of the French, received me with great politeness. He approved of my idea, and agreed that, at the expense of a thousand piasters, a bridge of 12 feet in breadth, which all Peru would come to see through curiosity, might be constructed in that place. But as I set out three days after, I do not know whether this project, with which this worthy man seemed highly pleased, was ever carried into execution.

It may here be remarked, that it would be easy to arrange the voussoirs of a bridge of this kind, in such a manner that, in case of necessity, any one of them might be taken out, in order to substitute another in its stead: which would afford the means of making all the necessary repairs.

PROBLEM XIII.

Is it possible to construct a Plat-band or Frame which shall have no lateral thrust?

It would be of great advantage to be able to execute a work of this kind; for one of the obstacles which architects experience, when they employ columns, arises from the thrust of their architraves, which requires that the lateral columns should be strengthened by different means. This embarrassment they are particularly liable to, when they make detached porches to project before an edifice, like that of Sainte-Genevieve: the two frames, that of the face and the side, exert such a push on the angular column or columns, that it is very difficult to secure them; and it is even sometimes necessary to renounce them, if stones cannot be found sufficiently large to make architraves of one piece, from column to column, at least in the spaces nearest the angles.

These difficulties would be obviated, if frames could be made without any thrust. This we do not think impossible; and we propose the problem to architects in the hope that some of them will be able to solve it.

PROBLEM XIV.

Is it a perfection, in the Church of St. Peter at Rome, that those who see it, for the first time, do not think it so large as it really is; and that it appears of its real magnitude after they have gone over it?

Though we announced, in the beginning of this work, that we meant to exclude from it whatever was mere matter of taste; as the above question is connected with physical and metaphysical reasons, we are of opinion that it may be admitted.

The impression which the church of St. Peter at Rome makes, on the first view, has been boasted of as a perfection. Every person, as far as we have heard or read, who enters this edifice, for the first time, conceives the extent of it to be far less than it is generally accounted to be by public report. To have a just idea of its grandeur, one must have seen, and in some measure studied, every part of it.

Before we venture to say any thing decisive on this subject, it may perhaps be of some use to examine the causes of this first impression. In our opinion, it arises from two sources.

The first is the small number of principal parts into which this immense edifice is divided; for, from the entrance to the middle, which constitutes the dome, there are

* This is an Indian bridge, the very idea of which is enough to make one shudder. A man is placed in a large basket, fastened by a pulley to a rope which is extended from the one side of a torrent to the other. The basket and rope are both constructed of those creeping plants, which the inhabitants of America employ in almost all their works. As soon as the man has got into the machine, it is drawn over to the opposite side, by means of a rope fastened to the pulley. If the rope, used for dragging over the machine, should break, the man must remain suspended for some hours, until means have been found to relieve him from his painful situation.

only three lateral arcades. But, though dividing a large mass into many small parts tends, in general, to diminish its effect, there is still a medium to be observed; and it appears to us that Michael Angelo kept too far below it.

The second cause of the impression which we here examine, is the excessive size of the figures and ornaments, which serve as appendages to the principal parts. We can indeed judge of the size of objects beyond our reach, only by comparing them with neighbouring objects, the dimensions of which are familiar to us. But if these objects, the dimensions of which are known, or are nearly given by nature, accompany others to which they have a ratio that approaches too near to equality, it must necessarily follow that the latter, in the imagination of the spectator, will lose a part of their magnitude. Such is the case with the church of St. Peter at Rome: the figures placed in niches, which decorate the spaces between the pillars of the arcades; those between the pilasters, and those which ornament the tympana of the lateral arcades, are truly gigantic; but they are human figures; they are besides, for the most part, raised very high; consequently they appear less, and make the principal parts which they accompany to appear less also.

By some people, this illusion is considered to be a master-piece of the art and genius of the celebrated architect, the principal author of this monument. Shall we be permitted to differ from them? For what is the object which the constructors of this immense edifice had in view; and which will be the aim of all those who raise edifices that exceed the usual measures? Doubtless to excite astonishment and admiration. We are convinced that Michael Angelo would have been much mortified, had he heard a stranger, just arrived at Rome, and entering St. Peter's for the first time, say publicly: "This is the church respecting the immensity of which we have heard so much: it is a large building, but not so large as generally reported."

In our opinion, it would display much more ingenuity to construct an edifice which, though of a moderate size, should immediately excite in the mind the idea of considerable extent; than to construct an immense one which, on the first view, should appear of a moderate size. We do not think that on this subject there can be any difference of opinion. Whatever then may be the perfection, which it must be allowed the church of St. Peter possesses, so far as harmony of proportion, beauty and magnificence of architecture, are concerned, we are of opinion that Michael Angelo missed his aim in regard to the object in question; and it is probable that he would have approached much nearer to it, had he employed less gigantic appendages. If the children, for example, which support the *bénitiers** had been of less size; if the figures which accompany the archivaults of his lateral arcades, as well as those which decorate the niches between the pilasters, had been on a scale not so enormous; a comparison of the one with the other would have made the principal parts appear much greater. Those who turn their eyes from these gigantic objects, and direct them towards a man near the middle, or at the extremity of the church, experience this effect: it is then, by comparing their own size with that of the principal parts of the edifice in the neighbourhood, that they begin to form an idea of its extent, and are struck with astonishment: but this second impression is the effect of a sort of reasoning; and the sensation, when produced in this manner, has not the same energy, as when it is the effect of a first view.

While we are on this subject, we shall take the liberty of offering a few observations on the means of enlarging, as we may say, any space by the help of the imagination. In our opinion, nothing contributes more to produce this effect, than insulated columns; that is to say; columns not regularly connected; for, when coupled or grouped, they always produce this effect, more or less, though it would doubtless be much better to employ them single. The result is, that every time a spectator

* *Benitiers* are vases for holding holy-water.

changes his position, different openings occur; and a variety of aspects which astonish and deceive the imagination.

But when columns are employed, they ought to be large; for in the same degree as they have a majestic appearance when constructed on a grand a scale, they are, in our opinion, mean and diminutive when small, and particularly when supported on pedestals. The court of the Louvre, though in other respects beautiful, would have a much more striking effect, if the columns, instead of being mounted on meagre pedestals, rose from the ground supported merely by a socle, like those in some of the vestibules of that palace. One might almost say, and there is some reason to think, that pedestals were invented to render fit for use, columns collected at hazard, and which have not the requisite dimensions.

If Michael Angelo then, instead of forming his lateral spaces of immense arcades supported by pillars, decorated with pilasters, had employed groups of columns; if, instead of placing only three rows of lateral arcades, between the entrance and the part of the dome, he had placed a greater number, which this arrangement would have allowed him to do; and if the figures employed amidst this decoration had not far exceeded the natural size, we entertain no doubt that the spectator would have been struck with astonishment on the first view, and that the edifice would have appeared much larger.

But it is to be observed, at the same time, that the knowledge which we now possess, in regard to the resistance of materials, and the philosophy or mechanical part of architecture, was not known at the time when Michael Angelo lived. It is probable that he durst not venture to load columns, even when grouped, with a weight so considerable as that which he had to raise upon these pillars. But it is proved, by late experiments in regard to stones, that there is no weight that an insulated column, six feet in diameter, made of very hard stone, well chosen and prepared, is not capable of supporting. Our ancient churches, called improperly *Gothic*, are a proof of it; for there are some of them, the whole mass of which rests on pillars scarcely six feet in diameter, and often less: they therefore in general convey an idea of extent which the Greek architecture, employed in the same places, does not excite.

PART TENTH.

CONTAINING THE MOST CURIOUS AND AMUSING OPERATIONS IN REGARD TO PYROTECHNY.

WHY it has been usual to consider Pyrotechny as a branch of the mathematics, we do not know. The least reflection will readily shew, that it is an art by no means mathematical, though dimensions, proportions, &c., are employed in it. There are a great number of other arts which have a much better claim to be included among these sciences.

However, as we might be blamed for omitting an art which affords a considerable field for amusement, and as it is connected, at least, with natural philosophy, we shall make it the subject of one of the divisions of this work. But as we do not intend to give a complete treatise of Pyrotechny, we shall confine ourselves to those parts which are most common and most curious: we shall also avoid every thing that relates to the fatal art of destroying men. We can see no amusement in the motion of a bullet, which carries off files of soldiers, nor in the action of a bomb or shell that sets fire to a town. The preceding editors and continuators of Ozanam, seem to have possessed a very military spirit, if they considered all these things as harmless recreation. For our part, having imbibed other principles in that happy country, Pennsylvania, we shudder even at the idea of introducing such atrocities under the form of amusement.

Pyrotechny, as we consider it in this work, is the art of managing fire, and of making, by means of gunpowder and other inflammable substances, various compositions agreeable to the eye, both by their form and their splendour. Of this kind are rockets, serpents, sheaves of fire, fixed or revolving suns, and other pieces employed in decorations and fire-works.

Gunpowder being the most common ingredient in Pyrotechny, we shall begin with an account of its composition.

ARTICLE I.

Of Gunpowder.

Gunpowder is a composition of sulphur, salt-petre, and pounded charcoal: these three ingredients mixed together, in the proper quantities, form a substance exceedingly inflammable, and of such a nature, that the discovery of it could be owing only to chance. A single spark is sufficient to inflame, in an instant, the largest mass of this composition. The expansion, suddenly communicated either to the air, lodged in the interstices of the grains of which it consists, or to the nitrous acid which is one of the elements of the saltpetre, produces an effort which nothing can resist; and the most ponderous masses are driven before it with inconceivable velocity. We must however observe that this invention, to which the epithet of *diabolical* is frequently applied, is not so destructive to the human race as it might at first appear: battles seem to have been attended with less slaughter since gunpowder began to be used; and, as is remarked by the celebrated Marshal Saxe, the noise and smoke

produced by fire arms, during a battle, are more terrible than the execution they make. We must however except cannon when well directed. But let us return to our subject, and give an account of the process for making gunpowder.

Sulphur is found ready formed, and almost in its last degree of purity, in volcanic productions. It is found also, and much more frequently, in the state of sulphuric acid; that is to say, combined with oxygen: it is in this state that it is found in argil, gypsum, &c. It may be extracted likewise from vegetable substances, and animal matters.

To purify sulphur, melt it in an iron pan: by which means the earthy and metallic parts will be precipitated; and then pour it into a copper kettle, where it will form another deposit of the foreign matters, with which it is mixed. After keeping it in fusion some time, pour it into cylindric wooden moulds, in order that it may be formed into sticks.

Saltpetre, or, as it is called in the modern chemistry, nitrate of potash, exists in a natural state, but in small quantities. It is found sometimes at the surface of the ground, as in India, and sometimes on the surface of calcareous walls, the roofs of cellars, under the arches of bridges, &c.

To extract the saltpetre from the lime of walls, or other earths impregnated with it, the earths are put into casks, placed on timbers, and water is poured over them to the height of about three inches. When the water has remained in that state five or six hours, it is suffered to run off by apertures made in the bottom of the casks, from which it falls into a gutter that conveys it to a common reservoir sunk in the earth. When the sediment has been deposited, the clear liquor is drawn off into a proper vessel, in order to be evaporated.

When the liquor is in a state of ebullition, in proportion as it evaporates, there is precipitated calcareous earth, and then muriate of soda. To know when it is sufficiently evaporated, put a drop of it on a piece of cold iron, and if it becomes fixed, and assumes a white solid globular form, it is time to slacken the fire. The liquor must then be left at rest for twenty-four hours, after which it is run off and set to crystallize.

It is needless to describe charcoal, as it is every where known. We shall only observe, that the charcoal found by experience to be the fittest for the composition of gunpowder, is that made from the alder, willow, or black dog-wood.

To make gunpowder, mix together 6 parts of pounded nitre, well purified, 1 part of pounded sulphur, exceedingly pure, and 1 part of pounded charcoal, adding a quantity of water sufficient to reduce them to a soft paste. Put the whole into a wooden or copper mortar, and with a pestle of the same materials, to prevent inflammation, pound these ingredients for twenty-four hours, to mix them thoroughly; taking care to keep them always moderately moist. When they are well incorporated, pour the mass upon a sieve pierced with small holes of the size which you intend to give to the grains of powder. If it be then pressed, shaking the sieve, it will pass through in grains, which must be dried in the sun, or over a stove without fire. When dry, it ought to be put into vessels capable of preserving it from moisture.

Every one knows that, in consequence of the great consumption of gunpowder, certain machines, called *powder-mills*, have been invented. These machines consist of a beam turned by means of a water-wheel, and furnished with a great number of projecting arms, which raise up and let fall in succession a series of pestles or stampers, below which are placed copper vessels or mortars containing the matter to be pounded or incorporated. These mills however are exceedingly disagreeable neighbours; for notwithstanding the precautions taken there are few of them which do not some time or other blow up. On this account they ought always to be erected at a distance from towns or dwellings.

As the enlarged state of chemistry has introduced some improvements in the art of making gunpowder, we shall here, in addition to what has been above said, give the following account of the process employed for this purpose in some of the English manufactories.

Gunpowder is made of three ingredients; saltpetre, charcoal, and brimstone; which are combined in the following proportions: for each 100 parts of gunpowder, saltpetre 75 parts, charcoal 15, and sulphur 10.

The saltpetre is either that imported principally from the East Indies, or that which has been extracted from damaged gunpowder. It is refined by solution, filtration, evaporation, and crystallization; after which it is fused; taking care not to use too much heat, that there may be no danger of decomposing the nitre.

The sulphur used is that which is imported from Sicily, and is refined by melting and skimming: the most impure is refined by sublimation.

The charcoal formerly used in this manufacture, was made by charring wood in the usual manner. This mode is called charring in pits. The wood is cut into pieces of about three feet in length; it is then piled on the ground, in a circular form, three, four, or five cords of wood making what is called a pit, and then covered with straw, fern, &c. kept down by earth or sand: and vent holes are made, as may be necessary, in order to give it air. As this method is uncertain and defective, the charcoal now used in the manufacturing of gunpowder, is made in the following manner. The wood to be charred is first cut into pieces of about nine inches in length, and put into an iron cylinder placed horizontally. The front aperture of the cylinder is then closely stopped: at the other end there are pipes connected with casks. Fire being made under the cylinder, the pyro-ligneous acid, attended with a large portion of hydrogen gas, comes over. The gas escapes, and the acid liquor is collected in the casks. The fire is kept up till no more gas or liquor comes over, and the carbon remains in the cylinder.

The several ingredients, being thus prepared, are ready for manufacturing. They are first ground separately to a fine powder; they are then mixed together in the proper proportions; and the composition in this state is sent to the gunpowder mill, which consists of two stones placed vertically, and running on a bed-stone. On this bed-stone the composition is spread out, and moistened with as small a quantity of water as will reduce it to a proper body, but not to a paste: after the stone runners have made the proper revolutions over it, it may then be taken off.

A powder mill is a slight wooden building, with a boarded roof. Only about 40 or 50 lbs. of composition is worked here at a time, as explosions may happen by the runners and bed-stone coming into contact, and even from other causes. These mills are worked either by water or by horses.

The composition, when taken from the mill, is sent to the corning house, to be corned or grained. Here it is first formed into a hard and firm mass; it is then broken into small lumps, and afterwards grained, by these lumps being put into sieves, in each of which is a flat circular piece of lignum vitæ. The sieves are made of parchment skins, having round holes punched through them. Several of these sieves are fixed in a frame, which by proper machinery has such a motion given to it, as to make the lignum vitæ runner in each sieve go round with great velocity, so as to break the lumps of powder, and by forcing it through the holes to form it into grains of several sizes. The grains are then separated from the dust by sieves and reels made for that purpose.

The grains are next hardened, and the rougher edges are taken off by shaking them a sufficient time in a close reel, moved in a circular direction with a proper velocity.

The powder for guns, mortars, and small arms, is generally made at one time, and

always of the same composition. The only difference is in the size of the grains, which are separated by sieves of different fineness.

The gunpowder thus corned, dusted, and reeled, which is called glazing, as it gives it a small degree of gloss, is then sent to the stove and dried; care being taken not to raise the heat so much as to decompose the sulphur. The heat is regulated by a thermometer placed in the door of the stoves, if dried in a gloom-stove.*

A gunpowder stove dries the powder either by steam or by the heat from an iron gloom, the powder being spread out on cases placed on proper supports around the room.

If gunpowder is injured by damp in a small degree, it may be recovered by again drying it in a stove; but if the ingredients are decomposed, the nitre must be extracted, and the gunpowder re-manufactured.

There are several methods of proving and trying the goodness and strength of gunpowder. The following is one by which a tolerably good idea may be formed of its purity, and also some conclusion as to its strength.

Lay two or three small heaps, about a dram or two of the powder on separate pieces of clean writing paper; fire one of them by a red hot wire; if the flame ascends rapidly, with a good report, leaving the paper free from white specks, and without burning holes in it; and if sparks fly off and set fire to the adjoining heaps, the goodness of the ingredients and proper manufacture of the powder may be safely inferred; but if otherwise, it is either badly made, or the ingredients are impure.

Dr. Hutton, the former editor of the English edition of the Recreations, was fortunate enough to succeed in constructing the most convenient and most accurate eprouvette that has perhaps ever been contrived for accurately determining the comparative strength of gunpowder. It consists of a small cannon, or gun, suspended freely, like a pendulum, with the axis of the gun horizontal. This being charged with the proper charge of powder and then fired, the gun swings, or recoils backward, and the instrument itself shews the extent of the first or greatest vibration, which indicates the strength to the utmost nicety.

Having thus given an account of almost every thing necessary to be known in regard to the process for making gunpowder, we shall now say a few words respecting the physical causes of its inflammation and exploding.

Gunpowder being composed of the above ingredients, when a spark, struck from a piece of flint and steel, falls on this mixture, it sets fire to a certain portion of the charcoal, and the inflamed charcoal causes the nitre with which it is mixed or in contact, to detonate, and also the sulphur, the combustibility of which is well known. Portions of the charcoal contiguous to the former take fire in like manner, and produce the same effect in regard to the surrounding mass: thus the first portion inflamed, inflames a hundred others; these hundred communicate the inflammation to ten thousand; the ten thousand to a million, and so on. It may be easily conceived that an inflammation, the progress of which is so rapid, cannot fail to extend itself in the course of a very short time, from the one extremity to the other of the largest mass.

We shall observe, in support of this inflammation, that granulated powder inflames with much more rapidity than that which is not granulated. The latter only puffs away slowly, while the other takes fire almost instantaneously; and of the granulated kinds of gunpowder, that in round grains, like the Swiss powder,

* This kind of stove consists of a large cast-iron vessel, projecting into one side of a room, and heated from the outside, till it absolutely glows. From the construction it is hardly possible that fire can be thrown from the gloom as it is called; but stoves heated by steam passing through steam-tight tubes, or otherwise, ought certainly to be preferred; for the most cautious workman may stumble, and if he has a case of powder in his hand, some of it may be thrown upon the gloom: and it is not improbable that some of the accidents which have happened to powder mills may have been occasioned in this manner.

inflames sooner than that in oblong irregular grains, like the French. The reason of this is, that the former leaves to the flame of the grains first inflamed, larger and freer interstices, which produced the inflammation with more rapidity.

In regard to the expansion of inflamed gunpowder, is it occasioned by the air interposed between its grains, or by the aqueous fluid which enters into the composition of the nitre? We doubt much whether it be the air, as its expansibility does not seem sufficient to explain the phenomenon; but we know that water, when converted into vapour by the contact of heat, occupies a space 14000 times greater than its original bulk, and that its force is very considerable.

In the foregoing account however Montucla seems to have missed the true cause of the expansive force of fired gunpowder, the discovery of which is chiefly due to the English philosophers, and particularly to the learned and ingenious Mr. Robins. This author apprehends that the force of fired gunpowder consists in the action of a permanently elastic fluid, suddenly disengaged from the powder by the combustion, similar in some respects to common atmospheric air, at least as to elasticity. He shewed, by satisfactory experiments, that a fluid of this kind is actually disengaged by firing the powder; and that it is *permanently* elastic, or retains its elasticity when cold, the force of which he measured in this state. He also measured the force of it when inflamed, by a most ingenious method, and found its strength in that state to be about a thousand times the strength or elasticity of common atmospheric air. This however is not its utmost degree of strength, as it is found to increase in its force when fired in larger quantities than those employed by Mr. Robins; so much so indeed, that, by more accurate and effectual experiments, we have found its force rise as high as 1500 or 1600 times the force of atmospheric air in its usual state. Much beyond this it is not probable it can go, nor indeed possible, if there be any truth in the common and allowed physical principles of mechanics. With an elastic fluid, of a given force, we infallibly know, or compute the effects it can produce, in impelling a given body; and on the other hand, from the effects or velocities with which given bodies are impelled by an elastic fluid, we as certainly know the force or strength of that fluid. And these effects we have found perfectly to accord with the forces above mentioned. If any gentleman therefore thinks he has discovered that fired gunpowder is 50 or 60 times as strong as above stated, he must have been deceived by mistaking or misapplying his own experiments; and we apprehend it would not be difficult, if this were the proper place, to shew that this has actually been the case.

Mr. Robins's discovery and opinion have also been corroborated by others, among the best chemists and philosophers. Lavoisier was of opinion that the force of fired gunpowder depends, in a great measure, on the expansive force of uncombined caloric, supposed to be let loose, in a great abundance, during the combustion or deflagration of the powder; and Bouillon Lagrange, in his Course of Chemistry, says, when gunpowder takes fire, there is a disengagement of azotic gas, which expands in an astonishing manner, when set at liberty; and we are even still ignorant of the extent of the dilatation occasioned by the heat arising from the combustion. A decomposition of water also takes place, and hydrogen gas is disengaged with elasticity; and by this decomposition of water there is formed carbonic acid gas, and even sulphurated hydrogen gas, which is the cause of the smell emitted by burnt powder.

Remarks.—I. It is ridiculous therefore to believe in the existence of *white gunpowder;* that is, a kind of powder which impels a ball without any noise; for there can be no force without sudden expansion, nor sudden expansion without a concussion of the air, which produces sound.

II. It was childish to give precepts, as in the preceding editions of this work, for making red, blue, green, &c. gunpowder; as they could answer no good purpose.

We shall now proceed to our principal object, the construction of the most common and curious pieces of fire-works.

ARTICLE II.

Construction of the Cartridges of Rockets.

A rocket is a cartridge or case made of stiff paper, which being filled in part with gunpowder, saltpetre, and charcoal, rises of itself into the air, when fire is applied to it.

There are three sorts of rockets: small ones, the calibre of which does not exceed a pound bullet; that is to say, the orifice of them is equal to the diameter of a leaden bullet which weighs only a pound; for the calibres, or orifices of the moulds or models used in making rockets, are measured by the diameters of leaden bullets:— middle sized rockets, equal to the size of a ball of from one to three pounds; and large rockets, equal to a ball of from three to a hundred pounds.

To give the cartridges the same length and thickness, in order that any number of rockets may be prepared of the same size and force, they are put into a hollow cylinder of strong wood, called a mould. This mould is sometimes of metal; but at any rate it ought to be made of some very hard wood.

This mould must not be confounded with another piece of wood, called the former or roller, around which is rolled the thick paper employed to make the cartridge. If the calibre of the mould be divided into 8 equal parts, the diameter of the roller must be equal to 5 of these parts. See Fig. 1, where A is the mould, and B the roller. The vacuity between the roller and the interior surface of the mould, that is to say, $\frac{3}{8}$ of the calibre of the mould, will be exactly filled by the cartridge.

As rockets are made of different sizes, moulds of different lengths and diameters must be provided. The calibre of a cannon is nothing else than the diameter of its mouth; and we here apply the same term to the diameter of the aperture of the mould.

The size of the mould is measured by its calibre; but the length of the moulds for different rockets does not always bear the same proportion to the calibre, the length being diminished as the calibre is increased. The length of the mould for small rockets ought to be six times the calibre, but for rockets of the mean and larger size it will be sufficient if the length of the mould be five times or even four times the calibre of the mould.

At the end of this section we shall give two tables, one of which contains the calibres of moulds below a pound bullet; and the other the calibres from a pound to a hundred pounds bullet.

Fig. 1.

For making the cartridges large stiff paper is employed. This paper is wrapped round the roller B, (Fig. 1.), and then cemented by means of common paste. The thickness of the paper when rolled up in this manner, ought to be about one-eighth and a half of the calibre of the mould, according to the proportion given to the diameter of the roller. But if the diameter of the roller be made equal to $\frac{3}{4}$ the calibre of the mould, the thickness of the cartridge must be a twelfth and a half of that calibre.

When the cartridge is formed, the roller B is drawn out, by turning it round, until it is distant from the edge of the cartridge the length of its diameter. A piece of cord is then made to pass twice round the cartridge at the extremity of the roller; and into the vacuity left in the cartridge, another roller is introduced, so as to leave some space between the two. One end of the packthread must be fastened to something fixed, and the other to a stick conveyed between the legs, and placed in such a man-

ner, as to be behind the person who choaks the cartridge. The cord is then to be stretched by retiring backwards, and the cartridge must be pinched until there remains only an aperture capable of admitting the piercer D E. The cord employed for pinching it is then removed, and its place is supplied by a piece of pack-thread, which must be drawn very tight, passing it several times around the cartridge, after which it is secured by means of running knots made one above the other.

Besides the roller B, a rod C (Fig. 1.), is used, which being employed to load the cartridge, must be somewhat smaller than the roller, in order that it may be easily introduced into the cartridge. The rod C is pierced lengthwise, to a sufficient depth to receive the piercer D E, which must enter into the mould A, and unite with it exactly at its lower part. The piercer, which decreases in size, is introduced into the cartridge through the part where it has been choaked, and serves to preserve a cavity within it. Its length, besides the nipple or button, must be equal to about two-thirds of that of the mould. Lastly, If the thickness of the base be a fourth part of the calibre of the mould, the point must be made equal to a sixth of the calibre.

It is evident that there must be at least three rods, such as C, pierced in proportion to the diminution of the piercer, in order that the powder which is rammed in by means of a mallet, may be uniformly packed throughout the whole length of the rocket. It may be easily perceived also, that these rods ought to be made of some very hard wood, to resist the strokes of the mallet.

In loading rockets, it is more convenient not to employ a piercer. When loaded on a nipple, without a piercer, by means of one massy rod, they are pierced with a bit, and a piercer fitted into the end of a bit-brace. Care however must be taken to make this hole suited to the proportion assigned for the diminution of the piercer. That is to say, the extremity of the hole at the choaked part of the cartridge, ought to be about a fourth of the calibre of the mould; and the extremity of the hole which is in the inside for about two-thirds of the length of the rocket, ought to be a sixth of the calibre. This hole must pass directly through the middle of the rocket. In short, experience and ingenuity will suggest what is most convenient, and in what manner the method of loading rockets, which we shall here explain, may be varied.

After the cartridge is placed in the mould, pour gradually into it the prepared composition; taking care to pour only two spoonfuls at a time, and to ram it immediately down with the rod C, striking it in a perpendicular direction with a mallet of a proper size, and giving an equal number of strokes, for example 3 or 4, each time that a new quantity of the composition is poured in.

When the cartridge is about half filled, separate with a bodkin the half of the folds of the paper which remains, and having turned them back on the composition, press them down with the rod and a few strokes of the mallet, in order to compress the paper on the composition.

Fig 2.

C

A

Then pierce three or four holes in the folded paper, by means of a piercer, which must be made to penetrate to the compcsition of the rocket, as seen at A (Fig. 2.) These holes serve to form a communication between the body of the rocket and the vacuity at the extremity of the cartridge, or that part which has been left empty.

In small rockets this vacuity is filled with granulated powder, which serves to let them off: they are then covered with paper, and pinched in the same manner as at the other extremity. But in other rockets, the pot containing stars, serpents, and running rockets, is adapted to it, as will be shewn hereafter.

It may be sufficient however to make, with a bit or piercer, only one hole, which must be neither too large nor too small, such as a fourth part of the diameter of the rocket, to set fire to the powder, taking care that this hole

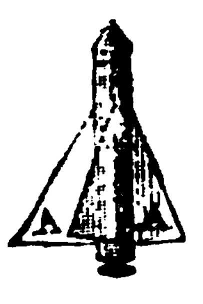
be as straight as possible, and exactly in the middle of the composition. A little of the composition of the rocket must be put into these holes, that the fire may not fail to be communicated to it.

It now remains to affix the rocket to its rod, which is done in the following manner. When the rocket has been constructed as above described, make fast to it a rod of light wood, such as fir or willow, broad and flat at the end next the rocket, and decreasing towards the other. It must be as straight and free from knots as possible, and ought to be dressed, if necessary, with a plane. Its length and weight must be proportioned to the rocket; that is to say, it ought to be six, seven, or eight feet long, so as to remain in equilibrium with it, when suspended on the finger, within an inch or an inch and a half of the neck. Before it is fired, place it with the neck downwards, and let it rest on two nails, in a direction perpendicular to the horizon. To make it ascend straighter and to a greater height, adapt to its summit A a pointed cap or top, as C, made of common paper, which will serve to facilitate its passage through the air.

These rockets, in general, are made in a more complex manner, several other things being added to them to render them more agreeable, such for example as a petard, which is a box of tin-plate, filled with fine gunpowder, placed on the summit. The petard is deposited on the composition, at the end where it has been filled; and the remaining paper of the cartridge is folded down over it to keep it firm. The petard produces its effect when the rocket is in the air, and the composition is consumed.

Stars, golden rain, serpents, saucissons, and several other amusing things, the composition of which we shall explain hereafter, are also added to them. This is done by adjusting to the head of the rocket an empty pot or cartridge, much larger than the rocket, in order that it may contain serpents, stars, and various other appendages, to render it more beautiful.

Fig. 3.



Rockets may be made to rise into the air without rods. For this purpose four wings must be attached to them in the form of a cross, and similar to those seen on arrows or darts, as represented at A (Fig. 3.) In length these wings must be equal to two-thirds that of the rocket; their breadth towards the bottom should be half their length, and their thickness ought to be equal to that of a card.

But this method of making rockets ascend is less certain, and more inconvenient than that where a rod is used; and for this reason it is rarely employed.

We shall now shew the method of finding the diameters or calibre of rockets, according to their weight; but we must first observe that a pound rocket is that just capable of admitting a leaden bullet of a pound weight, and so of the rest. The calibre for the different sizes may be found by the two following tables, one of which is calculated for rockets of a pound weight and below; and the other for those from a pound weight to 50 pounds.

I.—TABLE OF THE CALIBRE OF MOULDS OF A POUND WEIGHT AND BELOW.

Ounces.	Lines.	Drams.	Lines.
16	19½	14	7¼
12	17	12	7
8	15	10	6⅓
7	14¾	8	6¼
6	14¼	6	5⅔
5	13	4	4⅘
4	12⅓	2	3¾
3	11⅔		
2	9⅙		
1	6⅔		

The use of this table will be understood merely by inspection; for it is evident that a rocket of 12 ounces ought to be 17 lines in diameter; one of 8 ounces, 15 lines; one of 10 drams, 6⅓ lines; and so of the rest.

On the other hand, if the diameter of the rocket be given, it will be easy to find the weight of the ball corresponding to that calibre. For example, if the diameter be 13 lines, it will be immediately seen, by looking for that number in the column of lines, that it corresponds to a ball of 5 ounces.

II.—TABLE OF THE CALIBRE OF MOULDS FROM 1 TO 50 POUNDS BALL.

Pounds.	Calibre.	Pounds.	Calibre.	Pounds.	Calibre.	Pounds.	Calibre.	Pounds.	Calibre.
1	100	11	222	21	275	31	314	41	344
2	126	12	228	22	280	32	317	42	347
3	144	13	235	23	284	33	320	43	350
4	158	14	241	24	288	34	323	44	353
5	171	15	247	25	292	35	326	45	355
6	181	16	252	26	296	36	330	46	358
7	191	17	257	27	300	37	333	47	361
8	200	18	262	28	304	38	336	48	363
9	208	19	267	29	307	39	339	49	366
10	215	20	271	30	310	40	341	50	368

The use of the second table is as follows: If the weight of the ball be given, which we shall suppose to be 24 pounds, seek for that number in the column of pounds, and opposite to it, in the column of calibres, will be found the number 288. Then say, as 100 is to 19½, so is 288 to a fourth term, which will be the number of lines of the calibre required; or multiply the number found, that is 288, by 19½, and from the product 5616, cut off the two last figures: the required calibre will therefore be 56·16 lines, or 4 inches 8 lines.

On the other hand, the calibre being given in lines, the weight of the ball may be found with equal ease: if the calibre, for example, be 28 lines, say as 19½ is to 28, so is 100 to a fourth term, which will be 143·5, or nearly 144. But in the above table, opposite to 144, in the second column, will be found the number 3 in the first: which shews that a rocket, the diameter or calibre of which is 28 lines, is a rocket of a 3 pounds ball.

ARTICLE III.

Composition of the Powder for Rockets, and the manner of filling them.

The composition of the powder for rockets must be different, according to the different sizes; as that proper for small rockets would be too strong for large ones. This is a fact respecting which almost all the makers of fire-works are agreed. The quantities of the ingredients, which experience has shewn to be the best, are as follow:

For rockets capable of containing one or two ounces of composition.

To one pound of gunpowder, add two ounces of soft charcoal; or to one pound of gunpowder, a pound of the coarse powder used for cannon; or to nine ounces of gunpowder, two ounces of charcoal; or to a pound of gunpowder, an ounce and a half of saltpetre, and as much charcoal.

For rockets of two or three ounces.

To four ounces of gunpowder, add an ounce of charcoal; or to nine ounces of gunpowder, add two ounces of saltpetre.

For a rocket of four ounces.

To four pounds of gunpowder, add a pound of saltpetre, and four ounces of charcoal: you may add also, if you choose, half an ounce of sulphur; or to one pound two ounces and a half of gunpowder, add four ounces of saltpetre, and two ounces of charcoal; or to a pound of powder, add four ounces of saltpetre, and one ounce of charcoal; or to seventeen ounces of gunpowder, add four ounces of saltpetre, and the same quantity of charcoal; or to three ounces and a half of gunpowder, add ten ounces of saltpetre, and three ounces and a half of charcoal. But the composition will be strongest, if to ten ounces of gunpowder, you add three ounces and a half of saltpetre, and three ounces of charcoal.

For rockets of five or six ounces.

To two pounds five ounces of gunpowder, add half a pound of saltpetre, two ounces of sulphur, six ounces of charcoal, and two ounces of iron filings.

For rockets of seven or eight ounces.

To seventeen ounces of gunpowder, add four ounces of saltpetre, and three ounces of sulphur.

For rockets of from eight to ten ounces.

To two pounds and five ounces of gunpowder, add half a pound of saltpetre, two ounces of sulphur, seven ounces of charcoal, and three ounces of iron filings.

For rockets of from ten to twelve ounces.

To seventeeen ounces of gunpowder, add four ounces of saltpetre, three ounces and a half of sulphur, and one ounce of charcoal.

For rockets of from fourteen to fifteen ounces.

To two pounds four ounces of gunpowder, add nine ounces of saltpetre, three ounces of sulphur, five ounces of charcoal, and three ounces of iron filings.

For rockets of one pound.

To one pound of gunpowder, add one ounce of sulphur, and three ounces of charcoal.

For a rocket of two pounds.

To one pound four ounces of gunpowder, add two ounces of saltpetre, one ounce of sulphur, three ounces of charcoal, and two ounces of iron filings.

For a rocket of three pounds.

To thirty ounces of saltpetre, add seven ounces and a half of sulphur, and eleven ounces of charcoal.

For rockets of four, five, six, or seven pounds.

To thirty-one pounds of saltpetre, add four pounds and a half of sulphur, and ten pounds of charcoal.

For rockets of eight, nine, or ten pounds.

To eight pounds of saltpetre, add one pound four ounces of sulphur, and two pounds twelve ounces of charcoal.

We shall here observe that these ingredients must be each pounded separately, and sifted: they are then to be weighed and mixed together for the purpose of loading the cartridges, which ought to be kept ready in the moulds. The cartridges must be made of strong paper, doubled, and cemented by means of strong paste, made of fine flour and very pure water.

Of Matches.

Before we proceed farther, it will be proper to describe the composition of the matches necessary for letting them off. Take linen, hemp, or cotton thread, and double it eight or ten times, if intended for large rockets; or only four or five times, if to be employed for stars. When the match has been thus made as large as necessary, dip it in pure water, and press it between your hands, to free it from the moisture. Mix some gunpowder with a little water, to reduce it to a sort of paste, and immerse the match in it; turning and twisting it, till it has imbibed a sufficient quantity of the powder; then sprinkle over it a little dry powder, or strew some pulverised dry powder upon a smooth board, and roll the match over it. By these means you will have an excellent match, which, if dried in the sun, or on a rope in the shade, will be fit for use.

ARTICLE IV.

On the cause which makes rockets ascend into the air.

As this cause is nearly the same as that which produces recoil in fire-arms, it is necessary we should first explain the latter.

When the powder is suddenly inflamed in the chamber, or at the bottom of the barrel, it necessarily exercises an action two ways at the same time; that is to say, against the breech of the piece, and against the bullet or wadding, which is placed above it. Besides this, it acts also against the sides of the chamber which it occupies; and as they oppose a resistance almost insurmountable, the whole effort of the elastic fluid produced by the inflammation is exerted in the two directions above mentioned. But the resistance opposed by the bullet being much less than that opposed by the mass of the barrel or cannon, the bullet is forced out with great velocity. It is impossible, however, that the body of the piece itself should not experience a movement backwards; for if a spring is suddenly let loose, between two moveable obstacles, it will impel them both, and communicate to them velocities in the inverse ratio of their masses: the piece therefore must acquire a velocity backwards nearly in the inverse ratio of its mass to that of the bullet. We make use of the term nearly, because there are various circumstances which give to this ratio certain

modifications; but it is always true that the body of the piece is driven backwards, and that if it weighs with its carriage a thousand times more than the bullet, it acquires a velocity which is a thousand times less, and which is soon annihilated by the friction of the wheels against the ground, &c.

The cause of the ascent of a rocket is nearly the same. At the moment when the powder begins to inflame, its expansion produces a torrent of elastic fluid, which acts in every direction; that is, against the air which opposes its escape from the cartridge, and against the upper part of the rocket; but the resistance of the air is more considerable than the weight of the rocket, on account of the extreme rapidity with which the elastic fluid issues through the neck of the rocket to throw itself downwards, and therefore the rocket ascends by the excess of the one of these forces over the other.

This however would not be the case, unless the rocket were pierced to a certain depth. A sufficient quantity of elastic fluid would not be produced; for the composition would inflame only in circular coats of a diameter equal to that of the rocket; and experience shews that this is not sufficient. Recourse then is had to the very ingenious idea of piercing the rocket with a conical hole, which makes the composition burn in conical strata, which have much greater surface, and therefore produce a much greater quantity of inflamed matter and fluid. This expedient was certainly not the work of a moment.

ARTICLE V.

Brilliant fire and Chinese fire.

As iron-filings, when thrown into the fire, inflame and emit a strong light, this property, discovered no doubt by chance, gave rise to the idea of rendering the fire of rockets much more brilliant than when gunpowder, or the substances of which it is composed, are alone employed. Nothing is necessary but to take iron-filings, very clean and free from rust, and to mix them with the composition of the rocket. It must however be observed, that rockets of this kind will not keep longer than a week; because the moisture contracted by the saltpetre rusts the iron-filings, and destroys the effect they are intended to produce.

But the Chinese have long been in possession of a method of rendering this fire much more brilliant and variegated in its colours; and we are indebted to Father d'Incarville, a jesuit, for having made it known. It consists in the use of a very simple ingredient: namely, cast iron reduced to a powder more or less fine; the Chinese give it a name, which is equivalent to that of *iron sand.*

To prepare this sand, take an old iron pot, and having broken it to pieces on an anvil, pulverise the fragments till the grains are not larger than radish seed: then sift them through six graduated sieves, to separate the different sizes, and preserve these six different kinds, in a very dry place, to secure them from rust, which would render this sand absolutely unfit for the proposed end. We must here remark, that the grains which pass through the closest sieve, are called sand of the first order; those which pass through the next in size, sand of the second order; and so on.

This sand, when it inflames, emits a light exceedingly vivid. It is very surprising to see fragments of this matter no bigger than a poppy seed, form all of a sudden luminous flowers or stars, 12 and 15 lines in diameter. These flowers are also of different forms, according to that of the inflamed grain, and even of different colours according to the matters with which the grains are mixed. But rockets into which this composition enters cannot be long preserved, as those which contain the finest sand will not keep longer than eight days, and those which contain the coarsest, fifteen. The following tables exhibit the proportions of the different ingredients for rockets of from 12 to 36 pounds.

For red Chinese fire.

Calibres.	Saltpetre.	Sulphur.	Charcoal.	Sand of the 1st order.
Pounds.	Pounds.	Ounces.	Ounces.	oz. dr.
12 to 15	1	3	4	7 0
18 to 21	1	3	5	7 8
24 to 36	1	4	6	8 0

For white Chinese fire.

Calibres.	Saltpetre.	Bruised Gunpowder.	Charcoal.	Sand of the 3d order.
Pounds.	Pounds.	Ounces.	oz. dr.	oz. dr.
12 to 15	1	12	7 8	11 0
18 to 21	1	11	8 0	11 8
24 to 36	1	11	8 8	12 0

When these materials have been weighed, the saltpetre and charcoal must be three times sifted through a hair sieve, in order that they may be well mixed: the iron sand is then to be moistened with good brandy, to make the sulphur adhere, and they must be thoroughly incorporated. The sand thus sulphured must be spread over the mixture of saltpetre and charcoal, and the whole must be mixed together by spreading it over a table with a spatula.

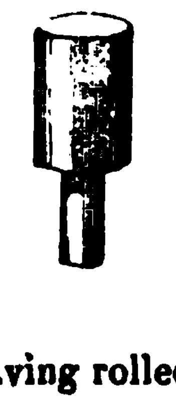
ARTICLE VI.

Of the Furniture of Rockets.

The upper part of rockets is generally furnished with some composition, which taking fire when it has reached to its greatest height, emits a considerable blaze, or produces a loud report, and very often both these together. Of this kind are saucissons, marroons, stars, showers of fire, &c.

To make room for this artifice, the rocket is crowned with a part of a greater diameter called the pot, as seen in Fig. 4. The method of making this pot, and connecting it with the body of the rocket, is as follows.

Fig. 4.



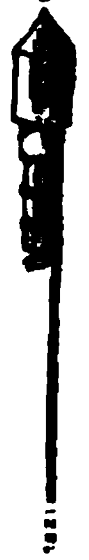
The mould for forming the pot, though of one piece, must consist of two cylindric parts of different diameters. That on which the pot is rolled up must be three diameters of the rocket in length, and its diameter must be three fourths that of the rocket; the length of the other ought to be equal to two of these diameters, and its diameter to $\frac{1}{8}$ that of the rocket.

Having rolled the thick paper intended for making the pot, and which ought to be of the same kind as that used for the rocket, twice round the cylinder, a portion of it must be pinched in that part of the cylinder which has the least diameter; this part must be pared in such a manner as to leave only what is necessary for making the pot fast to the top of the rocket, and the ligature must be covered with paper.

To charge such a pot, attached to a rocket; having pierced three or four holes in the double paper which covers the vacuity of the rocket, pour over it a small quantity of the composition with which the rocket is filled, and by shaking it, make a part enter these holes; then arrange in the pot the composition with which it is to be

charged, taking care not to introduce introduce into it a quantity heavier than the body of the rocket.

Fig. 5.



The whole must then be secured by means of a few small balls of paper, to keep every thing in its place, and the pot must be covered with paper cemented to its edges: if a pointed summit or cap be then added to it, the rocket will be ready for use.

We shall now give an account of the different artifices with which such rockets are loaded.

I.—*Of Serpents.*

Serpents are small flying rockets, without rods, which instead of rising in a perpendicular direction, mount obliquely, and descend in a zig-zag form without ascending to a great height. The composition of them is nearly the same as that of rockets; and therefore nothing more is necessary than to determine the proportion and construction of the cartridge, which is a follows.

Fig. 6.

C B A

The length A C (Fig. 6), of the cartridge may be about 4 inches; it must be rolled round a stick somewhat larger than the barrel of a goose quill, and after being choaked at one of its ends, fill it with the composition a little beyond its middle, as to B; and then pinch it so as to leave a small aperture. The remainder B C, must be filled with grained powder, which will occasion a report when it bursts. Lastly, choak the cartridge entirely towards the extremity C; and at the other extremity A place a train of moist powder, to which if fire be applied, it will be communicated to the composition in the part A B, and cause the whole to rise in the air. The serpent, as it falls, will then make several small turns in a zig-zag direction, till the fire is communicated to the grained powder in the part B C; on which the serpent will burst with a loud report before it falls to the ground.

If the serpent be not choaked towards the middle, instead of moving in a zig-zag direction, it will ascend and descend with an undulating motion, and then burst as before.

The cartridges of serpents are generally made of playing cards. These cards are rolled round a rod of iron or hard wood, a little larger, as already said, than the barrel of a goose quill. To confine the card, a piece of strong paper is cemented over it.

The length of the mould must be proportioned to that of the cards employed, and the piercer of the nipple must be three or four lines in length. These serpents are loaded with bruised powder, mixed only with a very small quantity of charcoal. To introduce the composition into the cartridge, a quill, cut into the form of a spoon, may be employed: it must be rammed down by means of a small rod, to which a few strokes are given with a small mallet.

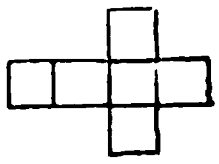
When the serpent is half loaded, instead of pinching it in that part, you may introduce into it a vetch seed, and place granulated powder above it to fill up the remainder. Above this powder place a small pellet of chewed paper, and then choak the other end of the cartridge. If you are desirous of making larger serpents, cement two playing cards together; and, that they may be managed with more ease, moisten them a little with water. The match consists of a paste made of bruised powder, and a small quantity of water.

II.—*Marroons.*

Marroons are small cubical boxes, filled with a composition proper for making them burst, and may be constructed with great ease.

Cut a piece of pasteboard, according to the method taught in geometry to form the

Fig. 7.



cube, as seen Fig. 7.; join these squares at the edges, leaving only one to be cemented, and fill the cavity of the cube with grained powder; then cement strong paper in various directions over this body, and wrap round it two rows of pack-thread, dipped in strong glue: then make a hole in one of the corners, and introduce into it a match.

If you are desirous to have luminous marroons, that is to say marroons which, before they burst in the air, emit a brilliant light, cover them with a paste, the composition of which will be given hereafter for stars; and roll them in pulverised gunpowder, to serve as a match or communication.

III.—*Saucissons.*

Marroons and saucissons differ from each other only in their form. The cartridges of the latter are round, and must be only four times their exterior diameter in length. They are choked at one end in the same manner as a rocket; and a pellet of paper is driven into the aperture which has been left, in order to fill it up. They are then charged with grained powder, above which is placed a ball of paper gently pressed down, to prevent the powder from being bruised; the second end of the saucisson being afterwards choaked, the edges are pared on both sides, and the whole is covered with several turns of pack-thread, dipped in strong glue, and then left to dry.

When you are desirous of charging them, pierce a hole in one of the ends; and apply a match, in the same manner as to marroons.

IV.—*Stars.*

Stars are small globes of a composition which emits a brilliant light, that may be compared to the light of the stars in the heavens. These balls are not larger than a nutmeg or musket bullet, and when put into the rockets must be wrapped up in tow, prepared for that purpose. The composition of these stars is as follows.

To a pound of fine gunpowder well pulverised, add four pounds of saltpetre, and two pounds of sulphur. When these ingredients are thoroughly incorporated, take about the size of a nutmeg of this mixture, and having wrapt it up in a piece of linen-rag, or of paper, form it into a ball; then tie it closely round with a pack-thread, and pierce a hole through the middle of it, sufficiently large to receive a piece of prepared tow, which will serve as a match. This star, when lighted, will exhibit a most beautiful appearance; because the fire as it issues from the two ends of the hole in the middle, will extend to a great distance, and make it appear much larger.

If you are desirous to employ a moist composition in the form of a paste, instead of a dry one, it will not be necessary to wrap up the star in any thing but prepared tow; because, when made of such paste, it can retain its spherical figure. There will be no need also of piercing a hole in it, to receive the match; because when newly made, and consequently moist, it may be rolled in pulverised gunpowder, which will adhere to it. This powder, when kindled, will serve as a match, and inflame the composition of the star, which in falling will form itself into tears.

Another method of making Rockets with Stars.

Mix three ounces of saltpetre, with one ounce of sulphur, and two drams of pulverized gunpowder; or mix four ounces of sulphur with the same quantity of saltpetre, and eight ounces of pulverized gunpowder. When these materials have been well sifted, besprinkle them with brandy, in which a little gum has been dissolved, and then make up the star in the following manner.

Take a rocket mould, eight or nine lines in diameter, and introduce into it a nipple, the piercer of which is of a uniform size throughout, and equal in length to the height of the mould. Put into this mould a cartridge, and by means of a pierced

rod load it with one of the preceding compositions; when loaded, take it from the mould, without removing the nipple, the piercer of which passes through the composition, and then cut the cartridge quite round into pieces of the thickness of three or four lines. The cartridge being thus cut, draw out the piercer gently, and the pieces, which resemble the men employed for playing at drafts, pierced through the middle, will be stars, which must be filed on a match thread, which, if you choose, may be covered with tow.

To give more brilliancy to stars of this kind, a cartridge thicker than the above dimensions, and thinner than that of a flying-rocket of the same size, may be employed; but, before it is cut into pieces, five or six holes must be pierced in the circumference of each piece to be cut. When the cartridge is cut, and the pieces have been filed, cement over the composition small bits of card, each having a hole in the middle, so that these holes may correspond to the place where the composition is pierced.

Remarks.—I. There are several other methods of making stars, which it would be too tedious to describe. We shall therefore only shew how to make *étoiles à pet*, or stars which give a report as loud as that of a pistol or musket.

Make small saucissons, as taught in the third section; only it will not be necessary to cover them with pack-thread: it will be sufficient if they are pierced at one end, in order that you may tie to it a star constructed according to the first method, the composition of which is dry; for if the composition be in the form of a paste, there will be no need to tie it. Nothing will be necessary in that case, but to leave a little more of the paper hollow at the end of the saucisson which has been pierced, for the purpose of introducing the composition; and to place in the vacuity, towards the neck of the saucisson, some grained powder, which will communicate fire to the saucisson when the composition is consumed.

II. As there are some stars which in the end become petards, others may be made which shall conclude with becoming serpents. But this may be so easily conceived and carried into execution, that it would be losing time to enlarge further on the subject. We shall only observe, that these stars are not in use, because it is difficult for a rocket to carry them to a considerable height in the air: they diminish the effect of the rocket or saucisson, and much time is required to make them.

V.—*Shower of Fire.*

To form a shower of fire, mould small paper cartridges on an iron rod, two lines and a half in diameter, and make them two inches and a half in length. They must not be choaked, as it will be sufficient to twist the end of the cartridge, and having put the rod into it to beat it, in order to make it assume its form. When the cartridges are filled, which is done by immersing them in the composition, fold down the other end, and then apply a match. This furniture will fill the air with an undulating fire. The following are some compositions proper for stars of this kind.

Chinese fire.—Pulverised gunpowder one pound, sulphur two ounces, iron sand of the first order five ounces.

Ancient fire.—Pulverised gunpowder one pound, charcoal two ounces.

Brilliant fire.—Pulverised gunpowder one pound, iron filings four ounces.

The Chinese fire is certainly the most beautiful.

VI.—*Of Sparks.*

Sparks differ from stars only in their size and duration; for they are made smaller than stars; and are consumed sooner. They are made in the following manner.

Having put into an earthern vessel an ounce of pulverised gunpowder, two ounces of pulverised saltpetre, one ounce of liquid saltpetre, and four ounces of camphor

reduced to a sort of farina, pour over this mixture some gum-water, or brandy in which gum-adraganth or gum-arabic has been dissolved, till the composition acquire the consistence of thick soup. Then take some lint which has been boiled in brandy, or in vinegar, or even in saltpetre, and then dried and unravelled, and throw into the mixture such a quantity of it as is sufficient to absorb it entirely, taking care to stir it well.

Form this matter into small balls or globes of the size of a pea; and having dried them in the sun or the shade, besprinkle them with pulverised gunpowder, in order that they may more readily catch fire.

Another method of making Sparks.

Take the saw-dust of any kind of wood that burns readily, such as fir, elder-tree, poplar, laurel, &c., and boil it in water in which saltpetre has been dissolved. When the water has boiled some time, take it from the fire, and pour it off in such a manner that the saw-dust may remain in the vessel. Then place the saw-dust on a table, and while moist besprinkle it with sulphur, sifted through a very fine sieve: you may add to it also a little bruised gunpowder. Lastly, when the saw-dust has been well mixed, leave it to dry, and make it into sparks as above described.

VII.—*Of Golden Rain.*

There are some flying-rockets which, as they fall, make small undulations in the air, like hair half frizzled. These are called *fusées chevelues*, bearded rockets; they finish with a kind of shower of fire, which is called golden rain. The method of constructing them is as follows.

Fill the barrels of some goose quills with the composition of flying-rockets, and place upon the mouth of each a little moist gunpowder, both to keep in the composition, and to serve as a match. If a flying-rocket be then loaded with these quills, they will produce, at the end, a very agreeable shower of fire, which on account of its beauty has been called golden rain.

ARTICLE VII.

Of some Rockets different in their effect from common rockets.

Several very amusing and ingenious works are made by means of simple rockets, of which it is necessary that we should here give the reader some idea.

I.—*Of Courantins, or Rockets which fly along a rope.*

A common rocket, which however ought not to be very large, may be made to run along an extended rope. For this purpose, affix to the rocket an empty cartridge, and introduce into it the rope which is to carry it; placing the head of the rocket towards that side to which you intend it to move: if you then set fire to the rocket, adjusted in this manner, it will run along the rope without stopping, till the matter it contains is entirely exhausted.

If you are desirous that the rocket should move in a retrograde direction; first fill one half of it with the composition, and cover it with a small round piece of wood, to serve as a partition between it and that put into the other half; then make a hole below this partition, so as to correspond with a small canal filled with bruised powder, and terminating at the other end of the rocket: by these means the fire, when it ceases in the first half of the rocket, will be communicated through the hole into the small canal, which will convey it to the other end; and this end being then kindled, the rocket will move backwards, and return to the place from which it set out.

Two rockets of equal size, bound together by means of a piece of strong pack-thread, and disposed in such a manner that the head of the one shall be opposite

to the neck of the other, that when the fire has consumed the composition in the one it may be communicated to that in the other, and oblige both of them to move in a retrograde direction, may also be adjusted to the rope by means of a piece of hollow reed. But to prevent the fire of the former from being communicated to the second too soon, they ought to be covered with oil-cloth, or to be wrapped up in paper.

Remark.—Rockets of this kind are generally employed for setting fire to various other pieces when large fire-works are exhibited; and to render them more agreeable, they are made in the form of different animals, such as serpents, dragons, &c.; on which account they are called *flying dragons*. These drágons are very amusing, especially when filled with various compositions, such as golden rain, long hair, &c. They might be made to discharge serpents from their mouths, which would produce a very pleasing effect, and give them a greater resemblance to a dragon.

II.—*Rockets which fly along a rope, and turn round at the same time.*

Nothing is easier than to give to a rocket of this kind a rotary motion around the rope along which it advances; it will be sufficient for this purpose, to tie to it another rocket, placed in a transversal direction. But the aperture of the latter, instead of being at the bottom, ought to be in the side, near one of the ends. If both rockets be fired at the same time, the latter will make the other revolve around the rope, while it advances along it.

III.—*Of rockets which burn in the water.*

Though fire and water are two things of a very opposite nature, the rockets above described, when set on fire, will burn and produce their effect even in the water; but as they are then below the water, the pleasure of seeing them is lost; for this reason, when it is required to cause rockets to burn as they float on the water, it will be necessary to make some change in the proportions of the moulds, and the materials of which they are composed.

In regard to the mould, it may be eight or nine inches in length, and an inch in diameter: the former, on which the cartridge is rolled up, may be nine lines in thickness, and the rod for loading the cartridge must as usual be somewhat less. For loading the cartridge, there is no need of a piercer with a nipple.

The composition may be made in two ways; for if it be required that the rocket, while burning on the water, should appear as bright as a candle, it must be composed of three materials mixed together, viz., three ounces of pulverised and sifted gunpowder, one pound of saltpetre, and eight ounces of sulphur. But if you are desirous that it should appear on the water with a beautiful tail, the composition must consist of eight ounces of gunpowder pulverised and sifted, one pound of saltpetre, eight ounces of pounded and sifted sulphur, and two ounces of charcoal.

When the composition has been prepared according to these proportions, and the rocket has been filled in the manner above described, apply a saucisson to the end of it; and having covered the rocket with wax, black pitch, rosin, or any other substance capable of preventing the paper from being spoilt in the water, attach to it a small rod of white willow, about two feet in length, that the rocket may conveniently float.

If it be required that these rockets should plunge down, and again rise up; a certain quantity of pulverised gunpowder, without any mixture, must be introduced into them, at certain distances, such for example, as two, three, or four lines, according to the size of the cartridge.

Remarks.—I. Small rockets of this kind may be made, without changing the mould or composition, in several different ways, which, for the sake of brevity, we are obliged to omit. Such of our readers as are desirous of further information on this

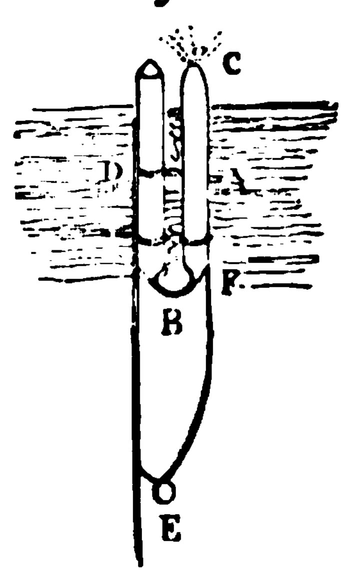
subject, may consult those authors who have written expressly on pyrotechny, some of whom we shall mention at the end of the 12th section.

II. It is possible also to make a rocket which, after it has burnt some time on the water, shall throw out sparks and stars; and these after they catch fire shall ascend into the air. This may be done by dividing the rocket into two parts, by means of a round piece of wood, having a hole in the middle. The upper part must be filled with the usual composition of rockets, and the lower with stars, which must be mixed with grained and pulverised gunpowder, &c.

III. A rocket which takes fire in the water, and, after burning there half the time of its duration, mounts into the air with great velocity, may be constructed in the following manner.

Take a flying rocket, furnished with its rod, and by means of a little glue attach it to a water rocket, but only at the middle A (Fig. 8.) in such a mannner that the latter shall have its neck uppermost, and the other its neck downward. Adjust to their extremity B a small tube, to commuicate the fire from the one to the other, and cover both with a coating of pitch, wax, &c., that they may not be damaged by the water.

Fig. 8.



Then attach to the flying rocket, after it has been thus cemented to the aquatic one, a rod of the kind described in the 2d article, as seen in the figure at D; and from F suspend a piece of packthread, to support a musket bullet E, made fast to the rod by means of a needle or bit of iron wire. When these arrangements have been made, set fire to the part C after the rocket is in the water ; and when the composition is consumed to B, the fire will be communicated through the small tube to the other rocket: the latter will then rise and leave the other, which will not be able to follow it on account of the weight adhering to it.

IV.—*By means of rockets, to represent several figures in the air.*

If several small rockets be placed upon a large one, their rods being fixed around the large cartridge, which is usually attached to the head of the rocket, to contain what it is destined to carry up into the air; and if these small rockets be set on fire while the large one is ascending, they will represent, in a very agreeable manner, a tree, the trunk of which will be the large rocket, and the branches the small ones.

If these small rockets take fire when the large one is half burned in the air, they will represent a comet; and when the large one is entirely inverted, so that its head begins to point downwards, in order to fall, they will represent a kind of fiery fountain.

If the barrels of several quills, filled with the composition of flying rockets, as above described, be placed on a large rocket; when these quills catch fire, they will represent, to an eye placed below them, a beautiful shower of fire, or of half frizzled hair if the eye be placed on one side.

If several serpents be attached to the rocket with a piece of pack-thread, by the ends that do not catch fire; and if the pack-thread be suffered to hang down two or three inches, between every two, this arrangement will produce a variety of agreeable and amusing figures.

V.—*A rocket which ascends in the form of a screw.*

A straight rod, as experience shews, makes a rocket ascend perpendicularly, and in a straight line: it may be compared to the rudder of a ship, or the tail of a bird, the effect of which is to make the vessel or bird turn downwards that side to which it is inclined: if a bent rod therefore be attached to a rocket, its first effect will be to make

the rocket incline towards that side to which it is bent; but its centre of gravity bringing it afterwards into a vertical situation, the result of these two opposite efforts will be that the rocket will ascend in a zig-zag or spiral form. In this case indeed, as it displaces a greater volume of air, and describes a longer line, it will not ascend so high, as if it had been impelled in a straight direction; but, on account of the singularity of this motion, it will produce an agreeable effect.

ARTICLE VIII.

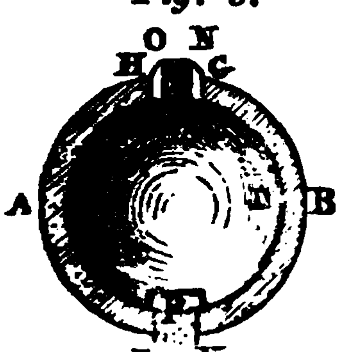
Of Globes and Fire Balls.

We have hitherto spoken only of rockets, and the different kinds of works which can be constructed by their means. But there are a great many other fireworks, the most remarkable of which we shall here describe. Among these are globes and fire balls; some of which are intended to produce their effect in water; others by rolling or leaping on the ground: and some, which are called *bombs*, do the same in the air.

I.—*Globes which burn on the water.*

These globes, or fire balls, are made in three different forms; spherical, spheroidal, or cylindrical; but we shall here confine ourselves to the spherical.

Fig. 9.



To make a spherical fire ball, construct a hollow wooden globe of any size at pleasure, and very round both within and without, so that its thickness A C or B D (Fig. 9.), may be equal to about the ninth part of the diameter A B. Insert in the upper part of it a right concave cylinder E F G H, the breadth of which E F may be equal to the fifth part of the diameter A B; and having an aperture, L M or O N, equal to the thickness A C or B D, that is, to the ninth part of the diameter A B. It is through this aperture that fire is communicated to the globe, when it has been filled with the proper composition, through the lower aperture I K. A petard of metal, loaded with good powder, is to be introduced also through the lower aperture, and to be placed horizontally as seen in the figure.

When this is done, close up the aperture I K, which is nearly equal to the thickness E F or G H, of the cylinder E F G H, by means of a wooden tompion dipped in warm pitch; and melt over it such a quantity of lead that its weight may cause the globe to sink in water, till nothing remain above it but the part G H; which will be the case if the weight of the lead, with that of the globe and the composition, be equal the weight of an equal volume of water. If the globe be then placed in the water, the lead by its gravity will make the aperture I K tend directly downwards, and keep in a perpendicular direction the cylinder E F G H, to which fire must have been previously applied.

To ascertain whether the lead, which has been added to the globe, renders its weight equal to that of an equal volume of water, rub the globe over with pitch or grease, and make a trial, by placing it in the water.

The composition with which the globe must be loaded, is as follows: to a pound of grained powder, and 32 pounds of saltpetre reduced to fine flour, 8 pounds of sulphur, 1 ounce of scrapings of ivory, and 8 pounds of saw-dust previously boiled in a solution of saltpetre, and dried in the shade or in the sun.

Or, to 2 pounds of bruised gunpowder, add 12 pounds of saltpetre, 6 pounds of sulphur, 4 pounds of iron filings, and 1 pound of Greek pitch.

It is not necessary that this composition should be beaten so fine as that intended for rockets: it requires neither to be pulverised nor sifted; it is sufficient if it be well mixed and incorporated. But to prevent it from becoming too dry, it will be proper to besprinkle it with a little oil, or any other liquid susceptible of inflammation.

II.—*Of Globes which leap or roll on the ground.*

Fig. 10.

I. Having constructed a wooden globe A, (Fig. 10.) with a cylinder C, similar to that above described, and having loaded it with the same composition, introduce into it four petards, or even more, loaded with good grained gunpowder to their orifices, as A B; which must be well stopped with paper or tow. If a globe, prepared in this manner, be fired by means of a match at C, it will leap about, as it burns, on a smooth horizontal plane, according as the petards are set on fire.

Instead of placing these petards in the inside, they may be affixed to the exterior surface of the globe; which they will make to roll and leap as they catch fire. They may be applied in any manner to the surface of the globe, as seen in the figure.

II. A similar globe may be made to roll about on a horizontal plane, with a very rapid motion. Construct two equal hemispheres of pasteboard, and adjust in one of them, as A B, (Fig. 11.), three common rockets C, D, E, filled and pierced like flying rockets which have no petard: these rockets must not exceed the interior breadth of the hemisphere, and ought to be arranged in such a manner, that the head of the one shall correspond to the tail of the other.

Fig. 11.

The rockets being thus arranged, join the two hemispheres, by cementing them together with strong paper, in such a manner, that they shall not separate, while the globe is moving and turning, at the same time that the rockets produce their effect. To set fire to the first, make a hole in the globe opposite to the tail of it, and introduce into it a match. This match will communicate fire to the first rocket; which, when consumed, will set fire to the second by means of another match, and so on to the rest; so that the globe, if placed on a smooth horizontal plane, will be kept in continual motion.

It is here to be observed, that a few more holes must be made in the globe, otherwise it will burst.

The two hemispheres of pasteboard may be prepared in the following manner: construct a very round globe of solid wood, and cover it with melted wax; then cement over it several bands of coarse paper, about two inches in breadth, giving it several coats of this kind, to the thickness of about two lines. Or, what will be still easier and better, having dissolved, in glue water, some of the pulp employed by the paper makers, cover with it the surface of the globe; then dry it gradually at a slow fire, and cut it through in the middle; by which means you will have two strong hemispheres. The wooden globe may be easily separated from the pasteboard by means of heat; for if the whole be applied to a strong fire the wax will dissolve, so that the globe may be drawn out. Instead of melted wax, soap may be employed.

III.—*Of Aërial Globes, called Bombs.*

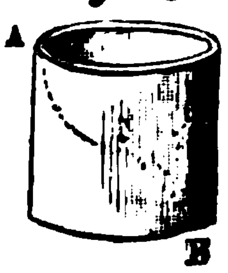
These globes are called aërials, because they are thrown into the air from a mortar, which is a short thick piece of artillery of a large calibre.

Though these globes are of wood, and have a suitable thickness, namely, equal to the twelfth part of their diameters, if too much powder be put into the mortar, they will not be able to resist its force; the charge of powder therefore must be proportioned to the globe to be ejected. The usual quantity is an ounce of powder for a globe of four pounds weight; two ounces for one of eight, and so on.

As the chamber of the mortar may be too large to contain the exact quantity of powder sufficient for the fire ball, which ought to be placed immediately above the

powder, in order that it may be expelled and set on fire at the same time, another mortar may be constructed of wood, or of pasteboard with a wooden bottom, as A B, (Fig. 12.) It ought to be put into a large iron mortar, and to be loaded with a quantity of powder proportioned to the weight of the globe.

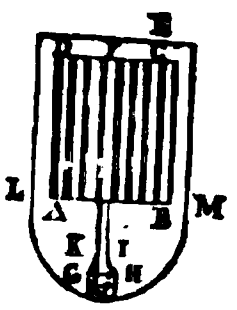
Fig. 12.



This small mortar must be of light wood, or of paper pasted together, and rolled up in the form of a cylinder, or truncated cone, the bottom excepted; which, as already said, must be of wood. The chamber for the powder A C must be pierced obliquely, with a small gimblet, as seen at B C; so that the aperture B, corresponding to the aperture of the metal mortar, the fire applied to the latter may be communicated to the powder which is at the bottom of the chamber A C, immediately below the globe. By these means the globe will catch fire, and make an agreeable noise as it rises into the air; but it would not succeed so well, if any vacuity were left between the powder and the globe.

A profile or perpendicular section of such a globe is represented by the right-angled parallelogram A B C D, (Fig. 13.), the breadth of which A B is nearly equal to the height A D. The thickness of the wood, towards the two sides, L, M, is equal, as above said, to the twelfth part of the diameter of the globe; and the thickness, E F, of the cover, is double the preceding, or equal to a sixth part of the diameter. The height G K or H I of the chamber, G H I K, where the match is applied, and which is terminated by the semicircle L G H M, is equal to the fourth part of the breadth A B; and its breadth G H is equal to the sixth part of A B.

Fig. 13.



We must here observe that it is dangerous to put wooden covers, such as E F, on aërial balloons or globes: for these covers may be so heavy, as to wound those on whom they happen to fall. It will be sufficient to place turf or hay above the globe, in order that the powder may experience some resistance.

The globe must be filled with several pieces of cane or common reed, equal in length to the interior height of the globe, and charged with a slow composition, made of three ounces of pounded gunpowder, an ounce of sulphur moistened with a small quantity of petroleum oil, and two ounces of charcoal: and in order that these reeds or canes may catch fire sooner, and with more facility, they must be charged at the lower ends, which rest on the bottom of the globe, with pulverised gunpowder moistened in the same manner with petroleum oil, or well besprinkled with brandy, and then dried.

The bottom of the globe ought to be covered with a little gunpowder half pulverised and half grained; which, when set on fire, by means of a match applied to the end of the chamber G H, will set fire to the lower part of the reed. But care must have been taken to fill the chamber with a composition similar to that in the reeds, or with another slow composition made of eight ounces of gunpowder, four ounces of saltpetre, two ounces of sulphur, and one ounce of charcoal; the whole must be well pounded and mixed.

Instead of reeds, the globe may be charged with running rockets, or paper petards, and a quantity of fiery stars or sparks mixed with pulverised gunpowder, placed without any order above these petards, which must be choaked at unequal heights, that they may perform their effect at different times.

These globes may be constructed in various other ways, which it would be tedious here to enumerate. We shall only observe that, when loaded, they must be well covered at the top; they must be wrapped up in a piece of cloth dipped in glue, and

a piece of woollen cloth must be tied round them, so as to cover the hole which contains the match.

ARTICLE IX.

Jets of Fire.

Jets of fire are a kind of fixed rockets, the effect of which is to throw up into the air jets of fire, similar to jets of water. They serve also to represent cascades; for if a series of such rockets be placed horizontally on the same line, it may be easily seen that the fire they emit will resemble a sheet of water. When arranged in a circular form, like the radii of a circle, they form what is called a *fixed sun*.

To form jets of this kind, the cartridge for brilliant fires must, in thickness, be equal to a fourth part of the diameter; and for Chinese fire, only to a sixth part.

The cartridge is loaded on a nipple, having a point equal in length to the same diameter, and in thickness to a fourth part of it; but as it generally happens that the mouth of the jet becomes larger than is necessary for the effect of the fire, you must begin to charge the cartridge, as the Chinese do, by filling it to a height equal to a fourth part of the diameter with clay, which must be rammed down as if it were gunpowder. By these means the jet will ascend much higher. When the charge is completed with the composition you have made choice of, the cartridge must be closed with a tompion of wood, above which it must be choaked.

The train or match must be of the same composition as that employed for loading; otherwise the dilatation of the air contained in the hole made by the piercer, would cause the jet to burst.

Clayed rockets may be pierced with two holes near the neck, in order to have three jets in the same plane.

If a kind of top, pierced with a number of holes, be added to them, they will imitate a bubbling fountain.

Jets intended for representing sheets of fire ought not to be choaked. They must be placed in a horizontal position, or inclined a little downwards.

It appears to us that they might be choaked so as to form a kind of slit, and be pierced in the same manner; which would contribute to extend the sheet of fire still farther. A kind of long narrow mouths might even be provided for this particular purpose.

PRINCIPAL COMPOSITIONS FOR JETS OF FIRE.

1st. *For Jets of 5 lines or less, of interior diameter.*

Chinese fire.—Saltpetre 1 pound, pulverised gunpowder 1 pound, sulphur 8 ounces, charcoal 2 ounces.

White fire.—Saltpetre 1 pound, pulverised gunpowder 8 ounces, sulphur 3 ounces, charcoal 2 ounces, iron sand of the first order 8 ounces.

2d. *For Jets of from* 10 *to* 12 *lines in diameter.*

Brilliant fire.—Pulverised gunpowder 1 pound, iron-filings of a mean size, 5 ounces.

White fire.—Saltpetre 1 pound, pulverised gunpowder 1 pound, sulphur 8 ounces, charcoal 2 ounces.

Chinese fire.—Saltpetre 1 pound 4 ounces, sulphur 5 ounces, charcoal 5 ounces, sand of the third order 12 ounces.

3d. *For Jets of* 15 *or* 18 *lines in diameter.*

Chinese fire.—Saltpetre 1 pound 4 ounces, sulphur 7 ounces, charcoal 5 ounces, of the six different kinds of sand mixed 12 ounces.

Father d'Incarville, in his memoirs on this subject, gives various other proportions for the composition of these jets; but we must confine ourselves to what has been here said, and refer the reader to the author's memoirs, which will be found in the "Manual de l'Artificier."

The saltpetre, pulverised gunpowder, and charcoal, are three times sifted through a hair sieve. The iron sand is besprinkled with sulphur, after being moistened with a little brandy, that the sulphur may adhere to it; and they are then mixed together: the sulphured sand is then spread over the first mixture, and the whole is mixed with a ladle only; for if a sieve were employed, it would separate the sand from the other materials. When sand larger than that of the second order is used, the composition is moistened with brandy, so that it forms itself into balls, and the jets are then loaded: if there were too much moisture, the sand would not perform its effect.

ARTICLE X.

Of Fires of different Colours.

It is much to be wished that, for the sake of variety, different colours could be given to these fire-works at pleasure; but though we are acquainted with several materials which communicate to flame various colours, it has hitherto been possible to introduce only a very few colours into that of inflamed gunpowder.

To make white fire, the gunpowder must be mixed with iron or rather steel-filings.

To make red fire, iron sand of the first order must be employed in the same manner.

As copper filings, when thrown into a flame, render it green, it might be concluded, that if mixed with gunpowder, it would produce a green flame; but this experiment does not succeed. It is supposed that the flame is too ardent, and consumes the inflammable part of the copper too soon. But it is probable that a sufficient number of trials have not yet been made; for is it not possible to lessen the force of gunpowder in a considerable degree, by increasing the dose of the charcoal?

However, the following are a few of those materials which, in books on Pyrotechny, are said to possess the property of communicating various colours to fire-works.

Camphor mixed with the composition, makes the flame to appear of a pale white colour.

Raspings of ivory give a clear flame of a silver colour, inclining a little to that of lead; or rather a white dazzling flame.

Greek pitch produces a reddish flame, of a bronze colour.

Black pitch, a dusky flame, like a thick smoke, which obscures the atmosphere.

Sulphur, mixed in a moderate quantity, makes the flame appear bluish.

Sal ammoniac and verdigrise give a greenish flame.

Raspings of yellow amber communicate to the flame a lemon colour.

Crude antimony gives a russet colour.

Borax ought to produce a blue flame; for spirit of wine, in which sedative salt, one of the component parts of borax, is dissolved by the means of heat, burns with a beautiful green flame.

Much, however, still remains to be done in regard to this subject; but it would add to the beauty of artificial fireworks, if they could be varied by giving them different colours: this would be creating for the eyes a new pleasure.

ARTICLE XI.

Composition of a Paste proper for representing animals and other devices in fire.

It is to the Chinese also that we are indebted for this method of representing

figures with fire. For this purpose, take sulphur reduced to an impalpable powder, and having formed it into a paste with starch, cover with it the figure you are desirous of representing on fire: it is here to be observed, that the figure must first be coated over with clay, to prevent it from being burnt.

When the figure has been covered with this paste, besprinkle it while still moist with pulverised gunpowder; and when the whole is perfectly dry, arrange some small matches on the principal parts of it, that the fire may be speedily communicated to it on all sides.

The same paste may be employed on figures of clay, to form devices and various designs. Thus, for example, festoons, garlands, and other ornaments, the flowers of which might be imitated by fire of different colours, could be formed on the frieze of a piece of architecture covered with plaster. The Chinese imitate grapes exceedingly well, by mixing pounded sulphur with the pulp of the jujube, instead of flour paste.

ARTICLE XII.

Of Suns, both fixed and moveable.

None of the pyrotechnic inventions can be employed with so much success, in artificial fire-works, as suns; of which there are two kinds, fixed and revolving: the method of constructing both is very simple.

For fixed suns cause to be constructed a round piece of wood, into the circumference of which can be screwed twelve or fifteen pieces in the form of radii; and to these radii attach jets of fire, the composition of which has been already described; so that they may appear as radii tending to the same centre, the mouth of the jet being towards the circumference. Apply a match in such a manner, that the fire communicated at the centre may be conveyed, at the same time, to the mouth of each of the jets, by which means, each throwing out its fire, there will be produced the appearance of a radiating sun. We here suppose that the wheel is placed in a position perpendicular to the horizon.

These rockets or jets may be so arranged as to cross each other in an angular manner; in which case, instead of a sun, you will have a star, or a sort of cross resembling that of Malta. Some of these suns are made also with several rows of jets: these are called *glories*.

Revolving suns may be constructed in this manner. Provide a wooden wheel, of any size at pleasure, and brought into perfect equilibrium around its centre, in order that the least effort may make it turn round. Attach to the circumference of it fire-jets placed in the direction of the circumference; they must not be choaked at the bottom, and ought to be arranged in such a manner that the mouth of the one shall be near the bottom of the other, so that when the fire of the one is ended, it may immediately proceed to another. It may be easily perceived, that when fire is applied to one of these jets, the recoil of the rocket will make the wheel turn round, unless it be too large and ponderous: for this reason, when these suns are of a considerable size, that is when they consist for example of 20 rockets, fire must be communicated at the same time to the first, the sixth, the eleventh, and the sixteenth; from which it will proceed to the second, the seventh, the twelfth, the seventeenth and so on. These four rockets will make the wheel turn round with rapidity.

If two similar suns be placed one behind the other, and made to turn in a contrary direction, they will produce a very pretty effect of cross-fire.

Three or four suns, with horizontal axes passing through them, might be implanted in a vertical axis, moveable in the middle of a table. These suns, revolving around the table, will seem to pursue each other. It may be easily perceived that, to make them turn around the table, they must be fixed on their axes, and these axes, at the place where they rest on the table, ought to be furnished with a very moveable roller.

We shall say nothing farther on artificial fireworks; because it is not possible in this work to give a complete treatise of Pyrotechny. We shall therefore content ourselves with pointing out, to those who are fond of this art, a few of the best authors on the subject. One is, "Traité des Feux d'Artifice de M. Frezier," a new edition of which was published in 1745. We shall mention also the work of M. Perrinet d'Orval, entitled "Traité des Feux d'Artifice, pour le Spectacle et pour la Guerre." To these we may add "Le Manuel de l'Artificier," Paris 1757, 12mo. which contains, in a very small compass, the whole substance of the art of making artificial fireworks: it is an abridgment of the latter work, augmented with several new and curious compositions, in regard to the Chinese fire, by Father d'Incarville.

ARTICLE XIII.

Of Ointment for Burns.

It is proper that we should terminate a treatise on pyrotechny by some remedy for burns; as accidents must often take place in handling such a dangerous element as fire. We shall therefore not hesitate to follow the example of Ozanam, who in this respect is himself a follower of Siemienowitcz, and the greater part of those who have written on this subject: we shall even confine ourselves to the remedy he proposes.

Boil fresh hog's lard in common water, over a slow fire; skim it continually till no more scum is left, and let the melted lard remain in the open air for three or four nights. Melt it again in an earthen vessel, over a slow and moderate fire, and strain it into cold water through a piece of linen cloth; then wash it well in pure river or spring water, to free it from its salt, and to make it become white; then press it into a glazed earthern vessel and preserve it for use.

It generally happens, in cases of burning, that the skin rises in blisters, which however must not be opened till the third or fourth day after the ointment has been applied.

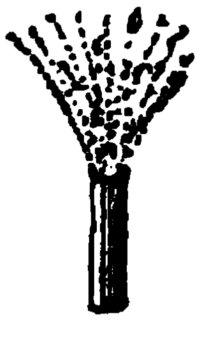
ARTICLE XIV.

Pyrotechny without fire, and merely Optical.

As the inventions which we have here described, are necessarily attended with considerable expense, and are besides dangerous, attempts have been made in modern times, and with a considerable degree of success, to imitate the different kinds of fire-works by optical effects, and to give them the appearance of motion, though in reality fixed. By means of this invention, the spectacle of artificial fire-works may be exhibited at a very small expense, and if the pieces employed are constructed with ingenuity, if the rules of perspective are properly observed, and if, in viewing the spectacle, glasses which magnify the objects and render them somewhat less distinct be employed, a very agreeable illusion will be produced.

The artificial fire-works imitated with most success by this invention, are fixed suns, gerbes and jets of fire, cascades, globes, pyramids and columns moveable around their axes. To represent a gerbe of fire, take paper blackened on both sides, and very opake, and having delineated on a piece of white paper the figure of a gerbe of fire, apply it to the black paper, and with a point of a very sharp penknife make several slashes (Fig. 14.) in it, as 3, 5 or 7, proceeding from the origin of the gerbe: these lines must not be continued but cut through at unequal intervals. Pierce these intervals with unequal holes made with a pinking iron, (Fig. 14.), in order to represent the sparks of such a gerbe. In short you must endeavour to paint, by these lines and holes, the well known effect of the fire of

Fig. 14.



inflamed gunpowder, when it issues through a small aperture.

Fig. 15.

According to the same principles, you may delineate the cascades (Fig. 15) and jets of fire which you are desirous of introducing into this exhibition, which is purely optical; and those jets of fire which proceed from the radii of suns, either fixed or moveable. It may easily be conceived that in this operation taste must be the guide.

If you are desirous of representing globes, pyramids, or revolving columns, draw the outlines of them on paper, and then cut them out in a helical form; that is, cut out spirals with the point of a penknife, and of a size proportioned to that of the piece.

Fig. 16.

It is to be observed also, that as these different pieces have different colours, they may be easily imitated by pasting on the back of the paper, cut as here described, very fine silk paper coloured in the proper manner. As jets, for example, when loaded with Chinese fire, give a reddish light, you must paste to the back of these jets transparent paper, slightly tinged with red; and proceed in the same manner in regard to the other colours by which the different fire-works are distinguished.

When these preparations have been made, the next thing is to give motion, or the appearance of motion, to this fire, which may be done two ways according to circumstances.

If a jet of fire, for example, is to be represented, prick unequal holes, and at unequal distances from each other, in a band of paper, (Fig. 17.), and then move this band, making it ascend between a light and the above jet: the rays of light which escape through the holes of the moveable paper will exhibit the appearance of sparks rising into the air. It is to be observed that one part of the paper must be whole, that another must be pierced with holes thinly scattered; that in another place they must be very close, and then moderately so: by these means it will represent those sudden jets of fire observed in fire-works.

Fig. 17.

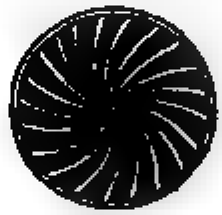
To represent a cascade, the paper pierced with holes, instead of moving upwards, must be made to descend.

This motion may be easily produced by means of two rollers, on one of which the paper is rolled up while it is unrolled from the other.

Suns are attended with some more difficulty; because in these it is necessary to represent fire proceeding from the centre to the circumference. The artifice for this purpose is as follows.

On strong paper describe a circle, equal in diameter to the sun which you are desirous to exhibit, or even somewhat larger; then trace out on this circle two spirals, at the distance of a line or half a line from each other, and open the interval between them with a penknife, in such a manner, that the paper may be cut from the circumference, decreasing in breadth to a certain distance from the centre, (Fig. 18.); cut the remainder of the circle into spirals of the same kind, open and close alternately, then cement the paper circle to a small iron hoop, supported by two pieces of iron, crossing each other in its centre, and adjust the whole to a small machine, which will suffer it to revolve round its centre. If this moveable paper circle, cut in this manner, be placed before the representation of your sun, with a light behind it, as soon as it is made to move towards that side to which the convexity of the spirals is

Fig. 18.



turned, the luminous spirals, or those which afford a passage to the light, will give, on the image of the radii or jets of fire of your sun, the appearance of fire in continual motion, as if undulating from the centre to the circumference.

The appearance of motion may be given to columns, pyramids, and globes, cut through in the manner above described, by moving upwards, in a vertical direction, a band of paper cut through into apertures inclined at an angle somewhat different from that of the spirals. By these means the spectators will imagine that they see fire continually circulating and ascending along these spirals; and the result will be a sort of illusion, in consequence of which the columns or pyramids will seem to revolve with them.

But we shall not enlarge farther on this subject; it is sufficient to have explained the principle on which this cheap kind of pyrotechny can be exhibited; the taste of the artist may suggest to him many things to give more reality to this representation, and to render the deception stronger.

We shall however add a few words respecting illuminations which form a part of pyrotechny.

Take some prints respecting a castle, or palace, &c.; and having coloured them properly, cement paper to the back of them, in such a manner that they shall be only semi-transparent; then, with pinking irons of different sizes, prick small holes in the places and on the lines where lamps are generally placed, as along the sides of the windows, on the cornices, or balustrades, &c. But care must be taken to make these holes smaller and closer, according to the perspective diminution of the figure. With other irons of a larger size, cut out, in other places, some stronger lights; so as to represent fire-pots, &c. Cut out also the panes in some of the windows, and cement to the back of them transparent paper of a green or red colour, to represent curtains drawn before them, and concealing an illuminated apartment.

When the print is cut in this manner, place it in the front of a sort of small theatre, strongly illuminated from the back part, and look at it through a convex glass of a pretty long focus, like that used in those small machines called optical boxes If the rules of perspective have been properly observed in the prints, and if the lights and shades have been distributed with taste, this spectacle will be highly agreeable. It may be intermixed with some of the pyrotechnic artifices above described; as such illuminations are in general accompanied with fire-works.

PART ELEVENTH.

CONTAINING EVERY THING MOST CURIOUS IN PHILOSOPHY IN GENERAL, AND IN ITS VARIOUS BRANCHES.

Having gone through the different parts of the Mathematics, and of the sciences or arts comprehended under that head, we now enter the field of Philosophy, which presents as many objects worthy of exciting our curiosity as the mathematics; or, rather, which is indeed still more fertile in that respect, and affording matter still better adapted to the comprehension of the generality of readers. This matter is even so abundant, that we can scarcely establish divisions in it; so that this part of our work will be a kind of a miscellany, without much order, of every thing that belongs to philosophy in general. We shall successively review in it the principal properties of bodies; the inventions, whether useful or amusing, to which these properties have given birth; various questions relating to the system of the world, to meteors, and the origin of springs, with other objects, which it would be too tedious to enumerate. But before we enter this vast field, it is necessary that we should establish some general principles, which we shall do in the following account of what philosophers have called the four elements, viz., fire, air, water, and earth.

PRELIMINARY DISCOURSE,

On the Elements of Bodies.

In analyzing any material, when we have arrived at its last component parts, and cannot decompose them farther, we ought to regard them as its elements. Now every one knows that all or most bodies, submitted to analysis, furnish a fixed matter; also something that is inflammable; an invisible fluid, which manifests itself only by its expansibility and its resistance; and lastly, another which heat raises into vapours, and which afterward re-unites under a visible form. These four component parts have been named *earth*, *fire*, *air*, and *water*. These enter into the composition of most bodies; but cannot themselves be decomposed. They ought therefore to be considered as the elements of all other bodies; which justifies the common denomination, which has been established almost from the first dawn of philosophy, according to which there are in nature the four elements, fire, air, water, and earth.

I.—*Of Fire, both elementary and material.*

What is Fire? This is perhaps one of the most obscure questions in philosophy, and the least susceptible of a satisfactory answer. The most probable account, however, which its known properties enable us to give, is the following.

Fire is a fluid universally diffused throughout nature; which penetrates all bodies with more or less facility; is susceptible of being accumulated in some of them, and this accumulation produces, in regard to us, that sensation which we call heat. When this accumulation is carried to a higher degree, it produces inflammation and combustion, which are always accompanied with light. In every state, this fluid

dilates bodies in proportion to the greater or less quantity of it present; and it at length separates their parts, which we call fusing, burning, calcining.

That fire is a fluid, there can be no reason to doubt: for if it were not, how could it be diffused throughout the atmosphere, and throughout water, without forming an obstacle to the motion of bodies? How could it penetrate the densest and most compact bodies, such for example as metals?

Nay, fire is not only a fluid, but it is even the principle of all fluidity: without its influence, all the fluids with which we are acquainted, would be reduced to masses absolutely solid. Metals become fixed at a degree of heat far superior to that of boiling water. Water loses its fluidity as soon as the heat or quantity of fire has been diminished to a certain degree. Spirit of wine, and even mercury, are congealed by the progressive diminution of heat. There is a degree of cold, or privation of heat, perhaps, which would convert air into a fluid-like water, and even into a solid body; but we are at a prodigious distance from that term.

Fire penetrates all bodies with more or less facility. This follows from the communication of heat from a hot to a cold body. It is with greater or less, a moderate facility, and not with extreme facility, that heat is communicated: for it is well known that this communication is not instantaneous: if the point of a pretty long needle be presented to the flame of a taper, both its ends do not become equally hot at the same time. One body receives this heat more readily than another; or, as we may say, has a greater affinity for heat.

The accumulation of the igneous fluid produces on our bodies that sensation which we call heat. This requires no proof; but the sensation is only relative. As long as the palm of the hand, for instance, is hotter than the body with which it is in contact, the latter seems to us cold; but it will, on the contrary, seem warm, if the hand be colder, or contains less of the igneous fluid; or if that fluid tends to pass gradually, as it does, from that body into the hand. Every person almost is acquainted with the following experiment: heat one hand in a very high degree, and cool the other almost to the temperature of ice; if you then immerse both of them into tepid water, the one will experience a sensation of cold, and the other of heat.

This accumulation, carried to a considerable degree, produces inflammation, always accompanied with light. It results from some experiments made by Buffon, that iron exposed, without being in contact, to the action of another body in a state of inflammation, becomes itself inflamed and red. But an ignited body is nothing else than a body in which the ingneous fluid is accumulated to such a degree as to become luminous. All light indeed is not accompanied with heat; but all heat, carried to a certain degree, produces light.

Has fire weight? It appears to us that there can be no doubt that fire is heavy: it is matter, since it acts upon matter; and therefore it must possess weight. But the question is, to know whether this weight is perceptible, and can be indicated by the instruments which we employ. S'Gravesande and Muschenbroeck made some experiments on this subject; but they found no difference between masses of ignited iron, or iron penetrated with fire, and the same masses when cold. They however concluded from them, that as ignited iron, which by its increasing in volume ought to weigh somewhat less in air, weighed the same in that state as when cold, this equality must have arisen from the addition of the weight of the fire present in it. But their experiments were not made with the necessary degree of care.

Buffon, who, by means of the forges belonging to him, was enabled to make a much greater number of experiments, and on a larger scale, always found that pieces of forged iron, brought to a state of ignition, weighed a little more than when cold; and he fixed the diminution at a 600th part of the weight of the ignited body. But it must be allowed, and Buffon was sensible of it himself, that this experiment could not be decisive: for he has shewn, that iron kept for some time at a red heat, con-

tinually loses a part of its weight, because it gradually burns: on this account he made further experiments on a substance very common in furnaces, namely slag. He first assured himself that slag retains its weight, or loses only an insensible portion of it, in consequence of being heated and cooled again. He then weighed some of this slag cold, by a very delicate balance; he next brought it to a white heat, and then weighed it a second and a third time after it had cooled. Five experiments of this kind always gave an excess of weight in the ignited slag, above that which it had when cold, both before and after. This difference amounted to a 580th or a 600th part of that of the piece of slag.

But it may be said, if this be the case, fire is heavier than air; for the specific gravity of slag is to that of water, as 2½ to 1; therefore this gravity is to that of air as 2125 to 1. But the fire contained in a piece of ignited slag, is about a 600th part of its weight; consequently it is to the weight of an equal volume of air, as 3½ to 1, which is not credible. So great is the tenuity of fire, that we can hardly allow ourselves to think that its specific gravity approaches even near to that of air.

But it must be observed, that in an ignited mass brought to a white heat, a large quantity of fire is accumulated the weight therefore of fire, in its ordinary state, and in bodies heated to the mean temperature of our atmosphere, may be utterly insensible; but when five or six hundred times, or more than that quantity, has been accumulated, and to such a degree as to produce ignition, its gravity may then become sensible. Let us suppose, for example, that the fire diffused throughout air, heated to 1 degree of the thermometer, weighs only the 300th part of the weight of that air; when five or six hundred parts more have been introduced into it, to produce ignition, its weight may then equal, and even surpass, the weight of such air as we breathe. We do not know whether this would have been Buffon's answer; but such, in our opinion, is that which might be given.

Those persons however are mistaken, who consider the increase of weight which metals acquire by calcination, as a proof of the heaviness of fire, which by this operation they suppose to become fixed, and in some measure *solidified* along with the metallic calces. It is now known that fire has no part in this augmentation of weight.

Fire dilates bodies: by dilating them, it separates their constituent parts, and at length liquefies them. This phenomenon, in regard to the effect, is well known. That fire dilates bodies, is well known, as will be shewn hereafter by means of a very ingenious machine, which serves to determine the degree and ratio of this dilatation. But it cannot produce this effect without separating the constituent parts of these bodies; and this is the mechanism by which it is afterwards able to liquefy, and even to volatilize them; for the solidity of a body is the effect of the mutual adhesion of its constituent parts; an adhesion which, in all probability, is produced by the contact of these moleculæ in large surfaces. But when fire, introduced between them, produces a separation, and causes them scarcely to touch each other, the bod has then attained to such an extreme degree of fluidity, as to be volatilized.

These particles, having no longer any adhesion, can be carried away by the least effort, such as that of heat, which continually exercises an action to extend itself in every direction.

There are some bodies, however, which fire at first tends to contract: but this is because they contain principles which the fire dissipates: of this kind is clay, which at first shrinks in the fire; but, if exposed to a greater degree of heat, it dilates, liquefies, and is changed into glass.

II.—*Of Air.*

Air is an elastic, heavy fluid, susceptible of compression; which expands by heat,

and contracts by cold. It is necessary for maintaining life to all the animals with which we are acquainted; it becomes charged with, and combines with, water, as water combines with it. Such are the principal properties of air, of which we shall here give a general view, and which we shall prove hereafter by some curious experiments.

Air is a heavy fluid. To discover this property in air, and to prove its existence, requires only a slight knowledge of philosophy. It may be demonstrated by a very simple experiment. Take a glass globe, 6 inches in diameter, furnished with a tube that can be opened and shut by a stop-cock; exhaust it of air by means of a pneumatic machine, and then shut it, so as to exclude the external air; weigh the globe thus exhausted of air by a very nice balance; if you then admit the external air, by turning the cock, the equilibrium will be immediately destroyed, and that end of the balance which supports the globe will preponderate. For a globe of the size above mentioned, 45 or 50 grains must be added to the weight, to restore the equilibrium.

Air is an elastic fluid. This may be proved by the following very simple experiment. Introduce air into a bladder, but in such a manner as not entirely to fill it. If the bladder be then carried, in that state, to the summit of a mountain, it will be more and more distended; and by carrying it to the top of a very high mountain, such as the Cordilleras of Peru, it might be made to distend so much as to burst.

The same effect will be produced if the bladder be placed under the receiver of an air-pump. For if the receiver be then exhausted of air, on the first stroke of the piston, the bladder will swell out, even if it contains only an inch of air; and when the external air is suffered to re-enter the receiver, it will resume its former state.

There can be no doubt that this effect is produced by the elastic force of the air; which, when the pressure of the external air is removed, increases in volume; and when the pressure is restored, it assumes its former state. It is like a spring, more or less compressed by a weight, and which extends itself in a greater or less degree, as the weight is heavier or lighter.

Air is a fluid susceptible of compression. This is a consequence of its elasticity. It is proved, by experience, that a double weight compresses it in such a manner as to occupy only one half of its former volume; a quadruple weight reduces it to a fourth part of that volume, and so on. So that it may be said in general that the same mass of air, the temperature remaining the same, occupies a volume which is in the inverse ratio of the compressing weight.

Air expands by heat, and contracts by cold. This property of air may be proved also by a very simple experiment. In an apartment, brought to a mean degree of heat, introduce air into a bladder, but in such a manner as not entirely to fill it. If it be then brought near the fire, so as to be exposed to a degree of heat greater than the mean temperature, we see the bladder distend, and occupy a larger volume. By exposing it to colder air, a contrary effect is produced.

Air is necessary for maintaining animal life. This truth is well known. It may be proved in the most evident manner, by shutting up animals in the receiver of an air-pump: for as soon as you begin to exhaust it of air, the animals shew every sign of uneasiness; they pant for breath, and at length expire, when only a small quantity of air remains. If the air be gradually re-admitted, before they are quite dead, they recover life and motion.

Air becomes charged with water, and combines with it; as water, on the other hand, becomes charged and combines with air. The first part of this proposition is sufficiently proved by facts, with which every one is acquainted. Air is sometimes more and sometimes less humid. Air charged with moisture deposits it in certain bodies,

capable of attracting and absorbing it in a great degree; such as salt of tartar, which becomes so much impregnated with it, that it resolves itself into a liquid, merely by the contact of common air, though it has been dried by a violent heat. It is water, disengaged from the air with which it was combined, that occasions the moisture which deposits itself on stones, marble, &c., and during weather distinguished by the appellation of *damp*. The contact of the air alone gradually diminishes the water contained in any vessel, especially if the air be in motion; because a new portion of air is every moment applied to the surface of the water. It is by this mechanism that those winds which have passed over a large extent of sea, as is the case with our west and south-west winds, become charged with water, and are mostly attended with rain.

Water, in its turn, becomes charged with air. This is proved by a curious experiment, made by Mariotte. Take a certain quantity of water, and having freed it as much as possible from air, put it into a small bottle, leaving no vacuity in it but a space of the size of a pea; at the end of twenty-four hours the water will occupy the whole capacity of the bottle. What can have become of the air, which was in the vacuity, if it has not been absorbed by the water, which was in contact with it?

This property, which air has of combining with water, of even becoming saturated with it, and of afterwards quitting it, is the cause of various physical effects, such as the production of clouds and rain, the rising or falling of the barometer, &c. But these phenomena we shall explain more at length in another place.

III.—*Of Water.*

The principal properties of this common and well-known fluid are as follows: it is transparent, insipid, and inodorous; it always tends to put itself in equilibrio, that is, to assume a form the surface of which is concentric with the earth, a property it possesses in common with all the other heavy and non-elastic fluids; it is incompressible: can be reduced to vapour by heat, carried to a certain degree, and in that state is endowed with a very great elastic force. When exposed to a certain degree of cold, it is transformed into a solid and transparent body: it dissolves salts, and a multitude of other substances; and by these means it becomes the vehicle of the nourishing particles both of animals and vegetables, which renders it so essentially necessary in the animal economy, that it is in some measure more difficult to live without water, or without some fluid of which it forms the basis, than without solid aliment.

Such are the properties of water, of which we shall here give a few proofs, till we come to another part of this work, where we shall have an opportunity of enlarging farther on the same subject.

It is needless to adduce any proof that water is transparent, inodorous, and insipid. When this fluid possesses either taste or smell, it is because it holds in solution some foreign bodies. People ought therefore to be suspicious of water which is said to be agreeable to the taste, as it is certain that it is not pure.

Water always arranges itself in such a form that its surface is concentric with the earth. Every body is acquainted with this property of water, which it possesses in common with all the other non-elastic fluids, and which is the basis of the art of levelling. When two masses of water communicate with each other we may rest assured that their surfaces are level, or at an equal distance from the centre of the earth. Those persons then are mistaken, who believe that the water of the Mediterranean is more or less elevated than that of the Red Sea, at the bottom of the Gulf of Suez, which, as is said, caused the plan for cutting through the isthmus to be abandoned, lest the Mediterranean should run into the Red Sea, or the latter into the former. Nothing can be more absurd, since these two seas have a communication

with each other by the ocean. Had they been originally created on a different level, they would not have failed soon to assume the same level.

Water is incompressible. The members of the Academy *del Cimento*, the first who it appears adopted the true method of philosophizing, namely, by subjecting every thing to the test of experiments, made a very curious one, which proves this property of water. They inclosed a quantity of water in a hollow ball of gold, of a certain considerable thickness, taking care to ascertain that the cavity was completely filled; they then subjected the ball to the blows of a hammer, by which means its capacity was diminished; but the water, instead of suffering itself to be compressed, passed through the pores of the gold, though exceedingly small. This experiment was repeated by Mr. Boyle and by Muschenbroeck, who both attest the truth of it.

Water by a certain degree of heat is reduced into highly elastic vapour. This truth may be proved also by very simple experiments. If a small quantity of water be thrown upon a strong fire, you will immediately see it transformed into vapour.

If water be kept in a state of violent ebullition in a close vessel, there arises from it an elastic vapour of so great force, that unless a vent be opened for it, or if the vessel be not sufficiently strong to resist its action, it will undoubtedly burst: for this reason, in the boiler of the steam-engine there is a valve, which must be opened when the steam has acquired a certain force, otherwise it would be shivered to pieces.

This vapour, according to the calculations made by philosophers, occupies a space 14000 times greater than the water which produced it. Hence arises the prodigious force it acquires, when confined in a much less space.

Water, when exposed to a certain degree of cold, is transformed into a solid transparent body, which we call ice. This fact is so well known, that it is needless to prove it. We shall therefore confine ourselves to an explanation of this singular effect.

It is fully proved, by the formation of ice, that the primitive state of water was that of a solid body. It is a solid fused by a degree of heat far below that which, according to our sensations, we call *temperate;* for it would be a strange error to imagine, that what we call zero of the thermometer, is the absence of all heat. Since spirit of wine, and various other liquors, remain fluid at degrees of cold much greater than that which freezes water, it is evident that the degree called zero, which is marked 0, is merely a relative term, the commencement of the division.

Water then is only a liquefied solid, which keeps itself in a liquid state at a degree of heat very little more than that marked 0, on our common thermometers, and which in that of Fahrenheit is marked 32°. The reason of this we shall explain when we come to speak of thermometers.

Let us now take a short view of water in its solid state. When heated to a certain degree, the matter of fire, with which it is then impregnated, raises up and separates from each other the moleculæ of which it is composed; for as these moleculæ do no longer touch each other by so large surfaces, though still within the limits of adhesion, they easily run one over the other. Thus we have ice brought to a state of fusion, as lead is by a heat of 226 degrees. The matter of fire escapes to diffuse itself in equilibrium in other bodies, which have less, for it is in this manner that cooling is effected; these moleculæ approach each other; they come in contact by the small facets which they reciprocally present, thus adhere and form a solid body. What is here said, in regard to the small facets of the particles of water, seems to be proved by the ramifications of ice, for these ramifications, both in ice and in snow, are always formed under angles of 60 or 120 degrees; which indicates planes uniformly inclined. We shall enlarge further, in another place, on this phenomenon, which depends on crystallization.

It would be ridiculous, at present, to explain the formation of ice by the supposed frigorific particles, the existence of which seems to rest on no foundation. Water freezes at a degree of heat which can no longer keep it in fusion: for the same reason, and by the same mechanism, that lead becomes fixed at a degree of heat less than 226 of Reaumur. But the same philosophers who, to explain the congelation of water, have recourse to the frigorific particles diffused throughout the atmosphere, do not recur to them in the present case: they well know that the fixation of lead arises only from the particles, which the fire does not keep sufficiently separated, approaching each other. Why then should we recur to any thing else in regard to the congelation of water?

It is indeed true, that in the congelation of water there is one phenomenon exceedingly singular, which is, that water decreases in volume in proportion as it cools, at least to a certain degree; but at the moment when ice is formed, this volume increases very sensibly; hence the philosophers above mentioned conclude, that some foreign matters, or their supposed frigorific particles, have been introduced into it. But we shall observe, 1st. That the case is the same with iron. 2d. That this is the effect of crystallization; for we must here repeat, that the congelation of water is merely a crystallization, by which its moleculæ assume an arrangement which is determined by their primitive form. But this arrangement cannot be effected without producing an increase of volume, as happens in regard to iron when it becomes fixed, or loses its fluidity, merely by the diminution of heat, which kept it in fusion. This will become more evident when we have explained the phenomena of crystallization.

Water dissolves salts and a variety of other substances.—Every one knows that all saline bodies, whether acids, alkalis, or neutral salts, are soluble in water, in a greater or less quantity; and a very singular phenomenon in this respect is, that water which holds in solution as much of a certain salt as it can contain, will still dissolve some other salt. But, for the most part, it abandons one of them when it becomes charged with the other, if it has a greater affinity for the latter.

Of the other substances, which water dissolves, we shall mention in particular the gummy or mucilaginous part of animals or vegetables, which forms the nourishment of the former. It is in consequence of this property, that water is so useful to the animal economy; for the nutritive part of aliment must be dissolved and diluted in water, or some other fluid of the same kind, before it is swallowed, or this solution must be effected in the stomach after deglutition. Hence it is that water, in some measure, is the first aliment of man and of animals. It is not an aliment itself, but it is the vehicle of every thing that serves as aliment.

Finally, water is the base of all the other aqueous fluids, such as spirits, oils, &c.; for water may be extracted from all of them by a very simple process, namely distillation. Combustion produces the same effect by disengaging the matter purely aqueous

IV.—*Of Earths.*

Earth is that part of compound bodies, which remains fixed after they are analyzed. When by the action of fire, we have consumed or raised in exhalations the inflammable part, have expelled or driven the air into the atmosphere, have raised the water in vapours, there remains a solid and fixed body, not farther alterable by fire, that is the elementary earth, the different kinds of which it is that commonly constitutes the nature of the mixture.

It must indeed be acknowledged, at least till we arrive at a decomposition beyond that of the fixed body, that this elementary earth is not all of the same kind; contrary to which, it is found that all water, all respirable air, is homogeneous: for where, by calcination, for instance, we have reduced a metal to calx, which is vitrifiable, that calx or earth is not necessarily homogeneous, neither to another metallic calx.

nor to caput mortuum, or to the earth of another body, as the calx of stone, or the earth of any vegetables or animal calces. The proof of this is simple; for metallic calx being revivified by the addition of phlogiston, produces only the same metal which had given the calx; and, by whatever way we proceed, the earth of any other compound will not yield a metal, however we may combine it. This property of metallic calx, is the basis of the art of separating the metals from the earths and stones with which they are mineralized; for as soon as their calces, vitrified by the violence of fire, comes in contact with the carbonic matter, those of metals regain their metallic form, and disengage themselves by their weight from the vitrified calces of the other heterogeneous bodies with which they were confounded.

It has been usual to distinguish earths into calcareous, vitrifiable, and refractory. Calcareous earths are those which, burned in the fire, reduce into a calx. The properties of this calx are well known, the principal characteristic of which is that of attracting and absorbing moisture violently, and of effervescing with water. But it is not necessary to subject them to that test to know them: they are easily distinguished by exposing them to the action of any gentle acid. Calcareous earths dissolve with more or less effervescence; whereas other earths suffer no dissolution.

Vitrifiable earths are those which, exposed to a fire more or less active, suffer a fusion, and become more or less fluid.

The refractory earths, are those on which the most violent heat excited in our furnaces produces no effect or alteration.

We say the most violent heat excited in our furnaces; for perhaps, if all the earths are not found to be vitrifiable, this happens only because we have not been able to produce a sufficient degree of heat. In fact, in proportion as we have succeeded in producing more considerable degrees of heat, we also are able to vitrify materials which had resisted the former degrees of fire. But it is a remarkable circumstance that some earths which separately are unfusible, on being mixed together become fusible and vitrifiable. Thus, for example, calcareous earth, mixed with argil, runs and becomes glass. Usually, metallic matters, mixed either with calcareous earths, or with refractory, as pure argil, communicate to them also fusibility, which these have not separately.

We shall limit ourselves here as to what might be said concerning the elements; what has been now said being the most solid and best proved part of the subject. We shall now pass on successively through all the branches of physics, in selecting what they offer the most curious and interesting. We have already said we shall hardly regard much order in these observations: from the bowels of the earth we shall sometimes suddenly raise ourselves to the upper regions of the atmosphere; from a problem in the celestial physics, we shall pass to a question in mineralogy. We shall treat apart on electricity, on magnetism, and on chemistry, because these branches of philosophy are extremely fertile in curious experiments, and present each of them materials enough for separate treatises.

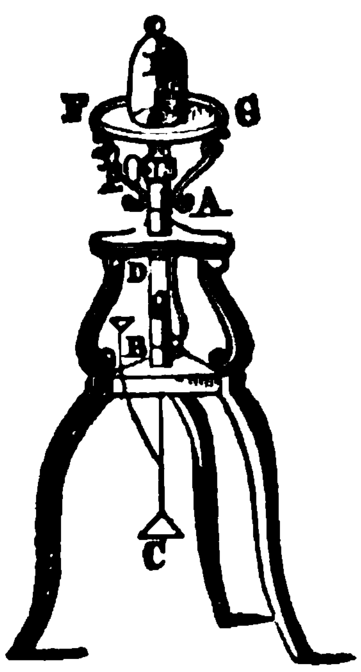
PROBLEM I.

Construction of the Pneumatic Machine, or Air-Pump, with an account of the principal experiments in which it is employed.

Air being an elastic fluid, it may be easily conceived that if it be shut up in a close vessel, and if to this vessel be adapted a pump, made to communicate with it, when the piston is drawn up the air contained in the vessel will enter the body of the pump. If the communication between the vessel and the body of the pump, be then intercepted, and if that between the latter and the external air be opened, by pushing down the piston, the air contained in the body of the pump will be expelled. If the communication between the body of the pump and the external air be then shut, and that between the body of the vessel be opened, when the piston

is drawn up, the air in the vessel will again rush into the body of the pump; and by thus repeating the same operation as before, the whole air contained in the vessel will be evacuated. If the body of the pump be equal, for example, to the capacity of the vessel with which it communicates, the first operation will reduce the air to one half of its density; the second to the half of that half, or to a fourth, and so on in succession: hence a very few strokes of the piston will reduce the air contained in the vessel to a very great degree of tenuity.

Fig. 1.



Such is the mechanism of the air-pump, of which the following is a more minute description. A B (Fig. 1.) is a cylindric pump or barrel, in which the piston D is made to play by means of the branch or handle D C, having at its extremity a stirrup for receiving the foot, by means of which it can be forced downwards. The body of the pump is fitted into a collar, from which proceed three or four branches that form a sort of stand. From the top of the pump A there arises a tube, about an inch in diameter, to the upper part of which is adapted a circular plate, with a small raised border, or rim around it. On this plate is placed the receiver, in the form of a bell. The small tube above mentioned, which serves to establish a communication between the vessel and the body of the pump, generally passes through this plate, and has a screw at the end, in order that the tube of another vessel, such as a bell or small balloon, from which it is required to evacuate the air, may be screwed upon it. Beneath the plate, and between it and the body of the pump, is a stop-cock I, so constructed that, by turning it to one side, a communication is established between the body of the pump and the receiver, while all communication is prevented between it and the exterior air; and by turning it in a contrary direction a contrary effect is produced. Such is the form of the pneumatic machine; at least of certain simple kinds of it, for there are others more complex. One kind, for example, consists of two cylinders, the pistons of which are alternately worked by means of a crank; so that one of them always becomes filled with air from the receiver, while the other throws out into the atmosphere the air it contained. But it is needless to enter into these details: those who are desirous of seeing the newest improvements in regard to air-pumps, may have recourse to the different treatises on natural philosophy, where they will find a description and figures of the different additions made to this machine by mechanics and philosophers, to render the use of it more convenient or more general.

By combining this description with what has been said in regard to the air, it will be easy to conceive in what manner this machine is employed. When a bell-formed receiver is used, a piece of oiled leather, with a hole in the middle of it, to afford a passage to the tube H, is sometimes placed on the plate F G. This wet leather causes the contact of the edges of the receiver to be more exact, than if it rested on metal; for some aperture or cleft would often remain, through which the exterior air would introduce itself. The receiver is then placed upon the plate, with or without the leather, and the cock is turned in such a manner as to open a communication between the body of the pump and the receiver. The piston, which we suppose raised up to the top, is then forced down by pressing the foot on the stirrup, and when it is as low as possible, the cock is turned in such a manner as to intercept the first communication, and establish that between the body of the pump and the exterior air; the piston being then raised, the air in the body of the pump is expelled, and the cock is turned in the contrary direction which shuts the second communication, and opens the former; the piston is then forced down again, and the

same effect takes place. Or the pump is otherwise worked in the manner peculiar to its form and construction. Every stroke of the pump expels a portion of the air originally contained in the receiver, and in a decreasing geometrical progression. Thus, for example, if the body of the pump is equal in capacity to the receiver, the first stroke of the piston will expel one half of the air contained in the receiver; the second will expel the fourth part; the third the eighth part; the fourth the sixteenth part, &c.; so that it may with truth be said that it can never be entirely evacuated; but after fourteen or fifteen strokes of the piston it will be so rarefied, that there will remain only a part infinitely small; for, on the above supposition, the quantity of air remaining after the first stroke of the piston will be $\frac{1}{2}$, after the second $\frac{1}{4}$, after the third $\frac{1}{8}$, and so on; after the fifteenth then it will be only the 32768th part, which in general is equivalent to a perfect vacuum, for experiments such as those that are usually made.

After these observations on the form and use of the pneumatic machine, we shall proceed to a few of the most curious experiments.

Experiment 1.

Place on the plate of the machine a receiver in the form of a bell; if you try to remove it you will experience no resistance; but if you give only one stroke with the piston, it will adhere to the plate with considerable force: after 2, 3, or 4, it will adhere with more force; and after 18 or 20, with the force of several hundred pounds weight. If the base of the receiver be, for example, a circle a foot in diameter, the adhesive force will be about 1617 pounds.

This experiment is a proof of the gravity of the air of the atmosphere; for the air is the only body which, by pressing on the receiver, can produce the adhesion experienced. When the air under the receiver is as dense as the external air, there is no adhesion, the air within and without being then in equilibrio with each other; but when that within is evacuated, either in whole or in part, the equilibrium is destroyed, and the external compresses the receiver against the plate on which it rests, with the excess of the weight it has over the force of the internal air. It will be found that this force is equal to that of a cylinder of water 33 feet in height, and having a base equal to that of the receiver. It was by these means that we found the result of 1617 pounds; for a cylindric foot of water weighs 49 pounds; consequently 33 feet weighs 1617.

Experiment 2.

Place under a receiver an apple, much shrivelled, or a very flaccid bladder, in which there remains but a small quantity of air. If the receiver be then exhausted, you will see the skin of the apple become distended, so that it will assume almost the same form, and have as fresh an appearance, as it had when plucked from the tree. The bladder will, in like manner, swell up, and may even be distended to such a degree as to burst. When the air is re-admitted into the receiver, they will both resume their former contracted state.

We have here an evident proof of the elasticity of the air. While the wrinkled apple or flaccid bladder is immersed in atmospheric air, its weight counteracts the elastic force of the air contained in both; but when the latter is freed from the weight of the former, its elasticity begins to act, and by these means it distends the sides of the vessel which contains it. When the air is re-admitted, the elasticity is counteracted as before, and the apple and bladder resume their former shape.

Experiment 3.

Place under the receiver a small animal, such as a cat, or a mouse, &c. If you then pump out the air, you will immediately see the animal become troubled, swell up, and

at length expire, distended and foaming at the mouth. These phenomena are the effect of the air contained in the animal's body, which being no longer compressed by the external air, exercising its elasticity, it distends the membranes, and throws out the humours which it meets with in its way.

Experiment 4.

If butterflies or common flies be placed under the receiver, you will see them fly about as long as the air contained in the machine is similar to the external air; but as soon as you have given a few strokes with the piston, they will in vain make efforts to rise, as the air has become too much rarefied to support them.

Experiment 5.

Adapt to a flat bottle a small tube, so constructed that it can be screwed upon the end of the tube, which rises above the plate of the machine. On the second or even the first stroke of the piston, you will see the bottle burst; for this reason it ought to be covered with a piece of wire netting, to prevent the fragments of it from doing mischief by flying about.

The same effect is not produced on a receiver, because its spherical form gives it strength, in the same manner as an arch, to resist the pressure of the exterior air.

Experiment 6.

Provide a small machine consisting of a bell and hammer, the latter of which can be put in motion by wheel work, so as to strike the bell and make it sound. Wind up this small machine, and having put it in motion, place it below the receiver and exhaust the air. As the air is exhausted you will hear the sound of the bell always become weaker; and if you continue to exhaust the air, the sound will at length cease entirely, or be scarcely heard. On the other hand, if you begin to re-admit the air, the sound will be revived, and will increase more and more.

This experiment, which we have mentioned in another place, fully proves that air is absolutely necessary for the transmission of sound, and that it is the vehicle of it.

Experiment 7.

Provide a receiver with a hole in the top, and through this aperture introduce the tube of a barometer, so that the bulb shall be in the inside of the receiver: then close the remaining aperture with mastic, or with a metal plate, so as to exclude the external air. Place the receiver thus prepared on the plate of the instrument, and begin to exhaust it of air. On the first stroke of the piston you will see the mercury fall considerably: a second stroke will make it still fall, but a quantity less than the former; and so on in a decreasing proportion. In short, as the air in the receiver becomes less, the mercury will descend more and more towards the level of that in its bason.

Experiment 8.

Provide two hollow hemispheres of brass or copper, two feet in diameter, more or less, with very smooth edges, so that they can be fitted to each other in such a manner as to form a hollow globe. To one of these hemispheres let there be adapted a tube passing into the inside of it, furnished with a stop-cock, and constructed in such a manner that it can be screwed on the end of the tube H of the pneumatic machine. Each of these hemispheres must have affixed to it a ring, or handle, by one of which the globe can be suspended, while a weight is attached to the other.

When these arrangements are made, adapt the two concave hemispheres to each other, so that the edges may be in perfect contact. Screw upon the end of the tube H of the pneumatic machine, the end of that which communicates with the inside of the globe, and exhaust it of air as much as possible, by forty, fifty, or more strokes of the piston. Then shut the communication between the inside of the globe and the external air, by turning the stop-cock, and remove the globe from the machine. If you then suspend the globe by one of the rings, and attach a considerable weight to the other, you will find that the weight will not be able to separate the two hemispheres. If the globe indeed be two feet in diameter, and well exhausted of air, the force with which the edges are pressed together will be equal to about 6500 pounds.

This is what is called the celebrated experiment of Magdeburgh, because first made by Otto Guerik, a burgomaster of that town. He applied to the globe several pairs of horses, some dragging in one direction and some in another, without their being able to separate the two hemispheres: and in this there is nothing astonishing, for though six horses draw a waggon, loaded with a weight equal to several thousand pounds, it is well known that, one with another, they do not exert a continued effort greater than about 180 pounds, and, dragging by jerks, their exertion does not exceed perhaps 4 or 500 pounds. The effort of six horses, therefore, is equal to no more than 3000 pounds. We shall even suppose it to be 4 or 5000 pounds; but if the six horses draw in different directions, they do not double that force; they only oppose to the first the resistance necessary to make it act, and do nothing more than what would be done by a fixed obstacle to which the globe might be attached. It needs therefore excite no surprise that, in the experiment of Magdeburgh, twelve horses were not able to disjoin the hemispheres; for according to this disposition, these twelve horses were equivalent only to six; and it has been shewn that the effort of these six horses, according to the above calculation, was very inferior to that which they had to overcome.

PROBLEM II.

To invert a glass full of water, without spilling it.

Pour water or any liquor into a glass, till it is full to the edge, and place over it a square bit of pretty strong paper, so as to cover the mouth of it entirely; and above the paper place any smooth body, such as the bottom of a plate, or a piece of glass, or even your hand. If you then invert the whole, and afterwards raise it up, you will see the paper adhere to the glass, and the water will not fall out.

This effect is produced by the gravity of the air, for as the air presses on the paper, which covers the mouth of the glass, with a weight superior to that of the water, it must necessarily support it. But as the paper becomes moist, and affords a passage to the air, it at length suddenly falls down.

Remark.—In consequence of the same principle, water or any other liquor may be drawn from a vessel, by means of a pipe open at both ends. For, let A B, (Fig. 2.), be a tube, thick in the middle, and tapering towards both ends, which terminate in two pretty narrow apertures. Immerse in any liquor with both ends open until it is full; and then place your finger on the upper end, so as to close the aperture; if you then draw it from the fluid, the liquor it contains will remain suspended in it, though the lower end be open; and it will not flow out till you remove your finger from the upper orifice.

Fig. 2.

Instead of employing a tube like that above described, you may use a vessel, such as A B, (Fig. 3.), made like a bottle, and having its bottom pierced with a great number of small holes. If you immerse this vessel in water with the

Fig. 3.

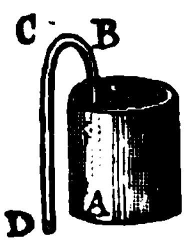
bottom downwards, the liquor will enter through the holes and fill it; and if you then place your finger on the mouth of it, and draw it from the fluid, the water will remain suspended in it as long as your finger continues in that situation; but as soon as it is removed, the water will run out.

This is what is called the *clepsydra* or *watering pot* of Aristotle; but neither Aristotle, nor any of those philosophers who followed him, till the time of Torricelli, assigned a better reason for this effect, than the horror which nature, as they said, had of a vacuum.

PROBLEM III.

To draw off all the liquor contained in a vessel, by means of a syphon.

Fig. 4.



The name *syphon* is given to a tube or pipe, consisting of two branches, A B and C D, (Fig. 4.), united by a crooked part B C. Whether this part be straight or bent is of no importance. It has sometimes an aperture which serves for filling the two branches, or for sucking up the liquor in which the shorter branch is immersed, while the other is shut. It is employed in the following manner, for solving the problem proposed.

Having filled the two branches of the syphon with liquor, close them with your fingers, and immerse the shorter one in the vessel, so that the end of it shall almost touch the bottom; then remove your finger from the end of the longer branch, which will be lower than the bottom of the vessel to be emptied, and you will see the liquor run out at the extremity D of this branch, till the vessel is entirely emptied. Or the syphon may be filled and set a-running after it is placed in the liquid, by sucking the air out at the lower end with your mouth.

This phenomenon is also an effect of the gravity of the air: for when the syphon is full of liquor, and placed as above described, the air by its weight exercises a pressure on the surface of the liquor to be emptied, and at the same time on the orifice of the lower branch. The latter pressure indeed is for this reason somewhat superior to the former; but as this branch is full of a liquor heavier than air, the advantage must be in its favour, and the column ought to fall down. At the same time the air pressing on the surface of the fluid in the vessel, forces the liquor into the branch of the syphon immersed in it; which furnishes a new supply to the longer one, and so on in succession, till the whole liquor is exhausted.

Remarks.—I. All the wine contained in a cask might easily be drawn off in this manner by the bung-hole; and this indeed is the method employed in some places for transferring liquor from one cask to another, without disturbing the lees, which are at the bottom.

II. By the same means, water may be conveyed from any place to another on a lower level, making it pass over an obstacle higher than either, provided the place which the water has to surmount is not higher than 32 or 33 feet above the level where it begins to ascend; for it is well known that the gravity of the atmosphere cannot support a column of water greater than 32 or 33 feet. It is even necessary that the obstacle should be several feet less in height than 32 feet above the level of the water to be raised; otherwise the water will move in a very slow manner, unless the orifice of the longer branch be much lower than that level.

This is a very economical kind of pump, and might be employed to convey water from one place to another, when it is impossible or inconvenient to pierce an intervening obstacle, to establish a pipe of communication. As we have never made the

experiment, we cannot venture to give this as a very certain method, on account of the air which might lodge itself at the summit of the bending of the pipe.

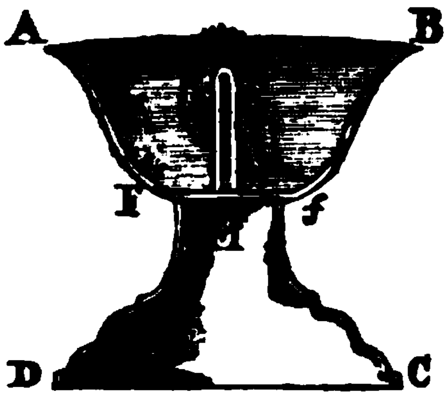
It is on the property of the syphon also that the following hydraulic amusements depend.

PROBLEM IV.

To construct a vessel which, when filled to a certain height with any liquor, shall retain it; and which shall suffer the whole to escape, when filled with the same liquor to a height ever so little greater.

Those who may be desirous of giving to this small hydraulic machine a more mysterious air, add to it a small figure, which they call Tantalus, because in the attitude of drinking; but as soon as the water has reached its lips, it suddenly runs out. The construction of this machine is as follows.

Fig. 5.



Let there be a metallic vessel A B C D, (Fig. 5.) divided into two parts by a partition I *f*: in the middle of this partition is a small round hole, to receive a tube M S, about two lines in diameter, the lower orifice of which must descend a little below the partition. This tube is covered by another somewhat larger, closed at the top, and having on one side, at the bottom, an aperture, so that when water is poured into the vessel, it may force itself between the two tubes, and rise to the upper orifice S of the first. This mechanism must be concealed by a small figure in the attitude of a man stooping to drink, and having its lips a little above the orifice S.

If water be poured into this vessel, as soon as it reaches the lips of the figure, being above the orifice S, it will begin to run off; and a sort of syphonic motion will take place, in consequence of which the whole of the water will run into the lower cavity, which ought to have in the side, towards the partition, an aperture to let the air escape at the same time.

This hydraulic machine might be rendered still more agreeable, by constructing the small figure in such a manner, that when the water has attained to its utmost height, it shall cause the figure to move its head, in order to approach it; which would represent the gestures of Tantalus, endeavouring to catch the water to quench his thirst.

PROBLEM V.

Construction of a vessel which while standing upright retains the liquor it contains; but which, if inclined, as for the purpose of drinking, immediately suffers it to escape.

Pierce a hole in the bottom or side of the vessel to which you are desirous of giving this property, and insert in it the longer branch of a syphon, the other extremity of which must reach nearly to the bottom, as seen Fig. 6.

Fig. 6.

Then fill the vessel with any liquor, as far as the lower side of the bent part of the syphon: it is evident that when inclined, and applied to the mouth, this movement will cause the surface of the water to rise above the bending, and from the nature of the syphon, the liquor will then begin to flow off; and if the vessel is not restored to its former position, will continue doing so till it becomes empty.

This artifice might be concealed by means of a double cup, as it appears Fig. 7; for the syphon *a b c*, placed between the two sides, will produce the same effect. If the vessel be properly presented to the person whom you

Fig. 7.

are desirous of deceiving, that is to say in such a manner as to make him apply his lips to the side *b*, the summit of the syphon, the inclination of the liquor will cause it to rise above that summit, and it will immediately escape at *c*. Those persons however who are acquainted with the artifice will apply their lips to the other side, and not meet with the same disappointment.

PROBLEM VI.

Method of constructing a fountain, which flows and stops alternately.

This fountain, the invention of M. Shermius, is exceedingly ingenious, and affords a very amusing spectacle, because it seems to flow and stop at command. It depends on the operation of a syphon, which, by the peculiar mechanism of this machine, is sometimes obstructed and sometimes left free, as will appear by the following description.

Fig. 8.

G

A B (Fig. 8.) is a vessel shaped like a drum, and close on all sides, except a hole in the middle of the bottom F, into which is soldered a tube C D, open at both its extremities, C and D; but the upper one C ought not to touch the top of the cylinder, in order that the water may have a free passage. When this vessel is to be filled, it must be inverted, and the water is then introduced through the aperture D, till it is nearly full.

From the centre of the bottom of a cylindric vessel G H, somewhat larger, rises a tube D E, a little narrower, so that it can be fitted exactly into the former. Its height also ought to be somewhat less; and its summit E must be open. These two tubes, C D and E D, have two corresponding holes, I, *i*, at an equal height above the bottom of the lower vessel, so that when the one tube is inserted into the other, the holes may be made to correspond; and establish a communication between the external air and that in the upper vessel. Lastly, there must be two or four holes, as K L, in the bottom of the vessel A B, through which the water may flow into the lower vessel G H; and in this vessel also there ought to be one or two holes, M N, of a smaller size, through which the water may escape into another large vessel placed below the whole apparatus.

To make the machine play, pour water into the vessel A B, till it is almost entirely full; having then stopped the pipes K and L, introduce the tube D E into C D, so that the vessel G H shall serve as a base, and make the two holes I and *i* correspond with each other; if the holes or pipes K and L be then unstopped, as the external air will have a communication, by the apertures I, *i*, with that which is above the water in the vessel A B, the water will flow readily into the vessel G H: but the quantity which escapes from G H being less than that which falls from the upper vessel A B, it will soon rise above the apertures I, *i*, and intercept the communication between the external air and at that of vessel A B; consequently the water will soon after cease to flow. But as the water will continue to flow from the lower vessel, while no more falls into it from the upper one, the apertures I, *i*, will soon be uncovered, and the above communication will be re-established: the water therefore will again begin to flow through the pipes K and L, and, rising above the apertures I, *i*, will soon after begin to escape again; and this play will take place alternatly, till no more water is left in the vessel A B.

The time when the air is about to be introduced through the apertures, I, *i*, into the top of the vessel A B, will be known by a small gurgling noise; and at that moment you must command the fountain to flow. When you see the water begin to

rise above the same apertures I, *i*, you must command it to stop. Hence the name given to this machine, *the fountain of command.*

PROBLEM VII.

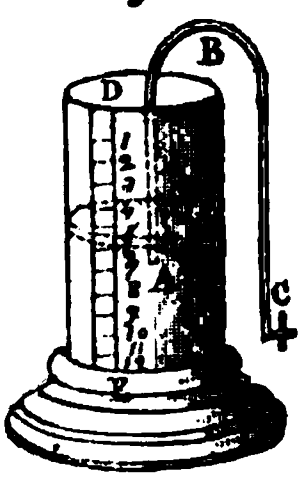
How to construct a clepsydra, which indicates the hours by the uniform efflux of water.

We have shewn, in the Mechanics, that if a vessel has a hole in its bottom, the water flows out faster at first than it does afterwards, so that if we wished to employ the efflux of water to indicate the hours, as the ancients did, it would be necessary to make the divisions very unequal; because, if the whole height were divided into 144 equal parts, the highest, if the vessel were cylindrical, ought to contain 23, the second 21, &c., and the last only 1.

Are there any means then of causing the water to flow off in a uniform manner? This is a problem which naturally presents itself in consequence of the preceding observation. We have already solved it in Mechanics, by shewing what form ought to be given to the vessel, that the efflux of the water through a hole in its bottom may be uniform. But we shall here give a more perfect solution, as it is equally exact whatever may be the law of the retardation of the water.

This solution is founded on the property of the syphon, and is very old, since it was described by Hero of Alexandria. It is as follows.

Fig. 9.



Provide a syphon A B C (Fig. 9.) and affix to the shorter branch A B a piece of cork, capable of keeping the whole syphon in a vertical situation, as seen in the figure. When this apparatus is made to play, and the water begins to flow off through the longer branch, it will continue to escape with the same velocity, whatever may be the height of the water: for, in this machine, the efflux takes place in consequence of the inequality of the force with which the atmosphere presses on the surface of the liquid, and on the orifice of the longer branch; since the syphon then sinks down as the surface of the liquid falls, it is evident that the velocity of its efflux will be uniform.

If the height of the vessel D E be therefore divided into equal parts, these divisions will indicate equal intervals of time. To render this clepsydra more curious, the branch A B might be concealed by a small light figure made to float on the surface of the water in the vessel, and indicating the hour with a rod, or its finger, on a small dial-plate.

On the other hand, the water might be made to flow from any vessel whatever, through a similar syphon, into another vessel of a prismatic or cylindric form, from which might arise a small figure floating on the water, to indicate the hour as above described.

PROBLEM VIII.

What is the greatest height to which the Tower of Babel could have been raised, before the materials carried to the summit lost all their gravity.

To answer this mathematical pleasantry, which belongs as much to the physical part of astronomy as to mechanics, we must observe:

1st. That the gravity of bodies decreases in the inverse ratio of the square of their distance from the centre of the earth. A body, for example, raised to the distance of a semi-diameter of the earth above its surface, being then at the distance of twice the radius, will weigh only $\frac{1}{4}$ of what it weighed at the surface.

2d. If we suppose that this body partakes with the rest of the earth in the rotary motion which it has around its axis, this gravity will be still diminished by the centrifugal force; which on the supposition that unequal circles are described in the

same time, will be as their radii. Hence at a double distance from the earth this force will be double, and will deduct twice as much from the gravity as at the surface of the earth. But it has been found, that under the equator the centrifugal force lessens the natural gravity of bodies $\frac{1}{289}$th part.

3d. In all places, on either side of the equator, the centrifugal force being less, and acting against the gravity in an oblique direction, destroys a less portion, in the ratio of the square of the cosine of the latitude to the square of the radius.

These things being premised, we may determine at what height above the surface of the earth a body, participating in its diurnal motion in any given latitude, ought to be to have no gravity.

But it is found by analysis that under the equator, where the diminution of gravity at the surface of the earth, occasioned by the centrifugal force, is about $\frac{1}{289}$, the required height, counting from the centre of the earth, ought to be $\sqrt[3]{289}$, or 6 semi-diameters of our globe plus $\frac{61}{100}$, or 5 semi-diameters and $\frac{61}{100}$ above the surface.

Under the latitude of 30 degrees, which is nearly that of the plains of Mesopotamia, where the descendants of Noah first assembled, and vainly attempted, as we learn from the Scriptures, to raise a monument of their folly, it will be found that the height above the surface of the earth ought to have been $6\frac{27}{100}$ semi-diameters of the earth.

Under the latitude of 60 degrees, this height above the surface of the earth ought to have been $9\frac{47}{100}$ semi-diameters of the earth.

Under the pole this distance might be infinite; because in that part of the earth there is no centrifugal force, since bodies at the pole only turn round themselves.

PROBLEM IX.

If we suppose a hole bored to the centre of the earth, how long time would a heavy ball require to reach the centre, neglecting the resistance of the air?

As the diameter of the earth is about 7930 miles, the semi-diameter will be 3965 miles, or 20935200 feet. If the acceleration were uniform, the solution of the problem would be attended with no difficulty; for nothing would be necessary but to say, according to Galileo's rule, As $16\frac{1}{12}$ feet are to 20935200 feet, so is the square of 1 second, which is the time employed by a heavy body in falling $16\frac{1}{12}$ feet, to a fourth term, which will be the square of the number of seconds employed in falling 20935200 feet. But this fourth term will be found to be 1301940; and if we extract the square root of it, we shall have the required number, that is 1140 seconds, or 19 minutes. Such, according to this hypothesis, would be the time employed by a heavy body in falling to the centre of the earth.

But it is much more probable that a body, proceeding along the radius of the earth, would lose its gravity, as it approached the centre; for at the centre it would have no gravity at all; and it can be demonstrated, supposing the density of the earth to be uniform, and that attraction is in the inverse ratio of the squares of the distances, that the gravity would decrease in the same proportion as the distance from the centre. The problem therefore must be solved in another manner, founded on the following proposition demonstrated by Newton:

If a quadrant be described with a radius equal to that of the earth, an arc which has $16\frac{1}{12}$ feet for versed sine, will be to the quadrant, as 1 second employed to pass over in falling these $16\frac{1}{12}$ feet, is to the time employed to fall through the whole semi-diameter of the earth.

But an arc of the earth corresponding to $16\frac{1}{12}$ feet of fall, or versed sine, is 4′ 16″ 5‴; and this arc is to the quadrant, as 1 to 1265·2. Consequently we have this proportion, as 4′ 16″ 5‴ are to 90°, or as 1 to 1265·2, so is 1 second, employed in falling $16\frac{1}{12}$ feet at the surface of the earth, to 1265^s 12 , or 21^m 5^s 12^{th}. This will be the time employed by a heavy body in falling from the surface of the earth to the

centre, according to the second supposition, which is more consistent with the principles of philosophy than the former.

PROBLEM X.

What would be the consequence, should the moon be suddenly stopped in her circular motion; and in what time would she fall to the earth?

As the moon is maintained in the orbit which she describes around the earth, only by the effect of the centrifugal force which arises from her circular motion, and which counterbalances her gravitation towards the earth, it is evident that, if the circular motion were annihilated, the centrifugal force would be annihilated also; the moon then would be abandoned to a tendency towards the earth, and would fall upon it with accelerated velocity.

But this motion would not be accelerated according to the law discovered by Galileo; for this law supposes that the force of gravity is uniform, or always the same. In the present case, the gravity of the moon towards the earth would vary, and be increased in the inverse ratio of the square of the distance, according as she approached the centre; which renders the problem much more difficult.

Newton however has taught us the method of solving it: this philosopher has shewn, that this time is equal to the half of that which the same planet would employ to make a revolution around the same central body, but at half her present distance from it. Now, it is well known that the lunar orbit is nearly a circle, the radius of which is equal to 60 semi-diameters of the earth, and her revolution is 27 days 7 hours 43 minutes;* hence it is found, by the celebrated rule of Kepler, that if she were distant from the earth only 30 of its semi-diameters, she would employ in her revolution around it no more than 9 days 15 hours 51 minutes. Consequently her semi-revolution would be 4 days 19 hours 55½ minutes, which is therefore the time the moon would employ in falling to the centre of the earth.

Remark.—If we examine, by the same method, in what time each of the circum-solar planets, under the like circumstances, would fall into the sun, it will be found that

	D.	H.
Mercury would fall in	15	13
Venus	39	17
The Earth	64	13
Mars	121	10
Ceres	297	6
Pallas	301	4
Juno	354	19
Vesta	405	0
Jupiter	765	19
Saturn	1901	0
Georgium Sidus	5425	0

PROBLEM XI.

What would be the gravity of a body transported to the surface of the sun, or any other planet than the earth, in comparison of that which it has at the surface of our globe?

It can be demonstrated, to all those capable of comprehending the proofs, that the

* We make the revolution of the moon 27 days 7 hours 43 minutes, and not 29 days 12 hours 44 minutes; for the revolution here meant, is from any point of the heavens to the same point again, and not a synodical revolution, which is longer; because when the moon has described a complete circle, she has still to come up with the sun, which in the course of 27 days has advanced in appearance 27 degrees, or nearly.

gravity of a body at the surface of the earth, is nothing else than the tendency of that body towards every part of the earth; the result of which must be a compound tendency passing through the centre, provided the earth be a perfect globe, which we here suppose, on account of the small difference between its figure and that of a sphere. It can be demonstrated also, that as attraction takes place in the direct ratio of the masses, and the inverse ratio of the square of the distances, a particle of matter placed on the surface of a sphere, which exercises on it a power of attraction, will tend towards it with the same force as if its whole mass were united in its centre.

It thence follows, that if we suppose two spheres, unequal both in their diameters and masses, the gravity of the particle on the one, will be to that of the same particle on the other, in the compound ratio of their masses taken directly, and of the squares of their semi-diameters taken inversely.

But it has been demonstrated by astronomical observations, that the sun's semi-diameter is equal to about 111 of the earth's semi-diameters, and that his mass is to that of the earth, as 341908 to 1: the gravity then of a body at the surface of the sun will be to that of the same body at the surface of the earth, in the compound ratio of 341908 to 1, and of the inverse of the square of 111 to 1, that is of 12321 to 1.

If the number 341908 be therefore divided by 12321, we shall have $27\frac{3}{4}$ nearly; consequently a body of a pound weight, transported to the surface of the sun, would weigh $27\frac{3}{4}$ pounds.

But we shall endeavour to illustrate this subject by a reasoning still simpler. If the whole mass of the sun, which is 341908 times as great as that of the earth, were compressed into a globe equal in size to the earth, the body in question, instead of weighing one pound, would weigh 341908. But as the surface of the sun is 111 times as far from his centre as that of the earth is from its centre, it thence follows that the above weight must be diminished in the ratio of 12321, or of the square of 111, to the square of unity; that is, we must take only the 12321st part of the weight above found, which gives the preceding result, viz. $27\frac{3}{4}$.

By a similar reasoning it will be found, that a body of a pound weight carried to the surface of Jupiter, would weigh $2\frac{7}{10}$ pounds; to that of Saturn $1\frac{1}{100}$, and to that of the moon only $2\frac{1}{2}$ ounces.

The masses of Mercury and the other planets which have no satellities can only be guessed at from the effect which they produce in disturbing the motions of the other planets.

The mass of the Moon as compared with that of the sun may be deduced by comparing their influence in producing the tides, and the precession of the equinoxes.

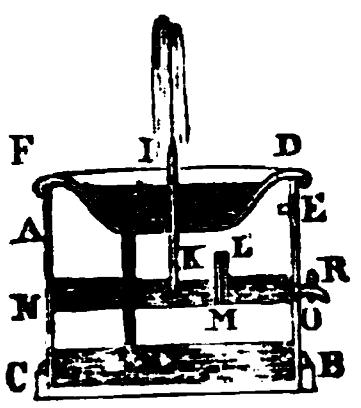
Tabular view of the comparative light and heat, volumes, mass, density, and gravity at the surface of the sun and principal planets.

	Volume.	Mass.	Density.	Gravity.	Light & Heat.
Mercury	0·063	$\frac{1}{2025810}$	....	1·03	6·680
Venus	0·927	$\frac{1}{405871}$	....	0·98	1·911
Earth	1·000	$\frac{1}{354936}$	3·9326	1·00	1·000
Mars	0·139	$\frac{1}{2546320}$	....	0·33	·431
Jupiter	1280·900	$\frac{1}{1070 \cdot 5}$	·9926	2·72	·037
Saturn	995·000	$\frac{1}{3312}$	·5500	1·01	·011
Georgium Sidus..	80·490	$\frac{1}{17918}$	1·1000	....	·003
Sun............	1384472·000	1	1·0000	27·90	....
Moon	·020	$\frac{1}{26820000}$	2·4815	0·16	....

PROBLEM XII.

To construct a fountain which shall throw up water by the compression of the air.

Fig. 10.



Let there be a vessel, a section of which is represented (Fig. 10.), namely, composed of a cylindric pedestal or parallelopipedon, crowned with a kind of cup F A E D. This pedestal is divided, by a partition N O, into two cavities, the lower one of which must be somewhat smaller than the other.

A tube G H, passing through the partition, reaches nearly to the bottom C B; while another tube L M has its upper orifice near the bottom of the cup, and its lower M near the partition N O. A third tube I K, which, like the first, passes through the bottom of the cup, tapers to a point at the upper end, and with the other reaches nearly to the partition.

When the vessel has been thus constructed, pour water into the upper cavity, through a lateral hole, till it reaches nearly to the orifice L of the tube M L; then carefully stop the lateral hole, and pour water into the cup; this water, flowing into the cavity N B, will compress the air in it, and will force it, in part, to pass through M L into the space above the water in the upper cavity, where it will be more and more condensed, and force the water to spout out through the orifice I, especially if it be some time confined, either by keeping the finger on the orifice I, or by means of a small stopcock, which can be opened when necessary.

Remarks.—I. This small fountain may be varied different ways. Thus, for example, if the weight of the water which flows through G H into the lower cavity N B, be not sufficient to give the necessary force to the water which issues through I, water might be introduced by means of a syringe, or even air by means of a pair of bellows adapted to the orifice G, and furnished at the nozzle with a stopcock.

Quicksilver might also be poured into it: this fluid would enter it notwithstanding the resistance of the air, and force it to exercise a powerful action on the fluid contained in the upper cavity.

Fig. 11.

II. This fountain might be constructed in a manner still simpler. Provide a bottle, such as A B (Fig. 11.), and introduce into it, through the cork, a tube C D, the lower orifice of which reaches nearly to the bottom, while the upper one terminates in a narrow aperture. The communication between the external air and that in the bottle ought to be completely intercepted at A. Let us now suppose that this bottle is three-fourths filled with water: if you breathe with all your force into the tube through the orifice C, the air in the space A E F will be condensed to such a degree as to press on the surface of the water E F, which will make it issue with impetuosity through the orifice C, and even force it to rise to a considerable height. When the play of the machine has ceased, if any water remains in it, to make it recommence its play, nothing will be necessary but to blow into it again.

PROBLEM XIII.

To construct a vessel, into which if water be poured, the same quantity of wine shall issue from it.

The solution of this problem is a consequence, or rather a simple variation of the preceding. Let us suppose that the small tube I K (Fig. 10.), is suppressed, and that the cavity A O is filled with wine; if a small cock R be inserted into the

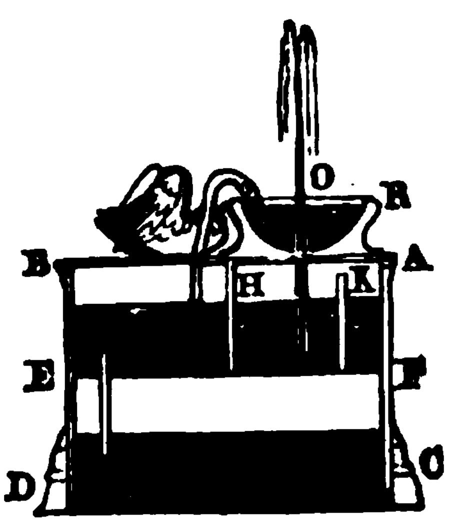
machine near the bottom N O, it is evident that when water is poured into the cup F A E D, the air, being forced into the upper cavity, will press on the surface of the wine, and oblige it to flow through the cock, until it be in equilibrium with the weight of the atmosphere: if more water be then poured into the cup F D, nearly as much wine will issue through the cock: so that the water will appear to be converted into wine.

Hence, if it be allowable to make allusion to a celebrated event recorded in the Sacred Scriptures, were this machine constructed in the form of a wine-jar, it might be called the pitcher of Cana.

PROBLEM XIV.

Method of constructing an hydraulic machine, where a bird drinks up all the water that spouts up through a pipe and falls into a bason.

Fig. 12.



Let A B D C (Fig. 12.), be a vessel, divided into two parts by a horizontal partition E F; and let the upper cavity be divided into two parts also by a vertical partition G H. A communication is formed between the upper cavity B F and the lower one E C, by a tube L M, which proceeds from the lower partition, and descends almost to the bottom D C. A similar communication is formed between the lower cavity E C and the upper one A G, by the tube I K, which rising from the horizontal partition E F, proceeds nearly to the top A B. A third tube, terminating at the upper extremity in a very small aperture, descends nearly to the partition E F, and passes through the centre of a bason R S, intended to receive the water which issues from it. Near the edge of this bason is a bird with its bill immersed in it; and through the body of the bird passes a bent syphon Q P, the aperture of which P is much lower than the aperture Q. Such is the construction of this machine, the use of which is as follows.

Fill the two upper cavities with water through two holes, made for the purpose in the sides of the vessel, and which must be afterwards shut. It may be easily seen that the water in the cavity A G ought not to rise above the orifice K of the pipe K I. If the cock adapted to the pipe L M be then opened, the water of the upper cavity B G will flow into the lower cavity, where it will compress the air, and make it pass through the pipe K I into the cavity A G; in this cavity it will compress the air which is above it, and the air, pressing upon it, will force it to spout up through the pipe N O, from whence it will fall down into the bason.

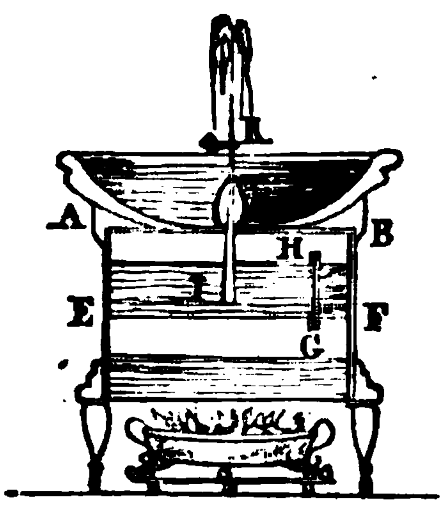
But at the same time that the water flows from the cavity B G into the lower one, the air will become rarefied in the upper part of that cavity: hence, as the weight of the atmosphere will act on the water already poured into the bason through the orifice O of the ascending pipe N O, the water will flow through the bent pipe Q S P, into the same cavity B G; and this motion, when once established, will continue as long as there is any water in the cavity A G.

PROBLEM XV.

To construct a fountain, which shall throw up water, in consequence of the rarefaction of air dilated by heat.

Construct a cylindric or prismatic vessel, a section of which is represented (Fig. 13.) raised a little on four feet, that a chaffing dish with coals may be placed beneath it. The cavity of this vessel must be divided into two parts by a partition E F, having in it a round hole about an inch in diameter: into this hole is inserted a pipe G H, which rises nearly to the top, and over the top is placed a vessel in

Fig. 13.



the form of a bason, to receive the water furnished by the jet. Another pipe I K passing through the top, into which it is soldered or cemented, descends nearly to the partition E F: this pipe may be made a little wider at the lower extremity, but the upper end ought to be somewhat narrow, that the water may spout up to a greater height. It will be proper to adapt to the upper part of this pipe a small stop-cock K, by means of which the water can be confined till the air is sufficiently rarefied to produce the jet.

When the machine is thus constructed, pour water into the upper cavity till it reaches the orifice H of the tube G H; then place a chafing-dish of burning coals, or a lamp with several wicks, below the bottom of the vessel. By these means the air contained in the lower cavity will be immediately rarefied, and passing through the pipe G H, into the space above the water contained in the upper cavity, will force it to rise through the orifice I of the pipe I K, and to spout up through the aperture K.

To render the effect more sensible and certain, it will be proper to put a small quantity of water into the lower cavity; for when this water begins to boil, the elastic vapour produced by it, passing into the upper cavity, will exert a much greater pressure on the water, and force it to rise to a more considerable height.

Care however must be taken, if the steam of boiling water be employed, not to heat the machine too much; otherwise the violent expansion of the water might burst it.

PROBLEM XVI.

To measure the degree of the heat of the atmosphere, and of other fluids. History of thermometers, and the method of constructing them.

One of the most ingenious inventions, by which the revival of sound philosophy was distinguished in the beginning of the 17th century, was that of the instrument known under the name of the thermometer, so called because it serves to measure the temperature of bodies, and particularly that of the atmosphere, and of other fluids, into which it can be immersed. This invention is generally ascribed to the Academy del Cimento, which flourished at Florence, under the protection of the grand-dukes of the house of Medici, and which was the first in Europe that applied to experimental philosophy. It is asserted also, that Cornelius Drebbel of Alcmaer in north Holland, who lived at the court of James I. king of England, had a share in this invention. But we shall not here enter into a discussion of this point in the history of Natural Philosophy, as it is foreign to our design.*

The invention of the thermometer is founded on the property which all bodies, and particularly fluids, have of dilating by the heat which pervades them. As spirit of wine possesses this property in an eminent degree, this liquid was employed in preference to any other. A very narrow glass tube, terminating in a bulb of about an inch in diameter, was filled with this liquor, after it had been coloured red by means of tincture of turnsol, or orchilla weed, in order to render it more visible,

* The first description of a thermometer ever published, is that of Solomon de Caux, a French engineer, in his book "Des Forces Mouvantes," printed in 1624, in folio, but written, as appears, prior to that period, for the dedication to Louis XIII. is dated 1615, and the privilege granted by that monarch is of 1614. The thermometer here alluded to, acts by the dilatation of air confined in a box, which, pressing against water, forces it to rise in a tube. As Drebbel's thermometer was of the same kind, it may be asked whether his invention was prior to that of Solomon de Caux? This is a question which seems difficult to be determined.—*Note of the French Censor.*

It may be easily conceived, that the size of the bulb being considerable, compared with that of the tube, as soon as the liquor became in the least dilated, it would be in part forced to pass into the tube: the liquor therefore would be obliged to ascend. On the other hand, when condensed by cold, it would of course descend. It was only necessary to take care, that during very cold weather the liquor should not entirely descend into the bulb; and that during the greatest degree of heat to be measured, it should not entirely escape from it. Towards the lower part, some degrees of temperature were inscribed by estimation: such as *cold*, and a little lower *great cold;* towards the middle *temperate*, and at the top *heat*, and *great heat*.

Such is the construction of that thermometer called the Florentine, which was used for almost a century; and such are those still sold in many country places by itinerant venders, and which are purchased with confidence by the ignorant.

This thermometer, though its form and the greater part of its construction have been retained, is attended with this fault, that it indicates the variations of heat only in a very vague and uncertain manner. By its means we may indeed know that one day has been hotter or colder than another; but that degree of heat or cold cannot be compared with another degree, nor with the heat or cold in another place: besides, the words *heat* and *cold* are merely relative. An inhabitant of the planet Mercury would probably find one of our hottest summers exceedingly cool, and perhaps very cold; while an inhabitant of Saturn, if transported to the frigid zone of our earth during winter, would perhaps find it intolerably hot. We ourselves at the close of a fine day in summer, experience a sensation of cold, when removed into air much less hot, and *vice versâ*.

On this account, attempts have been made to construct thermometers, by which the degrees of heat and cold could be compared to a degree of heat or cold invariable in nature; so that all thermometers constructed according to this principle, though by different artists, in different places and at different times, should correspond with each other, and indicate the same degree when exposed to the same temperature. This was the only method of making experiments that could be of utility.

This was at length accomplished by means of the two following principles, which were discovered by experience.

The first is, that the degree of the temperature of pounded ice, beginning to melt, or of water beginning to freeze, is constantly the same, at all times and in all places.

The second is, that the degree of the temperature of boiling water is also constant. We here speak of fresh water; and we suppose also that the height of the atmosphere does not vary; for we know that when water is pressed with a greater weight, it requires a degree of heat somewhat greater than when it is less pressed. This is proved by the pneumatic machine, from which if a part of the air be exhausted, water boils at a less degree of heat than when exposed to the open air. Hence arises a sort of paradox, that at the summit of a mountain, the same quantity of heat is not required to boil water as at the bottom of it. But when the gravity of the air is the same, and when the water holds no salt in solution, it begins to boil at the same degree of heat; and when it once attains to that state, it never acquires a greater degree, however long it may be boiled.

These two constant degrees of heat and cold, so easy to be obtained, have therefore appeared to philosophers very proper for being employed in the construction of thermometers. The simplest method for this purpose is as follows:—

Provide a tube, one of the ends of which is blown into a bulb of about an inch in diameter; if the tube be a capillary one, the bulb may be smaller. By a process which we shall describe hereafter, pour quicksilver into the tube, till it rises to the height of a few inches above the bulb; and then immerse the bulb into pounded ice put into a bason. When the mercury ceases to fall, make a mark on the tube, in

order that this point may be known; then immerse the thermometer into boiling water, and mark the point where the mercury ceases to rise, which will be that of boiling water. Nothing then will be necessary but to divide that interval, between these two marks, into any equal number of parts at pleasure, such as 100, for instance, which appear to us to be the most convenient. For this purpose affix the tube to a small piece of board, having a piece of paper cemented to it, and divide the interval between the marks into the number of parts you have chosen; if 8 be inscribed at the point of freezing, and if a few degrees be marked below it, your thermometer will be constructed.

Care however must be taken, to ascertain whether the diameter of the tube is uniform throughout: for it may be easily seen that a tube of unequal calibre would render the motion of the mercury irregular. For this purpose, introduce a small drop of mercury into the tube, and make it pass from the one end to the other; if it every where occupies the same length, it is evident that no part of the tube is narrower than another; if the drop becomes lengthened or shortened, the tube must be rejected as faulty.

Several of the modern philosophers, with a view to improve the construction of thermometers, have entered into minute details in regard to the increase of volume which mercury and spirit of wine acquire, when they pass from the degree of freezing to that of boiling water; but it appears to us, that as these two terms have been found to be invariable, they might have saved themselves the trouble of entering into these considerations, which tend only to render their processes more difficult.

It now remains that we should describe the method of filling the bulb and tube with the fluid intended to form the thermometer; and which, for reasons to be mentioned hereafter, we shall suppose to be mercury; for this operation is attended with some difficulty, especially when the tube is very small.

The first thing to be attended to, is to clean very well the inside of the tube; which, if it be not a capillary one, may be done by means of a very dry plug fixed to the end of a wire, and then drawn up and down the tube. If the tube be capillary, it must first be heated, and then the bulb: the air issuing from the latter will expel any dirt that may be adhering to it.

The mercury ought to be exceedingly pure, or revived by means of cinnabar; it must also be boiled to expel the air which may be diffused through it.

When these preparations have been made, attach to the summit of the tube a small paper funnel; apply the tube to a chafing-dish in such a manner as to heat it gradually, and then heat the bulb in the same manner, till it cannot be held in the hand without a thick glove. When the thermometer has acquired this degree of heat, if the small funnel above mentioned be filled with heated mercury, in proportion as the glass cools the air will become rarefied, and afford a passage to the mercury into the bulb, till it be in equilibrium with it. Repeat the same operation to introduce a new quantity of mercury, and so on till the tube is full; and then graduate the thermometer, expelling from it, by the means of heat, what is more than necessary to make it reach the highest point marked towards the upper extremity of the tube. when immersed in boiling water. When this point has been fixed mark it by means of a thread, or by notching the tube with a file, and having suffered the thermometer to cool, immerse it in melting ice, which will give the freezing point.

It may be readily conceived, that if the whole of the mercury, during this operation, should enter the bulb, it will be necessary to introduce a little more, in order to carry the point of boiling water somewhat higher.

Melt and draw out the upper end of the tube, by applying it to an enameller's lamp, and heat the mercury to such a degree, as to make it ascend near to the summit; then seal it hermetically at the lamp, and by these means nothing will

remain in the upper end of the tube but a quantity of air imperceptible, or exceedingly small.

Then affix the tube to a board, made of some wood which has the property of expanding but a very little in length by heat: fir has this property as well as that of lightness; the bulb must be insulated from the board that the air may surround it more freely, and that it may not be affected by the heat which the wood may acquire.

A question here naturally arises: What kind of liquor is the best, and most convenient for constructing accurate and durable thermometers—Spirit of wine or mercury?

In our opinion, this question is attended with no difficulty; for all philosophers must agree that mercury is the fittest fluid for constructing thermometers. No doubt of its superiority over spirit of wine can be entertained, by those who consider:—

1st. That spirit of wine, unless well dephlegmated, is not always the same; and who can assert that, in its different states, its progress is always the same, or that its dilatation is not different at the same degree of heat? This point has been determined by experience; and therefore no certain comparison can be made between different spirit of wine thermometers.

2d. If spirit of wine be well dephlegmated, as it then becomes highly spirituous and volatile, is it not to be apprehended that its volume may be gradually diminished? It is indeed true, that to prevent this inconvenience, the tube is hermetically closed at the top; but this precaution will not prevent the most volatile part from being exhaled in the upper part of the tube; and in that case the spirit of wine, becoming less expansible, will remain below the degree at which it ought to be; and the same thing will take place in every state of the spirit of wine, whether it be employed with water, as is usual, in order to moderate its dilatibility.

3d. Spirit of wine boils at a degree of heat less than that of boiling water; consequently it is not proper for examining degrees of heat which are greater; for beyond ebullition the progress of the dilatation of any liquor does not follow the same laws; because after that term it becomes volatilized, or is suddenly reduced into vapour of a volume a thousand times greater.

On the other hand, spirit of wine, when united with water, is susceptible of freezing at a degree of cold not much less than that at which water congeals; and therefore it is very improper for measuring degrees of cold much below that term.

Mercury is attended with none of these defects. This substance, as far as chemists have been able to ascertain, is of an uniform nature when pure; to make it boil requires a degree of heat six times as far distant from zero, or the term 0, as that at which water boils; and it does not freeze but at a degree of cold very far indeed below that of the congelation of water.

Another advantage of mercury, whether employed in thermometers or barometers, is, that while in the act of rising, the small column assumes a convex form at the top, and when it falls a concave form; for this reason, when the summit assumes a convex form, we can say that the mercury is in the act of rising; and when it becomes concave, we may conclude that it begins to fall; which is very convenient for prognosticating heat, and for ascertaining whether it increases or has become stationary, or has begun to decrease.

PROBLEM XVII.

Description of the most celebrated thermometers, or those chiefly used: Method of reducing the degrees of one to corresponding degrees of another.

Several thermometers, different in the division of their scale, though constructed on the same principle, are employed in Europe. As the division of the scale is

altogether arbitrary, it is necessary that we should point out the method of reducing degrees of the one to corresponding degrees of another.

These thermometers are that of Fahrenheit, that of Reaumur, that of Celsius, and that of Delisle.

The first of these thermometers is constructed with mercury, and is graduated in a manner which, on the first view, may appear rather whimsical. The freezing point corresponds to the 32d degree; and between this point and that of boiling water there are 180 degrees; so that the heat of boiling water corresponds to the 212th degree. The reason of this division is that Fahrenheit assumed, as the Zero of his thermometer, the greatest degree of cold which he could produce by a mixture of snow and spirit of nitre; having then immersed his instrument in melting ice, and afterwards in boiling water, he divided the interval between these two points into 180 degrees, which gave him 32 between the above artificial cold and that of common freezing. Experience has since shewn that it is possible to produce an artificial cold much more intense than that produced by Fahrenheit.

This thermometer is that generally used in England; but it appears that the scale is not the most commodious. It might however be improved by transposing zero to the place of the 32d degree, in which case there would be 180 degrees between the freezing point and that of boiling water; and the degree now marked 0 in this thermometer, would be — 32, denoting the degrees below freezing by the negative sign *minus*. Fahrenheit, it appears, was the first person who employed mercury in the construction of this instrument.

Reaumur's thermometer is generally made with spirit of wine, and is graduated in such a manner, that the degree of melting ice is marked 0, and that which corresponds to boiling water is marked 80; consequently there are 80 degrees between these two points. The scale below 0 is marked 1, 2, 3, 4, &c.; and when these degrees are used, the words *below freezing* are added, or for the sake of brevity the sign —.

Delisle's thermometer is much used in the North; and for this reason it is necessary we should make known the manner in which it is divided. Delisle begins his scale at the point of boiling water, and proceeds downwards to the freezing point; between which and that of boiling water there are 150 degrees: 150 degrees of his thermometer correspond therefore to 80 of Reaumur, or 180 of Fahrenheit.

Celsius of Upsal, and Christin of Lyons, sensible of the defects of spirit of wine, and finding the division into 80 degrees inconvenient, endeavoured to remedy these faults by constructing a thermometer with mercury, and dividing the interval between the freezing point and that of boiling water into 100 degrees. The only difference between this thermometer and that of Reaumur, is, that mercury is used instead of spirit of wine, and that 100 divisions are employed in the same space in which Reaumur employs 80: one degree of the thermometer of Celsius is therefore equal to $\frac{4}{5}$ of a degree of that of Reaumur; consequently, to convert the degrees of Celsius's thermometer to corresponding degrees of Reaumur, multiply by 4 and divide by 5: to convert Reaumur's degrees to those of Celsius, a contrary operation must be employed. To convert the degrees of Celsius's thermometer to those of Fahrenheit's, multiply by 9, then divide by 5, and to the product add 32. To convert Fahrenheit's degrees to those of Celsius, subtract 32 from the number of degrees proposed, and having multiplied the remainder by 5, divide the product by 9.

To convert degrees of Fahrenheit into degrees of Reaumur, the following process must be employed: if the degrees of Fahrenheit are above 32, subtract 32 from them, then multiply the remainder by 4, and divide the product by 9, the quotient will be the corresponding degree of Reaumur's division. Let the proposed degree of Fahrenheit,

for example, be 149: if 32 be subtracted from this number, the remainder will be 117, which multiplied by 4, gives for product 468; and if this product be divided by 9, we shall have for quotient 52, which is the corresponding degree of Reaumur's thermometer.

If the degree of Fahrenheit be between 0 and 32, it must be subtracted from 32; then multiply the remainder by 4, and divide the product by 9: the quotient will be the corresponding degree of Reaumur's thermometer. In this manner it will be found, that 12 degrees of Fahrenheit correspond to $8\frac{8}{9}$ degrees of Reaumur, below freezing.

Lastly, when the proposed degree is below 0, add it to 32, and then proceed as above directed: the quotient will be the corresponding degree of Reaumur's thermometer. Thus, it will be found that the 45th degree of Fahrenheit, below 0, corresponds to $34\frac{2}{9}$ degrees below 0 of Reaumur.

It may here be readily seen, that to convert degrees of Reaumur's scale to the corresponding degree of Fahrenheit's, the reverse of this operation must be performed.

In regard to Delisle's thermometer; it is evident, from its construction, that the 150th degree in its scale, corresponds to the zero of Reaumur's scale. If the proposed degree of Delisle's thermometer then be less than 150, it must be subtracted from 150: if you then multiply by 8, and divide by 15, the quotient will be the corresponding degree of Reaumur, above freezing.

Let the proposed degree of Delisle's thermometer, for example, be 120: if this number be subtracted from 150, the remainder will be 30; then say, as 150 to 80, or as 15 to 8, so is 30 to a fourth term, which will be 16 = the degree of Reaumur's thermometer, above 0, or the freezing point.

If the degree of Delisle's thermometer exceeds 150, as if it be 190, for example, subtract 150 from it, which will leave for remainder 40; then make use of this proportion: As 15 is to 8, so is 40 to $21\frac{1}{3}$, which will be the degree of Reaumur's thermometer, below 0, corresponding to the 190th degree of Delisle's thermometer.

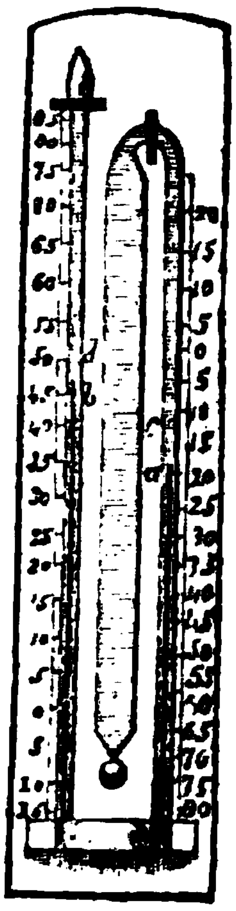
As it will be easy to perform the reverse of this operation, in order to convert the degrees of Reaumur's thermometer into those of Delisle's, more examples are needless.

It is certainly much to be wished that all philosophers would agree to employ only one kind of thermometer, that is to say, constructed in the same manner, with mercury, and having the same scale. In regard to the latter, there can be no doubt that the division of 100 parts, between the freezing point and that of boiling water, would be preferable to any other, as decimal divisions are attended with many advantages in regard to facility of calculation; and this mode of division has since been adopted in France. The thermometer so divided is called the *centigrade thermometer*.

ON REGISTER THERMOMETERS.

Many contrivances have been proposed for making the thermometer register the variations of temperature. One invented by Mr. Six of Colchester, and named after the inventor, is represented in Fig. 14. It is nothing but a spirit-of-wine thermometer, with a long cylindrical bulb; and its tube bent in the form of a syphon with parallel legs, and ending in a small cavity. A part of both legs, as from *a* to *b*, is filled with mercury; and the ramainder of the legs, and a small portion of the cavity, are filled with highly rectified spirit of wine. The double column of mercury gives motion to two indices, *c* and *d*. Each index consists of a bit of iron wire inclosed in a glass tube, capped at each end with a button of enamel. They are of such size that they would move freely in the tube, were it not for a thread of glass drawn from the

Fig. 14.



upper cap of each, and inclined so as to press against one side of the tube with sufficient force to retain the index attached at any part of the tube to which it is raised by the mercurial column.

The instrument, then, effects its object in the following manner. When the spirit in the bulb is expanded by heat, it depresses the mercury in the limb *a*, and raises it in the limb *b*, of the syphon. When the spirit *d* in the bulb contracts by cold, the mercury in the limb *b* descends, and causes a proportional rise of the column in *a*. Now it will be seen that when the mercury in either column rises, it will carry the index in that column with it; and when it begins to fall, it will leave the index attached to the side of the tube, by the small glass spring above adverted to; the lower part of the index marking the highest point to which the mercury in that tube had risen.

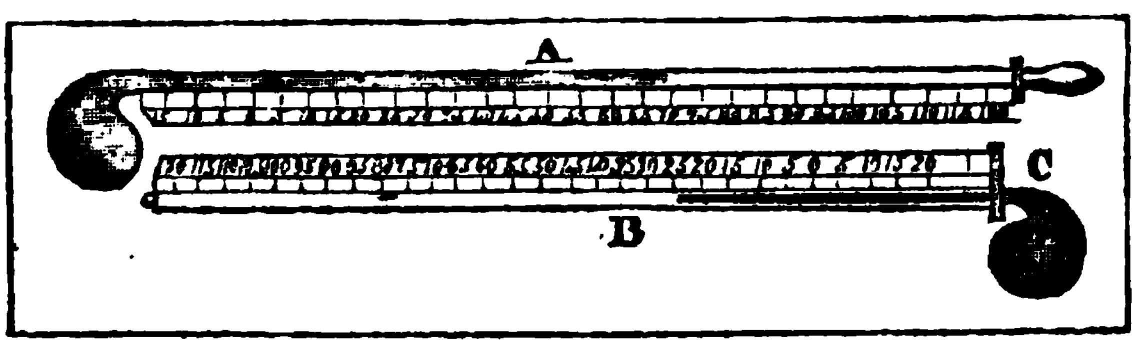
In this way the highest and lowest temperatures are seen that have occurred between any two times of observation. To prepare the instrument for a new observation, both indices are brought to the surface of the mercury by the attraction of a magnet.

It is obvious, from this description, that there must be an ascending scale attached to *b* to measure the expansion of the spirit by heat, and a descending scale to *a* to mark the contraction of the spirit by cold.

Some makers, instead of the glass spring, insert a *bristle* into the cap of the index; but a fine wire of silver or platina would be preferable.

Another invention of Dr. John Rutherford's, called the day and night thermometer, has, from its cheapness, and the simplicity of its construction, in some measure superseded that of Six.

Fig. 15.



This instrument is represented in Fig. 15, where A represents a spirit, and B a mercurial, thermometer; both placed horizontally on the same piece of wood or ivory.

The index of B is a piece of steel wire, which is pushed before the mercury, and left where the mercury had attained its greatest expansion; and marking therefrom the highest temperature. The index of A is a piece of glass about half an inch long, with a small knob at each end. It lies *in* the spirit, which passes freely beyond it, when expanded by heat; and when contracted by cold, the last film of the column of spirit is enabled, by the attraction between the spirit and the glass, to carry the index back towards the bulb; leaving it at the point which marks the greatest cold that has taken place since the setting of the index.

From the position of the thermometers, it is obvious that, to bring both the indices to the surface of the respective fluids, it is only necessary to incline the instrument, making the end towards C the lowest.

For ordinary purposes this instrument is very convenient; it is not easily deranged, and it can be adjusted in a moment.

For a description of other instruments of the kind, see the treatise on the Thermometer and Pyrometer, in the "Library of Useful Knowledge."

PROBLEM XVIII.

Construction of another kind of thermometer, which measures heat by the dilatation of a bar of metal.

The property which all metals have of dilating by heat, serves as a principle for the construction of another thermometer, exceedingly useful, as much greater degrees of heat can be measured by it than by other thermometers; for a spirit-of-wine thermometer cannot measure a degree of heat greater than that acquired by spirit of wine in a state of ebullition; and a mercurial thermometer cannot measure any degree of heat greater than that of boiling mercury. It was perhaps for this reason that Newton employed, in his thermometer, linseed oil; for it is well known that fat oils, before they are brought to ebullition, require a degree of heat much greater than that which fuses the greater part of the metals and semi-metals, such as lead, tin, bismuth, &c.

Muschenbroeck is the inventor of this new kind of thermometer, called also *Pyrometer*. Its construction is as follows.

If we suppose a small bar of metal, 12 or 15 inches in length, made fast at one of its extremities, it is evident, that if it be dilated by heat, it will become lengthened, and its other extremity will be pushed forwards. If this extremity then be affixed to the end of a lever, the other end of which is furnished with a pinion adapted to a wheel, and if this wheel move a second pinion, the latter a third, and so on, it will be evident that, by multiplying wheels and pinions in this manner, the last one will have a very sensible motion; so that the moveable extremity of the small bar cannot pass over the hundredth or thousandth part of a line, without a point of the circumference of the last wheel passing over several inches. If this circumference then have teeth fitted into a pinion, to which an index is affixed, this index will make several revolutions, when the dilatation of the bar amounts only to a quantity altogether insensible. The portions of this revolution then may be measured on a dial-plate, divided into equal parts; and by means of the ratio which the wheels bear to the pinions, the absolute quantity which a certain degree of heat may have made the small bar to dilate, can be ascertained; or, by the dilatation of the bar, the degree of heat which has been applied to it may be determined.

Such is the construction of Muschenbroeck's pyrometer. It is necessary to observe, that a small cup is adapted to the machine, in order to receive the liquid or fused matters, subjected to experiment, and in which the bar to be tried is immersed.

When it is required to measure, by this instrument, a considerable degree of heat, such as that of boiling oil or fused metal, fill the cup with the matter to be tried, and immerse the bar of iron into it. The dilatation of the bar, indicated by the turning of the index, will point out the degree of heat it has assumed, and which must necessarily be equal to that of the matter into which it is immersed.

This machine serves to determine the ratio of the dilatation of metals; for by substituting, in the room of the pyrometric bar, other metallic bars of the same length, and then exposing them to an equal degree of heat, the ratios of their dilatation will be shewn by the motion of the index.

A TABLE *of the different degrees of heat at which different matters begin to melt, to freeze, or to enter into ebullition, according to the thermometers of Fahrenheit, Reaumur, and Celsius.*

Names of the Matters.	Degrees of Fahrenheit.	Degrees of Reaumur.	Degrees of Celsius.
Mercury congeals	— 39	— 31½	— 39½
Mercury boils	708	300	375
Water freezes	32	0	0
Water boils	212	80	100
Rectified spirit of wine freezes	— 33	— 29	— 36
The same boils	175	63½	79
Brandy consisting of equal parts spirit and water freezes	— 7	— 17½	— 21⅞
The same boils	190	70	87½
Water saturated with marine salt boils	218	82⅔	103⅓
Lixivium of wood ashes boils	240	92⅓	114
Burgundy, Bourdeaux, &c., wine freezes ...	— 20	— 5⅘	— 7¼
Spirit of nitre freezes	— 40	— 23	— 40
The same boils	242	93⅓	116
Wax melts	142	49⅘	62¼
Butter melts	80 to 90	21 to 26	26 to 32
Oil of turpentine begins to boil	560	234	292
Olive oil becomes fixed	43	5	6¼
Rape-seed oil boils, and is ready to inflame ...	714	298	372
Tin fused	408	167	309
Lead fused	540	226	282
Bismuth ditto	460	190	238
Regulus of anatomy ditto	805	344	430

TABLE *of the different degrees of heat or cold observed in various parts of the earth, or in certain circumstances, or in consequence of certain operations, according to Reaumur's thermometer.*

	Degrees.
Constant heat of the vaults below the observatory of Paris	9¼
Heat at which chickens are hatched	35
—— at which silk-worms are hatched	19
—— for an orangery	15
—— for pine-apples	18
—— for the chamber of a sick person	17
—— for a stove	12
—— of the human skin	29 to 30
—— of the interior of the human body	31
Fever heat	32 to 40
Heat observed at Paris in 1753	30⅛
———— at Senegal	37
———— in Syria in 1736	35
———— at Martinico	32
Cold observed at Paris in 1766	— 9¾
———————— in 1740	— 10½
———————— in 1754	— 12
———————— in 1767	— 13
———————— in 1768	— 14½
———————— in 1709	— 15½

		Degrees.
Cold observed at Paris in 1776	—	16¾
——— at Petersburgh December 1759	—	33½
——— December 1772	—	50
——— at Torneo in 1737	—	37
——— at Quebec	—	37
——— at Epsal in 1733	—	40
——— at Kiringa in Siberia in 1738 (See *Flora Siberica*)	—	70
Artificial cold with spirit of nitre and snow cooled to 33 degrees	—	170

Table of the ratio of the dilatation of metals by heat, according to Mr. Ellicot.

Names of Metals.	Respective Dilatation.	Names of Metals.	Respective Dilatation.
Gold	73	Iron	60
Silver	103	Steel	56
Copper	89	Lead	149
Similor	95	Tin	148

OBSERVATIONS ON THE PRECEDING TABLES.

I. The first observation we shall here make, is on the congelation of mercury by an extraordinary degree of cold. This singular experiment was made, for the first time, at Petersburgh in the month of December 1759, and deserves that we should here give a particular account of it.

The cold having become very intense in that city, in the month of December 1759, Mr. Braun embraced that opportunity of making some experiments on the artificial cold that could be produced assisted by its means. He put into a glass vessel snow already cooled to 208 degrees of Delisle's thermometer, or 31 degrees of Reaumur, and having cooled to the same degree good fuming spirit of nitre, he poured it upon the snow. He then immersed in the mixture the bulb of a thermometer, so constructed that the scale of it extended about 6000 degrees, both above and below zero, which in Delisle's thermometer is the point of boiling water, and saw with astonishment the mercury rapidly descend to the 470th degree below that term. The mercury having then stopped, Mr. Braun shook the thermometer, and found that the mercury had no motion. He broke the bulb, and found that the mercury was completely frozen. This experiment was repeated either the same day, or on the 26th of December, when the natural cold was still more intense, and the mercury fell to the 212th degree of Delisle's thermometer, or the 33d of Reaumur. Several of the Academicians of Petersburgh were present at the latter experiment, and confirmed the truth of it. The small ball of congealed mercury was hammered, and it appeared to have the ductility of lead.

One thing very singular, and which Mr. Braun remarks with astonishment, is, that in several of these experiments, the mercury fell with moderate velocity from the point of the temperature of the air to that of 470 degrees below zero; but when it reached that term it fell at once below the 600th degree, without the bulb of the thermometer being broken.

This phenomenon, in our opinion, is nearly the inverse of that which takes place in the congelation of water. It is well known that in proportion as water cools, it diminishes in volume; but when it reaches the degree of congelation, it suddenly increases in volume, so that if a thermometer were constructed with pure water, it is probable the water would first fall, and then burst the ball of the thermometer. This is an effect of the new arrangement of the parts which takes place, with a force almost irresistible, at the moment when they are all in contact.

But there is reason to think, that in mercury the contrary is the case; that is to

say, when cooled to such a degree that its component particles are almost in contact, they suddenly arrange themselves in a certain form by their mutual attraction; and this form is apparently such, that in this disposition they must occupy less volume, as those of water occupy more.

But however this may be, it is confirmed by the experiment of Mr. Braun, that mercury is only a metal kept in a state of fusion by a degree of heat much less than that which freezes water, and a multitude of other liquors. We must even remove it from the class of semi-metals, and rank it among the number of real metals.

We find also, in this experiment, the reason why mercury is the most volatile of the metals. Since the degree of heat necessary to keep it in fusion, is so far below that which melts ice, it needs excite no astonishment that at the 300th degree of Reaumur's thermometer it begins to be volatilised; for this degree is about the same as the 600th would be to lead, or the 1200th to copper, &c.

II. Another remark is, that the degree of water beginning to freeze is indeed fixed; but the case is not entirely the same with that of boiling water. It has been found that the more water is charged with the weight of the atmosphere, the greater is the degree of heat necessary to make it boil. This was remarked by M. le Mounier, who found at the summit of the Canignou that boiling water raised the thermometer only to the 78th degree. This has been since proved by other philosophers, such as M. de Secondat, the son of the celebrated Montesquieu, on the Pic du Midi, one of the highest mountains of the Pyrenees, and by Mr. Deluc on a mountain still higher. Water has also been made to boil under the receiver of an air pump, at a degree much below the 80th of the thermometer: this effect may be produced by partly evacuating the air.

It is therefore necessary that this degree of the thermometer should be fixed, taking into consideration the height of the barometer; and in rectified and comparative thermometers, of which we have heard, the 80th degree is that which indicates boiling water when the height of the barometer is 27 inches Paris measure, or 28·8 English inches: this is what we ought to understand by the degree of boiling water.

It has been found also that the thinnest liquors boil at a degree of heat less than water; but that fat oils require a much greater degree.

III. We have rectified, according to the observations of Deluc, or observations made at his request, the temperature of the vaults of the observatory at Paris, which is not 10 as commonly said, but 9½ at most. We have rectified also, by the observations of M. Braun, the degree of boiling mercury, which is generally placed at the 600th degree of Fahrenheit, but which, according to that philosopher, is the 708th or 709th.

IV. In the table of the dilatation of metals, it is seen that steel is that which dilates the least by heat; the next is iron, and the next to that is gold. Lead and tin dilate the most. It appears also by this table, that the dilatability does not follow the ratio of the specific gravities, nor that of the ductility, nor of the strength of these metals: there are even irregularities in their dilatations, on which account it is to be wished that a greater number of experiments were made on this subject, and in a more correct manner.

Remark.—It is rather matter of surprise, that M. Montucla has taken no notice of the accounts of the freezing and fixing of mercury, in the Philos. Trans. for the year 1783, especially as the errors of M. Braun concerning this matter are there corrected, and the degree of cold at which it freezes is ascertained by many different persons, both by natural cold, and by artificial mixtures, with perfect satisfaction. It is there proved that the degree of cold at which mercury freezes, is — 39 of Fahrenheit, or 39° below 0 in that scale. It is also shewn that the extraordinary degree of depression of thermometers accompanying frozen mercury, which deceived M. Braun and some

other persons, is owing to the sudden contraction of mercury in the act of freezing, and after it; contrary to the nature of water, which expands and enlarges in the same circumstances. Hence it happens that congealed mercury, becoming more dense and compact, sinks in fluid mercury; while common ice, or congealed water, floats in that fluid.

PROBLEM XIX.

What is the cause of the intense and almost continual cold experienced on the tops of high mountains, and even of those situated in the torrid zone; while it is hot in the neighbouring plains or valleys?

The cold experienced on high mountains, while the neighbouring plains are exposed to the most violent heat, is a phenomenon which has long excited the attention of philosophers. It is now known that one of the hottest climates in the world is the coast of Peru, and yet those who gradually ascend the Cordilleras from it, observe that the heat progressively decreases; so that when they have got to the valley of Quito, at the height of about 1400 toises above the level of the sea, the thermometer, in the course of the whole year, scarcely rises 13 or 14 degrees above zero. If they ascend still higher, this temperature is succeeded by a severe winter, and when they get to the perpendicular height of about 2400 toises, they meet with nothing, even under the equinoctial line, but eternal ice.

Some philosophers have asked how this is possible. In proportion as they rise above the surface of the earth, they approach the sun, consequently his rays ought to be warmer; and yet they experience the contrary. Some have thence concluded that the rays of the sun are not the principle of the heat which we experience; for if they are, say they, how comes it that they have less activity exactly in the place where they ought to have more? This paradox we shall endeavour to explain.

It must first be observed, that when people ascend to the height of some thousands of yards above the surface of the earth, it is wrong for them to conclude that the rays of the sun ought to have more activity there than at the surface. This difference would be insensible even if they should ascend to a height equal to the earth's semi-diameter, or some thousands of miles, for the sun being at the distance of 22000 semi-diameters of the earth, and as the heat of the sun's rays increases in the inverse ratio of the squares of the distances, the direct heat of the sun at the height of a semi-diameter of the earth will be to that experienced at the surface, as the square of 21999 to the square of 21998; a ratio which will be found to be that of 10999 to 10998 or of 11000 to 10999; so that the heat would be only one 11000th part less at the surface than at the distance of the earth's semi-diameter above it, a difference quite insensible. What then can be the difference at the height of four or five thousand feet above the surface of the earth? certainly nothing, and therefore no attention ought to be paid to it.

But there are very sensible physical causes, in consequence of which bodies are less susceptible of heat, and while on those elevated parts of the earth retain it a shorter time than when they are nearer the surface. It is certain that the heat which we experience at the surface of the earth is not merely the effect of the direct heat of the sun, but of several causes united.

These causes are, 1st. The mass of the heated bodies, which retain longer the heat they have received, according as they are denser and more voluminous: hence the terrestrial bodies retain, even in a great measure throughout the night, the heat communicated to them during a fine summer's day. The day after this they receive another accession of heat by the presence of the sun; and so on in succession.

2d. The air being more dense in the plains and valleys, it retains a greater portion of the heat it receives in the day-time, and prevents the dissipation of the heat communicated to the earth. For this reason, the heat continually increases in the lower

grounds, as the sun rises above the horizon; but on the summits of the mountains the case is not the same.

In the first place, the air in those high regions is much rarer than at the surface of the earth. No sooner is the sun sunk below the horizon, than it loses the heat it received in the course of the day; for every person must have observed, that a dense body, such as a piece of money, retains heat longer than a body of little density, such as a bit of cloth. If you approach a large fire, and stand some time before it, you will find the money in your pocket burning hot; if you retire, it will be in this state for a long time, while your clothes will retain only a common degree of heat. Hence, the small quantity of heat which the thin air of the mountains has received in the course of a summer's day is soon dissipated; it is not accumulated there as in the lower regions, where the contact also of dense terrestrial bodies, violently heated, contributes to maintain it in that state. In the second place, the exceedingly high insulated peaks of these mountains are only small masses, when compared with the whole of the terrestrial bodies in the plains and the valleys. If they are heated to a certain point, the heat they have received is speedily evaporated; and this evaporation is promoted by the coolness of the surrounding air, which is lowered to the temperature of ice, almost as soon as the sun has set.

Hence it may be easily conceived that the air, which surrounds high mountains, acquires only in a very transient manner a certain degree of heat; that it is almost always below the temperature even of ice; that on this account all the aqueous meteors there formed are converted into snow and ice, that when a certain mass is once formed, it will oppose the introduction of heat, either into the surrounding air, or into the parts which it covers, and this new obstacle will tend to increase the cold and the mass of the ice. In this manner have been formed those accumulated masses of snow and ice which cover the summits of the Cordilleras, as well as some parts of the Alps and the Appennines; in short, all those mountains of the universe whose height exceeds a certain limit, which in the torrid zone is about 2400 toises perpendicular elevation above the level of the sea.

We must here remark that this height is less as the latitude is greater: thus, in the torrid zone, you must ascend to the height of 2400 or 2500 toises to arrive at those regions of perpetual ice; but in the temperate zone, for example, these eternal glaciers will be met with at the height of 1400 or 1500 toises. The commencement of those found in Switzerland, according to the measurement of Mr. Deluc, is at the height of 1500 toises above the level of the Mediterranean; and on proceeding farther north they will be found near the level of the sea. The glaciers of Norway are certainly less elevated than those of Switzerland. In short, in the frigid zone that region of continual ice is at the surface of the earth. Hence it happens that in those regions the ice, as is well known, never melts. Both the arctic and antarctic poles are surrounded, to the distance of several hundreds of leagues, with circular bands of ice; which, according to every probability, exclude all hopes of ships being ever able to traverse the frozen ocean, in order to proceed through the seas of China and Japan to the passage known to exist between Asia and America.

When the sun's rays pass through a perfectly transparent medium, they are not found to impart to it the slightest increase of temperature, and hence if our atmosphere were perfectly transparent, it could receive no heat from the sun by direct radiation. It is only when the solar rays meet with dark or opaque substances that they give out or impart *sensible heat*. The surface of the earth being thus warmed by the immediate influence of the sun's rays, the temperature thus acquired is slowly imparted to the air in contact with it; which, being rarefied, ascends upwards, gradually giving off its heat during its ascent, till it reaches a situation where its specific gravity is the same as that of the surrounding air; when its tendency to rise ceases; and, for a moment, it becomes stationary. The descending portions of air are affected

in a way the reverse of this; giving out a portion of their latent heat as they sink, till they arrive at the surface, where, being heated by coming again in contact with the earth, they re-ascend, to begin anew the same round of changes in their thermal condition.

The heat at the surface of the earth then gives rise to a succession of ascending and descending currents; and the principal cause of the difference of temperature at the top and bottom of a column of atmosphere is the exhalation and absorption of heat, caused by the alternate condensation and rarefaction of the air. The imperfect manner in which heat is conducted through fluids secures the lower strata of the atmosphere from loss of heat by transmission; and hence a diminution of temperature is observed as we ascend above the surface of the earth.

The law which regulates the diminution of heat as we ascend in the atmosphere, varies with the latitude as well as the season of the year. In ascending from Geneva to Chamouni, Saussure observed that Fahrenheit's thermometer fell 1° for 236 feet; and on another occasion 1° for 266 feet. And from such observations he assigned, as a mean, 1° Fahrenheit for 292 feet in summer, and for 419 in winter. M. Ramond gives 299, M. d'Aubuisson 315, and Guy Lussac 341, for 1° Fahrenheit. At the time of the experiment from which the last-named result was obtained, the heat at the surface of the earth was very great.

From the observations made by Humboldt among the Andes, we learn that the heat does not decrease uniformly as the height increases; and also that the rate of diminution is greatly affected by local circumstances. He observed, for instance, on one occasion, that the decrease became slower between 1000 and 3000 metres of ascent; and was more particularly retarded from 1000 to 2500; but that it afterwards increased between 3000 and 4000 metres. The following table shews a few of his results.

	Height in Metres.		Change for 1 deg. Fahrenheit. English feet.
From	0	to 1000	309
	1000	2000	536
	2000	3000	423
	3000	4000	239
	4000	5000	328

His mean result for the whole ascent is 346 feet for 1° Fahrenheit. He ascribes the slow decrease of heat between 1000 and 3000 metres of height to the absorption of light by the clouds, to the formation of rain, and to the dispersion of heat by radiation from the lower strata of the clouds.

It may however be inferred from the above observations, that though the diminution of heat as we ascend is not perfectly uniform, yet it may be considered as nearly so for all heights that we can have access to.

Lagrange is inclined to adopt the hypothesis of uniform decrease; but Euler considers that a harmonic progression is more in accordance with appearances.

The late Professor Leslie has given a formula deduced from the capacity of air for heat under different degrees of density. His formula is this: b denoting the pressure of the barometer at the lower station, and β that at the higher; then the difference of temperature in degrees of the centigrade thermometer is represented by $25 . \left\{ \frac{b}{\beta} - \frac{\beta}{b} \right\}$.

Mr. Leslie admits that the co-efficient (25) may require alteration, and that in many places it may better to take 30 for the multiplier in summer and 25 in winter.

It is evident that in every climate a point of elevation may be reached, where it will be continually freezing; and that the height of this point will depend both on

the local situation of the place and the season of the year. Near the equator, Bouguer noticed that it began to freeze on the sides of the lofty mountain Pinchincha, at the height of 15577 feet above the level of the sea; whereas perpetual congelation was found by Saussure to take place on the Alps at the height of 13428 feet.

A curve traced on the meridian through the points at which it constantly freezes, is called the line of *perpetual congelation.*

The following table exhibits the vertical heights of this curve as computed by Kirwin, for every 5° of latitude up to 80°; and a little way beyond that latitude the curve probably coincides with the surface of the earth..

Lat.	Mean height of curve of perp. cong.	Lat.	Mean height of curve of perp. cong.	Lat.	Mean height of curve of perp. cong.
	feet.		feet.		feet.
0	15577	30	11592	60	3684
5	15457	35	10664	65	2516
10	15067	40	9016	70	1557
15	14498	45	7658	75	748
20	13719	50	6260	80	120
25	13030	55	4912		

Dr. Brewster, on the supposition established by Humboldt, that the line of constant freezing is different from that of perpetual snow, has given the following formulæ.

Let t = the mean temperature in degrees of Fahrenheit, and b = the latitude in degrees; then the height in English feet of the curve of perpetual congelation is $310 . t - 32°$, and of the line of perpetual snow the height is $310 . \overline{t - 32°} + 48\, l$.

For the Alps and the west of Europe in general t may be taken as equal to $81\frac{1}{2}° . \cos. l$; and thence the above formulæ may be transformed into others, depending on the latitude only.

The curve of perpetual congelation must evidently be higher in summer and lower in winter; and though the difference is not great in tropical climates, in higher latitudes it is very considerable.

PROBLEM XX.

Of the attenuation of which some matters are susceptible: Calculation of the length to which an ingot of silver may be wire-drawn, and of the thickness of gilding.

We shall not here examine the question which has so much engaged the attention of philosophers, whether matter be divisible or not *in infinitum.* To resolve this question, it would be necessary to be acquainted with the ultimate moleculæ or elements of bodies, which in all probability are placed beyond our reach. But nature and art present to us some instances of the attenuation of matter, which, if they do not prove its divisibility *in infinitum*, prove at least that the boundaries of this division are removed beyond what the imagination can conceive.

The ductility of silver and gold supplies us with two of those examples furnished by art. An ounce of gold is a cube of $5\frac{1}{4}$ lines on each side; so that one of its faces will consequently cover about 27 square lines. This cube a gold-beater reduces into leaves, which, all together, would cover 146 square feet. But 27 square lines are contained 111980 times in 146 square feet, consequently the thickness of this gold leaf is the 111980th part of $5\frac{1}{4}$ lines, or the 21534th part of a line.

But we can go still farther; for this attenuation is nothing in comparison of the following.

A cylindric ingot of silver, weighing 45 marcs, about 22 inches in length, and 15 in breadth, is covered with six ounces of gold reduced to gold-leaf. The thickness of the gold in this state, called gilding, is about the 15th part of a line. But only one ounce of gold may be employed; and in this case the thickness of the gilding will be only the 90th part of a line.

The ingot thus gilt is made to pass through several holes in succession, each smaller than the other, till it is reduced to a wire of the thickness of a hair. M. Reaumur took a wire of gilt silver, drawn out in this manner, and having weighed half a gros of it, with the greatest nicety, measured its length, which he found to be 202 feet: whence it is easy to conclude that the gros must have been 404 feet in length; the ounce 2232, the marc 23856, and the 45 marcs 1163520 or 96 leagues of 2000 toises each. Here then we have an ingot of silver, 22 inches in length, drawn out in such a manner as to form a wire of 96 leagues in length.

Nay more, this gilt wire is made to pass between two rollers of polished steel, to flatten it and reduce it to a thin plate. This operation, by rendering it $\frac{1}{8}$ of a line in breadth, lengthens it a seventh part more at least; so that the wire by these means is converted into a thin plate 110 leagues in length, with the thickness of the 256th part of a line. In regard to the gold it will be found that its thickness is only the 59000th part, and even the 60000th part of a line.

Thus, if we suppose the ingot of silver to have been gilt with two ounces of gold, its thickness would be the 175000th part of a line; and supposing only one ounce of gold, the thickness would be the 350000th. But as there are some places of the plate unequally gilt, if we suppose that these are a half less than the rest, it will be found that the thickness of the latter will be only one 525000th of a line.

Lastly, it is well known that this plate may be made to pass a second time under the steel rollers, bringing them nearer to each other, in such a manner as to render its breadth double; hence it follows that in the latter state there are parts of the gilding where its thickness is only the 1000000th part of a line; which is in the same proportion as a line is to the length of 1200 toises, or half a league.

It is however certain that these gold particles have mutual adhesion and continuity; for if this silver wire be immersed in aquafortis, the silver will be dissolved and the gold will remain like a small hollow tube. Lastly, if the gilding be viewed through a microscope, no trace of discontinuity will be observed.

As the ductility of gold is far greater than that of silver, a much longer wire might be made with an ingot of gold of the same weight. But can we believe what is related by Muschenbroeck on this subject? This philosopher says that an artist of Augsbourg made a gold wire weighing only a grain, which however was 500 feet in length. He could therefore have made a gold wire a league in length, and weighing only a dram, or the third of a gros; a wire 24 leagues in length would have weighed only one ounce; and with a pound of gold he could have made a wire 192 leagues in length. A wire of this size, capable of encompassing the globe of the earth, would have weighed only about 50 pounds.

But we can shew that a thread, the work of an insect, surpasses in fineness the wire ascribed to the artist of Augsbourg. It has been observed that a single thread of silk 360 feet in length, weighs a grain; 24 grains therefore will give 1440 toises, and 36 grains a league of 2160 toises: an ounce of this thread will extend 16 leagues, and a pound 128: in short, a thread of this kind capable of encompassing the globe of the earth would weigh no more than 70 marcs, or 35 pounds. We shall here add, that the thread of a spider's web, which is much finer and lighter than the thread of a silk-worm of the same length as the above, would weigh only two marcs, or a a pound.

PROBLEM XXI.

Continuation of the same subject: Division of matter in the solution of bodies, and in odours and light.

But new subjects of admiration present themselves to us in the prodigious smallness of some parts of matter: these we shall here add, with an account of their affinity.

Metallic solutions afford the first example. Dissolve a grain of copper in a sufficient quantity of volatile alkali; and you will obtain a liquor of a blue colour. If you pour this solution into three pints of water, the whole water will be sensibly coloured blue. But three French pints make 144 cubic inches; and as each inch in length may be divided into lines, then into tenths of a line visible to the eye, it will be found that in these 144 cubic inches, there are 248832000 of such parts, every one of which is coloured blue. A grain of copper is divided then, by these means, into at least 248832000 parts. But we shall go still farther; each of these parts may be seen by a microscope that magnifies objects 100 times in length, consequently 10000 times in surface; and every one of them will be found to be coloured: if we therefore multiply the above number by 10000, or add to it four ciphers, we shall have a grain of copper divided into 2488320000000 parts, visible to the eye, at least when assisted by the microscope.

Let us now proceed to odours. It is said that a grain of musk is capable of perfuming, for several years, a chamber 12 feet in every direction, without sustaining any sensible diminution in its volume, or its weight. But a space such as the above contains 1728 cubic feet, each of which contains 1728 cubic inches, and each of these 1728 cubic lines; so that the number of cubic lines is the third power of 1728. It is probable, that every one of these cubic lines contains some of the odorous particles; the air of the chamber may in the course of several years be renewed 1000 times; and the grain of musk, without sensible alteration, may furnish new odorous particles. In calculating the tenuity of each of these, the imagination is lost.

However, notwithstanding the tenuity of these odorous particles, they do not pass through glass and metals; and there are certain *effluvia* which penetrate them; such as those of luminous bodies or light, magnetism, and electricity. How great then must be the tenuity of the particles of which these consist! But we shall confine our observations to light.

If those particles, the emission of which are supposed to constitute light, were not of a smallness almost infinite, there is no body which could resist the action of the weakest light; for their multitude, and the rapidity with which they proceed from the luminous body, are such, that without this prodigious tenuity light would break to pieces every body on which it might fall, instead of exciting in it that gentle vibration, that insensible tremulous motion, in which heat consists, when it has only the density of the light of the sun.

Light, indeed, in a second passes over 128880 leagues, or 257760000 toises; consequently, if a particle of light were only equal to the 257760000th part of a grain of lead, a line in diameter, it would make on our organs the same impression as a similar grain of lead impelled with the velocity of a toise per second. There is no doubt that such an impression would be very sensible to the delicate parts of our bodies. But what would it be if millions of millions of such globules should strike against it, and be followed at an interval of time infinitely small by a like quantity of others, as is the case when our body is exposed to the light! No human being could resist it.

The tenuity of a particle of light then is still far below that which we have assigned to it as its first limits. Let us endeavour to determine another, that may approach nearer to the truth.

The density of the sun's light, such as it is when it reaches the earth, in our climates, is of such a nature, that if diffused throughout a space 250000 times greater, it would have a splendour equal to that of the full moon.

It is probable that the latter, diffused in the like manner, would be equal, at least, to that of a glow-worm, which enlightens an object at the distance of 10 feet; consequently the latter will be found by calculation to be 26500000000 times weaker. It is besides very probable, that in the pupil of the eye, which at that distance beholds the light of the glow-worm, there is no part which is not itself sensibly enlightened: let us suppose it to be a square line of surface, and that this square line is divided into 10000 sensible parts; every moment therefore there are 10000 globules of light, which reach the retina, united in one imperceptible point, and with the velocity of 257760000 toises per second, without producing however a sensible impression, and even scarcely the perception of light.

If we suppose the same quantity of globules of light thrown by the weakest light on a square line of surface, it will be found that in a line square of the sun's light, there are 625000000000000, and in an inch 90000000000000000. This quantity of globules, moved with the velocity of 257760000 toises per second, and renewed perhaps a thousand times in that interval, would produce however in the palm of the hand but a slight sensation of heat; and hence there is reason to conclude, that 900 thousand millions of millions of these particles, moved with the above velocity, make less impression than the shock of a leaden ball, a line in diameter, which falls from the height of three feet. And hence arises this new consequence, that if we suppose the particles of light to have the same density as lead, each of them, compared with a ball of lead a line in diameter, is in a less ratio than a 257760000th by 90000000000000000, or a 23198400000000000000000000th part to unity.

Such, then, at least, is the tenuity of the particles of light; and by other reasoning perhaps we might prove that it is still much rarer; so that in all probability the above ratio must be reduced to that of unity to a comparative number of 30 or 35 figures. But we shall confine ourselves to what has been already said, because it is sufficient for our purpose, and to shew, as we have done elsewhere, that the sun, for several successive ages, may furnish, without any sensible diminution, sufficient matter for the emission of the light which proceeds from him; and this may serve to answer an objection which has been made to the Newtonian theory of light.

PROBLEM XXII.

What velocity ought to be given to a cannon bullet, in a horizontal direction, to prevent it from falling to the earth, and to make it circulate around it like a planet, supposing the resistance of the air to be destroyed?

If a cannon bullet be fired off in a horizontal direction, from the top of a mountain, it will fall to the earth, as is well known, at a certain distance. If we now suppose that the velocity communicated to this bullet is more and more increased, it will fall at a greater and greater distance; for the parabola, or rather ellipsis, it describes, will be broader and broader. We may therefore conceive the velocity to be so great, that the bullet shall fall to the earth at the point diametrically opposite to that from which it was fired. In this case, if the velocity were increased ever so little, the bullet would not touch the earth, but would return to the point from which it set out; describing a line similar to that which it before described. It would then continually move in an elliptical line, around the earth, and really be a small planet, performing its revolution around it.

The question then is, to find what would be the periodical time of this revolution; for by knowing this time, we could easily find the velocity of the small planet,

or that with which the bullet set out; because nothing would be necessary but to divide the space passed over, which in this case is nearly the circumference of the earth, by the time employed in passing over it.

The solution of this problem may be easily deduced from the celebrated rule of Kepler; for if we suppose our small planet in motion, it must, compared with the moon, perform its revolutions in such a manner that the squares of the periodical times shall be as the cubes of the distances. But the mean distance of the moon from the earth is 60 semi-diameters, and that of the small planet will be equal to the earth's radius, or 1 semi-diameter. We shall consequently have this ratio, as the cube of 60 or 216000 is to 1, so is the square of the periodical time of the moon to the square of the periodical time of the small planet. But the periodical time of the moon is 27 days 8 hours, or 656 hours, the square of which is 430336; if we then say, as 216000 is to 1, so is 430336 to a fourth term; we shall have for this fourth term $\frac{430336}{216000}$; or in decimals 1·9923; the square root of which 1·41, will express the number of hours employed by the small planet in its revolution. But 1·41 in hours and minutes is equal to 1h. 24m. 36s. The small planet, therefore, would perform its revolution in that time; which, supposing a great circle of the earth to be 24000 miles, gives nearly 282 miles per minute, or 4·7 miles per second.

If a velocity greater than the above, but less than 149½ leagues, were given to this body, it would describe an ellipsis, the perigeum of which would be in the point of departure. If the velocity of the projection were 149½ leagues per minute, or greater, the body would not return to the earth; for in the first case it would describe a parabola, the summit of which would be in the point of projection, and in the second it would describe an hyperbola.

PROBLEM XXIII.

Examination of a singular opinion respecting the Moon and the other planets.

It has been said, and the singularity of the conjecture has given it some importance, that the moon may be nothing else than a comet, which in approaching to or receding from the sun, and passing at the proper distance from the earth, may have been diverted from its course, and thus have become that secondary planet which accompanies our earth. For, if we suppose that such a comet, having only the projected motion necessary for describing a circle around the earth, at the distance of 60 semi-diameters from its centre, really passed at that distance from our globe, and in a plane inclined to its orbit, it must necessarily, say some philosophers, have become our moon.

This conjecture is supported by some remarks which seem to give it a certain degree of probability. The moon, say they, when viewed through a telescope, presents the appearance of a body which has been torrefied; the cavities interspersed over its surface seem to be fissures, occasioned by the intense heat which caused the moisture it contained to escape in vapours; and they add, that no appearance of humidity now remains in the moon, since it has no atmosphere. All this agrees exceedingly well with a comet, which has passed very near the sun.

It is also to be observed, say they, that the largest planets, such as Jupiter and Saturn, have several satellites; for as their attraction extended much farther than that of the earth, they had a far greater power over the comets which passed in their neighbourhood, the motion of these comets having been besides lessened in consequence of their distance from the sun. The small planets, such as Mercury, Venus, and Mars, have no satellites, on account of the smallness of their size, and the velocity with which comets pass them, in advancing towards, or receding from the sun.

These ideas are ingenious; but this assertion or conjecture, when examined according to the principles of geometry, cannot be maintained.

It is found, indeed, by calculation, that whatever may be the position or magnitude of the orbit of a comet, it cannot, when it passes near the orbit of the earth, have the velocity necessary to make it become a satellite to it, whatever may be the proximity at which it passes; for it can be demonstrated that every comet, when it approaches the sun within a distance equal to that of the earth, has at that moment a velocity in its orbit, which is to that of the earth, as $\sqrt{2}$ is to 1, or as 1414 to 1000. But this velocity is far greater than that of the moon in her orbit, and even greater than that of a planet which should circulate almost at the surface of the earth, as the following calculation will shew.

The earth in about 365 days passes over an orbit of 597 millions of miles in circumference: its velocity then in its orbit is such, that it passes over in a day 1635616 miles; in an hour 68150; and in a minute 1136; therefore, if we multiply the last number by $\frac{1414}{1000}$, we shall have nearly 1606 miles for the space which every comet, when it arrives at the distance of the earth from the sun, necessarily passes over in a minute.

Let us now examine that of the moon in her orbit. The mean diameter of the moon's orbit is about 60 times the earth's diameter; consequently its circumference will be 188 of these diameters; which, estimating the earth's diameter at 8000 miles, gives for the circumference of the lunar orbit 1504000 miles. This space the moon passes over in 27 days 8 hours, wanting a few minutes; or $27\frac{1}{3}$ days: the moon therefore in her orbit passes over in a day 55024 miles; in an hour 2293, and in a minute 38. Hence it is evident, that if a comet should pass at a distance from the earth equal to that of the moon, which the comet transformed into our satellite might do, it could have a velocity of no more than 38 or 40 miles per minute, instead of 1606, which every comet necessarily has at that distance from the sun. The moon then could not be a comet, which passing too near the earth was, as we may say, subdued and carried away by it.

Let us now see whether the comet in question, by passing much nearer the earth, and even close to its surface, could be attracted by it. We shall find, by a similar calculation, that it could not circulate around the earth; for we have already seen that a body, to circulate round our globe near its surface, would require a velocity of almost 300 miles per minute. But this is far below the velocity which a comet passing very near the earth would necessarily have; for if a body should be projected from the summit of a mountain, towards the East or West, with the velocity of 1600 miles per minute, it would recede from the earth without ever returning to it, that velocity being much greater than is necessary to make it describe around the earth any ellipsis whatever, or even a parabola.

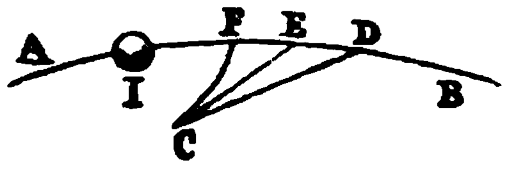
Here then the Earth, and no doubt Mars, is excluded from the privilege of ever being able to obtain a satellite in that manner, and this will hold good much more in regard to Venus and Mercury. But is this the case with Jupiter and Saturn? We shall examine this question also, by employing the same kind of calculations.

The velocity of Jupiter's revolution around the sun, is 494 miles per minute; consequently the velocity of every comet advancing to, or receding from the sun, when at the same distance as Jupiter from that luminary, will be about 700 miles in the same time. It is found that the velocity of the first satellite of Jupiter, in its orbit, is 37366 miles per hour, or 622 per minute: the velocity then of every comet which should pass Jupiter at the distance of his first satellite, would necessarily be rather more considerable, being about a seventh part more. Hence it follows, that neither the first satellite nor any of the rest was originally a comet, which this large planet appropriated to itself; for the other satellites have a velocity still less than that of the first.

It now remains to determine whether a comet, in passing near Jupiter, could be stopped by it. This indeed does not appear to be absolutely impossible: for a sa-

tellite which should perform its revolution near the surface of Jupiter would employ a little more than three hours; which gives a velocity of 1557 miles per minute. But we have already seen that the velocity of the comet would be no more than 700; which therefore is not too great to make a body describe even a circle around Jupiter, very near his surface. If a comet then, in advancing to or returning from the sun, should fall into the system of Jupiter, between him and his first satellite, it might continue to circulate around that planet in an orbit, if not circular, at least elliptical, and more or less elongated.

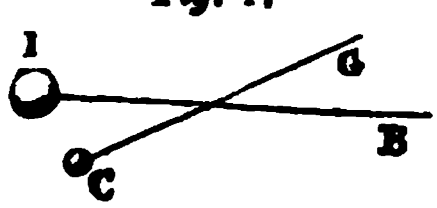
Fig. 16.



For let us suppose A B (Fig. 16.), to be the orbit of Jupiter; and that this planet is at I, proceeding towards B, and that the comet is in C, proceeding towards D, at an angle of about 45 degrees; and that C D denotes the velocity of the comet, which we have found to be greater than that of Jupiter in his orbit. If D E be made equal to the velocity of Jupiter, C E will be the velocity of the comet, and even its route, in regard to Jupiter, supposed to be fixed and without any action on the comet. But on account of this action it would describe an inflected route, such as C F, which would make it fall almost in a perpendicular direction on the orbit of Jupiter, and with a velocity but little greater than that of the first satellite. If at the moment then when Jupiter was at the point I, so situated as to make I F less than the distance of Jupiter from his first satellite, we do not see what could prevent the comet from assuming around him that circular or elliptical motion suited to its projectile force; and if it should perform the revolution once, it is evident that it ought to continue it.

We however confess, that we have not yet examined this point so far as to be able to assert that it is fully demonstrated. To be assured of it, the following question, which is only a branch of that of the three bodies, but which would require too much intricate analysis for this work, must be answered. Two bodies, I and C,

Fig. 17



(Fig. 17.), which attract each other in the inverse ratio of the squares of the distances, and in the direct ratio of their masses, being projected from the points I and C, according to the directions I B and C G, with given velocities, to find the curves which they would describe. To simplify the problem, we might even suppose one of them, I, to be so large in regard to the second, as to be scarcely turned aside from its course.

PROBLEM XXIV.

How far is there reason to apprehend the shock of a Comet; and what devastation would be thence occasioned on the earth?

What has been said respecting comets in the preceding problem, naturally leads us to examine a question become celebrated by the alarm it occasioned at Paris some years ago. In the year 1774, a Memoir was written by one of the Academicians, of which an incorrect account was propagated, and in which it was said the author announced the speedy approach of a comet to the earth, and that the effect of this approach would be at least a rising of the waters of the ocean, so as to overwhelm our continent. In consequence of these reports, the people of that capital were thrown into the utmost consternation and uneasiness. I knew some women who were so terrified, that they did not close their eyes for several nights successively; and I was even obliged, in order to quiet one of them, to assure her that a very great error had been found in the Academician's calculation, and that, on this account, he had fallen into disgrace with the Society to which he belonged. The motive, I hope, will plead my excuse with that illustrious astronomer. I am certain

that in so good a cause he would have indulged in the same innocent deception; for the object was nothing less than to restore rest, and their former lustre, to two eyes capable of deranging the observations of the most insensible astronomer. But however, as I have always been devoted, notwithstanding my taste for the abstract sciences, to that charming portion of the human race, I shall endeavour to tranquillize them, and to prove that the danger of being crushed or inundated by a comet, is not so great as to disturb their repose.

Astronomers long ago conjectured that a comet might become fatal to the earth. The celebrated Whiston, whose imagination was rather too powerful for his reasoning faculty, observing a comet, viz. that of 1680, accompanied with an immense tail, began to conjecture that if any of the planets should happen to meet with this tail, it might by its attraction condense its vapours, and be inundated by it. He supposed farther, that the deluge had been produced by the same cause: and added, that a comet, such as that above mentioned, if it approached near the sun in returning, might acquire a degree of heat several thousand times greater than that of red hot iron; and consequently might consume our earth. He was of opinion also, that the general conflagration, which is one day to destroy the globe we inhabit, will be occasioned in this manner.

These ideas, in which there is more of singularity than truth, are a sufficient proof of what we have already observed in regard to Whiston's disposition. It is impossible to say what might be the case, if a comet, heated to such a violent degree, should pass very near us. It is probable, considering the rapidity with which it would move, when at its least distance from the earth, that we should not be much incommoded by it. In regard to the danger of being inundated by the vapours of its tail, it is entirely void of foundation; for it may be easily demonstrated, that these vapours which float in a medium as thin as ether, must themselves be exceedingly rarefied. There is some reason to believe that all this immense tail, reduced to a fluid, such as water, would scarcely furnish enough for an abundant shower. In short, the comet alluded to, returns only about every 575 years; consequently it will not appear again till about 448 years have elapsed.

Dr. Halley considered this danger in another manner. This philosopher observed, that if the comet of 1680 had passed through the ecliptic 31 days later, its distance from the earth would not have been greater than the sun's semi-diameter, which is about 441623 miles; and he adds, there can be no doubt that such a proximity between these two bodies would have occasioned a considerable derangement in the motion of the earth: such as a change of its eccentricity and periodical time. May the Author of nature, adds he, preserve us from the shock of these enormous masses, or even from their contact, which is but too possible! He however remarks, that the highly varied position of the orbits of comets, and their inclination to the ecliptic, which in general is very great, seem to be arranged by the Author of nature to secure us from so fatal a catastrophe.

As the astronomy of comets, since the time of Halley, has been enriched with the knowledge of very many new ones, it was natural to examine whether there was any of them which, by some change in their position, and the magnitude of their orbits, might become dangerous to our earth. This labour was undertaken by De la Lande, in consequence of the comet seen in 1770; and he found that there are some of them which, by changing their elements a little, might approach very near to the orbit described by our earth. He shewed, at the same time, that there is no great cause for being alarmed at this supposed danger, as several thousands might be betted to one, that if a comet should even pass through the earth's orbit, these two bodies would not fall in with each other.

This danger, as may be seen, was at a sufficient distance to give no great cause of apprehension; but he added, that if we should suppose such a comet to pass at the

distance of 45000 miles, it would raise the waters of the ocean, and according to its position occasion a flux capable of covering our whole continent, and of sweeping away all its inhabitants, together with their habitations. This augmented the danger in a considerable degree; for if 10000 could be betted to 1 that the earth and the comet would not be at the same time in the ecliptic, at the distance of a diameter of our globe, no more than 2000 could be betted to 1 that they might not be at the distance of 5 diameters from each other, and consequently that we might not be drowned. But the stake is so great, that even this small chance cannot be considered without some uneasiness: and there are people who would not hold a chance in a lottery where there is only one blank to a hundred thousand prizes.

But fortunately all these calculations are founded on suppositions which, though they may be realized in the course of ages, cannot take place in the present state of the universe; for the orbit of no comet hitherto known falls in with the path described by the earth in the ecliptic. It is indeed true, that, as the orbits of the planets and comets are subject to insensible variations, it may happen hereafter that the orbit of a comet will intersect that of the earth; but unless it should absolutely coincide with the plane of the ecliptic, that position can be only momentary; and as the revolutions of the comets are exceedingly long, there is a great probability that this position will be changed when the comet passes the ecliptic.

But let us suppose that this position is so constant, that a comet, when it passes the ecliptic, shall be exactly in the same plane, and in the path of the earth; and let us examine, by consulting the laws of probability, what chance there is, that at the moment when the comet is in the ecliptic, the earth will be in a point sufficiently near to come in contact with it. The calculation is as follows:—

At the moment when the comet is in the ecliptic, there are so many positions for the earth in the same circle as there might be terrestrial diameters; but only three of these positions are absolutely critical; for there is one which would give a central shock, and the other two, at the distance of a diameter before or behind the place of the comet, would give merely a superficial shock. But it is found, that the circumference of the earth's orbit contains the diameter of the earth 72450 times; and if this number be divided by 3, the quotient will be 24150. Hence if we suppose a comet to be in the path of the earth, 24150 might be betted to 1, that the latter would not be exposed to any shock, even of the most superficial kind. We may add also, that this dangerous position of the comet is, as we may say, the affair of a moment; for in crossing the earth's orbit, it has a velocity of 4000 miles per minute; consequently the danger would not last above 3 minutes. Some danger certainly might be apprehended if the earth, when a comet is in its proximity, should move in so irregular a manner as to fall in with it, and to block up its way.

The danger of our globe being inundated by the rising of the waters of the ocean is still more unfounded, even if the comet should pass at a very moderate distance from it, such as that of 36000 or 40000 miles, which is about a sixth part of the distance of the earth from the moon. It is indeed true, that if we suppose a comet to fall in exactly with the orbit of the earth, it is only 1 to about 7200, that our globe may not be at a greater distance from it than four or five times its diameter; but the rapidity with which the approach would take place, and with which the two globes would afterwards recede, would not allow the waters sufficient time to rise so as to inundate our continent; for a certain period would be required to communicate to the enormous mass of the waters of the ocean such a movement as that of the flux and reflux. A proof of this is, that the flux, even in the open seas, does not happen till some time after the moon's passage of the meridian; and that the high tides, at the new and full moons, do not occur on those days, but on the following ones. But if a comet should arrive at the earth's orbit, it would traverse

our lunar system nearly in the course of an hour; consequently it could produce only a very slight motion in the open seas, such as the Pacific Ocean. Some of the small islands interspersed in it, which are almost on a level with the water, might be overwhelmed; but our continent would absolutely be sheltered from such a misfortune.

The most singular circumstance, in regard to the terror spread throughout Paris, in consequence of an incorrect account being propagated of M. de la Lande's Memoir, was, that the greatest danger to which the earth had been exposed in the course of several ages, was then past; for of all the comets hitherto known, that of the year 1770 approached nearest to the earth. On the first of July it was at the distance of 2250000 miles, which is only about nine times the moon's distance from the earth.

We shall here observe, that a comet, nearly as large as the earth, and traversing the heavens with a velocity equal to that above mentioned, would be a grand and magnificent spectacle to astronomers. What a noble phenomenon, a new star, of nearly nine degrees apparent diameter, passing over by its own motion about 180 degrees of the heavens in the course of two hours! What astronomer would not wish to behold such an uncommon phenomenon, were it even to occasion some catastrophe to the small uninhabited and already half inundated islands of the vast ocean?

It has however been calculated, that this can never take place, without some great derangement in the motion of our globe. M. de Sejour has found, that if a comet, as large as the earth, should pass it, at the distance of about 40000 miles, it would change its periodical revolution; and this revolution, instead of 365 days 6 hours and some minutes, would become 367 days and some hours. But no physical evil would thence result to the universe. Astronomers indeed would have to recalculate their tables, which would be thus rendered useless; chronologists would be under the necessity of altering their method of computing time, and states would be obliged to change their calendars; but this would only furnish matter for new speculations, and afford more occupation to the learned.

Recent observations on comets shew that their density is in general exceedingly small. Stars invisible to the naked eye were seen through the *centre* of Biela's comet; and though the motions of some comets have been sensibly deranged by the attraction of planets near which they have passed, when in the vicinity of the sun; the motions of the planets or of their satellites have not been affected in the slightest observable degree by the attraction of the comets. It is probable, therefore, that a direct concussion between a comet and a planet would not, to the latter, be an affair of serious moment.

THEOREM

A pound of cork weighs more than a pound of lead or of gold.—A body weighs more in summer than in winter.

These two propositions, on the first view, may appear to some of our readers a paradox; but when they have read the following reflections, the paradox will vanish.

When bodies are weighed in air, which is commonly the case, they are weighed in a fluid which, according to the laws of hydrostatics, always destroys a portion of their weight equal to that of a similar volume of the fluid: hence a cubic inch of lead or of gold, for example, when weighed in air, loses of its absolute weight a quantity equal to the weight of a cubic inch of air; and the case is the same with all other bodies. A pound of cork, under the same circumstance, loses a quantity of its weight equal to that of a volume of air of the same size as the cork. But the volume of a pound of cork is much greater than that of a pound of gold or of lead; consequently a pound of cork, when weighed in air, has a greater absolute

weight than a pound of gold; because, though the weight of the former be diminished by the weight of a greater quantity of air than the latter, they still remain equal.

This reasoning is confirmed by experience; for if a pound of gold or of lead, and a pound of cork, suspended from a good balance, be brought into equilibrium, and if the whole be then covered with the receiver of an air-pump; when the air is exhausted, the cork will be immediately seen to preponderate. In this case, indeed, the weight of the cork is increased by the weight of an equal volume of air; and that of the gold is also increased by the weight of a volume of air equal to itself. But the former is much greater; consequently the equilibrium must be destroyed, and the cork must preponderate. Having thus explained the first paradox, we shall now proceed to the second.

In summer the air is dilated by the heat, and its density being thus lessened, the necessary result is, that the same volume of air is lighter; consequently each of the bodies, brought into equilibrium, loses less of its weight than when the air is denser. But this effect is not produced in the same proportion: the pound of cork, for example, in common air loses 4 grains of its weight, and therefore has an absolute weight of 1 pound 4 grains; while the pound of gold, losing only half a grain, weighs in reality 1 pound and half a grain. In air, dilated so far as to weigh a half less, a volume of air equal to the volume of cork will weigh only 2 grains; and the volume of air equal to that of the gold will weigh no more than a quarter of a grain; hence the pound of cork weighed in common air will weigh in this rarefied air 1 pound 2 grains; and the pound of gold one pound and a quarter of a grain:* the cork therefore will preponderate.

Corollaries.—I. From what has been said this consequence may be deduced: that can weights in equilibrio at the surface of the earth, will not be so when carried to the summit of a mountain. For, on the summit of a mountain the air is more dilated, and therefore, according to the above reasoning, the equilibrium will be deranged, and the most voluminous body will preponderate.

II. The contrary will be the case if the bodies be in equilibrio at the summit of the mountain, and be then weighed at the bottom of it; or if they be weighed at the surface of the earth, and be then carried to the bottom of a mine. In this case, the most voluminous will become the lightest.

III. It would therefore be attended with advantage to purchase gold in summer, and sell it in winter; or to purchase it in a cold place and to sell it in a stove. For gold is generally weighed with copper or brass weights, which in summer lose less of their absolute weight than they do in winter: hence it follows, that in summer they weigh more. By these means, therefore, a larger quantity of gold will be obtained in summer than in winter; consequently, by selling it in winter the buyer will get less.

In purchasing diamonds, a contrary method ought to be pursued, because they are weighed with copper weights, which are specifically heavier. If a weight of copper then be in equilibrio with a weight of diamonds, in air of a mean temperature; on transporting them into cold air, the copper will preponderate; and the contrary will be the case when they are transported into warmer air. Diamonds therefore ought to be purchased in cold air, or in winter, and to be sold in summer, or in warm air.

The difference in both cases is however so small, that it would be a poor speculation to purchase diamonds in winter, with a view of selling them in summer, or to buy gold in summer in order to dispose of it in winter. But the spirit of the mathematics

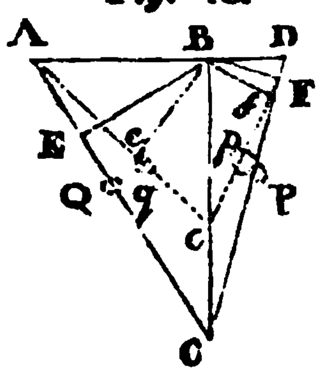
* The weights here given by way of example, are French.

is capable of shewing and appreciating the difference; and though this phenomenon may be of little use in traffic, it is nevertheless a physical and a mathematical truth.

THEOREM II.

Two homogeneous weights which are in equilibrium at the surface of the earth, when suspended from a balance with unequal arms, will not be so when carried to the summit of a mountain, or to the bottom of a mine.

Fig. 18.



Let us suppose a balance (Fig. 18.) with unequal arms, A B and B D, to be loaded with the two weights P and Q, which are in equilibrio, and therefore unequal: if the balance be in a horizontal situation, these weights, as they tend to the centre of the earth, which we suppose to be C, will form with the balance the unequal angles C A B and C D B: consequently the angle A, at the larger arm, will be the least. From the point B, let fall on the lines of direction A C and D C, the perpendiculars B E B F: by the laws of mechanics these perpendiculars will be in the reciprocal ratio of the weights, so that B E will be to B F, in the same ratio as the weight P to the weight Q: that is to say, the product of P by B F will be equal to that of Q by B E.

Now let the balance be removed nearer to the centre of direction; or what amounts to the same thing, let the centre be brought nearer, as to *c* for example, by which means the new directions will be A *c* and D *c*; and let B *e* and B *f* be the new perpendiculars to these lines of direction: if the ratio of B *f* to B *e* be the same as that of B F to B E, or of Q to P, there will still be an equilibrium; but it may be easily demonstrated that this ratio is no longer the same: consequently the product of Q by B *e*, will not be equal to that of P by B *f*; and therefore there will no longer be an equilibrium. It can even be shewn that, when the centre is brought nearer, the ratio of B *e* to B E will be less than that of B *f* to B F; hence it follows that B *e* will be less than is required to make these ratios equal; and in this case the weight nearest the point of suspension will preponderate.

For the same reason the contrary effect will be produced, if the balance be removed farther from the centre, by transporting it, for example, to the summit of a mountain.

It may here be asked, why does the equilibrium subsist, notwithstanding this demonstration? The reason is plain: the centre of the earth is always at so great a distance compared with the length of such a balance, that the lines of direction are sensibly parallel, at whatever height or depth above or below the surface of the earth they may be placed. The difference therefore from an exact equilibrium is so small, that it cannot be observed with the most perfect balances constructed by the art of man.

PROBLEM XXV.

Of the Central Fire.

Those acquainted with the phenomena observed by different philosophers, in the interior parts of the earth, cannot help acknowledging that the surface, even in our climates, is subject to vicissitudes of heat and cold which we experience. At a certain depth, not very great, for it is sufficient to descend only about a hundred feet, the heat is constantly the same, that is to say, about 10 degrees of Reaumur's thermometer, or 54½ of Fahrenheit. This is observed the same in all climates and in all countries.

It is evident, therefore, that the earth, independently of the variable heat of the sun, has a source of heat peculiar to itself, from whatever cause it may arise.

Nay, we shall here shew that the degree of heat which the presence of tne sun, during several months of the year, adds to the internal heat of the earth, or that which it loses by his absence, is only a small part of the internal heat.

To suppose indeed that the degree of cold which freezes water is tne zero, or the 0 degree of heat, would, as before observed, be erroneous; for heat and cold are merely relative terms. If the common liquors of our earth were of the nature of spirit of wine, as the fluids of our bodies would then be proof against congelation, unless they were exposed to a diminution of heat beyond that at which spirit of wine freezes, it is more than probable that we should experience no disagreeable sensation by living in a temperature similar to that which congeals water; on the other hand, if our liquors were of such a nature as to freeze at the degree at which wax begins to become fixed, we should probably experience at this temperature the same sensation as we experience at that which congeals water. Every degree above that term would be heat, and every degree below it would be cold.

Besides, there is no doubt that an absolute degree of cold would congeal all liquors. But spirit of wine congeals only at 29 degrees below zero of Reaumur's thermometer: there is still heat therefore at 28 degrees, though, on account of the disagreeable sensation which we experience, we call it severe cold. We cannot however suppose that this is the ultimate degree of cold. Several reasons, which it would be too tedious to explain here, give reason to think that this absolute degree of cold is a thousand degrees, at least, below the zero of Reaumur's thermometer.

But let us confine ourselves to the 240th degree, which we shall assume as that of the absolute privation of heat, and let us suppose a thermometer the zero of which is placed at that term; or let us substitute, in one of our common thermometers, the degree 240 for that usually marked zero, which is only the degree of the congelation of water: in this case we shall have 250 degrees for the term which we call temperate. But taking the mean degree of heat during summer in our hemisphere, it will be found, that it does not exceed 26 degrees above that of the congelation of water, and consequently 16 above temperate: hence we have for this degree of heat the absolute degree of 266. The thermometer therefore will vary from temperate to the greatest heat 16 degrees in 250, which is somewhat less than the 15th part.

It will be found, in like manner, that the mean degree of the cold of winter, in our northern hemisphere, is 6 degrees below congelation, according to the rate of Reaumur's thermometer; that is to say, 16 degrees below temperate: hence the mean diminution of heat below temperate, occasioned by the absence of the sun, is about a 15th part of the heat marked by the degree 10. Hence it follows, that from winter to summer, the variation of the heat is at most only $\frac{1}{8}$, or as 7 to 8. But it is highly probable, as M. de Mairan has shewn, in the Memoirs of the Academy, for 1765, and Buffon in the supplement to his Natural History, that the ratio of this variation is much less.

The former fixes it at $\frac{1}{32}$, or as 31 to 32; and the latter at $\frac{1}{51}$, or as 50 to 51. But let us confine ourselves to the ratio we have formed, in order that we may set out from a principle fully proved.

The conclusion we thence form, and it is a consequence which cannot be denied, is as follows: In the globe of the earth there is a degree of constant heat, which is at least 7 or 8 times as great as that produced by the presence of the sun while he illuminates it in the most advantageous manner for heating it. Here then we have a fire, or source of heat, which may be called central. It now remains that we should examine the cause of it.

According to some philosophers, this fire is merely the effect of the continual effervescence occasioned by the mineral matters inclosed in the bowels of the earth, when they meet and become mixed with each other. Iron, which appears to be universally diffused throughout nature, and which communicates its colour to argillaceous earths, produces, as is well known, a violent effervescence with the vitriolic acid, which is also very abundant. Hence, say they, is the cause which excites and maintains in the bowels of the earth that continual fire by which it is heated, and which often manifests itself by the eruptions of volcanoes, dispersed in considerable numbers over its surface: volcanoes according to these philosophers, are the chimneys or spiracles of this central fire.

It would be difficult to shew the absolute falsity of this opinion: but it does not appear that a fire of this nature can be general throughout the bowels of the earth. The number of the volcanoes, which exist at its surface, is too small to have a cause so general: there are even very few of them that burn without interruption. The central fire, however, if it be real, must be constant and perpetual; and therefore it is necessary that we should recur to some other cause.

Another, which has long appeared to possess a great degree of probability, is as follows. The central heat, say some philosophers, is nothing else than the heat which the body of the earth, continually warmed by the sun, has acquired in consequence of the presence of that luminary. But let us render this idea more familiar by an experiment.

If a globe of iron, which revolves round its axis in a determinate time, and which has been cooled to the degree of ice, as well as the surrounding air, be exposed before a fire, the impression of the fire will first heat the surface, and the heat will gradually penetrate to its interior parts; so that after a great number of revolutions the globe will acquire such a degree of internal heat, that it will be incapable of receiving more; and the presence of the fire will only serve to make it retain that which has already been communicated to it.

It may be readily conceived also, that the nature of the globe, or its distance from the fire, may be such, that this constant degree of heat shall not be very remote from that of the congelation of water.

In this case, what will be the result? as it is the surface of bodies that always begins to lose the heat they have acquired, because it loses more by its contact with the air than is furnished to it by the interior parts, it will necessarily happen, if the surrounding air be nearly at the degree of congelation, that the part of the surface which is illuminated obliquely, or that which by a slower revolution is opposite to the side next to the fire, will lose a little of its heat; and as we suppose the mean heat, which the globe has acquired, to be not far distant from the degree of congelation, like the temperature of the earth, the surface, in those parts less favourably exposed to the action of the heat, may assume a degree of cold equal to that of ice. Consequently, if there were on the surface of this globe some matter, such as wax or water, susceptible of melting and congealing alternately, it would certainly experience these alternations: it might even happen that it would remain constantly frozen in the neighbourhood of the Poles; thus it would alternately melt and be congealed in the mean parts between the Poles and the equator, and it would always remain fluid in the environs of the equator.

But this is exactly what takes place at the surface of the earth: exposed for a great number of ages to the benign influence of the sun, the heat has been communicated to its most interior parts, and this internal heat is what is called the *central fire*: it is continually receiving an additional quantity, and this makes up for the loss of that dissipated at its surface, by the contact of the air which is less heated. In a word, as the iron globe above mentioned, would possess to the depth of several

lines below its surface, a neat nearly constant, the degree of heat which prevails to some depth below the surface of the earth is, in like manner, almost invariable.*

But it is difficult to believe that the mass of the earth, if deprived of all heat, and exposed to the sun, could ever acquire that heat which it seems to possess. How many ages, or millions of ages, would be necessary, before a heat, so feeble as that of the sun, could melt an ocean entirely congealed, and insinuate itself into its bowels! In our opinion, the ice melted at the line by the presence of the sun would have been again congealed during the twelve hours of his absence; so that the globe exposed in this state to the sun, would have remained in it to eternity, had not some other powerful cause suddenly communicated to it that fund of heat, which, by vivifying nature, renders the earth habitable, and susceptible of vegetation.

A third cause of the central heat remains to be examined: it is that of Buffon. According to this celebrated philosopher, the earth and other circum-solar planets were formerly a part of the sun, and were detached from its surface by a comet, which entering in to some depth, projected the fragments to different distances. While they were in a state of fusion, each of them, in consequence of the laws of universal gravitation, must necessarily have assumed a globular form. The more considerable masses, such as Venus, the Earth, Mars, Jupiter, and Saturn, being projected in this manner, flew off in a tangent, which, together with the attractive force of the sun, made them describe, around that luminary, orbits more or less elongated. Such of these new planets as had smaller fragments accidentally in their neighbourhood, overcame them in some measure; and these fragments, turning around the larger ones, in consequence of the same laws, became their satellites. In this manner, the Earth, Saturn, and Jupiter, acquired those moons by which they are accompanied.

If this generation of the earth and circum-solar planets be admitted, it is evident that these globes were at first fluid; and this may serve to explain their formation into oblate spheroids; for the earth and other planets must have necessarily been, during the course of some time, either in a state of fusion or of semi-fluid paste; otherwise their diurnal motion could not have given them that form which they possess. But let us set out from their supposed state of fusion. Masses of so considerable a size as Venus, the Earth, &c., could not certainly cool in a day or a year, nor even in twenty centuries. They first passed from a state of fusion to that of solidity; they remained long impregnated with a quantity of fire, which rendered them uninhabitable. At length their surface has gradually cooled, till they retained only that degree of heat necessary for animal life, and to render them susceptible of vegetation. The interior parts of the earth still possess a more considerable degree of heat than the surface, and this heat must go on increasing towards the centre. Such is the central fire.

But by a necessary consequence of the cause of this fire, it must always decrease, so that a small portion of it is every day lost. It appears indeed, that the fertility of the earth daily decreases, and that mankind degenerate, both in size and in strength. This diminution, however, cannot be proved; we have not been long enough possessed of an instrument proper for measuring heat; it is not much above half a century since comparative thermometers were invented. But if it be found, 500 years hence, for example, that the constant heat in the caverns below the observatory of Paris, is not more than 7 or 8 degrees, instead of 9¾, which it is at present, the progressive cooling of the mass of the earth will be a fact which can no longer be doubted, whatever may be the cause of that heat, and of its decrease.

* We say *almost* invariable, because we are acquainted with no observations of the thermometer in subterranean places, but those made in the caverns below the observatory at Paris.

We must however observe, notwithstanding our respect for the illustrious philosopher who is the author of this idea, that there are many difficulties in regard to this formation of the earth and planets, which it is not easy to resolve.

1st. If the planets were formed in this manner, it is difficult to conceive how the comets could have a different origin: and if the latter were planets circulating around the sun, the Sovereign Cause, who arranged the universe, could, with equal ease, have formed the planets in the same manner.

2nd. It seems difficult to reconcile with the laws of motion and universal gravitation, the position and dimension of the orbits of these new planets; for according to what has been demonstrated by Newton and others, since they proceeded from the sun, in a line nearly a tangent to his surface, and from a point of his surface, they ought at each revolution to pass through the same point: this however is not the case; on the contrary, the orbits of the planets are nearly circular.

It appears also, that in this projection the largest masses could not go to the greatest distances, and describe the largest circles; it would seem that the smallest planets ought to be the most distant from the sun; for if several bodies are thrown promiscuously by any force whatever, the smallest will be projected with the greatest velocity.

In short, the effect of such a projection is beyond calculation; and it may be said also that the comet in question, while it ploughed the surface of the sun, communicated to it an impulse which made it change its place. This comet indeed, which could carry with it at once such masses as the planets, must have been of an enormous size, and impinging against the sun with immense velocity, could not fail to cause a small displacement of that luminary, which is in the centre of our system, in a sort of inert state.

Remark.—Whatever may be the fate of these ideas, the following are some of the consequences which Buffon deduces from his system on the formation of the earth, and which are too curious to be omitted in a work of this kind.

Setting out from his principles on the formation of the earth and the planets, Buffon made a series of very curious experiments, to determine in what ratio the refrigeration of different masses of matter takes place, according to their nature and size; and from these experiments he concludes:

That a globe, such as Mercury, must have required 2127 years to be consolidated to the centre; 24813 to become so cold that it could be touched; 54192 to be reduced to its present temperature; and in the last place, that it would require 187775 to become so cold as to have only the 25th part of its present temperature: for the sake of brevity we shall call these the 1st, 2d, 3d, and 4th epochs.

That Venus must have employed 3596 years in the first epoch; 41900 in the second; 91600 in the third; and that 228540 would be required for the fourth.

That the earth employed in the first epoch 2936 years; in the second 34270; in the third 74800; and that 168125 will be necessary before its temperature is reduced to a 25th part of what it is at present.

The earth therefore has existed 112 thousand years; and hence it follows, that Mercury passed the degree of the present temperature of the earth 30000 years ago; and that it has even lost already six of the 25 degrees which remained to it.

That the moon employed only 644 years in the first epoch; 7515 in the second; 16409 in the third; and 72514 in the fourth

Hence the moon, 15000 years ago, was reduced to such a degree of coolness as to have only a 25th part of the heat of our earth. It needs therefore excite no astonishment that she should appear to us as an accumulation of ice, and that she exhibits no signs of living nature. If she had inhabitants, they must long ago have been congealed.

Mars employed 1130 years in becoming solid to the centre; 13000 in the second epoch; 28538 in the third; and 60,300 in the fourth: consequently this planet has been useless for 9 or 10 thousand years.

In regard to Jupiter, the case is different: he must have employed 9400 years in the first epoch, and will require 110,000 for the second. But it is only 112000 years since the earth and Jupiter were formed; consequently 7 or 8 thousand years will be necessary, before Jupiter can be cooled to such a degree as to admit placing the foot upon it, without being burnt. When it attains to this epoch, it will require 240400 years before it be reduced to our present temperature; and then 483000 to lose nearly the whole of its heat. This globe then will begin to be habitable, when we are rendered absolutely torpid with cold.

In the last place, Saturn employed 5140 years in becoming fixed to the centre; and required 59900 before he was fit to be touched: the duration of his third epoch will be 130800 years; and of this epoch above 47000 years have already elapsed; so that it will be above 84000 years before his temperature is reduced to that of the earth.

In regard to the satellites, we shall only observe that the greater part of them are in a habitable state and fit for vegetation, the fourth of Jupiter excepted, which is already advanced in its fourth epoch: the third of Saturn is nearly at the same degree of temperature as the earth, but rather somewhat warmer; the fourth is considerably advanced in its fourth epoch; and the fifth must have been a mass of ice for nearly 50000 years

PROBLEM XXVI.

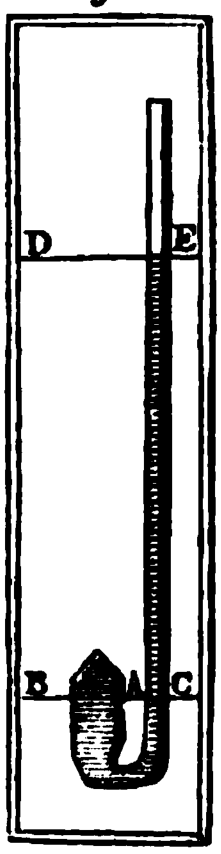
Construction of the Barometer.—To measure the Variations of the Gravity of the Air.

The barometer is one of those instruments for the discovery of which we are indebted to the 17th century, a period that gave birth to a great many happy ideas. This instrument, which serves to determine the variations that take place in the gravity of the air, derives its name from two Greek words, βαρος and μετρειν, the former of which signifies *weight*, and the latter *to measure*. It was the invention of Torricelli, a disciple of the celebrated Galileo, who employed it chiefly to prove the gravity of the air in which we live and breathe. But it was Pascal who first discovered its variations, by means of the famous experiment which he caused his brother-in-law to make on the Puy-de-Dome, a mountain in the neighbourhood of Clermont. It enabled him to demonstrate, in the most evident manner, the gravity of the air, which some still persisted to deny, notwithstanding the experiment of Torricelli.

A barometer may be easily constructed without much expense. Provide a vessel, some inches in depth, filled with mercury, and a glass tube 30 or 35 inches in length, hermetically sealed at one end. Invert the tube, that is to say turn the sealed end downwards, and fill it with mercury; apply your finger to the top so as to keep it shut, and having turned the sealed end uppermost, immerse the open end into the mercury in the vessel, and remove your finger, so as to allow the mercury in the tube to have a communication with that in the vessel: the column of mercury contained in the tube will then fall, but in such a manner that its upper extremity will remain about 28 inches, more or less, above the level of the mercury in the vessel, if the experiment be performed at a small height only above the level of the sea. In this manner you will have a barometer constructed; and if by any contrivance you can fix the tube thus immersed in the mercury, the end of the column of mercury will be seen to fluctuate between the height of 27 and 28 inches, or 29 or 30 inches English, according to the different constitutions of the atmosphere.

This is a barometer of the simplest kind, and such as it was when it came from the hands of Torricelli. At present, a glass tube, from 33 to 36 inches in length, is employed: it is hermetically sealed at the one end, and bent at the other, after

Fig. 19.



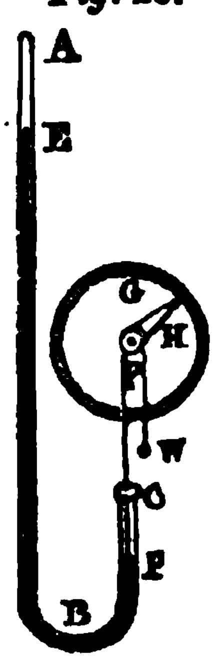
having been dilated at an enameller's lamp, so as to resemble a phial, as seen in the figure (Fig. 19.) This tube is filled by inclining it, and pouring in the mercury at different times, in such a manner, that when placed upright the mercury in the phial rises only to about half its height, as A B. The difference between the line C A B and the line D E, at which the mercury maintains itself, is the height of the column which counterbalances the pressure of the atmosphere, as may be easily conceived. This glass tube, when filled with mercury, is affixed to a piece of board, more or less ornamented; the interval between the 28th and 31st inch above C B, is divided into tenths, and the words *settled weather*, *fair*, *changeable*, *rain*, *stormy*, are inscribed at equal distances, beginning at the line of 28 inches. Such is the construction of the barometers commonly sold in the shops: but to render them good, some precautions are necessary.

1st. The diameter of the phial, or lower receptacle of the mercury must be considerably larger than that of the tube, otherwise the line A B, as may be easily perceived, will sensibly vary as the mercury rises or falls.

2d. The mercury must be purified from air as much as possible, or at least to a certain degree; and the tube ought to be heated and rubbed in the inside, to remove the moisture and dust which generally adheres to it; otherwise there will be a disengagement from it of air, which occupying the upper part of the tube, will, by its elasticity, form a counterpoise to the gravity of the atmosphere, and cause the column to remain lower than it ought to do. This air also, being dilated by heat, will produce on the column of mercury a much greater effect, so that its motions will depend both on the heat and gravity of the air, while it ought to depend on the latter cause alone.

Several expedients have been adopted for lengthening the scale of the barometer: sometimes the top of the tube has been bent aside; the diagonal instead of the perpendicular of the rectangle becoming the scale. But the most popular expedient is that adopted in what is called the *wheel barometer*.

Fig. 20.



This instrument consists of a bent tube, A B C (Fig. 20.) closed at A, the height of A above C not being less than 31 or 32 inches, the end C being open, and the tube filled with mercury. The mercury will fall in the leg A B till the difference of the levels of E and F be equal to the height of a column of mercury which balances the pressure of the atmosphere, and every change in the level of E will produce a corresponding but opposite one in that of F; so that the change in the height of the barometer column is double the change of the level F. On F a small iron ball floats, and a string attached to the ball passes over the pulley P, and to the end of the string is attached a weight W, somewhat lighter than the ball at F. The axis of the pulley passes through the centre of a large graduated circular plate G. This axis carries a hand H, which revolves when the pulley is turned.

When the mercury E rises, and F falls, the floating ball, not being completely balanced by W, falls with it; and the string, pressed by the weight on the wheel P, turns it, and with it the hand which points out, on the graduated plate, the inches and parts of the height of the barometer's column.

When E falls F rises; and the hand retrogrades to a corresponding point on the graduated arc.

PROBLEM XXVII.

Does the suspension of the mercury in the barometer depend on the gravity or the elasticity of the air?

We introduce this question, merely because it has been discussed in some books of natural philosophy, the authors of which have determined that this phenomenon ought to be ascribed to the elasticity, and not to the gravity of the air. The following analysis will shew how ill founded is the reasoning of those who entertain this opinion.

In this question there are two cases. In one of them the barometer is supposed to be placed in the open air; and this properly is the one which we here propose to examine. In the other, it is proposed to be shut up in a room so close, that no air can penetrate to it; or under the receiver of an air-pump, from which the air is excluded.

It is evident, in the second case, that the cause of the suspension of the mercury is the elasticity of the air alone; but to extend this to the case where the barometer is exposed to the open air, is reasoning, we will venture to say, in a manner unworthy of a philosopher.

To ascertain to which of the two causes the suspension of the mercury in the barometer, exposed to the open air, ought to be ascribed, let us suppose the air to be deprived of its weight or elasticity; and let us examine what would be the consequence.

If the air were deprived of its elasticity, it is evident that it would fall back upon itself, and form around the earth a kind of ocean of a peculiar fluid, the height of which would be much less than that of our atmosphere; but it would still have the same weight, for a ball of hair which has lost its elasticity, and is reduced to a less volume, weighs as much as it did, when in consequence of its elasticity it occupied a much larger space. The mercury in the barometer, if immersed to the bottom of this fluid, would sustain neither more nor less pressure, and consequently would maintain itself at the same height.

Let us now suppose, on the other hand, that the air, preserving its elasticity, has lost its gravity. In this case, as the parts of the air would experience no impediment to recede from each other; that is to say, as their elasticity would not be compressed by the weight resulting from the force exercised by the superior on the inferior parts, the air would be dissipated, without exercising any action on the column of mercury; unless we suppose, at the top of the atmosphere, a transparent arch to confine the elasticity of the air; for it is necessary that a spring, to exercise an action with one of its extremities, ought to rest against some fixed point with the other. But as such a supposition is ridiculous, it is evident that what confines the spring or elasticity of the air is its weight.

Since the air then, if deprived of its weight, and endowed with all the elasticity possible, would have no action on the mercury in the barometer; and, on the other hand, as it would still maintain the mercury at the same height, though deprived of its elasticity, provided it retained its weight, it may be asked to what cause this suspension ought to be ascribed? The answer is so easy that it is needless to mention it.

PROBLEM XXVIII.

Use of the Barometer to foretel the approach of fine or of bad weather. Precautions to be observed in this respect, in order to avoid error.

One of the principal uses of the barometer, is to foretel the approach of fine or of bad weather. Experience has indeed shown that the rise of the barometer, above its mean height, is generally followed by fine weather; and on the other hand, that when it falls below that height, it indicates the continuation or approach of rain.

These rules, however, are not absolutely general and infallible. Wind also has a great influence on the rise or fall of the mercury in the barometer; and therefore we think it necessary to give a few rules, founded on observation, which may enable those who have barometers, to form a more certain opinion respecting their indications.

1st. The rising of the mercury announces, in general, fine weather; and its fall is a sign of bad weather, as rain, snow, hail, or storms.

2d. During very hot weather, a sudden fall of the mercury indicates a storm and thunder.

3d. In winter, the rising of the mercury presages frost; and in the time of frost, if the mercury falls three or four lines, it announces a thaw; but if it rises during a continued thaw, snow will certainly follow.

4th. When bad weather takes place immediately after a fall of the mercury, it will not be of long duration; and the case will be the same in regard to fine weather, if it speedily follows a rise of the mercury.

5th. But during bad weather, if the mercury rises a great deal, and continues to do so for two or three days, before the bad weather is past, a change may be expected to fine weather, which will be of some duration.

6th. In fine weather, if the mercury falls very low, and continues so for two or three days before rain takes place, there is reason to conclude that the rain will be violent, of long duration, and accompanied with a strong wind.

7th. Irregularity in the motion of the mercury, announces uncertain and variable weather.

Such are the rules given by Desaguliers, according to a series of observations made by Mr. Patrick, a celebrated constructor of barometers, at London.

But there can be no doubt that they are liable to exceptions and variations.

It is known, for example, that in the countries situated between the tropics, the barometer scarcely varies; on the borders of the sea it always maintains itself within a few lines more or less of 28 inches.* This is a phenomenon difficult to be explained; and no reason ever yet assigned for it, appears satisfactory. Those therefore would be deceived who should apply the above rules to a barometer, transported to these countries.

It frequently happens, also, that the falling of the mercury takes place without any rain; but in that case a considerable degree of wind prevails, if not in the lower, at least in the upper part of the atmosphere; for Mr. Hawksbee contrived an experiment, by which he produced that effect on the barometer artificially.

PROBLEM XXIX.

How comes it that the greatest height of the barometer announces fine weather, and the least the approach of rain, or of bad weather?

Those not acquainted with the progress of the barometer, and who are ignorant that the mercury generally rises when the sky is serene and the air very pure, and that its fall, on the other hand, generally takes place before rain, would no doubt judge differently, and suppose that the mercury ought to fall when the air is serene and pure, and to rise when the air is charged and impregnated with vapours; for it is natural to believe, that pure and serene air is lighter than that which holds in solution a great deal of vapours. The progress of the mercury in the barometer is however quite the reverse; it is therefore a phenomenon which has been the subject of much discussion among philosophers, but without success; for all their explanations overturn themselves, and not one of them will bear examination.

* The natural mean height of the barometer is 30 English inches.

Some philosophers have said: the air is never more serene and more transparent than when well charged with vapours, or at least when they are perfectly dissolved and combined with it; for it is the property of perfect solutions to be transparent: it is not therefore astonishing that the mercury, being pressed down by a greater weight, should in this case rise. But when the aqueous vapours are separated from the air by any cause, they disturb its transparency, and begin to be precipitated: they no longer contribute to its weight, since they are not suspended in it; and as a proof of this, they quote the celebrated experiment of Rammazini, which is as follows.

Take a narrow vessel, several feet in height, and having filled it with water, place upon it a bit of cork, with a leaden weight suspended from it by a thread, so that the whole shall float. When the vessel has been thus prepared, put it into the scale of a balance, and load the other scale until an equilibrium is produced. If the thread by which the lead is attached to the cork, be then cut, it is observed that, while the lead is falling, this side of the balance is lightened, and the other preponderates. Hence it is evident, say the above philosophers, that while a weight is falling in a fluid, it exercises no pressure on the base; consequently, while the vapours, collected in the air, are precipitating themselves, or after they begin to be precipitated, the air is lighter, and the mercury becomes charged with a less weight.

This reasoning, which is that of Leibnitz, is exceedingly ingenious. Unfortunately, however, the experiment of Rammazini proves only that the scale of the balance is unloaded during the fall of the weight; but it does not prove that the bottom of the vessel is eased by the quantity of the weight which is falling; for these are two things very different. Recourse must therefore be had to another explanation.

For our part, we agree with M. de Luc,* that the only cause of the falling of the mercury in the barometer, on the approach of rain, is the diminution of the gravity of the air, when saturated with aqueous vapours. In our opinion, therefore, the air is never heavier than when it is exceedingly pure; and we are inclined to think so for various reasons.

The vapours seen floating in the atmosphere, under the form of clouds, are nothing but a solution of water in air: while this combination is imperfect, it is only semi-transparent, as is the case in regard to all solutions. The vapours, when in that state, are observed to rise in the atmosphere, and hence there is reason to conclude that they are lighter than air. The state of the air, in regard to gravity, when thus charged with vapours, may be deduced from the gravity of the vapours themselves; and since they are lighter than the air in which they ascend, we must infer that the air in which they are dissolved is lighter than pure air.

But it may be said, how can we conceive that air combined with a fluid heavier than itself should become lighter? It may be replied, that if the combination here meant were only the interposition of watery particles between those of the air, as might have been believed, before the improved state of chemistry had thrown light on a number of questions relating to the most common phenomena, this would be impossible. But this is not the mechanism of solutions, or of the combination of bodies with each other; each particle of the solvent combines with each particle of the dissolved body, and it is not improbable that this takes place here by the medium of fire, which is far lighter than either air or water. We can therefore form no conclusion respecting the weight of compound particles from that of the separated particles. Besides, in this state of combination, they may be endowed with a greater repulsive force; and this even seems to be very probable, since the expansibility of water, when reduced into vapour, is immense. There can be no absurdity

* "Traité des Barometres, Thermometres," &c. Geneve, 1770, 2 vols. 4to.

then in asserting that air charged with vapours is lighter than pure air. This will perhaps be demonstrated some day *à priori*, by chemical processes; and should this be the case, philosophers will be much surprised at the difficulty which has hitherto occurred in attempting to explain the falling of the mercury in the barometer, on the approach of rain.

PROBLEM XXX.

Of the Compound Barometer.

It has already been seen that a column of mercury of about 30 inches in height, is necessary to counterbalance the weight of the atmosphere; and hence it follows, that a simple barometer can never be at a less height, unless some fluid heavier than mercury be employed. As this length has been found inconvenient, attempts have been made to shorten it; with a view, as it would seem, of confining it within the same extent as the thermometer, which may be reduced to a much less size. The method in which this has been accomplished, is as follows:

The principle of the construction of these barometers, consists in opposing several columns of mercury to one of air; so that these columns taken together shall have the same length, viz. 30 inches, which one ought to have in order to be in equilibrium with the weight of the atmosphere. Consequently, 30 inches, or the common height of the mercury, must be divided by the length intended to be given to the barometer: the quotient will give the number of columns of mercury which must be opposed to the weight of the air.

Thus, if a barometer only 15 or 16 inches in length be required, it must be formed of three glass tubes, joined together by four cylindric parts of a larger size, as appears Fig. 21. Two of these tubes must be filled with mercury, and have a communication with each other, by means of the third, which ought to be filled with a lighter fluid. Thus the first branch, from D to E, is filled with mercury; the second, from E to F, is half filled with coloured oil of tartar, and half with carob-bean oil; and the third, from F to G, is filled with mercury. This arrangement therefore is the same thing, as if these two columns of mercury were placed one above the other; for it may be easily perceived that the column of mercury F G, presses on the first by means of the intermediate column F E, exactly in the same manner as if it were above it. In this kind of barometer, it is the separation of the two liquors, contained in the branch E F, that serves to indicate the variations of the weight of the atmosphere; and for this reason these two liquors must be of different colours, and of different specific gravities, to prevent them from mixing.

Fig. 21.

To fill this barometer, stop the aperture A, and pour mercury into the two lateral branches through the aperture B; then pour the two liquors into the middle branch, through the same aperture; after which it must be hermetically sealed.

Fig 22.

If a barometer only 9 or 10 inches in height were required, 30 must be divided by 9 or 10, which will give 3; consequently, three branches containing 9 or 10 inches of mercury, and two communicating branches filled with oil of tartar and carob oil, will be necessary. This barometer, consisting of five branches, is represented Fig. 22. It may be proper to observe, that the height of each branch ought to be estimated by the difference of the level of the liquor in the upper reservoir, and that of the liquor in the lower.

This construction, invented by M. Amontons, has the advantage of lessening the height of the barometer, which is sometimes inconvenient; and of rendering it fitter for being employed under certain circumstances as an ornament. But it is to be observed, that this advantage is gained at the expense of exactness. M. de Luc says that he never was able to obtain a tolerable instrument of this kind. The intermediate column acts as a thermometer; and those who have attempted to prove that this does not injure the accuracy of the instrument, did not reflect that their reasoning is true only when the line of the separation of the two colours is in the middle of the height of the tube

The Marine Barometer.

The barometer constructed in the ordinary way would evidently be useless on board a ship at sea, as the pitching and rolling of the vessel would keep the mercury in the tube in a state of perpetual oscillation. This inconvenience has been overcome by making the greater portion of the tube of very narrow bore, but terminated by a cylindrical portion of about three-tenths of an inch in diameter. The instrument is suspended by a spring and gimbals, at a point near the top, which is found in each case by trial. Thus improved, these instruments are in very great use both in the royal navy and in merchant ships; and they have rendered most important service to the maritime interests of the community, by giving notice of approaching storms, when frequently there have been no other indications which could have induced the mariner to make the necessary prepartions for encountering them.

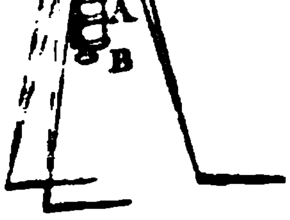
The Mountain Barometer.

The barometer in this form has been extensively used by travellers to make observations for the purpose of determining the height of mountains.

We extract the following description from "A Treatise on the principal Mathematical Instruments used in Surveying, Levelling, and Atronomy," by F. W. Simms, civil engineer; a work which we cordially recommend to all persons desirous to acquaint themselves with the use of such instruments.

In the brass box A (Fig. 23.) which covers the cistern of mercury, near the bottom of the tube, are two slits made horizontally, precisely similar and opposite to each other, the plane of the upper edges of which represents the beginning of the scale of inches, or the zero of the barometer. The screw B, at the bottom, performs a double office; first, it is the means of adjusting the surface of the mercury in the glass cistern to zero, by just shutting out the light from passing between it and the upper edge of the above-named slits; and secondly, by screwing it up, it forces the quicksilver upwards, and by filling every part of the tube, renders the instrument portable. By the help of a vernier the divided scale on the upper part is divided to the five-hundreth part of an inch; and even the thousandth part may be estimated. The screw C, at the top, moves a sliding piece on which the vernier scale is divided, the zero of which is at the lower end of the piece. In taking the height of the mercury, the sliding piece is brought down and set nearly by the hand, and the contact of the zero of the vernier with the top of the mercurial column is then perfected by the screw C, which moves the vernier the small quantity that may be required just to exclude the light from passing between the lower edges of the sliding piece, and the spherical surface of the mercury.

Fig. 23.



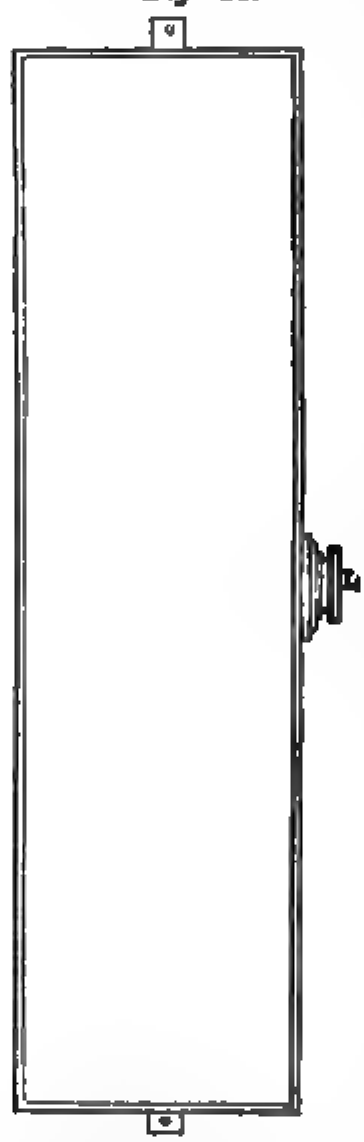
The barometer is attached to the stand by a ring, in which it turns round with a smooth and steady motion for the purpose of placing it in the best light for reading

off, &c.; and the tripod stand, when closed, forms a safe and convenient packing-case for the instrument.

A substitute for the barometer was invented a few a years ago by Mr. Adie, optician, of Edinburgh; it is thus described in the patent.

The principle of the instrument, which is called a *Sympiesometer*, consists in measuring the weight of the air by the compression of a column of gas. It consists of a tube of glass A B C D (Fig. 24.) about 18 inches long, and 0·7 of an inch diameter inside, terminated above by a bulb D, and having the lower extremity bent upward, and expanding into an oval cistern A, open at the top. The bulb D being filled with hydrogen gas, and a part of the cistern A and the tube B with almond oil, coloured with anchusa root, the enclosed gas, by changing its bulk according to the pressure of the atmosphere on the oil in the cistern, produces a corresponding elevation or depression of the oil in the tube, thereby indicating the variations in the weight of the atmosphere. The scale for measuring these changes is determined by placing the instrument, with an accurate barometer and thermometer, in an apparatus where the air can be rarefied or condensed, so as to make the barometer stand at 27, 30, or any other number of inches. The different heights of the oil in the tube of the sympiesometer, corresponding to those points, being marked on its scale E F, and the spaces between them being divided into 100 equal parts, these divisions represent hundredths of an inch in the mercurial column.

Fig. 24.



To correct the error that would arise from the change produced in the gas by variations of temperature, the principal scale E F is made to slide on another G H, so graduated, as to represent the amount of that change corresponding to the degrees of a thermometer, I K, attached to the instrument.

To use the instrument, note the height of the thermometer, and set the index *a*, which is upon the sliding scale, to the degree of temperature on the fixed scale; then the height of the oil, as indicated by the scale, is the pressure of the air, or the height of the mercury in the barometer at the time. The sliding scale is moved by means of the knob L.

This instrument has been thought to afford more delicate indications than the barometer, of *changes* in the atmospheric pressure; and the pressure determined by an instrument of this kind has been found to agree very well with those given by the barometer, not differing more than from the *eighth* to the *thirtieth* part of an inch.

PROBLEM XXXI.

What space would be occupied by a cubic inch of air, if carried to the height of the earth's semi-diameter?

We have already mentioned that air, in consequence of its elasticity, when charged with a double weight, is reduced to one half of its volume, and so on in proportion; at least as far as has hitherto been found by the experiments made on that subject. For the same reason, when freed from the half of the weight which it supports, it occupies a double space; and a quadruple space, when it has only a fourth part of

the weight to support. Thus, for example, on ascending a mountain, when it is found that the mercury has fallen half the height at which it stood at the bottom of the mountain, it is concluded that, being freed from half the weight which it supported when in the plain, it has been dilated to double the volume, or that the stratum of the surrounding air has only half the density of that at the bottom of the mountain; for the density is in the inverse ratio of the space occupied by the same quantity of matter.

This law of the dilatation of the air, in the inverse ratio of the weight with which it is loaded, has enabled geometricians to demonstrate, that as one rises in the atmosphere, the density decreases, or rarefaction increases, in a geometrical progression; while the heights to which one rises, increase in arithmetical progression. Hence, if it be known to what height we must rise to have the air rarefied one fourth, for example, or reduced to three fourths of the density which it has on the borders of the sea, we can tell that at a double height its density will be the square of $\frac{3}{4}$, or $\frac{9}{16}$; at a triple height it will be the cube of $\frac{3}{4}$, or $\frac{27}{64}$; in short, at a hundred times the height, it will be the 100th power of $\frac{3}{4}$, &c. Or, if the ratio of the density of the air, at the height of 1760 yards, or 1 mile, to the density of the air on the borders of the sea, has been determined, and if we call this ratio D, we shall have D^2 for the expression of that ratio at the height of 2 miles; at three miles it will D^3, &c: and at n miles, it will be D^n.

But, it is known by experiment that at the perpendicular height of a mile above the level of the sea, the mercury, which on the borders of the sea was at the height of 28 inches, or 336 lines, falls to 22 inches 4 lines, or 268 lines, or the height of the mercury at that elevation is expressed by the fraction $\frac{268}{336}$, unity being the whole height. Hence it follows, that the ratio of the density of the air at that height, to the density of the air on the borders of the sea, is expressed by that fraction: consequently to find what this ratio would be at the height of the earth's semi-diameter, we must first know how many miles are contained in that semi-diameter. Let us suppose that there are 3000. We must therefore raise the above fraction $\frac{268}{336}$, or $\frac{67}{84}$, to the 3000th power, which may be easily done by means of logarithms; for taking the logarithm of $\frac{67}{84}$, which is $-0{\cdot}0982045$, and multiplying it by 3000, we shall have for the logarithm of the required number $-294{\cdot}6135000$; which indicates that this number is composed at least of 295 figures. We may therefore say, that the density of the air which we breathe at the surface of the earth, is to that which we should find at the height of the earth's semi-diameter, as a number, consisting of 295 figures, is to unity. It is needless to make a calculation to prove that the sphere even of Saturn does not contain as many cubic inches as are expressed by that number; and consequently that a cubic inch of air, carried to the height of the earth's semi-diameter above its surface, would be extended in such a manner as to occupy a space greater than the sphere of Saturn.

We shall here just observe, that this rarity would be still greater, for the following reason. We have supposed the gravity uniform, which is not the case; for as gravity decreases in the inverse ratio of the distance from the centre of the earth, it thence follows, that in proportion as one rises above the surface, this gravity is diminished; so that at the distance of a semi-diameter from the earth, it is only a fourth part of what it was at the surface; every stratum of air then will be less loaded by the superior strata, since they will weigh less at the same height, than on the preceding supposition: consequently the air will be more dilated. Newton has shewn the method of making the calculation; but for the sake of brevity we shall omit it.

Remark.—The extreme rarity of the air, at a distance so moderate, may serve as a proof of the great tenuity of the matter with which the celestial space is filled. For if its density were every where the same as it is at the distance of the earth's

semi-diameter, it may be easily perceived how little the planetary bodies can lose of their motion by traversing it. The moon, during the many thousand years she has been revolving round the earth, cannot yet have displaced a quantity equal to a cubic foot of our air.

PROBLEM XXXII.

If a pit were dug to the centre of the earth, what would be the density of the air at the different depths, and at the bottom of it?

We shall begin our answer to this question by observing, that one could not proceed to a very great depth, without coming to air so highly condensed, that a person would float on it, in the same manner as cork does on mercury.

This is evident, if we suppose the gravity at the different depths of the pit to be uniform; for at the distance of a semi-diameter below the surface, the density must be to that of the air at the surface in the inverse ratio of the density of the latter to that of the air at the distance of a semi-diameter above it. But we have seen by what a number the rarity of the latter is expressed; and the same number will express the condensation at the centre.

Quicksilver is not quite 14000 times as heavy as the air which we breathe; and therefore the air at the centre would be thousands of millions of millions, &c. of times denser than mercury. But, for the sake of amusement, since we are on the subject of philosophical recreations, let us examine the most probable hypothesis of the gravity which prevails in the case stated in this problem. The gravity would not be uniform; it would decrease on approaching the centre, being exactly as the distance from the centre. But Newton has shewn that as the squares of the distances, from the centre, in this case decrease arithmetically, the densities would increase geometrically.

We must then first find what would be the density of the air at a determinate depth, such as 1000 toises, for example. But this is easy, on account of the proximity of that depth to the surface; for if the density at the surface be expressed by unity, that at the depth of 1000 toises, or a mile below it, will be the inverse of 1000 toises above it. But the latter was expressed by $\frac{67}{84}$, consequently the expression for the former will be $\frac{84}{67}$, or $1 + \frac{17}{67}$; hence the density being 1, at the distance of 3000 miles from the centre, the density at the distance of 2999 will be $\frac{84}{67}$. Let us then square 3000, which gives 9000000, and also 2999, which gives 8994001; the difference between these squares is 5999, by which if 9000000 be divided, we shall have the quotient 1500, for the number of squares decreasing arithmetically at the same rate that are contained in that square. If the logarithm of $\frac{84}{67}$, which is 0·0982045, be multiplied by 1500, the product will be 147·3067500, or the logarithm of the density at the centre, that at the surface being 1. But the number corresponding to this logarithm would contain 148 figures at least; whence it follows, that the density of the air, at the centre of the earth, would be to that at the surface, as a number consisting of 148 figures, or at least unity followed by 147 ciphers, to unity.

Were it required to determine at what depth the air would have the same density as water, it will be seen by a calculation founded on the same principles, that it would be at the distance of 30 miles below the surface.

It will be found, in like manner, that at the depth of 42 miles below the surface, the air would have the same density as quicksilver.

PROBLEM XXXIII.

Of the Air Gun.

This instrument, for the invention of which we are indebted to Otto Guerike, burgomaster of Magdebourg, so celebrated about the middle of the 17th century by

his pneumatic experiments, is a machine in which the elasticity of air, violently compressed, is employed to project a ball of lead, in the same manner as gunpowder. It consists of an air chamber, formed by the vacuity between two cylindric and concentric tubes, placed the one within the other; the bottom of this vacuity communicates with a pump, concealed in the butt end of the gun, and furnished with a piston which serves to introduce and condense the air, by means of valves properly adapted for the purpose. The ball is placed at the bottom of the inner tube, where it is retained by a little wadding, and at the bottom there is an aperture, closed by a valve, which cannot open until a trigger is pulled.

It may now be easily conceived, that when the air in the reservoir or chamber is compressed as much as possible, if the ball be placed at the bottom of the interior tube, and if the trigger, adapted to open the valve which is behind the ball, be pulled, the air violently compressed in the chamber will act upon it, and impel it with a greater or less velocity, according to the time it may have had to exert its action.

To make an air gun then produce the proper effect, it is necessary, 1st, that the opening of the valve should exactly occupy the same time that the ball does to pass through the length of the tube; for during that time the air will accelerate its motion, the expansion of the air being much more rapid than the motion of the ball. If the chamber should remain longer open, it would be a mere loss. 2d. The ball must be perfectly round, and exactly fitted to the calibre of the piece, in order that the air may not escape at its sides. As leaden balls are not always very regular, this defect may be remedied by wrapping a little tow around it.

Fig. 25.

When these requisites have been attended to, an air gun will discharge a ball with sufficient force to pierce a board two inches in thickness, at the distance of 50, and even of 100 paces. When the air chamber is once filled, it may be employed eight or ten times in succession. An English artist even invented a method of placing these balls in reserve in a small crooked channel, from which on discharging one ball, another issued to occupy its place; so that a person could discharge the air gun ten times running, much sooner than the most expert Prussian soldier could fire half the number of times. It must however be observed, that the force of the air gun decreases in proportion as the air chamber is emptied.

It may be easily conceived, that if this instrument, instead of being preserved in the cabinets of philosophers, should fall into the hands of certain persons, it would be a most formidable weapon, and the more dangerous as it makes no noise when discharged. But as gunpowder, after being a long time a mere ingredient in artificial fireworks, became the soul of a most destructive instrument, it is not improbable that the air gun, when brought to perfection, may in like manner be employed by armies to destroy each other, gloriously and without remorse.*

The air gun is represented Fig. 25, where the interval between the two cylinders, which serves to contain the air, may be easily distinguished; M N is the piston, by which the

* Within these few years the above anticipation of Montucla has been verified; guns, not indeed *air guns*, but *steam guns* on the principle of the *air gun*, have been made for the purposes of warfare: though, thanks to the peaceful temper of the times, they have not yet been brought into action.

At the Gallery of Practical Science, in the Lowther Arcade, London, the firing (!) of a stream of balls by the steam gun is daily exhibited.

air is introduced into that chamber; T L the valve, by which a communication is formed between the chamber and the cylinder; and o is the trigger. This mechanism may be so easily understood, that no further illustration is necessary.

PROBLEM XXXIV.

Of the Eolipyle.

The Eolipyle is a hollow vessel made of strong metal, and generally in the form of a pear, terminating in a long tail, somewhat bent. It is filled with water or some other liquor, by first exposing it to a strong heat, and then immersing it in the liquor to be introduced into it. While the interior air contracts itself to resume its former volume, the liquor, in consequence of the pressure of the external air, must necessarily enter to supply its place.

If the eolipyle, when filled in this manner, be placed on burning coals, the water it contains is reduced into vapour, which escapes by the narrow orifice in the tail: or if the fluid, by the position of the eolipyle, presents itself at the entrance, being pressed upon by the vapour, it issues through the orifice with force, and forms a pretty high jet.

If brandy has been employed instead of water, you may set fire to it with a taper; and, instead of a jet of water, you will have the agreeable spectacle of a jet of fire.

This experiment serves to shew, in a sensible manner, the strength of the vapour produced by a fluid exposed to a strong heat. For, in the first case, this vapour issues with impetuosity through the orifice of the eolipyle; and in the second the elastic force of the vapour, pressing on the fluid, makes it issue through the same orifice.

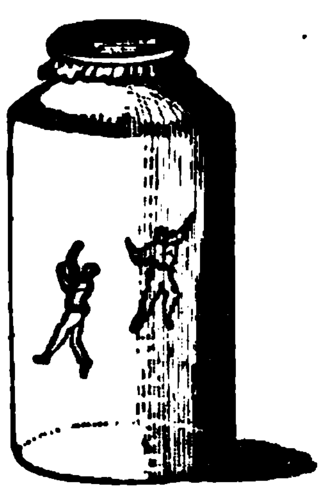
This experiment may be rendered still more amusing in the following manner. Provide a sort of small chariot, bearing a spirit of wine lamp, and place the belly of the eolipyle on the latter; close the orifice of the eolipyle with a stopper which does not adhere too firmly, and then kindle the lamp. Some time after, the stopper will fly out, and the fluid or vapour will issue through the orifice with great violence. The chariot being repelled, at the same time, by the resistance which the fluid or vapour experiences from the external air, receives a motion backwards; and if the axle-tree of the wheels be fixed to a vertical axis, the chariot will assume a circular motion, which will continue as long as the eolipyle contains any portion of the fluid.

It may be easily conceived, that this vessel must be made of very strong metal, otherwise it might burst, and either kill or wound the spectators.

PROBLEM XXXV.

To construct small figures, which remain suspended in water, and which may be made to dance, and to rise up or sink down, merely by pressing the finger against the orifice of the bottle or jar which contains them.

Fig. 26.



First construct two small hollow figures of enamel; but in the lower part, representing the feet, leave a small hole, through which a drop of water can be introduced, or apply to the back part of each a sort of appendage in the form of a tail (Fig. 26.), pierced at the end, so that a greater or less quantity of water may be made to enter into this tube. Then bring the figure into equilibrium in such a manner, that with this small drop of water it shall keep itself upright, and remain suspended in the fluid. Fill the bottle with water to the orifice, and cover it with parchment, which must be closely tied around the neck.

When you are desirous of putting the small figures in motion, press the parchment over the orifice with your finger, and the figures will descend; if you remove your finger they will rise; and if you apply and remove your finger alternately, the figures will be agitated in the middle of the liquor, in such a manner, as to excite the astonishment of those unacquainted with the cause.

The explanation of this phenomenon is as follows. When you press the water through the parchment which covers the orifice of the bottle, the water, being incompressible, condenses the air in the small figure, by causing a little more water, than what it already contains, to enter it. The figure having thus become heavier, must sink to the bottom; but when the finger is removed, the compresed air resumes its former volume, and expels the water introduced by the compression: the small figure, having by these means become lighter, must re-ascend.

PROBLEM XXXVI.

To construct a barometer, which shall indicate the variations of the atmosphere, by means of a small figure that rises or sinks in water.

The principles on which this small, curious barometer is constructed, have been explained in the foregoing problem. For since the pressure of the finger on the water, which contains the small figure in question, makes it descend, and as it rises again when the pressure is removed, it may be easily conceived that the weight of the atmosphere, according as it is greater or less, must produce the same effect. Hence, if the small figure be equipoised in such a manner as to remain suspended during variable weather, it will sink to the bottom when the weather is fine; because the weight of the atmosphere is then more considerable. The contrary will be the case when it threatens rain, and when the mercury in the barometer falls; for the weight of the atmosphere, which rests on the orifice of the bottle, being lessened, the small figure must of course rise.

PROBLEM XXXVII.

To suspend two figures in water, in such a manner that, on pouring in more water, the one shall rise up and the other sink down.

For this purpose, provide salt water, and suspend in it a small figure, or small glass bottle, of such a weight, that if the water contained a little less salt, it would fall to the bottom. Dispose, in the same manner, another small figure or bottle, open at the lower part; so that in the same water it shall keep at the bottom, by the mechanism described in the 35th problem.

When every thing is thus arranged, if fresh water, pretty warm, be poured into the salt water, which contains the figures, the first one will sink to the bottom, in consequence of a cause which may be easily conceived; and at the same time the other will rise to the surface: for the air in the second figure being dilated by the heat of the water, will expel, either in whole or in part, the drop of water which formed a portion of its weight: the figure, having thus become lighter, must consequently rise. These two small figures therefore will change places, merely by the effusion of more water; but the second, when the water cools, will re-descend.

PROBLEM XXXVIII.

Of Prince Rupert's Drops, or Batavian Tears.

This appellation is given to a sort of glass drops, terminating in a long tail, which possess a very singular property; for if you give one of them a pretty smart blow on the belly, it opposes a considerable resistance; but if the smallest bit be broken off from the tail, it immediately bursts into a thousand pieces, and is reduced almost to dust.

[illegible] by letting glass, in a state of fusion, fall drop by drop into [illegible] They are then found at the bottom completely formed. [illegible] however generally burst in the water, or immediately after [illegible] it. As these drops were first made in Holland, they are [illegible] larmes *Bataviques.*

[illegible] have been made with these glass drops, to discover the cause [illegible] These experiments are as follow:

[illegible] one of these drops be broken under the receiver of an air-pump, [illegible] may be easily concéived, it bursts in the same manner as it would [illegible] and if the experiment be performed in the dark, a flash of light [illegible] moment of rupture.

[illegible] of one of these drops be ground down gently on a cutler's wheel, [illegible] it sometimes bursts; but for the most part it does not.

[illegible] notch be made in the tail, by means of the same stone, the drop will [illegible]

[illegible] tail of one of these drops may be however cut off in the following manner [illegible] the place, at which you are desirous it should be cut, to an enameller's [illegible] these means it will be fused, and you may then separate the one part from [illegible] without fear of its bursting.

[illegible] if one of these drops be carefully heated on burning coals, and if it be then [illegible] to cool slowly, it will not burst, even when the tail of it is broken.

[illegible] philosophers have always been much embarrassed respecting the cause of this [illegible] ordinary phenomenon; and it must indeed be confessed that it is still very [illegible] We can only say, that it is not produced by air, as is proved by the first experiment. We think ourselves authorised to say also, from the fifth experiment, that it depends on the same cause which makes all articles of glass break, if care has not been taken to anneal them, that is to say if they are not subjected to a long heat that they may cool gradually, before they are exposed to the contact of the air. This appears to result from the last experiment; but it does not seem clear in what manner it is effected. It arises, in all probability, from the eruption of some fluid in the inside of the drop, which rushes through the broken part of the tail. It is perhaps an electrical phenomenon, and the drop may burst by the same mechanism that often cracks a glass jar, when it is discharged; that is, when the equilibrium is restored between its interior and exterior surface. Having explained the principal phenomena of these drops, we shall leave the rest to the sagacity and researches of our readers.

PROBLEM XXXIX.

To measure the quantity of rain which falls in the course of a year.

One of the meteorological objects which engage the attention of the modern philosophers, is, to observe the quantity of rain that falls on the earth in the course of a year. This observation may be easily made by means of an instrument which M. Cotte, in his Treatise on Meteorology, calls the *Udometer,** but which, in our opinion, ought rather to be called the *Uometer.*†

This instrument consists of a box of tin plate, or lead or tin, two feet square, which makes four feet of surface. Its sides are six inches in depth at least, and the bottom is a little inclined towards one of the angles, where there is a small pipe furnished with a cock. The water which flows through this pipe, falls into another square vessel, the dimensions of which are much less, and so proportioned, that the height of a line in the large vessel corresponds to three inches in the smaller. In the

* From ὑδωρ water, and μετρον a measure.

† From υω rain, and μετρον a measure.

present case, therefore, the base of this vessel ought to be only two inches six lines square. From this description it may be easily conceived that very small portions of a line of water, which has fallen into the large vessel, may be measured; since a line of height in the small one, will correspond to the thirty-sixth part of a line in the large one.

If the large vessel be properly placed, with the small one below the cock; and if the small one be covered in such a manner as to prevent the air from having access to the surface of the water it contains; it will not be necessary to examine the quantity of water which has fallen after each shower, or series of rain. It may be examined and measured every three, or four, or five days. It will, however, be better to do it after each fall of rain.

If a register be then kept of the quantity of water which falls every time that it rains, these quantities, added together, will give the quantity that falls in the course of the whole year.

It has been found in this manner, by a series of observations made at Paris, for 77 years, that the quantity of rain which falls there, one year with another, is 16 inches 8 lines.

But this quantity of water is not every where the same. In other places it is greater or less, according as they are situated near to the sea or to mountains. The following is a table of the principal places where observations of this kind have been made, and of the quantity of water which falls there annually.

	Inch.	Line.		Inch.	Line.
Paris	16	8	London	18	9
Bayeux	20	0	The Hague	26	6
Beziers	16	3	Rome	28	0
Aix in Provence	18	8	Padua	30	0
Toulouse	17	2	Petersburgh	16	1
Lyons	25	0	Berlin	19	6
Lille	23	0			

From an extensive collection of observations, Dr. Dalton concludes that, for 30 places in England and Wales, the average annual quantity of rain amounts to 35·2 inches; the greatest being 67·5 inches at Keswick, among the Cumberland hills; and the least at Uxminster, in the comparatively flat county of Essex. It is probable, however, that the annual quantity for the whole of England and Wales is below 30 inches.

It is certain that the quantity of rain which falls at the top of a hill is much greater than what falls at the bottom. Near Kinfauns, in Scotland, a rain gauge, on the summit of a hill 600 feet high, showed the mean annual fall of rain for five successive years to be about 41·5 inches; while a gauge at the bottom of the hill showed that for the same period the mean annual fall was not more than 25·7 inches.

Remark.—We think it necessary here to offer a remark which seems to have escaped all the philosophers who have made observations on the quantity of rain that falls. Every time it again rains, a small quantity of water is lost; namely, that which has served to moisten the bottom of the reservoir; for the water does not begin to run down till the bottom is moistened to a certain degree, and covered as we may say to a certain thickness with water, the quantity of which must be determined and taken into the account after every fall of rain. This quantity of water may be measured by the following process. Take a small sponge, moistened in such a manner that no water can be squeezed from it, even when pressed very hard; then fill the vessel, and having suffered the water to run from it, collect with the sponge what remains on the bottom, and squeeze it into a vessel,

the base of which is an inch square, and already moistened with water. It is evident that if a vessel, the base of which contains 4 square feet, gives in this manner water sufficient to rise to the height of an inch in the small vessel, there is reason to conclude that the pellicle of water which adhered to the metal was at least $\frac{1}{576}$ of an inch, or the 48th part of a line in thickness. At any rate, it may be safely estimated at the 30th or 36th of a line. If it has rained, therefore, two or three hundred times in the course of the year, 8 lines must be added to the quantity found.

PROBLEM XL.

Of the origin of Fountains. Calculation of the quantity of rain water sufficient to produce and to maintain them.

It would appear that the origin of fountains and springs ought not to have occasioned such a diversity of opinions, as has, for some time, prevailed among philosophers. An attentive consideration of these phenomena is sufficient to shew that the origin of them is entirely owing to the rains which continually moisten the surface of the globe, and which running over beds of earth, capable of preventing them from penetrating deeper, at length force a passage to places which are lower. Every person indeed must have observed, that the greater part of springs decrease in a considerable degree, when a long drought has prevailed; that some of them absolutely dry up when this drought continues too long; that when the surface of the earth has been moistened with snow or rain, they are renewed; and that they increase almost in the same progression as the waters become more abundant.

Some philosophers however have ascribed the origin of fountains to a sublimation of the waters of the sea, which, flowing into the bowels of the earth, rise up in vapour, in the fissures of the rocks, and thence trickle down into cavities and reservoirs prepared by nature, from which they make their way to the surface. Some have even gone so far as to imagine a sort of subterranean alembics.

But these conjectures are entirely void of foundation. If the water of the sea produced fountains in this manner, it would long ago have choaked up, with its salt, the subterranean conduits through which it is supposed to pass. Besides, the connection which is observed between the abundance of rain, and that of the water of the greater part of fountains, would not subsist; as the internal distillation would take place whether it rained or not. It is to be observed also, that the water of springs always distils from *above* beds of clay, and not from *below* them; but as these beds intercept the passage of vapours and water, the latter must necessarily come from above them. A sure method of destroying a spring, is to pierce this bed; but if the water came from below, a contrary effect would be produced.

What induced philosophers to have recourse to a cause so remote, and so false, no doubt was their imagining that rain water was not sufficient to feed all the springs and rivers. But they were certainly in an error; for instead of rain water being too small in quantity to answer that purpose, it seems rather difficult to conceive in what manner it is expended. This will be proved by the following calculation of Mariotte.

This author observes that, according to experiments which have been made, there falls annually on the surface of the earth about 19 inches of water. But to render his calculation still more convincing, he supposes only 15, which makes per square toise 45 cubic feet, and per square league of 2300 toises in each direction, 238050000 cubic feet.

But the rivers and springs which feed the Seine, before it arrives at the Pont-Royal at Paris, comprehend an extent of territory, about 60 leagues in length and 50 in breadth, which makes 3000 leagues of superficial content; by which if 238050000 be multiplied, we shall have for product 714150000000, for the cubic feet of water, which falls, at the lowest estimation, on the above extent of territory,

Let us now examine the quantity of water annually furnished by the Seine. This river, above the Pont-Royal, when at its mean height, is 400 feet in breadth, and 5 in depth. The velocity of the water, when the river is in this state, may be estimated at 100 feet per minute, taking a mean between the velocity at the surface and that at the bottom. If the product of 400 feet in breadth, by 5 in depth, or 2000 square feet, be multiplied by 100 feet, we shall have 200000 cubic feet, for the quantity of water which passes in a minute through that section of the Seine, above the Pont-Royal. The quantity then in an hour will be 12000000; in 24 hours, 288000000; and in a year, 105120000000 cubic feet. But this is not the seventh part of the water which, as above seen, falls on the extent of country that supplies the Seine.

But how shall we dispose of the remainder of this water? The answer is easy: the rivers, rivulets, and ponds lose a considerable quantity of water by evaporation; and a prodigious quantity is employed for the nutrition of plants.

Mariotte makes a calculation also of the water which ought to be furnished naturally by a spring that issues a little below the summit of Montmartre, and which is fed by an extent of ground 300 toises in length and 100 in breadth; making a surface of 30000 square toises. At the rate of 18 inches for the annual quantity of rain, the quantity which falls on that extent will amount to 1620000 cubic feet. But a considerable part of this water, perhaps three-fourths, immediately runs off: so that no more than 405000 forces its way through the earth and sandy soil, till it meets with a bed of clay at the depth of two or three feet, from which it flows to the mouth of the fountain, and feeds it. If 405000 therefore be divided by 365, the quotient will be 1100 cubic feet of water, which it ought to furnish daily, or about 38500 French pints; which makes about 1600 pints per hour, or 27 pints per minute. Such is nearly the produce of this spring.

An objection, founded on an experiment of M. de la Hire, described in the memoirs of the Academy of Sciences, for the year 1703, is commonly made to this idea respecting the origin of springs. This philosopher having caused a pit to be dug in a field, to the depth of 2 feet, found no traces of moisture: from which some conclude that the rain water flows only over the surface, and does not in any manner contribute to the origin of springs.

But this experiment is of no weight, as it is contradicted by a thousand contrary instances. Every one knows that water, in various places, oozes from the roofs of caverns and subterranean passages: it is this water which, after penetrating the earth, and flowing between the joints of stones, produces stalactites, and other stony concretions. It is therefore false that rain water never penetrates beyond the depth of a few feet. The fact, observed by M. de la Hire, was a particular case, from which it was wrong to deduce a general consequence.

It is objected also, that water is sometimes collected at heights at which it is impossible that rain water could give birth to a spring. To this it may be replied, that if the ground, where these collections of water exist, be examined, it will always be found that they are produced by rain or melted snow; that these places on the summits of mountains are only a kind of funnels, which collect the waters of some neighbouring plain, continually maintained by the rain or the snow, assisted by the small evaporation which takes place, in consequence of the rarity of the air. It is therefore evident to every rational mind, that the origin of springs and fountains can be ascribed to no other cause, than the rain water and snow which have been collected.

PROBLEM XLI.

The Water or Mercurial Mallet.

The water mallet, as it is called, is nothing else than a long glass flask, contain-

ing water, which when shaken in the flask, strikes it with a noise almost like that occasioned by a small blow with a mallet.

The cause of this phenomenon is the absence of the air, for as that fluid no longer divides the water in its fall, it proceeds to the bottom of the flask like a solid body, and produces the sound above mentioned.

To construct the water mallet, provide a long glass flask, pretty strong, and terminating in a neck that can be hermetically sealed; fill one fourth or one fifth of it with water; exhaust the air from it by means of an air pump, and then close the mouth of the flask hermetically. When the flask is taken out, if you fuse the neck of it gently at an enameller's lamp, in order to shut it more securely, the instrument will be completed.

If mercury be enclosed in the flask, instead of water, it will make a much greater noise or smarter blow; and you will even be astonished that it does not break the flask. If the mercury be well purified, it will be luminous; so that when made to run from the one end to the other, a beautiful stream of light will be seen in the dark.

Remark.—In our opinion, this property of mercury may be employed to construct what might be called a philosophical lantern. For this purpose, it would be necessary to dispose in a sort of drum a great number of small flasks, like the preceding, or spiral tubes, in which purified mercury should be kept in continual movement: which might be easily done if the drum were made to revolve by means of machinery; the result would be a continued light, which would have no need of aliment, or of being fed. Who knows, whether this idea may not enable us, at some future period, to dispense with the candles and lamps which we now employ to light our apartments? We are however afraid, that whatever be the number of flasks arranged in this manner, they will still afford too weak a light to supply the want of a single taper. But, perhaps, there are other useful purposes to which this invention may be applied.

PROBLEM XLII.

To make a Luminous Shower with mercury.

Place on the top of the air pump a small circular plate, pierced with holes, and supporting a small cylindric receiver, terminating in a hemisphere, and cover the whole with a larger receiver, having a hole in its summit, capable of admitting a glass funnel filled with mercury. This funnel must be so arranged, that it can be shut with a stopper, so as to be opened when necessary. Then exhaust the air, or nearly so, from the receiver, and open the funnel which contains the mercury; the mercury, in consequence of its weight, and of the pressure of the atmosphere, will run down, and, falling on the convex summit of the interior receiver, will be thrown up in small luminous drops, so as to resemble a shower of fire.

This experiment may be performed also in the following manner; provide a piece of pretty compact wood, and cut in it a small reservoir in the shape of a hemisphere, or of an inverted cone; apply it to the upper aperture of a receiver, and fill it with mercury. If you then exhaust the receiver, the pressure of the external air will force the mercury through the pores of the wood, so that it will fall down in small luminous drops.

PROBLEM XLIII.

What is the reason that in mines, which have spiracles, or air-holes, on the declivity of a mountain, at various heights, a current of air is established, which in winter has a direction different from what it has in summer? Explanation of a similar phenomenon, observed daily in chimneys.—Use to which a chimney may be applied in summer.

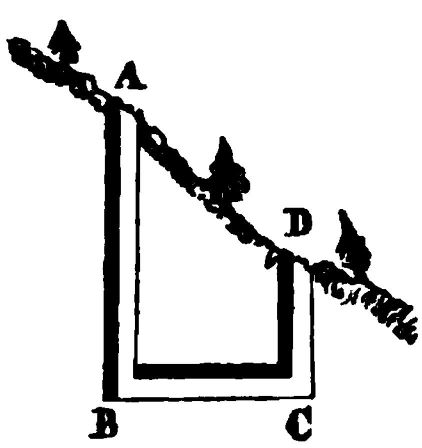
It is customary, in order to introduce air into a mine, at certain distances to sink

perpendicular wells, which terminate at the horizontal or somewhat inclined gallery, where the ore is dug up; and, in general, the mouths of these wells are at different heights, in consequence of the inclination of the side of the mountain. But in this case, a very singular phenomenon is observed: during the winter the air rushes into the mine through the mouth of the lowest well, and issues by that of the highest; the contrary is the case in summer.

To explain this phenomenon, it must be observed, that in the mine the temperature of the air is constantly the same, or nearly so, while without it is alternately colder and hotter; that is, colder in winter, and warmer in summer. It is to be remarked, on the other hand, that the well which has the mouth highest, the gallery, and the other well, form all together a bent siphon with unequal branches; the effect produced is as follows:

When the exterior air is colder than that in the mine, the column of air which presses on the lower orifice D (Fig. 27.), exerts a greater pressure on the whole air contained in the siphon D C B A, than that which presses on the upper orifice A; this air then must be expelled by circulating in the direction D C B A. But the cold air which enters by D, being immediately heated to the same degree as that in the mine, is impelled, like the former, by the column which rests on the orifice D.

Fig 27.



The contrary takes place in summer; for the exterior air, during that period, is warmer than the air in the mine. The latter then being heavier, that contained in the branch A B of the siphon, preponderates over the air in B C; so that the difference between the columns which press upon A and D, is not able to produce a counterpoise. The air contained in the siphon A B C D then must receive an impulse in that direction; and consequently must move in a direction contrary to the former. Such is the explanation of this phenomenon.

A similar one is observed daily in chimneys; and it is the more sensible as the flues of the chimneys are higher; for a chimney, with the chamber where it terminates, and the door or a window, form a siphon similar to the preceding. Besides, the exterior air from nine in the morning till eight or nine at night in summer, is warmer than the interior, and *vice versâ*. In the morning then, the air must descend the chimney, and issue through the door or the window; on the other hand, as the exterior air is colder in the night than in the day, it must, during the former, enter at the door or window, and ascend the chimney. About eight or nine in the morning, and at eight or nine in the evening, the air is, as it were, stationary: an effect which must necessarily take place at the time of the passage from one direction to another.

Dr. Franklin, who seems to have first observed this phenomenon, says that it might be applied to some economical uses during summer; and in that case the proverb, "as useless as a chimney in summer," would not be correct. One of these uses is, that the chimney might be applied as a safe; for if each of its mouths were closed by a piece of canvas, stretched on a frame, the alternate and almost continual current of air which would be established in it, could not fail to preserve meat from corruption.

This current might perhaps be employed also for some work that requires not so much a force as a continuance of it. For this purpose, it would be necessary to fix in the flue of the chimney a vertical axis with a helix, like the fly of a smoke-jack; the current of air would keep it in continual motion, sometimes in one direction, sometimes in the other; and in all probability with sufficient force to raise a small quantity of water per hour. And, as it would remain inactive only three or

four hours a day, it could not fail to produce a considerable effect daily. Besides, the moving power would cost nothing. It would however be necessary to have the wheels adjusted in such a manner, that to whatever side the axis furnished with the helix turned, the machine should always move in the same direction; which is not only possible, but was executed by M. Loriot at Paris.

Remark.—The same effect is easily experienced on a small scale, in a close room or chamber, which is very warm with several persons and candles in it, especially if there is no fire or no fire-place. For, by unlatching the door, and setting it a very little open, as an inch or half an inch, it will then be found that the air rushes strongly in near the bottom, but sets as strongly out near the top, and is quiescent near the middle parts. This is very easily tried by holding a candle in your hand, first near the bottom of the small opening, where the flame is violently blown inwards; then at the top, where it is carried strongly outwards; but held near the middle, the flame of the candle is quite still.

PROBLEM XLIV.

To measure the height of mountains by the barometer.

It is very difficult, and even sometimes impracticable, to measure the height of mountains by geometrical operations. A traveller, for example, who traverses a chain of mountains, and who is desirous of ascertaining the altitude of the principal points he has ascended, cannot have recourse to that method. The barometer, however, supplies a convenient and pretty exact one, provided it be employed with the necessary attention.

The principle on which this method is founded will be readily conceived, when it is recollected that if a barometer be carried to the top of a mountain, the quicksilver stands at a less height than at the bottom. 1st. Because it has a less column of air above it. 2d. Because this air has less density, as it is freed from the weight of a part of the air which it supported at the bottom of the mountain. Such is the foundation of the rules which have been invented for applying the height at which the mercury in the barometer stands to the purpose of measuring the height of mountains. But to give a very exact rule in regard to this operation, is attended with no small difficulty; for the height of the mercury in the barometer depends on a complication of so many physical causes, that it is exceedingly troublesome to subject them to calculation. M. de Luc of Geneva, who has considered this subject with the greatest care, by combining all these causes and circumstances, seems to have discovered a method which, if not absolutely perfect, is certainly more correct than any before given.

To proceed with exactness in this operation, it is necessary to have a good portable barometer, well freed from air; and a good thermometer, which we shall suppose to be that of Reaumur, though M. de Luc, to facilitate the calculation, proposes a particular kind of division. If great correctness be required, it will be necessary also that an observer should examine the progress of the barometer at the bottom of the mountain, or in one of the nearest towns, the height of which above the level of the sea is known.

When you have reached the summit of the mountain, or the place the altitude of which you are desirous of ascertaining, hold the barometer in a direction perfectly vertical, and examine the height of the mercury; suspend also the thermometer in some insulated place in the neighbourhood, and observe the degree to which the mercury rises.

Having then compared the height of the barometer observed on the mountain, with that of the corresponding barometer, observed at the same time at the bottom, take the logarithms of these two heights, expressed in lines, and cut off from them the four last figures: the remainder will be the difference of the heights expressed in French toises, the logs. being to seven places of decimals.

But this altitude requires a correction; for it is only exact when the simultaneous temperature of the two places is 16¾, according to the scale of Reaumur's thermometer. For each degree then that the thermometer has remained below 16¾, at the upper station, one toise must be added for every 215, and the same must be deducted for every degree above that temperature.

The same correction,* but in the contrary sense, must be made by means of the thermometer left at the fixed station; that is to say, for each degree it remained above 16¾, one toise in 215 must be deducted, and *vice versâ*. The height, when twice corrected in this manner, will be the difference nearly between the height of the two places above the surface of the sea, or the height of the one above the other.

Let us suppose, for example, that at the lower station the barometer stood at the height of 27 inches 2 lines, or 326 lines; and that at the upper station it fell to 23 inches 5 lines, or 281 lines.

The logarithm of 326 is 2·5132176, and that of 281 is 2·4487063; their difference is 0·0645113; from which if the three last figures be cut off, to answer for division by 1000, the remainder will be 645 toises.

We shall suppose also, that at the top of the mountain Reaumur's thermometer stood at 6 degrees above freezing, and in the lower station at 12; that is, for the former 10¾ below 16¾, consequently 10¾ toises are to be added to the above number for every 215; and hence, by the rule of three, the number to be added will be found to be 32 toises.

It will be found, by the converse correction, that the height to be deducted is 20; consequently there will remain 12 toises to be added, and therefore the height twice corrected will be 657.

Mr. Needham, on Mount Tourné, one of the Alps, observed the height of the barometer to be 18 inches 9 lines, or 225 lines. Now if we suppose that it was observed at the same moment at the level of the sea to stand exactly at 28 inches or 336 lines, which is its mean height on the borders of the sea, the difference between the logarithm of 336 and that of 225, cutting off the last three figures, will be found to be 1742. It may therefore be concluded, that the height of Mount Tourné is 1742 toises. But as in this case we have no corresponding observation at the level of the sea, nor any observation of the thermometer made at the same time, we have employed this observation of Needham only as an example of calculation. It is however probable that the height of Mount Tourné is between 1700 and 1800 toises.

The late Professor Leslie of Edinburgh has given a very simple rule for computing approximately the heights of mountains from barometrical observations. It is this: *As the sum of the heights of the mercurial columns, is to their difference, so is* 52000 *to the approximate height in English feet.*

To find the height of the upper column at the temperature of the lower, multiply the height of the upper column by twice the difference of degrees in the attached centigrade thermometer; and the product, divided by 1000, will be the correction to be added to the upper column, for change of temperature; and this height must be used in the preceding rule.

To correct the approximate height for the expansion of the air: multiply the height by twice the sum of the degrees on the detached thermometer, and dividing the product by 1000, the quotient is the correction to be added to the approximate height.

* This second correction, though not mentioned by M. de Luc, appears to us necessary, for several reasons, which it would be too tedious to explain here.

A concise rule, with tables for computing the heights of mountains, is given in "A Collection of Tables and Formulæ, by Francis Baily, Esq."

Remarks.—As a portable barometer is an instrument difficult to be procured and preserved, it is almost necessary that a traveller, when he is desirous of making observations, should construct a barometer for himself; but in this case, as the mercury will not be freed from air, it will always stand a few lines lower than a barometer which has been constructed with every possible care. This difference may amount to two or three lines.

In regard to Reaumur's thermometer, it is easily carried; but in what manner must a traveller proceed to have corresponding observations, either on the borders of the sea, or in any other determinate place, which are necessary before he can employ his own with sufficient exactness? This difficulty, in our opinion, seems to limit, in a considerable degree, this method of determining the heights of mountains.

Besides, it appears that, even if a traveller had on the borders of the sea, or in any village situated for example in the centre of France, the height of which above the sea is known, a diligent observer, he would not be much farther advanced, for the temperature of the atmosphere may be different on the borders of the sea at Genoa, that is to say, it may rain, for example, while the weather is fine and serene on the Alps and the Appenines; or the contrary may be the case; and hence there is a new difficulty to be surmounted.

This difficulty however might be obviated in part, by knowing the greatest, the mean, and the least elevation, of the barometer on the borders of the nearest sea, and thence determining, by meteorological conjectures, the nature of the temperature on the mountain to be measured, though one only passed over it; thus, if an hygrometer on the mountain indicated, for example, great moisture, there would be reason to conclude that the weather was inclined to rain, and that the fixed barometer stood at its least height. On the other hand, if the air was very dry, it might with probability be concluded that the weather was serene, and that the fixed barometer stood at its greatest height: but it must be confessed that this is not sufficient to give a satisfactory exactness.

However, a great many barometrical observations have been made on the summits of mountains, and their heights have been thence deduced. Several of them have also been measured geometrically; and we here add a tabular view of the heights above the level of the sea, of the most remarkable mountains on the surface of the earth, compiled from the most recent authorities.

EUROPE.	Height. Feet.
Mont Blanc, Alps..........	15688
Monte Rosa, Alps..........	15084
Gros Klockner, Tyrol	12796
Mont Perdu, Pyrenees	11283
Pic d'Ossana, ditto........	11700
Etna, Sicily..............	10870
Monte Carno, Appenines....	9523
Glac. de Buct, Alps........	10124
Mulharen, Spain	11783
Pic du Midi, Pyrenees....	9300
Olympus, Greece	9754
Canigou, Pyrenees..........	9247
Mount Cenis, Alps	9212
Monte Velino, Appenines....	8397
Simplon, Pass of............	6562
The Dole, Mount Jura......	5523
Hecla, Iceland	4887
Snuhetta, Norway	8122
Mont Mezin, France........	6567
Puy de Sanca, France......	6200
Vesuvius, Italy	3932
ASIA.	
Dhawalagiri, Himalaya......	28007
Jewahir, ditto	25747
Mowna Rowa, Sandwich I...	15988

	Height. Feet.
Ophir, Sumatra	13849
Egmont, New Zealand	11430
Ararat, Armenia	17260
Lebanon, Palestine	9600
Awatska, Kamschatka	12000

AFRICA.

Peak of Teyde, Teneriff	12180
Atlas, highest peak of, more than	12000
Table Mountain, Cape of Good Hope	3520

AMERICA.

	Feet.
Nevado di Sonato	25250
Illimani, Gold Mount. Peru	24450
Chimboraço	21600
Cajambi	19360
Antisuna, Rocky Mountains.	19136
Catapani, ditto	18867
Popocatepetl, ditto	17903
Mount St. Elie	17883
Orizaba	17390
Pinchincha, Rocky Mountains	15931
Mine of Chata, Peru	11830
Mean height of Andes	11820
Quito, City of	9515
El Pinal, City of	8362
Las Vigias, ditto	7820
Perote, ditto	7723
Mexico, ditto	7468
La Puebla, ditto	7200
St. Juan del Rio, ditto	6484
Mount Washington, Appalachian	6650

GREAT BRITAIN.

Benmacdouie, Scotland	4390
Ben Nevis, ditto	4370
Cairngorm, ditto	4080
Whernside, Yorkshire	4050
Ingleborough, ditto	3987
Ben Lawers, Scotland	3858
Ben More, ditto	3723
Ben Gloe, ditto	3690
Snowdon, Wales	3555
Shehalion, Scotland	3461
Helvellyn, England	3324
Skiddaw, ditto	3270
Ben Ledi, Scotland	3009
Ben Lomond, ditto	3240
Macgillicuddy's Reeks, Ireland	3404
Mourne Mountains, ditto	2500
Rippin Tor, Devonshire	1549

General Observation.—We shall here remark, that the very considerable differences often found between the barometric and geometrical measurement, must not be entirely imputed to the method. The latter is certain; but the observers of the barometrical heights have often employed imperfect instruments; in general, they have had no corresponding observations; and they have scarcely ever taken into account the difference of temperature at the posts of comparison; these differences need therefore excite no astonishment.

Remark.—We must observe that the French, in general, consider 28 Paris inches as the mean height of the barometer at the level of the sea; and as the following remarks on this subject by Mr. Kirwan, may be of use to the reader, we here subjoin them:—"Sir George Schuckburg has shewn, from 132 observations, made in Italy and in England, that the mean height of the barometer at the level of the sea, the temperature of the mercury being 55°, and of the air 62°, is 30·04 inches;* we may then assume the height of 30 inches as the natural mean height of the barometer at the level of the sea, in most temperatures between 32° and 82°; for if the mercury were cooled down to 32°, that is 23° below 55°, it would be lowered by that condensation only 0·07 of an inch; and if it were heated up to 80°, that is 25° above 55°, it would be raised only ·078 of an inch; quantities which, except in levelling, may be safely disregarded.

"The French have heretofore considered 28 Paris inches as the mean height of the barometer at the level of the sea, that is 29·84 English inches. But from 1400

* Phil. Transact. 1777 p. 586.

observations, made at Rochelle by Fleurieu de Bellevue, and from five years' observations made at Port Louis, in the Isle of France, he concludes the mean height of the barometer at the level of the sea to be 28 inches and two lines and $\frac{11}{16}$ of a line, in the temperatures of from 52° to 55° Fahrenheit, or 30·08 English inches.* Hence we may consider, in round numbers, 30 inches as the standard height of barometers at the level of the sea. And knowing the true height of any part of the earth, we may, by subtracting that height, expressed in fathoms, from the log. of 30, viz. 7471 213, find the logarithm which indicates the number of inches at which, as its natural mean, the mercury should stand at that height about the level of the sea.

"Thus, supposing the height to be 87 feet, equal to 14·500 fathoms: then 4771·213 — 14·500 = 4755 713, which is the logarithm of 29·9; this therefore is the natural mean height of the barometer at the elevation of 87 feet above the level of the sea.

"Consequently, to all heights heretofore calculated by the French, above the level of the sea, 139·32 feet must be added English measure, when the mercurial height at the level of the sea was barely supposed to be 28 French inches." (On the Variation of the Atmosphere, by Richard Kirwan, Esq. LL.D., F.R.S., and P.R.I.A. Dublin, 1801.)

Rule to compute heights by the Barometerin English measures, by Dr. Charles Hutton.

To complete the foregoing account of the measurement of altitudes by the barometer, I shall here annex the method of performing that operation in English measures, either feet or fathoms, as extracted from my Philosophical Dictionary, article Barometer, or from my Course of Mathematics, vol. 2, p. 255, edit. 6th; which is as follows.

1. Observe the degree or height of the barometer, both at the bottom and top of the hill, or other place, the altitude of which is required, as also the degree of the thermometer, for the temperature of the air, in both the same places.

2. Take out, from a table of logarithms, the logs. of these two heights of the barometer in inches and parts, and subtract the less log. from the greater. If from the remainder there be cut off three figures on the right hand, where the logs. consist of seven places, the other figures on the left hand will give the altitude required in fathoms of 6 feet each.

3. The above result requires a small correction, when the medium temperature of the air is different from 31 degrees of Fahrenheit's thermometer, which may be thus found, when much accuracy is desired. Add the two observed heights of the thermometer together, and take half that sum for the mean temper of the air. Take the difference between this mean and the number or temper 31; then, as many units as this difference amounts to, take so many times the 435th part of the fathoms above found; to which add them when the mean temperature exceeds 31, but subtract them when it is less; and the result will be the more correct altitude of the hill, &c., as required.

A small correction for the temperature of the barometer is sometimes employed, as may be seen in the books above quoted; but it is so small as to be seldom necessary to be observed.

For an example, suppose at the foot of a mountain, the barometer be observed 29·68 inches, and the thermometer 57; at the same time at the top of the mountain the barometer was 25·28, and thermometer 42. Then the calculation will be as below.

* La Chappe thought it 28 inches 1·5 lines. See Beguelin's Memoir. Mem. Berl. 1769. 12 Coll. Acad. 494.

29·68	log.	4724639	57	
25·28	log.	4027771	42	
		696·868	2)99	
		or 697 nearly.	$49\frac{1}{2}$	mean.
			31	subtr.
			$18\frac{1}{2}$	

Then, as 435 : $18\frac{1}{2}$: : 697 : 29 the correction.

$$\begin{array}{r} 29 \\ \hline 726 \end{array} \text{ fathoms} = 4356 \text{ f.}$$

So that the required altitude is equal to 726 fathoms, or 4356 feet.

PROBLEM XLV.

To make an artificial Fountain, which shall imitate a natural spring.

We here suppose that those who intend to try this experiment, have at their command a piece of ground, somewhat inclined, the bottom of which is a bed of clay, not far distant below the surface of the earth. In this case, a spring absolutely similar to a natural one, and capable of answering every domestic purpose, may be constructed by the following process.

Uncover this bed of clay for the extent of an acre, or about 70 yards in length, and the same in breadth. A border of clay must be formed at the lower end, leaving one aperture at the lowest point, through which the water may issue. To this aperture adapt a stone with a hole in it, about an inch in diameter. Then collect pebbles of various sizes, and cover this area with the largest, leaving an interval of a few inches only between them. Place others, somewhat smaller, above the interstices left by the former; arranging several strata in this manner above each other, always diminishing the size, till the last are only very large gravel. Cover the whole, to the thickness of some inches, with coarse sand, and then with some that is finer; but if moss can be procured it will be proper to cover the very large gravel with it to the thickness of half an inch, to prevent the sand from falling into the interstices between the pebbles.

It is evident that the rain water, which falls on the surface of this area, will penetrate through the sand, flow into the interstices between the pebbles, which cover the bed of clay, and at last, in consequence of the inclination of the ground, will proceed towards the aperture at the bottom, through which it will issue in a stream of greater or less size, according to the abundance of the rain.

Now, if we suppose that the water which falls annually on this piece of ground is 18 inches in height, it will be found that the quantity of water collected will be 66150 cubic feet; and if we suppose one fourth wasted by evaporation, or remaining between the joints and interstices of the stones, sand, and moss, we shall still have about 49600 cubic feet of water in the year, or about 303800 gallons; that is to say, almost 1000 gallons per day, a quantity much more than is necessary for the largest family.

It will perhaps be said that such a spring of water would cost exceedingly dear. This we shall admit. But we much doubt whether the construction of it would cost so much as that of a large cistern, which, to confine the water, requires to be lined with clay or cement: besides, the water collected by a cistern is only the rain water which falls from the roofs of a few houses, and which is consequently impure.

Besides, it might be rendered much less expensive by preparing, in the above manner, a small portion of ground, such as twenty yards square; and, to increase the quantity of rain water thus obtained, which would not exceed 5400 cubic feet, that

which fell on the neighbouring ground might be conveyed thither by small drains, from the distance of some hundred yards. By these means, a very abundant reservoir of filtred water might be formed at a very small expense; and the proprietor would enjoy the pleasure of having a spring exactly similar to those furnished by nature.

We are only apprehensive that the water would flow off with too much rapidity; but this inconvenience might be prevented by making the aperture through which it escaped of such a size as to render it perpetual; or by adapting to it a cock, and keeping it shut till it might be necessary to draw water.

PROBLEM XLVI.

What is the weight of the air with which the body of a man is continually loaded?

Who would imagine that the human body is continually loaded with the weight of twenty or thirty thousand pounds, which compresses it in every direction? This, however, is a truth which has been placed beyoud all doubt by the discovery of the gravity of the air.

Every fluid presses on its base, in the ratio of the extent of that base, and of its height. But it has been proved that the weight of the air is equivalent to the weight of a column of water 32 feet in height; consequently every square foot, at the surface of the earth, is charged with a column of air equal to one of water of 32 cubic feet; that is to say 2000 pounds, as a cubic foot of water weighs 62 pounds and a half. The surface of the body in a man of moderate size is estimated at 14 square feet; and therefore, if 2000 be multiplied by 14, we shall have 28000 pounds for the weight applied to the surface of the body of a moderate sized man.

But how is it possible to withstand such a load? The answer is easy: the whole human frame is filled with air, which is in equilibrium with the exterior air. Of this there can be no doubt; for an animal placed under the receiver of an air-pump, swells up as soon as the machine begins to be evacuated of air; and if the operation be continued, it will distend so much that it will at length perish and even burst.

It is the difference of this gravity that renders us more active or oppressed, according as the body is more or less loaded. In the first case, the body being more contracted by the weight of the air, the blood circulates with greater rapidity; and all the animal functions are performed with more ease. In the second, the weight being diminished, the whole machine is relaxed, and the orifices of the vessels become relaxed also; the motion of the blood is more sluggish, and no longer communicates the same activity to the nervous fluid; we are dejected, and incapable of labour, as well as of reflection, and this is the case in particular when the air is at the same time damp: for nothing relaxes the fibres of our frail machine so much as moisture.

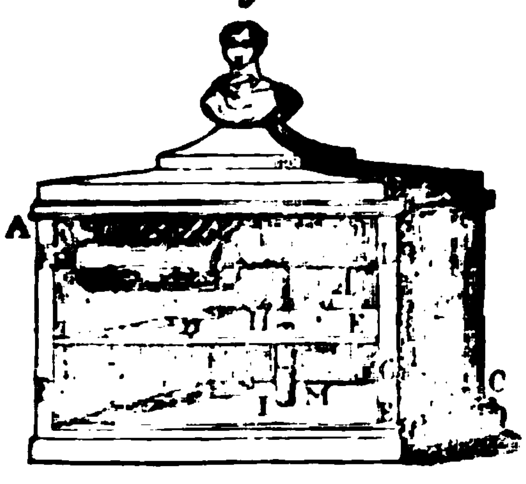
PROBLEM XLVII.

Method of constructing a small machine, which, like the statue of Memnon, shall emit sounds at sun-rise.

The story respecting the statue of Memnon, exhibited in one of the temples of Egypt, is well known. If we can credit the ancient historians, it saluted the rising sun by sounds, which seemed to proceed from its mouth. But however this may be, a similar effect can be produced in the following manner.

Provide a pedestal, in the form of a hollow parallelopipedon A B C (Fig. 28.); and let the cavity be divided into two parts by a partition D E. The lower part must be very close, and filled with water to a third of its height: the remainder must be filled with air. The partition D E must have a hole in it to receive a pipe, some lines in diameter, well soldered into it, and which reaches nearly to the bottom of the lower cavity. This tube must contain such a quantity of water, that when the air is cooled to the temperature of night, the water shall be nearly at the level of F G.

Fig. 28.



One of the faces of the pedestal must be so thin as to become easily heated by the rays of the sun. Of all metals, lead is the soonest heated in this manner; and therefore a thin plate of lead will be very proper for the required purpose.

K L is an axis which revolves freely on its pivots at K and L; round this axis is rolled a very flexible cord, which supports on the one hand the weight N, and on the other the weight M, which moves freely in the pipe H I. The ratio of these weights must be such, that M shall preponderate over N, when the former is left to itself; but N must preponderate when the former loses a part of its weight by floating in the water: this combination will not be attended with much difficulty.

In the last place, the axis K L supports a barrel, some inches in diameter, and a few inches in length, implanted with spikes, which, touching keys like those of a harpsichord, raise up quills and make them strike against strings properly attuned. The air must be finished in one or two revolutions of the barrel, and it must be exceedingly simple, and consist of a few notes only. All this mechanism may be easily inclosed in the upper cavity of the pedestal. On the top of it must be placed a figure or bust, representing that of Memnon, with its mouth open, and in the attitude of speaking. It would not be difficult to connect its eyes with the axis K L, in such a manner as to render them moveable.

From this description it may be readily conceived, that the side of the pedestal, exposed to the east, cannot receive the rays of the sun without becoming hot; and that, when heated, it will heat the air contained in the lower cavity; this air will make the water rise in the pipe H I, by which means the weight N will preponderate, and cause the axis K L to revolve, and consequently the cylinder furnished with spikes, which will raise the keys; and in this manner the air that has been noted will be performed. But for this purpose the diameter of the axis K L must be so proportioned, that the weight N by descending, two lines for example, shall cause the cylinder to revolve once or twice with sufficient rapidity to make the sounds succeed each other quick enough to form an air.

Father Kircher, it is said, had in his museum a machine nearly of the same kind; a description of which has been given by Father Schott; but we think ourselves authorised to assert, that it could not produce the desired effect; for Schott says that the air was impelled through a small pipe against a kind of vanes, implanted in a small wheel; but as the air, in this manner, could issue only very slowly, it is evident that no motion could be communicated to the wheel. If Kircher's machine then produced any effect, as said, the mechanism of it has not been properly described by Schott. We will not venture however to assert, that the one in question will answer the intended purpose, as we much doubt whether the rising sun would rarefy the air, contained in its lower cavity, in a manner sufficiently sensible in all climates.

Remark.—We shall say nothing farther in regard to the machines which may be put in motion by the compression, or the rarefaction, or condensation, &c. of the air; for if we should imitate Father Schott, we might find sufficient matter to fill a quarto volume. We shall therefore refer those who are fond of such machines to the "Mecanica Hydraulico-Pneumatica" of that Jesuit, printed at Frankfort, in 1657, 4to; and to his "Technica Curiosa, or Mirabilia Artis." Herbip. 1664, two vols. 4to.

The reader will find in these books abundance of such frivolous inventions,

extracted for the most part from the work of Father Kircher, who paid a good deal of attention to them; and from the "Spiritalia" of Hero; and from Alleoti, his translator and commentator; as also the "Philosophia Fontium," of Dobrezensky, &c. &c.

PROBLEM XLVIII.

The phenomena of Capillary Tubes.

Capillary tubes are tubes of glass, the interior aperture of which is very narrow, being only half a line, or less, in diameter. The reason of this denomination may be readily perceived.

These tubes are attended with some singular phenomena, in the explanation of which philosophers do not seem to have agreed. Hitherto it has been easier, in this respect, to destroy, than to build up. The principal of these phenomena are as follow:

I. It is well known that water, or any other fluid, rises to the same height in two tubes, which have a communication with each other; but if one of the branches be capillary, this rule does not hold good: the water in the capillary tube rises above the level of that in the other branch; and the more so, the narrower the capillary tube is.

It seemed very easy to the first philosophers, who beheld this pheuomenon, to give an explanation of it. They supposed that the air, which presses on the water in the capillary tube, experiences some difficulty in exercising its action, on account of the narrowness of the tube; and that the result must be an elevation of the fluid on that side.

This however was not very satisfactory; for what reason is there to think that the air, the particles of which are so minute, will not be at perfect freedom in a tube half a line, or a quarter of a line, in diameter?

But whether this explanation be satisfactory or not, it is entirely overturned by the second and third phenomena of the capillary tubes.

II. When mercury is employed, instead of water, this fluid, instead of rising in the capillary branch, to the level which it reaches in the other, remains below that level.

III. If the experiment be performed in vacuo, every thing takes place the same as in the open air. The cause of this phenomenon then is not to be sought for in the air.

IV. If the inside of the tube be rubbed with any greasy matter, such as tallow, the water, instead of rising above the level, remains below it. The case is the same, if the experiment be made with a tube of wax, or the quills of a bird, the inside of which is always greasy.

V. If the end of a capillary tube be immersed in water, this fluid immediatly rises above the level of that in the vessel, and to the same height to which it would rise in a siphon, if one of its branches were a capillary tube, and the other of the common size; so that if the surface of the water only be touched, it is immediately attracted, as it were, to the height above mentioned, and it remains suspended at that height when the tube is removed from the water.

VI. If a capillary tube be held in a perpendicular direction, or nearly so, and if a drop of water be made to run along its exterior surface, when the drop reaches its lower aperture, it enters the tube, if it be of sufficient size, and rises to the height at which it would stand, above the level, in the branch of a siphon, of that calibre.

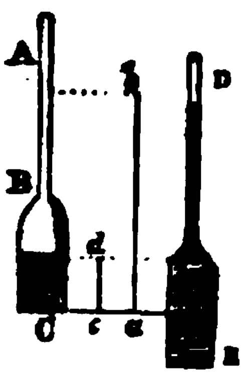
VII. The heights at which water maintains itself in capillary tubes, are in the inverse ratio of the diameters. Thus, if water rises to the height of 10 lines in a

tube one 3rd of a line in diameter, it ought to rise to the height of 10 lines in a tube one 6th of a line in diameter, and to the height of 100 in a tube one 30th of a line in diameter.

The falling of mercury below the level in such tubes, follows also the inverse ratio of the diameters of the tubes.

VIII. If a capillary tube be formed of two parts of unequal calibres, as seen Fig. 29, where the diameter of A B is much less than that of B C, and if a b be the height at which the water would maintain itself in a tube such as A B, and c d that at which it would remain in the larger one B C, when this tube is immersed in such a manner that the aperture of the smaller end B, shall be raised above the level, by a height greater than c d, the water will maintain itself as in D E, at that height c d above the level; but if the tube be immersed in such a manner that the water shall reach to B, it will immediately rise to the same height as if the tube were of the same calibre as that before mentioned.

Fig. 29.



The case is the same, if the capillary tube be immersed, beginning with the narrower branch.

IX. Those persons would be deceived who should imagine, that the lightest liquors rise to the greatest height in these tubes: of aqueous liquors, spirit of wine is that which rises to the least height. In a tube in which water rises 26 lines, spirit of wine rises only 9 or 10. The elevation of spirit of wine, in general, is only the half or a third of that of water.

This elevation depends also on the nature of the glass: in certain tubes, water rises higher than in others, though their calibres be the same.

To be convinced that these effects are not produced by anything without the tube or the liquor, it is necessary to see these phenomena, which are indeed the same in a vacuum, or in air highly rarefied, as in the air which we breathe. They vary also according to the nature of the glass of which the tube is formed; and they are different according to the nature of the fluid. The causes therefore must be sought for in something inherent in the nature of the tube, and in that of the fluid.

This cause is generally ascribed to the attraction mutually exercised between glass and water. This explanation has been controverted by Father Gerdil, a Barnabite and an able philosopher, who has done everything in his power to overturn it. On the other hand, M. de la Lande has stood forth in its defence, and is one of those modern writers who have placed this explanation in the clearest light. The reader may consult also, on this subject, a very learned and profound memoir by M. Weitbrecht, in the Memoirs of the Imperial Academy of Sciences at Petersburg.

PROBLEM XLIX.

Of some attempts to produce a Perpetual Motion, by means of capillary siphons.

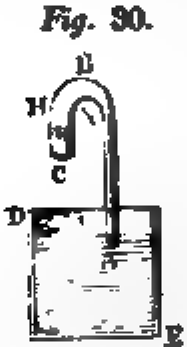
When philosophers saw water rise in a capillary tube, above the level of that in which it was immersed, or above that at which it stood in a wider tube, with which it formed an inverted siphon, they were induced to conjecture the possibility of a perpetual motion: for if the water, said they, rises to the height of an inch above that level, let us interrupt its ascent, by making the tube only three quarters of an inch in height: the water will then rise above the orifice, and falling down the sides into the vessel, the same quantity will again rise, and so on in succession. Or, if the water that rises in the capillary branch of a siphon be conveyed, by an inclined tube, into the other branch, a continual circulation of the fluid will take place; and hence a perpetual motion given by nature.

But, unfortunately, this idea was not confirmed by experiment. If the ascent

of water, in a capillary tube, be intercepted, by cutting the tube at half the height, for example, to which the water ought to rise, the latter will not rise above the orifice to trickle down the sides. And the case will be the same in the other attempt.

The following, however, is a very ingenious one; and it is difficult to discover the cause of its not succeeding.

Fig. 30.



Let A B C (Fig. 30.) be a capillary tube, the diameter of the long branch of which is much smaller than that of the other; it is supposed, that if the orifice A be immersed in the water contained in the vessel D E, it will rise to B, the summit of the bending of the tube; and that in the other branch B C, it will rise only to the height C H above the level.

If the siphon be filled with water, and if it be immersed to such a depth that the water can rise, as above said, to the bending B, it appears evident, and incontestable, that the water in the part B H, will force down that contained in C H. But this cannot take place without the water contained in A B following it; hence the water will continually ascend from A to B, and fall down into the vessel, through the branch B C. Here then we have a perpetual motion.

Nothing is more specious; but unfortunately this illusion is destroyed also by experience: the water does not fall through the branch B C; on the contrary, it rises till the branch A B alone is full.

We think it our duty to subjoin here another idea of a perpetual motion, by means of two siphons, though the siphons employed for this purpose are not altogether capillary. It deserves the more attention, as it was not proposed by an obscure person, but by one who is justly classed among the greatest mathematicians; we mean the celebrated John Bernoulli.

Fig. 31.

Let there be two liquors, said Bernoulli, susceptible of being mixed together, the specific gravities of which are as the lines A B and C D (Fig. 31.); it is well known that if two tubes, which communicate with each other, have their heights above the branch of communication in the same ratio, the shortest branch may be filled with the heaviest fluid, and the longest with the lightest, and these two fluids will remain in equilibrio: hence it follows, that if the longer branch be cut somewhere below the length it ought to have, the fluid contained in this branch will run into the lower one.

Let us now suppose that the shorter branch E F, is filled with a fluid composed of two liquors of different specific gravities, and that a filtre be placed in the point F, so as to afford a passage only to the lighter; let the tube F G be filled with the latter, and let its height be somewhat less, in order to establish an equilibrium between the liquor in the branch E F, and that in F G.

Things being in this state, as the filtre suffers only the lighter liquor to pass, the latter, in consequence of the equilibrium being destroyed, will be impelled outwards, through the orifice G; and consequently may be conveyed by a small pipe into the orifice E, where it will again mix with the liquor contained in E F: and this effect will always continue, because the column of liquor G F will be too light to counterbalance the compound column E F. Here then we have a perpetual motion; and this, says Bernoulli, is that which maintains the rivers, by means of the water of the sea; for, adhering to the old ideas, in regard to the origin of fountains, he imagined it was by a similar mechanism that the sea water, deprived of its salt, was conveyed to the summits of the mountains. He only rejected the idea of those who pretend that it

rises above its level, in consequence of the property of capillary tubes; for he remarked that in that case it would not flow down.

We will not venture to assert what might be the case, if it were possible to realise the suppositions of Bernouilli: we are however strongly inclined to believe that it would not succeed; and as the above reasoning, in regard to capillary tubes, though in appearance convincing, is belied by experience, we are of opinion that the case would be the same with this of Bernoulli.

PROBLEM L.

The prodigious force of moisture to raise burthens.

One of the most singular phenomena in physics, is the force with which the vapour of water, or moisture, penetrates into those bodies which are susceptible of receiving it. If a very considerable burthen be affixed to a dry and well stretched rope, and if the rope be only of such a length as to suffer the burthen to rest on the ground, on moistening the rope you will see the burthen raised up.

The anecdote respecting the famous obelisk erected by Pope Sixtus V., before St. Peter's, at Rome, is well known. The chevalier Fontana, who had undertaken to raise this monument, was, it is said, on the point of failing in his operation, just when the column was about to be placed on its pedestal. It was suspended in the open air; and as the ropes had stretched a little, so that the base of the obelisk could not reach the summit of the pedestal, a Frenchman cried out "Wet the ropes." This advice was followed; and the column, as if of itself, rose to the necessary height, to be placed upright on the pedestal prepared for it.

This story, however, though often repeated, is a mere fable. Those who read the description of the manœuvres which Fontana employed to raise his obelisk, will see that he had no need of such assistance. It was much easier to cause his capstans to make a few turns more than to go in quest of sponges and water to moisten his ropes. But the story is established, and will long be repeated in France, because it relates to a Frenchman.

However, the following is another instance of the power of moisture, in overcoming the greatest resistances: it is the method by which millstones are made. When a mass of this stone has been found sufficiently large, it is cut into the form of a cylinder, several feet in height; and the question then is, how to cut it into horizontal pieces, to make as many millstones. For this purpose, circular and horizontal indentations are cut out quite around it, and at proper distances, according to the thickness to be given to the millstones. Wedges of willow, dried in an oven, are then driven into the indentations by means of a mallet. When the wedges have sunk to a proper depth, they are moistened, or exposed to the humidity of the night, and next morning the different pieces are found separated from each other. Such is the process which, according to M. de Mairan, is employed in different places for constructing millstones.

By what mechanism is this effect produced? This question has been proposed by M. de Mairan; but in our opinion, the answer which he gives to it is very unsatisfactory. It appears to us to be the effect of the attraction by which the water is made to rise in the exceedingly narrow capillary tubes with which the wood is filled. Let us suppose the diameter of one of these tubes to be only the hundreth part of a line; let us suppose also that the inclination of the sides is one second, and that the force with which the water tends to introduce itself into the tube, is the fourth part of a grain: this force, so very small, will tend to separate the flexible sides of the tube, with a force of about 50000 grains; which make about 8¾ pounds. In the length of an inch let there be only 50 of these tubes, which gives 2500 in a square inch, and the result will be an effort of 21875 pounds. As the head of a wedge, of the kind above mentioned, may contain four or five square inches, the force it exerts will be

equal to about 90 or 100 thousand pounds; and if we suppose 10 of these wedges in the whole circumference of the cylinder, intended to form millstones, they will exercise together an effort of 900 thousand or a million of pounds. It needs, therefore, excite no surprise that they should separate those blocks into the intervals between which they are introduced.

PROBLEM LI.

Of Papin's Digester.

Papin's digester is a vessel, by means of which a degree of heat is communicated to water, superior to that which it acquires when it boils. Water indeed exposed to common air, or the mere pressure of the atmosphere, however strongly it boil, can acquire only a certain degree of heat, which never varies; but that inclosed in Papin's digester acquires such a degree, that it is capable of performing operations, for which common boiling water is absolutely insufficient. A proof of this will be seen in the description of the effects produced by this machine.

This vessel may be of any form, though the cylindric or spherical is best; but it must be made of copper or brass. A cover must be adapted to it, in such a manner as to leave no aperture through which the water can escape. To prevent the vessel from bursting, a hole is made in the side of it, or in the cover, some lines in diameter, with an ascending tube fitted into it, on which is placed the arm of a lever kept down by a weight. This lever serves as a moderator to the heat; for if there were no weight on the aperture of this regulator, the water, when it attains to a certain degree of common ebullition, would escape almost entirely through the aperture, either in water or in steam: if the weight be light, the water, in order to raise it, must assume a greater degree of heat. If there were no regulator of this kind, the machine might burst into pieces, by the expansive force of the steam. For this reason, it is proper that the machine should be of ductile copper, and not of cast iron; because the former of these metals does not burst like the latter, but tears as it were; so that it is not attended with the same dangerous consequences.

When the machine is thus constructed, fill it with water, and having fitted on the cover, let it be fastened strongly down by a piece of iron placed over it, which can be well secured by screws: then complete the filling it through the small tube which serves as a moderator or register, and set it over a strong fire. The water it contains cannot boil; but it acquires such a degree of heat that it is able, in a short time, to soften and decompose the hardest bodies; while the same effect could not be produced by ebullition continued for several weeks: it is even said that the heat may be carried so far, as to bring the machine to a state of ignition; in which case it is evident that the water must be in the same; but in our opinion this experiment is exceedingly dangerous.

However, the following are some of the effects of this heat, when carried only to three, four, or five times that of boiling water.

Horn, ivory, and tortoise-shell, are softened in a short time, and at length reduced to a sort of jelly.

The hardest bones, such as the thigh bone of an ox, are also softened, and at length entirely decomposed; so that the gelatinous part is separated from them, and the residuum is nothing but earthy matter. When no more than the proper degree of heat has been employed in this decomposition, the jelly may be collected: it coagulates as it cools, and may be made into nourishing soup, which would be equal to that commonly used, if it had not a taste somewhat empyreumatic. This jelly may be absolutely formed into dried cakes, which will keep exceedingly well, provided they be preserved from moisture. They may serve as a substitute for meat soup, &c.

Hence it may be conceived, how useful this machine might be rendered in the arts, for economy, and in navigation.

From these bones, thrown away as useless, food might be obtained for the poor in times of scarcity, or some ounces of bread, with soup made from the above cakes, would form wholesome and nourishing aliment.

Sailors might carry with them, during their long voyages, some of these cakes, preserved in jars hermetically sealed; they would cost much less than preparations of the same kind from meat, as the matter of which the former are made is of no value. The sailors, who are accustomed to live on salt provisions, would be less exposed to the scurvy. At any rate, these cakes might be reserved till a scarcity of fresh meat or of any other kind of provisions, which so often takes place at sea. It would be a great advantage to have collected into a small volume the nourishing part of several oxen; for since a pound of meat contains, at least, an ounce of gelatinous matter reduced to dryness, it thence follows, that 1500 pounds of the same meat, which is the whole weight of a bullock, would give only 94 pounds, which might be easily contained in an earthen jar.

In the last place, it would be of great use to the arts, to be able, with a machine of this kind, to soften ivory, horn, bone, and wood, so as to render them susceptible of being moulded into any form at pleasure.

PROBLEM LII.

What is the reason that in winter, when the weather suddenly becomes mild, the air in houses continues, even for several days, to be colder than the exterior air?

This question will present no difficulty to those who are acquainted with the phenomena of the communication of heat. It is well known, indeed, that the rarer a body is, the less time it requires to become hot, or to cool; and, on the other hand, that the denser it is, the more obstinately it retains the heat it has acquired.

Hence it may be easily conceived, that when cold has prevailed for some time, all the bodies of which our houses are composed, are cooled to the same degree as the exterior air. But when the exterior air, by any particular cause, becomes suddenly warmer, these bodies do not immediately assume the same temperature: they lose only gradually that which they had acquired; and during this time the interior air, which is surrounded by them, retains the same degree of cold.

Houses, strongly built—that is to say, constructed of good squared stone, which have thick walls—must, for this reason, retain much longer the cold they have received from the exterior air; and, for the same reason, the air within these houses will remain longer at a temperature below that of the atmosphere, than in houses built in a slighter manner: for the same reason, also, it will be less cold in such houses, at the commencement of winter, than in slighter houses.

PROBLEM LIII.

Of some natural signs, by which a change of the present temperature of the air can be predicted.

This part of philosophy, we must confess, is still in its infancy. No person has ever yet been able to make a series of observations, sufficient to shew the connection which subsists between the temperature of the air, and different physical signs which are commonly supposed to precede them. We shall here confine ourselves to a few of those signs, which are commonly considered as indications of good or bad weather, but we will not warrant them as infallible.

1st. When a strong hoar frost is seen on the ground in the morning, during winter, it will not fail to rain the second or third day after, at farthest.

2d. It has also been remarked, that it commonly rains on the day when the sun appears red or pale; or the next day when the sun at the time of setting is involved in a large cloud; in this case, if it rains, the next day is exceedingly windy. The same thing almost always takes place also, when the sun at setting appears pale.

3d. When the sun is red at the time of his setting, it is commonly a sign of fine weather the next day; on the other hand, if he rises red, rain or a strong wind commonly takes place the day after.

4th. When a white mist or vapour is seen to rise from the water or marshy places, after the sun has set, or a little before he rises, one may conjecture, with some degree of probability, that next day will be fine.

5th. When the moon is pale, it indicates rain; when red, wind may be expected; and when of a pure and silver colour, it is a sign of fair weather, according to this verse:

Pallida luna pluit, rubicunda flat, alba serenat.

6th. When small black clouds, detached from the rest, and lower, are seen wandering here and there, or when several clouds are seen collected in the west, at sun rise, it is a sign of a future tempest. If these clouds, on the other hand, disperse, it is a sign of fine weather. When the sun appears double or treble through clouds, it prognosticates a storm of a long duration. It is the sign of a great storm also, when two or three broken and spotted circles are seen around the moon.

7th. When an iris, or rather halo, is seen around the moon, it is a sign of rain; and if a halo is seen around the sun, during bright and serene weather, it is also a sign of rain: but if the halo appear in the time of rain, it is a sign of fine weather.

8th. If animals shew signs of fear and uneasiness, while the weather is exceedingly calm and close, it is almost certain that a storm will ensue. The barometer, in this case, falls exceedingly low all of a sudden.

9th. Indications of rain not being far distant, may be gathered from the actions of various animals, as follows:

When birds are seen more employed than usual in searching among their feathers, for the small insects which torment them;

When those which are accustomed to remain on the branches of trees, retire to their nests;

When the sea-gulls, and other aquatic fowls, and particularly geese, make a greater noise than usual;

When the swallows fly very low, and seem to skim over the surface of the earth;

When pigeons return to the pigeon-house before their accustomed time;

When certain fish, such as the porpoise, sport at the surface of the water;

When the bees do not quit their hives, or fly only to a very short distance;

When sheep bound in an extraordinary manner, and push each other with their heads;

When asses shake their ears, or are very much stung by the flies;

When flies and gnats sting more severely, and are more troublesome than usual;

When a great number of worms issue from the earth;

When frogs croak more than usual;

When cats rub their heads with their fore-paws, and lick the rest of their bodies with their tongue;

When foxes and wolves howl violently;

When the ants quit their labour, and conceal themselves in the earth;

When the oxen, lying together, frequently raise their heads, and lick each other's muzzles;

When the cocks crow before their usual hour;

When domestic fowls flock together, and squeeze themselves into the dust;
When toads are heard crying in elevated places.

10th. During the time of rain, if any small blue space of the heaven be observed, one may almost be assured that the rain will not be of long duration; this sign is well known to huntsmen.

11th. Very violent storms, when accompanied with earthquakes, are almost always preceded by an extraordinary calm in the air, and of that alarming kind which seems the silence of nature about to be convulsed. Animals, more sensible of these natural indications than man, are frightened by it, and hasten to their retreats. Sometimes a hollow subterranean noise is heard. When all these signs are united, the inhabitants of the unfortunate countries, subject to these destructive scourges, ought to fly from their houses, that they may avoid the danger of being buried under their ruins.

We shall not entertain our readers with the prolix description which Ozanam, or his continuator, here gives of one of these storms which spread devastation throughout the kingdom of Naples, in the time of the famous queen Jean.

12th. An English navigator says he observed that an aurora-borealis was always followed, in the course of a few days, with a violent gale from the south-east; and he gives this notice to navigators, about to enter the Channel, that they may be upon their guard.*

PROBLEM LIV.

To separate two liquors, which have been mixed together.

This operation is merely an application of the property of capillary tubes, and of that law of nature by which homogeneous fluids, when near each other, unite together. This is observed to be the case with two drops of mercury, or water, or oil, when they come into contact. It is even probable that, before they are in contact, they lengthen themselves, and mutually approach each other.

However, if you are desirous of separating water, for example, from the oil with which it has been mixed, take a bit of cloth or sponge, well moistened in water, and place it, immersing it by one end, in the vessel containing the liquors to be separated; the other end must be made to pass over the edge of the vessel, and to hang down much lower than the surface of the liquor: this end will soon begin to drop, and in this manner will attract and separate all the water mixed with the oil.

If it be required to draw off the oil, the rag or sponge must be first immersed in that liquid.

But those who should imagine that wine or alcohol can be separated in this manner from water, would be deceived: in order that the operation may succeed, the two liquors must be nearly immiscible together, otherwise they will both pass over at the same time. This process, therefore, cannot be employed for separating water from wine, though it has been given in the preceding editions of the Mathematical and Philosophical Recreations, with many others equally childish.

The colouring part of the wine appears indeed to remain behind, because it is less attenuated than the phlegm and spirit; but in reality these two liquors, of which wine essentially consists, are not separated from each other.

PROBLEM LV.

What is the cause of the ebullition of Water?

Though this question, on the first view, may appear as of little importance, it deserves to be examined; for those who might imagine that the bubbling observed

* See Philos. Transact. vol. LXV. p. 1.

in water which boils, is the necessary consequence of the heat it receives, would be deceived. That the contrary is the case may be proved by the following experiment.

Immerse, with the necessary care, any vessel, such as a bottle filled with water, for example, into kettle containing water in a strong state of ebullition; the water in the bottle will not fail to assume, in a short time, a degree of heat absolutely equal to that of the water which boils; this will be proved by means of a thermometer, yet the smallest sign of ebullition will not be observed in it.

What then is the cause of that observed in the water, which is immediately exposed to the action of the fire?

In our opinion, the boiling up is the effect of portions of the water, which touch the sides of the vessel, suddenly converted into vapour by coming into contact with these sides; for when a vessel rests on burning coals, its bottom tends to receive a degree of heat much greater than that necessary to convert immediately into vapour a drop that falls upon it. The pellicle of water which touches the bottom must, therefore, be continually converted into vapour; and this indeed is the case; for bubbles of an elastic fluid are continually seen rising from the bottom, and these bubbles, carried by an accelerated motion to the surface, in consequence of their lightness, produce there that bubbling which constitutes ebullition.

But the water contained in the bottle, immersed in the boiling liquid, cannot assume a degree of heat greater than that of boiling water; because, however strong the ebullition may be, the water does not acquire a greater degree of heat. On the other hand, a piece of metal, heated only to the degree of boiling water, does not convert the water it touches into vapour; the water therefore contained in the interior vessel, though become equally warm, cannot boil. Such is the explanation of the two phenomena; and their necessary connection with each other, as well as with the assigned cause, proves the truth of that cause.

PROBLEM LVI.

What is the reason that the bottom of a vessel, which contains water in a high state of ebullition, is scarcely warm?

Before we attempted to enquire into the cause of this phenomenon, we thought it proper first to assure ourselves of the fact, for fear of exposing ourselves to ridicule, like those who explain in so ingenious a manner the phenomenon of the child in Silesia with the golden tooth; a phenomenon however which was only a deception, as well as that which occurred to the marquis of Vardes, explained with so much sagacity by Regis, and which however was the trick of a servant. And the case is the same with many others, which ought first to be confirmed, before we attempt to explain them. We brought water therefore to a strong state of ebullition, in an iron vessel, and having touched the bottom of it, while the water was boiling, we indeed found that it had but a very moderate heat; it did not begin to be burning hot, till the moment when the ebullition had almost ceased.

In our opinion, this effect is produced in the following manner: we have already shewn, that the ebullition is occasioned by the pellicle of water, which touches the bottom of the vessel, being continually converted into vapour. This conversion into vapour cannot take place, without the bottom always losing some of that heat, which it acquires by the contact of the coals or fire. But during the interval between the moment when the vessel is taken from the fire, and that when it is touched, as no new igneous fluid reaches it, though it still continues to boil, it is probable that the remainder of this fluid is absorbed by the water which touches the bottom, and which is converted into vapour.

Without giving this explanation as absolutely demonstrative, we are strongly

inclined to think that such is the real case; and what seems to give it more probability is, that while the bottom of the vessel, from which the boiling proceeds, is but little hot, the sides have the heat of boiling water; so that the finger would be burnt, were it kept as long on them as it can be kept on the bottom. But no sooner has the boiling ceased, than the bottom itself receives part of the heat of the water, and the finger cannot then touch it without being burnt.

Remark.—The solution of the following little problem depends, in all probability, on a similar cause.

To melt lead in a piece of paper.

Wrap up a very smooth ball of lead in a piece of paper, taking care that there be no wrinkles in it, and that it be every where in contact with the ball; if it be held, in this state, over the flame of a taper, the lead will be melted without the paper being burnt. The lead, indeed, when once fused, will not fail in a short time to pierce the paper, and to run through.

PROBLEM LVII.

To measure the moisture and dryness of the air. Account of the principal Hygrometers invented for that purpose; their faults, and how to construct a comparative Hygrometer.

The air is not only susceptible of acquiring more or less heat, but also of becoming more or less humid. It belongs therefore to philosophy, to measure this degree of moisture; especially as this quality of the air has a great influence on the human body, on vegetation, and many other effects of nature.

This gave rise to the invention of the hygrometer, an instrument proper for measuring the humidity of the air.

But it must be allowed, that the instruments hitherto invented for this purpose do not give that result which might have been expected. We have hygrometers indeed, which indicate that the air has acquired more or less moisture than it had before; but they are not comparative, that is to say, they do not enable us to compare the moisture of one day or place with that of another.* It is, however, proper that we should make known the different kinds of hygrometers, were it only that we may be able to appreciate their utility.

I. As fir-wood is highly susceptible of participating in the dryness or humidity of the air, an idea has been conceived of applying this property to the construction of a hygrometer. For this purpose, a very thin small fir board is placed across between two vertical immoveable pillars, so that the fibres stand in a horizontal direction; for it is in the lateral direction, or that across its fibres, that fir and other kinds of wood are distended by moisture. The upper edge of the board ought to be furnished with a small rack, fitted into a pinion, connected with a wheel, and the latter with another wheel having on its axis an index. It may be easily perceived, that the least motion communicated by the upper edge of the board to the rack, by its rising or falling, will be indicated in a very sensible manner by the index; consequently, if the motion of the index be regulated in such a manner, that from extreme dryness to extreme moisture it may make a complete revolution, the divisions of this circle will indicate how much the present state of the atmosphere is distant from either of these extremes.

This invention is ingenious; but it is not sufficient. The wood retains its moisture a long time after the air has lost that with which it is charged; besides, the board

* This is not altogether correct. M. de Luc has described, in the Philosophical Transactions, the method of constructing a hygrometer, which approaches very near to what might be desired in this respect. We have added it to this article.

gradually becomes less sensible to the impressions of the air, and therefore produces little or no effect.

II. An hygrometer may be made also with the beard of a wild oat, fixed on a small column, placed in the centre of a round box: the other extremity of the beard passes through the centre of the cover of the box, the circumference of which is divided into equal parts; in the last place, a small index, made of paper, is adapted to the extremity of the beard. In order to afford access to the air, it is necessary that the sides of the box should be open, or cut into holes.

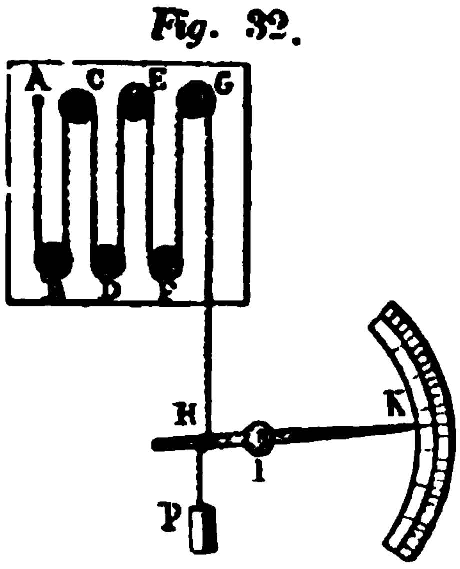
When this instrument is exposed to drier or moister air, the small index, by turning round, either in the one direction or the other, indicates the state of the atmosphere.

But this hygrometer, which is exceedingly sensible at first, gradually this loses this property: consequently, it is a very imperfect instrument, as well as the following.

III. Suspend a small circular plate by a string, or piece of catgut, fastened to its centre of gravity; and let the other end of the string be attached to a hook. According as the air is more or less moist, you will see the small plate turn round, in one direction or in another. This small machine may be covered by a bell-glass, to prevent its being deranged by the agitation of the air; but the bell must be elevated above the base on which it is placed, that the air may have access to the string.

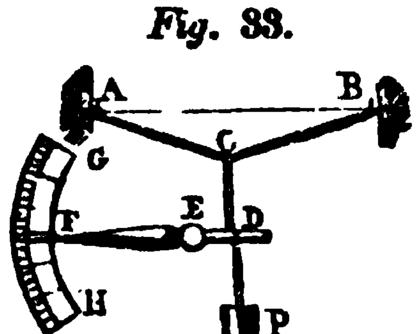
The hygrometers commonly sold, are constructed on this principle. They consist of a kind of box, the fore part of which represents an edifice with two doors. On one side of the metal plate which turns round, stands the figure of a man with an umbrella to defend him from the rain; and on the other, a woman with a fan. The appearance of the former of these figures, indicates damp, and that of the other, dry weather. This pretended hygrometer can serve for no other purpose than to amuse children; the philosopher must observe that, as the variations of humidity are transmitted to this instrument only by degrees, it will indicate moisture or drought, when the state of the atmosphere is quite contrary.

Fig. 32.



If a piece of cat-gut, made fast at one extremity, be conveyed over different pulleys, as A, B, C, D, E, F, G, &c., (Fig. 32.), so as to make several turns backwards and forwards; and if a weight P, be suspended from the other extremity, it may be easily seen that it must rise or fall in a more sensible manner, in consequence of the moisture or dryness of the air, according as the number of the turns backwards and forwards is greater. But this will be indicated better if an index H K, turning on a pivot I, and placed in such a manner that the part I K shall be much longer than I H, be made fast to the extremity of the cord H; the slightest change in the moisture of the air, will be manifested by the point K of the index.

Fig. 33.



V. An hygrometer may be constructed also in the following manner. Extend a cord, five or six feet in length, between the pegs A and B (Fig. 33), and suspend from the middle of it C, a weight P, by a thread P C. If an index D F, turning on the pivot, E, and having the part E F much longer than D E, be adapted to the thread P C, as seen at D; as the cord A C B will be shortened by moisture and lengthened

by drought, the weight P, as well as the point D, will rise or fall, and consequently make the index pass over a certain portion of the arc G H, the divisions on which will indicate the degree of moisture or dryness.

VI. Put into the scale of a balance any salt that attracts the moisture of the air, and into the other a weight, in exact equilibrium with it. During damp weather, the scale containing the salt will sink down, and thereby indicate that the state of the atmosphere is moist. An index, to point out the different degrees of drought or moisture, may be easily adapted to it.

This instrument, however, is worse than any of the rest; for salt, immersed in moist air, becomes charged with a great deal of humidity; but loses it very slowly when the air becomes dry: fixed alkali of tartar even imbibes moisture, till it is reduced to a liquid or fluid state.

VII. Music also may be employed to indicate the dryness or moisture of the air. The sound of a flute is higher during dry than during moist weather. If a piece of cat-gut then, extended between two bridges, be put in a state of vibration, it will emit a tone with which a tonometre must be brought into unison. When the weather becomes moister, the string will emit a lower sound; and the contrary will be the case when the air becomes drier.

VIII. M. de Luc of Geneva, to whom we are indebted for an excellent work on thermometers and barometers, attempted to construct a comparative hygrometer, and published a paper on that subject in the Philosophical Transactions. The description of this hygrometer is as follows.

Fig. 34.

It has a great resemblance to a thermometer. The first and principal part is a cylindric reservoir of ivory, about 2½ inches in length, the cylindric cavity of which is 2½ lines in diameter, and the thickness ¼ or $\frac{1}{16}$ of a line. This piece of ivory must be cut from about the middle of an elephant's tooth, both in regard to its thickness and length; and it is necessary that the cavity should be pierced in a direction parallel to that of the fibres. A representation of this piece is seen Fig. 34. where it is denoted by the letters A B C.

Fig. 34.
No. 2.

The second piece is a tube of turned copper, one end of which fits exactly into the ivory cylinder, while the other receives into its cylindric cavity a glass tube of about a quarter of a line internal diameter. A representation of it is seen Fig. 34, No. 2.

Fig. 34.
No. 3.

These three pieces are strongly fixed to each other, by introducing into the ivory cylinder the end of the copper tube destined to fill it, having first put a little fish glue between them. To unite these parts better together, the neck of the ivory cylinder ought to be surrounded by a virol of copper.

A glass tube of about 30 inches in length, and of such a size as to fit into the same cavity, is also introduced into it, as seen Fig. 34, No. 3, which represents the instrument completely constructed.

It is then filled with mercury, in such a manner that it shall rise to about the middle of the glass tube, and the ivory reservoir is immersed in water ready to freeze, taking care to maintain it at that temperature for several hours; for the ivory will require ten or twelve before it absorbs all the moisture it is capable of receiving. As soon as this reservoir is immersed in the water, the mercury is seen to rise, at first very quick, and then more slowly, until it at length remains stationary towards the

bottom of the tube. This place, which ought to be some inches above the insertion of the glass tube into the copper one, must be marked o, which signifies the zero of dryness or the greatest humidity. This point, as we have said, must be some inches higher than the copper tube; for it has been remarked, that if the instrument be immersed in hot water the mercury falls still lower, and this interval below zero is left for the purpose of marking these divisions.

We must here acknowledge, that we do not properly understand how M. de Luc proceeds in order to render his instrument comparative: something, in our opinion, still remains to be done to give it that property. We must therefore refer the reader to the original memoir, in the "Journal de Physique" of the abbé Rozier, for the year 1775. It will be sufficient to observe, that this hygrometer is very sensible; scarcely has it been placed in air more or less humid, than it gives indications of that sensibility, by the rise or fall of the mercury; but it requires, and always will require, to be accompanied with a thermometer; for the same degree of humidity has a greater effect on it during warm weather, than during cold; besides, the mercury rises or falls independently of moisture, merely by the effect of heat. This instrument, therefore, requires a double correction; the first to keep an account of the dilation which the mercury experiences by heat, a correction which will be minus whenever the heat exceeds the term of freezing: the second, to reduce the effect of the moisture observed, to what it would have been had the temperature been at freezing.

It may be readily conceived, of how great advantage it would be, in regard to the improvement of this hygrometer, to find a degree of dryness, or of less humidity, fixed and determinable in every country, to serve as a second fixed term, like that of water reduced to the temperature of melting ice, namely that of the greatest humidity; this would tend greatly to simplify the graduation of the instrument, which, according to the method of M. de Luc, appears to us to be complex and uncertain. But this is enough on the present subject, respecting which the nature of our work will not permit us to enter into farther details.

All the preceding contrivances have been superseded by the hygrometer invented by *Professor Daniel* of King's College, London.

We shall here give a concise abridgment of the account of this instrument, from the article on the subject in the Library of Useful Knowledge.

M. Le Roi having suggested the temperature at which dew *begins* to be deposited as a measure of the moisture of the air, *De Luc* proved that the quantity and force of vapour in vacuo, are the same as in an equal volume of air of the same temperature, or that these two elements of vapour depend on the temperature.

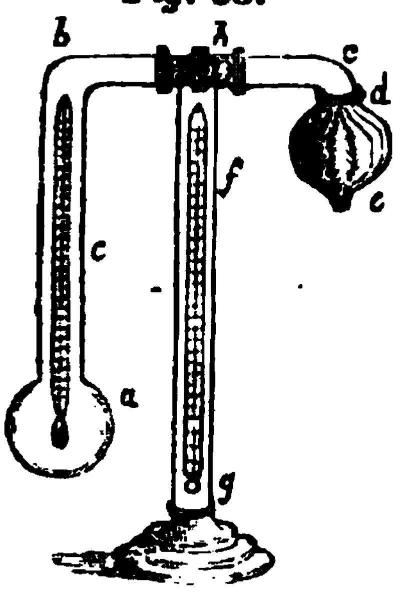
Dr. Dalton investigated the force of vapour at every degree of temperature from 0° to 212° Fahrenheit, and expressed this force by the height of the mercurial barometric column which it could support; and he has given the results in a tabular form, which are thus easily applied to hygrometric purposes. The *dew point* is found by pouring *cold* water into a glass, and noting the temperature at which, in the open air, dew ceases to be formed on the sides of the glass. This is the point at which, in air of that temperature, dew would just begin to be formed. Hence may be inferred not only the force exerted by the vapour, but its quantity in a perpendicular column of the whole atmosphere, and the force of evaporation.

Thus if the *dew point* be at 45° Fahrenheit, the force of vapour, by Dalton's table, is ·316 of an inch, or about the 95th part of the pressure of the air as measured by 30 inches of the mercurial column; or if the specific gravity of steam be ·70, the weight of the steam or vapour in a given volume of air will be 136th part of the whole. Now as the force of a whole atmosphere of steam, at the surface of the earth, would be the weight of a perpendicular column of it, and in a mixture of steam

and air the force exerted by each is as their relative weights: when the dew point is 45° the superincumbent column of vapour in the atmosphere, being one 95th part of the whole atmospheric pressure, is equivalent to the pressure of 4·3 of water; or the vapour, if condensed, would give that depth of water. Dalton has hence shewn how to find the force of evaporation at a given time; for the quantity of water evaporated from a given surface is proportioned to the maximum force of vapour at the temperature of the surface; the vapour continuing in contact with the surface of the water. Hence, as an example, if the dew point is 45° when the temperature of the air is 50°, we have by Dalton's table $\cdot375 - \cdot316 = \cdot059$ the force of evaporation.

On this principle is constructed Daniel's hygrometer, which in its most improved form is represented at Fig. 35.

Fig. 35.



The ball a is of black glass about 1·25 inch in diameter, and connected with a transparent glass ball d, of the same size, by a bent tube one eighth of an inch in diameter. A portion of sulphuric ether sufficient to fill about three fourths of the ball a is introduced; a small mercurial thermometer, with an elongated bulb, is fixed inside the limb $a\ b$, and the atmospheric air being expelled, the whole is hermetically sealed. The ball d is covered with muslin, and the whole is supported on a brass stem $f\,g$, on which is another delicate thermometer. The tube can be removed from the spring tube h; and the whole instrument, with a phial of ether, packed neatly in a box, which may be carried in the pocket.

The dew point is ascertained thus. The ether being all brought into the ball a by inclining the tube, the balls are placed perpendicularly, the temperature of the air is noted by the thermometer attached to the stand, and ether is gradually dropped on the muslin cover of d; and the cold produced by the evaporation of this ether, condenses the elastic etherial vapour within the ball, which produces a rapid evaporator from the ether in a, and lowers in consequence the temperature within the instrument. When the black ball is thus cooled to the dew point, a film of condensed vapour like a ring surrounds the ball; and if the thermometer inside the limb c be noted at that instant, we obtain the true dew point of air at the temperature shewn by the other thermometer.

Having thus found the dew point, and the temperature of the external air, the moisture contained in a cubic foot of air may be found from the following formula.

Weight in grains $= \dfrac{5656\cdot2}{448+t} \times p$, where t is the temperature of the external air, and p the elasticity of the aqueous vapour at the temperature shewn by the interior thermometer, which for every degree of the thermometer is given in Dr. Dalton's tables of the elastic force of steam. See the 5th vol. of the *Manchester Memoirs*.

PROBLEM LVIII.

On the supposition of what we have before shewn, in regard to the tenuity of the particles of light, and their great velocity; what loss of its substance may the sun sustain, in a determinate number of years?

One of the most specious objections made to the Newtonian theory of light, is, that if light consisted of a continual emanation of particles, thrown off from luminous bodies, the sun would have sustained such a loss of his substance, that he must have have been extinguished or annihilated, since the time at which he is commonly supposed to have been created. This objection we have always considered as of little weight; and we have long been convinced that, assuming as basis what can

be easily proved in regard to the tenuity of the particles of light, and their great velocity, a very probable hypothesis could be formed, from which it might be shewn, that no sensible diminution could have taken place in the sun, during the course of the 6000 years which he is commonly supposed to have existed.

We have since seen, in the Philosophical Transactions, a similar calculation by Dr. Horsley, to shew the frivolity of such an objection. But as there are different methods of considering the same question, our reasoning on the subject is as follows: it has nothing in common with that of the learned Englishman, but the prodigious tenuity of the particles of light.

To form this calculation, we suppose and require it may be granted, that at each instantaneous emanation of light from the sun, this luminary projects in every possible direction all the particles of light at its surface.

We require, it may be granted also, that this emanation is not absolutely continued, but composed of a multitude of instantaneous emanations or jets, which succeed each other with prodigious rapidity: we shall suppose that there are 10000 in a second. As the retina of the eye preserves for about $\frac{1}{8}$ of a second the impression it receives, it is evident that the impression made by the sun will be absolutely continued in regard to us.

We shall suppose also, what is almost proved, that the diameter of a particle of light is scarcely the 1000000000000th part of an inch.

According to these suppositions, it is evident that the sun, at each emanation, deprives himself of a luminous pellicle, the thickness of which is as before stated; consequently, in the course of a second, it will be the 100000000th part of an inch, and in 100000000 seconds this luminary therefore will have lost an inch in thickness. But 100000000 are nearly three years: in three years, then, the sun will have lost only an inch in thickness.

Hence it appears, that in the course of 3000 years, this loss will amount to 1000 inches, or $83\frac{1}{3}$ feet in depth; and during the 6000 years, which we suppose the sun to have existed, it will be $166\frac{2}{3}$. Hence it follows, that before the sun can lose one second only of his apparent diameter, forty millions of years must elapse; for the diminution of a second in the apparent diameter of the sun corresponds to 360000 yards: if in the course of 6000 years, the diminution is only about 54 yards in depth, it will be found, by the rule of proportion, that it will require 40 millions of years to make it extend to the depth of 360000 yards in thickness, or one second of apparent diameter.

We need therefore entertain no fear of the sun becoming extinct. Our children and grand children, at least, are secured from being witnesses of that fatal event.

We shall here add, that we have not taken the benefit of all the advantages we might have employed; for we might have extended this period much farther; and Dr. Horsley indeed finds a much greater interval between the present moment and the final consumption of the sun. But we have confined ourselves to those suppositions which are most admissible.

PROBLEM LIX.

To produce, amidst the greatest heat, a considerable degree of cold, and even to freeze water. On artificial congelations, &c.

It is a very singular phenomenon, and highly worthy of admiration, that a cold far exceeding that of winter can be produced even in the middle of summer; and what adds to the singularity is, that this production of cold does not take place unless the ingredients employed become liquid. Sometimes even by re-acting on each other they produce a strong effervescence. We shall first take a cursory view of the different means of producing cold; and then endeavour to give some explanation of the phenomenon.

I. Take water cooled only to the temperature of our wells, that is to say, to 10 degrees of Reaumur's thermometer, and for every pint throw into it about 12 ounces of pulverised sal ammoniac; this water will immediately acquire a considerable degree of cold, and even equal to that of congelation. If a smaller vessel then containing water be put into the one containing this mixture, the water in the former will freeze, either entirely or in part. If it freezes only in part, make a mixture in another vessel, similar to the first, and immerse in it the half-frozen water: by these means it will be entirely congealed.

If you employ this water half frozen, or at least greatly cooled in the interior vessel, and throw into it sal ammoniac, the cold produced will be much more considerable: a cold indeed several degrees below that of ice will speedily be the result.

If this mixture be made in a flat vessel on a table, with a little water placed between them, the ice formed below will make the vessel adhere to the table.

The solution of the salt must be accelerated as much as possible, by stirring the mixture with a stick; for the speedier the solution, the greater will be the cold.

II. Pulverise ice, and for one part of it mix two parts of marine salt; stir well the mixture, and a cold equal to that of the severest winter will be produced in the middle of the mass. By these means Reaumur was able to produce a cold 13 degrees below congelation.

Saltpetre, employed in the same quantity, will produce a cold only 3 or 4 degrees below freezing. It is a mistake therefore, as Reaumur observes, to imagine that saltpetre produces a greater effect than marine salt. Saltpetre is employed only because it is cheaper; and besides, when artificial cold is applied to domestic purposes, it is not necessary that it should be considerable.

Instead of saltpetre, Alicant soda, or the ashes of green wood, which contain an equivalent salt, might be employed: the same effect would be obtained, and at a much less expense.

III. A cold much greater, however, than any of the preceding, may be produced in the following manner. Take snow and well concentrated spirit of nitre, both cooled to the degree of ice; pour the spirit of nitre on the snow, and a cold 17 degrees below that of congelation will be immediately excited.

If you are desirous of producing a cold still more considerable, surround the snow and spirit of nitre with ice and marine salt; which will produce a cold 12 or 13 degrees below zero; if you then employ the snow and spirit of nitre cooled in this manner, a cold equal to 24 degrees below zero will be the result. This cold is much greater than that produced by Fahrenheit; for it did not exceed 8 degrees of his thermometer below zero, which amounts to 17⅞ degrees of Reaumur, below the same term.

But this is nothing in comparison of what the philosophers of Petersburgh performed, towards the end of the year 1759. Assisted by a cold of 30 degrees and more, they cooled snow and spirit of nitre below that temperature, and by these means obtained a degree of cold which, reduced to the scale of Reaumur's thermometer, was more than 170* degrees below zero. It is well known that at this term mercury freezes, and of the consequences of this experiment we have spoken elsewhere.

IV. There is still another method of producing a cold superior to that even which is necessary to freeze water. It is founded on a very singular property of evaporable fluids. Immerse the bulb of a thermometer in one of these fluids, such as well dephlegmated spirit of wine, and then swing it backwards and forwards in the air, to

* This number, when corrected, ought to be only 31½ below water-freezing on Reaumur; or 39, that is 71 below water-freezing, on Fahrenheit. See the remark at the end of Problem 18.

excite a current like that of the wind, which promotes the evaporation of the fluid; you will soon see the thermometer fall: by employing ether, the most evaporable of all liquors, you may even make the thermometer fall to 8 or 10 degrees below zero.

Very curious things might be said in regard to this property of evaporation; but to enlarge farther on the subject would lead us too far. We shall therefore only observe, that this method of cooling liquors is not unknown in the east. Travellers, who are desirous of drinking cool liquor, put their water into jars made of porous earthern ware, which suffers the moisture to ooze through it. These vessels are suspended on the sides of a camel, in such a manner as to be in continual motion, which answers the same purpose as if they were exposed to a gentle wind, and which causes the moisture to evaporate. By these means the remaining liquor is so much cooled, as to approach the degree of congelation.

We shall now offer a few observations on the cause of these singular effects, beginning with the means explained in the first three articles.

When ice and marine salt, or spirit of nitre and snow very much cooled, are mixed together, it is observed that cold is not produced unless these substances be dissolved. From this circumstance there is reason to conjecture, that the mixture absorbs the igneous fluid diffused throughout the surrounding bodies, or those surrounded by the mixture, which amounts to the same thing. The melting mixture produces, in this case, the same effect as a dry sponge applied to a moist body: as long as it is merely confined around it, no change will take place in it; but as soon as the sponge is at liberty to extend itself to its full volume, it will absorb a considerable part of the moisture contained in that body. It must be confessed, that we do not see the mechanism by which the frigorific mixture produces the same effect; but we may consider the above comparison as capable of giving some idea of it.

In regard to the reason why an evaporable liquor cools the bodies from which it evaporates, it appears that the most probable reason is the affinity which that liquor has to fire; so that each of its moleculæ, in flying off, carries with it some of those of the fire contained in that body. But how comes it that these moleculæ of the evaporable liquor do not combine rather with the fire which the air can furnish to it, and with which that element seems to have less adhesion than to solid bodies, since it cools more readily? This question we cannot answer; but we give the above explanation merely as conjecture.

Remark.—In addition to what has been given on this subject by Montucla, we shall here observe, that the best experiments yet made known on frigorific mixtures, without the aid of snow, are those of Mr. Walker, of Oxford: some of these are as follows:

Take strong fuming nitrous acid, diluted with water (rain or distilled water is best) in the proportion of 2 parts in weight of the former to one of the latter, well mixed, and cooled to the temperature of the air, 3 parts; of Glauber's salts 4 parts; of nitrous ammonia $3\frac{1}{2}$ parts,* each by weight, and reduced separately to fine powder. The Glauber's salt is to be first added to the diluted acid; the mixture must then be well stirred, and the powdered nitrous ammonia is immediately to be introduced, stirring the mixture again. The salts should be procured as dry and transparent as possible, and are to be used newly powdered.

These are the best proportions, when the common temperature is 50°. According as the temperature, at setting out, is higher or lower, the quantity of diluted acid must be proportionably diminished or increased. This mixture is little inferior

* A powder composed of sal ammoniac 5 parts, and nitre 4 parts, mixed together, may be substituted for the nitrous ammonia.

to one made by dissolving snow in nitrous acid; for it sunk the thermometer from 32° to 20°; that is in all 52°. In this experiment 4 parts diluted acid were used.

Crystallized nitrous ammonia, reduced to very fine powder, sunk the thermometer, during its solution in rain water, from 56° to 8°; when evaporated gently to dryness, and finely powdered, it sunk the thermometer to 49°. Mr. Walker has frequently produced ice by a solution in water of this salt alone, when the thermometer stood at 70°. If an equal weight of mineral alkali, finely powdered, be added to the mixture, the temperature will be lowered 10° or 11° more.

As it is evident that artificial frigorific mixtures may be applied to domestic purposes, in hot climates, especially where the inhabitants can scarcely distinguish summer from winter by the sense of feeling, it may not be amiss to give a few hints respecting the easiest method of using them.

In most cases, the following cheap one may be sufficient: Take any quantity of strong vitriolic acid, diluted with an equal weight of water, and cooled to the temperature of the air, and add to it an equal weight of Glauber's salt, in powder. This is the proportion when the temperature, set out with, is 50°; and will sink the thermometer to 5°; if the temperature be higher than 50°, the quantity of salt must be proportionally increased.

The obvious and best method of ascertaining the quantity of any salt necessary to produce the greatest effect by solution, in any liquid, at any given temperature, is to add the salt gradually, till the thermometer ceases to sink, stirring the mixture all the time. If a more intense cold be required, double aqua-fortis, as it is called, may be used. Glauber's salt, in powder, added, will produce very nearly as much cold as when added to diluted nitrous acid. A somewhat greater quantity of the salt is required. At the temperature of 50°, about three parts of the salt, to 2 of the acid, will sink the thermometer from that temperature to nearly 0°; and the consequence of more salt being added is, that it retains the cold rather longer. This mixture has one great advantage in its favour: it saves time and trouble. A little water in a phial immersed in a tea-cup full of this mixture, will be soon frozen, even in summer; and if the salt be added in crystals, not pounded, to double aqua-fortis, though in a warm temperature, the cold produced will be sufficient to freeze water or cream; but if diluted with one fifth of its weight of water, and cooled, it will be nearly equal to the diluted nitrous acid before mentioned, and will require the same proportion of the salt.

A mixture of Glauber's salt and diluted nitrous acid, sunk the thermometer from 70°, the temperature of the air and ingredients, to 10°.

The cold in any of these mixtures may be kept up a long time, by occasionally adding the ingredients in the proportions indicated.

Take equal parts of sal ammoniac and nitre, in powder; and cool them by immersing the vessel which contains them in pump water newly drawn, its temperature being generally 50°. On three ounces of this powder pour four ounces, wine measure, of pump water, at the above temperature, and stir the mixture; its temperature will be reduced to 14°, and consequently it will soon freeze the contents of any small vessel immersed in it. The cold may be continually kept up and regulated for any period of time, by occasionally pouring off the clear saturated liquor, and adding more water; taking care to supply it constantly with as much of the powder as it can dissolve. This is a convenient mixture; for if the solution be afterwards evaporated to dryness in an earthen vessel, and reduced to powder, it will answer the purpose as well as at first; as its power does not seem to be lessened by being repeatedly treated in this manner.

All the ingredients employed by Mr. Walker being taken at the temperature of 50°, the following table will exhibit the result of a great many experiments:

	Temperature.
*Sal ammoniac 5, nitre 5, water 16 parts	10°
Do. ——— 5, do. 5, Glauber's salt 8, water 16	4
*Nitrous ammoniac 1, water 1	4
Do. ——— 1, soda 1, water 1	7
†Glauber's salt 3, dilute nit. acid 2	3
Glauber's salt 6, sal ammoniac 4, nitre 2, dilute nit. acid 4	10
Do. ——— 6, nitrous ammoniac 5, dilute nit. acid 4	14
Phosphorated soda 9, dilute nit. acid. 4	12
Do. ——— 9, nitrous ammon. 6, dilute nit. acid 4	21
†Glauber's salt 8, marine acid 5	0
†Do. ——— 5, dilute vitriolic acid 4	3

The salts marked thus (*) may be recovered by evaporating the mixture, and may be used again repeatedly; those marked thus (†) may be recovered for use by distillation and crystallization: the dilute nit. acid was red fuming nitrous acid 2 parts, rain water 1 part: the dilute vit. acid was strong vitriolic acid and rain water, equal parts.

By a judicious management, frigorific mixtures, with the aid of snow or pounded ice, mercury even may be frozen into a solid mass. Mr. Walker immersed a half pint glass tumbler containing equal parts of vitriolic acid, the specific gravity of which was 1·5596, and strong fuming nitrous acid, in mixtures of nitrous acid and snow, until the mixed acids in the tumbler were reduced to — 30°: he then gradually added snow, which had been also previously cooled in a frigorific mixture to — 15°, to the mixture in the tumbler, stirring the whole, and found, after some minutes, that the mercury in a thermometer immersed in the fluid had become congealed or frozen.

Quicksilver may be congealed by adding newly fallen snow to strong fuming nitrous acid, previously cooled to between — 25° and — 30°, which may be easily and speedily effected by immersing the vessel containing the acid in a mixture of snow and nitrous acid.

But the most powerful frigorific mixture yet discovered, is produced by equal parts of muriate of lime and snow. An account of a very remarkable experiment of this kind is given in Tilloch's Philosophical Magazine, Vol. III. It was performed by Messrs. Pepys and Allen. Into a mixture of equal parts of muriate of lime at 33°, and snow at 32°, a bladder containing no less than 56 pounds of mercury was immersed, after the mixture had liquefied by stirring, and when its temperature was found to be — 42°; as soon as the cold mixture had deprived the mercury of so much of its heat that its own temperature was raised from — 42° to + 5°, the mercury was taken from it, and put into another fresh mixture, the same in every respect as the first.

In the mean time, the muriate of lime was kept cooling, by immersing the vessel which contained it into a mixture of the same ingredients: 5 pounds of the muriate were, by these means, reduced to — 15°; a mixture being made of this muriate and snow, at the temperature of 32°, in the course of three minutes it gave a temperature of — 62°, or 94° below the freezing point of water.

The mercury reduced to — 30° by immersion in the second mixture, and suspended in a net, was put into the new made mixture, and the whole was covered with a cloth to impede the passage of heat from the surrounding atmosphere. After an hour and forty minutes, the 56 pounds of mercury were found solid and fixed. The temperature of the mixture, at this time, was — 46°; that is 16° higher than when the mercury was put into it.

Several of those who were present at this experiment having, without attending to

the consequences, taken pieces of the frozen mercury into their hands, experienced a painful sensation, which they could compare to nothing but that produced by a burn or a scald, or by a wound inflicted with a rough edged instrument. The parts of the hand which were in contact with the metal lost all sensation, and became white, and to appearance dead: a phenomenon which alarmed the sufferers not a little: however, soon throwing away the pieces from them, as they would have done hot coals, the injury scarcely penetrated the skin; and in a little time the parts, by friction, resumed their usual sensation and colour.

The late Professor Leslie devised an elegant method of reducing the temperature sufficiently low to freeze water, in any climate, and at any season of the year. His method is shortly this: under the receiver of an air pump, place one vessel containing sulphuric acid, and another containing a small quantity of water. The air being partly withdrawn from the receiver by the air pump, vapour is raised abundantly from the water, and absorbed by the acid. Thus a degree of cold is produced which freezes the water in a very short time.

A saucer of porous earthenware is best adapted for holding the water, and instead of sulphuric acid, other absorbents may be used, such as parched oatmeal, the powder of mouldering whinstone, or the dry powder of pipeclay.

Mr. Leslie placed a hemispherical vessel of porous earthenware, containing a pound and a quarter of water, over a body of parched oatmeal, one foot in diameter and one inch deep; and by working the pump for some time, the whole of the water was frozen.

PROBLEM LX.

To cause water to freeze, by only shaking the vessel which contains it.

During very cold weather, put water into a close vessel, and deposit it in a place where it will experience no commotion; in this manner it will often acquire a degree of cold, superior to that of ice, but without freezing. If the vessel however be agitated ever so little, or if you give it a slight blow, the water will immediately freeze with singular rapidity. This will be the case, in particular, when the water is in vacuo.

This phenomenon is exceedingly curious; but in our opinion, it is susceptible of an explanation which must appear highly probable to those acquainted with the phenomena of congelation. Water does not congeal unless its moleculæ assume a new arrangement among themselves. When water cools, at perfect rest, its moleculæ approach each other, and the fluid which keeps it in fusion gradually escapes; but something more is necessary to determine the moleculæ to arrange themselves in a different manner, under angles of 60 or 120 degrees. This determination they receive by the shock given to the vessel; they were in equilibrio; the shock destroys that equilibrium, and they fall one upon another, forming themselves into groups, in such a manner as their approach to each other requires.

Another phenomenon of congelation is as follows. If you boil water, and then expose it to the frost, close to an equal quantity of unboiled water, the former will freeze sooner than the latter.

This is a fact proved by experiments, made at Edinburgh, by Dr. Black, and, in our opinion may be easily explained; as congelation is occasioned by the moleculæ of the water approaching each other: it must congeal the sooner, if these moleculæ, before being exposed to the frost, are already closer. But water which has boiled possesses, in this respect, an advantage over that which has not boiled; for the effect of boiling is to deprive it of a great deal of its combined air; these moleculæ then, *cæteris paribus*, must arrive sooner at the term of proximity, at which they adhere to each other, and form a solid body. We are convinced, that for the same

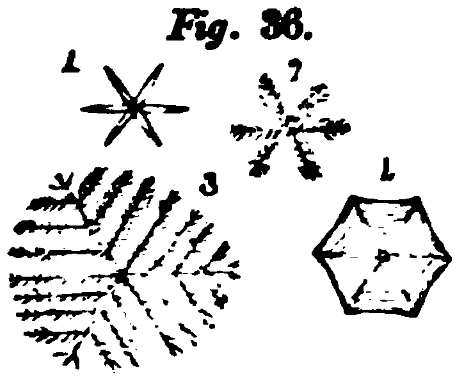
reason, water impregnated by artificial means with air, would be longer in freezing than common water.

PROBLEM LXI.

Of the figure observed sometimes in Snow: Explanation of that phenomenon.

It often happens, and it has long been remarked with admiration, that the small flakes of snow have a regular figure. Such is the case in particular, when the snow falls gently, and in very small flakes. This figure is hexagonal or stellated; sometimes it is a plain star with six radii; at other times the star is more complex, and resembles a cross of Malta, having six salient and six re-entering angles. It sometimes happens that each branch presents ramifications, like the barbs of a feather; but it would be too tedious to describe them all. We shall therefore confine ourselves to a representation of the most remarkable, as seen Fig. 36.

Fig. 36.



This phenomenon has always occasioned great embarrassment to philosophers, since the time of Descartes and Kepler, who seem to have been the first who remarked it. Bartholin wrote a dissertation "De Figura Nivis Hexangula," in which he reasons very badly on the subject. It was indeed difficult to reason justly on it,* until M. de Mairan observed, as he did with great sagacity, the phenomena of congelation, and until chemistry had discovered those of the crystallization of bodies, when they pass from a fluid to a solid state.

Chemistry indeed has taught us that all bodies, the elements of which, floating in a fluid, calmly approach each other, assume regular and characteristic figures. Thus sulphur, when it becomes fixed, forms long needles; regulus of antimony has on its surface the figure of a star. Salts, when they crystallize slowly, assume regular figures also. Marine salt forms cubes, alum octaedra, gypsum a kind of wedges, regularly irregular, the laminæ of which break into triangles of determinate angles; calcareous spar, called *Icelandic Crystal,* forms oblique parallelopipeda under invariable angles, &c.

On the other hand, M. de Mairan, while observing the progress of congelation, saw that the small needles of ice, which are formed, are implanted one into the other at regular and determinate angles, which are always 60 or 120 degrees.

Whoever is acquainted with these phenomena, will see nothing in ice and snow but a crystallization of water, condensed in cold air: one particle of frozen water meets another, and unites with it, at an angle of 60 degrees; a third is added, and is determined by the action of the point of the first angle, to unite itself in the same manner, &c. This is the simplest of the stars of snow, as represented by No. 1.

If new needles of ice are added, which will for the most part be the case, they must place themselves on the first radii, either by making an obtuse or an acute angle towards the centre. In the first case, the result will be a star, the radii of which have a kind of barbs like a feather, as in No. 2, or like a star, as No. 3. The last arrangement however is rare, and that of No. 2 is more common. Some are also seen, though in less number, much more complex; but whatever may be their composition, their elements are always angles of 60 or 120 degrees.

M. Lulolf of Berlin conjectured that these figures were occasioned by the sal ammoniac, or rather volatile alkali, with which snow is impregnated: and in support of this idea, he mentions a very pretty experiment. Having exposed some water to

* We find however that Gassendi referred the regular figure of snow to crystallization. See ad Diog. Laert. Not. opp. vol. I. p. 377.

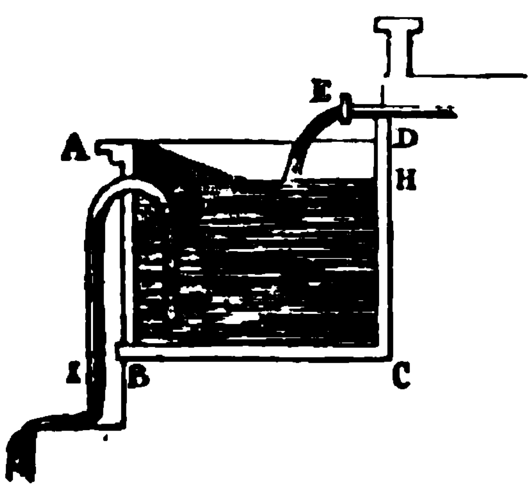
freeze near the common sewer, he found the surface of it entirely covered with small stars of ice, while the frozen water which was at a greater distance, exhibited nothing of the kind. He acknowledges however, that he was never able, by any process, to detect this principle in snow water, or snow melted in close vessels. No philosopher at present will indeed believe, that either sal ammoniac or volatile alkali exists in snow, unless accidentally; and there is no necessity of supposing it, in order to explain its crystallization in stars.

PROBLEM LXII.

To construct a fountain, which shall alternately flow and intermit.

We have already described the mechanism of a fountain which produces this effect, and which is well known to those acquainted with hydraulics; but as it cannot be adapted to the purposes we have here in view, we shall give another method of solving the problem.

Fig. 37.



Let A B C D (Fig. 37.) be a vessel of any form, which receives by the pipe E a continual influx of water, capable of filling it to the height G H, in the interval, for example, of two hours. Let F G I be a siphon, the upper orifice of which, immersed in the liquor, is F; let F G be the shorter, and G I the longer, branch, the orifice of which, I, must be considerably below the level of F; lastly, let the bore of this siphon be such, that it can draw off the liquor contained in the part G I, in the course of half an hour. These suppositions being made; if the vessel be empty, and if the water be suffered to run in by the pipe E, it will fill the vessel to the height G, in two hours, for example; but when it reaches the bending G, the siphon F G I will be filled, and the water flowing into it, in the course of somewhat more than half an hour,* it will empty, not only the water collected as far as G H, but that also which the pipe E may have furnished during that time; because this discharging pipe F G I will exhaust much more rapidly, than that which furnishes, viz. D E. The surface of the water, always descending, will at length fall to the level of the orifice F, and the air introducing itself, the play of the siphon will be interrupted: the water will then begin to rise again to the bending of the siphon at G, so that the play of the siphon will recommence, and this will be the case as long as the pipe E can furnish water.

Remark.—It is necessary to observe, that the siphon will not perform its effect, unless it be a capillary tube as far as the bending; for if its diameter, at this place, be 5 or 6 lines, the water, when it reaches to a little above the bending, will flow off without filling the whole pipe; and the pipe would run off a quantity of water equal only to that furnished by the pipe E. This observation was made, and with great justice, by the Abbé Para du Phanjas, who had recourse therefore, in this case, to several capillary tubes, uniting in one.

Another remedy consists in making the calibre of the discharging pipe capillary, throughout its whole length, and proportionally wide in a horizontal direction, in order that it may have the same surface, and that the same quantity of water may flow through it. By these means the discharging pipe, though single, will perform its office.

* This time will be exactly 40 minutes; for it is the sum of a sub-quadruple progression, the first term of which is 30 minutes, the second $7\frac{1}{2}$, &c.

PROBLEM LXIII.

To construct a fountain which shall flow and stop a certain number of times successively; and which shall then stop, for a longer or shorter period, and afterwards resume its intermitting course; and so on.

The solution of this problem depends on a very ingenious combination of two intermittent fountains, similar to the preceding. Let us suppose a similar fountain, the periodical flowing of which is exceedingly quick, that is to say, 2 or 3 minutes, and its intermission the same, making altogether an interval of 4 or 5 minutes: let this fountain be fed by another intermittent fountain, placed above it, the duration of the flowing of which is an hour, and the intermittence 2, or 3, or 4: it will thence follow, that the lower one will furnish water only while the upper one supplies it; that is to say, during an hour; and in the course of this hour the lower fountain will have 12 or 15 periods of flowing, interrupted by as many periods of cessation; after which time, as the fountain or pipe E of Fig. 37 will not furnish more water for two or three hours, the lower fountain will absolutely cease for one, or two, or three hours. Here then we have a fountain which will be doubly intermittent, as it will remain a considerable time without flowing, and when it flows it will be intermittent.

Remarks.—I. With three fountains of this kind, combined together, periods of flowing and intermission, so singular as to appear almost inexplicable, might be produced. But it may be readily conceived, that they would all depend on the same principle.

II. By means of these principles, a fountain to flow continually, but which should become larger and decrease alternately, might be easily constructed. Nothing would be necessary for this purpose, but to combine with the fountain of the preceding problem, a continued fountain: it is evident that it would become larger, when the water flowed through the siphon F G I; and that when it stopped, it would assume its usual state.

If this continued fountain were combined with the double intermittent one of this problem, the result would be a fountain uniform and continued for several hours of the day, and which would afterwards become larger and decrease alternately for an hour.

PROBLEM LXIV.

Construction of a fountain which shall cease to flow when water is poured into it; and shall not begin to flow again till some time after.

For this purpose, we must suppose a very close reservoir, half filled with water, as A B C D (Fig. 38.), having a discharging pipe E, some lines only in diameter. This reservoir forms part of another vessel H B F D, in which it is placed; and a portion of the vessel H G F remains empty: I K is a pipe which proceeds from the top of the interior reservoir, nearly to the bottom of the vessel, F D; the upper part of the vessel is funished with a rim, so as to resemble a cup, and the part H G, is pierced with a number of small holes; some moss, with coarse sand, or even grass, must be put into this cup, but in such a manner that the air may have access through the bottom of H G, into the cavity H C.

Fig. 38.

These things being supposed, let the small reservoir be half filled with water, which will flow out through the discharging pipe E; if water be then poured into the cup at the top, it will fall into

the lateral reservoir H C, and close the aperture K of the pipe K I. This aperture being closed, the air contained in that part above the interior reservoir, can no longer expand itself: the water flowing through E will fall at first slowly, and at length stop. But if a small pipe be inserted in the corner F, to afford a passage to the water which has fallen into the reservoir H C, when this water is discharged, that at E will again begin to flow.

If water be poured incessantly into the cup H G, and if its escape at F be concealed, this machine will excite great astonishment, as it will seem to flow only when no more water is poured into it.

This machine might be constructed in the figure of a rock, with a fountain issuing from the bottom of it; and the upper part might represent a meadow, or forest, &c. On pouring water over it from a watering pot, to represent rain, the small fountain would be seen to stop, and to continue in that state as long as water was poured over it. The use to which this idea might be applied will be seen hereafter.

PROBLEM LXV.

To construct a fountain which, after flowing some time, shall then sink down to a certain point; then rise again; and so on alternately.

Though we have not found anything satisfactory on this subject, it is nevertheless possible; for we shall mention hereafter some instances of fountains, the basons of which exhibit this phenomenon. We shall therefore content ourselves for the present with having proposed the problem to our readers.

REMARKS.—*Containing the history and phenomena of the principal intermittent fountains known, as well as of some lakes and wells which have similar properties. History of the famous lake of Tschirnitz.*

In the preceding problems we have explained the principles of the phenomena exhibited by a great number of fountains, or collections of water, the properties of which have at all times furnished matter of reflection to philosophers, and been a subject of admiration to the vulgar. But much is to be deducted from what the vulgar relate, or imagine they see, in regard to this subject. Many of these springs, when examined by philosophers, or accurate observers, lose the greater part of what they had of the marvellous. In several of them, however, there still remains enough to exercise the sagacity of the searchers into nature. The object of this work obliges us, in some measure, to make known the most remarkable of these fountains. But we shall confine ourselves to those, the facts respecting which are confirmed by good descriptions; for it is of no utility to repeat what is uncertain or incorrect.

I. The greater part of those springs which originate from accumulations of ice, are observed to be intermittent. Such are some of those seen in Dauphiné, on the road from Grenoble to Briançon. They flow, as we have been assured, more abundantly in the night than in the day time, which on the first view seems difficult to be reconciled with sound philosophy; but we shall shew that this may be explained without much difficulty.

The author of the Description of the Glacieres of Switzerland, speaks of a similar spring, at Engstler, in the canton of Berne: it is subject to a double intermittence, that is to say, an annual and a daily: it does not begin to flow till towards the month of May; and the simple peasants, in the neighbourhood, firmly believe that the Deity sends them this spring every year for the use of their cattle, which about that period they drive to the mountains. Besides, like those of which we have already spoken, it is during the night that it flows in the greatest abundance.

The annual re-appearance of this fountain, on the approach of spring, may be easily explained: for it is only towards this period that the mass of the earth, being sufficiently heated, begins to melt the ice from below. It is at this period, therefore,

that the fountain in question can flow. We make use of the expression from below; for it is in this manner that these enormous masses of ice are melted. No doubt indeed can be entertained of it, when it is observed that they continually give birth to large currents of water, even while their upper surface exhibits the strata of the preceding year scarcely altered. But how comes it that the greater part of these fountains furnish the largest quantity of water in the night time? This phenomenon deserves to be explained.

It arises, in our opinion, from the alternation of heat and cold, occasioned by the presence and absence of the sun, in the mass of the earth covered by this accumulation of ice. But as a certain time is necessary before the heat of the sun can produce its effect, and be communicated to the distant parts, it happens that the moment of their greatest heat is posterior, by several hours, to that of the greatest heat of the air, which takes place about three in the afternoon: it is only some hours then after sun-set, that the greatest liquefaction of the ice, which is in contact with the earth, can be produced; and if we take into consideration the space which the water thence arising must pass through, in confined channels between the valleys and under the ice, it will not seem astonishing that it should not make its appearance till towards night. It will therefore be about eleven o'clock, or midnight, that these streams, produced by the melting of the ice, will furnish the greatest quantity of water.

II. The intermittence in this case depends upon causes which may be easily discovered: it is not even a real intermittence: but the fountains we are about to describe are really intermittent.

A spring of this kind is seen at Fontainebleau, in one of the groves of the Park. It would probably be better known, and would not be inferior in celebrity to that of Laywell, if courts were more frequented by philosophers.

This fountain flows from a sandy bottom, into a bason six or eight feet square: there is a descent to it by several steps, in the last of which, or close to the water, is dug a small channel, which suffers it to run off. The following are the phenomena observed in this fountain.

The bason being supposed to be half full, as is the case when a large quantity of water has been drawn from it, the water rises to the edge of the last step, and runs off by the channel for some minutes. This discharge is followed by a bubbling, sometimes so strong as to be heard at a considerable distance. This is a sign of the speedy falling of the water. It immediately begins, indeed, to fall a few inches below the level of the channel; but this height is variable. It is then stationary for some time; but afterwards rises; and continues in this manner alternately. Each flux of this kind employs about seven or eight minutes. Sometimes however it seems to sport with the curious, and remains half an hour, or even a whole hour, without repeating the same play.

The description of a fountain, nearly similar to the preceding, may be seen in the Philosophical Transactions, Nos. 202 and 424; and in *Desaguliers's* Course, vol. 2: it is situated at one of the extremities of the small town of Brixham, near Torbay, in Devonshire: the people in the neighbourhood call it Lay-Well. It is on the declivity of a small hill, and distant from the shore a full mile; so that it can have no communication with the sea. The bason, according to the latest description, is eight feet in length, and four feet and a half in breadth. A current continually flows into the bason, and the water escapes at the other extremity, through an aperture, three feet broad, and of a proportionable depth.

Sometimes the water flows uniformly for several hours, without rising or falling; and hence some credulous people believe, that the presence of certain persons has an influence on this fountain, which interrupts its play. But, for the most part, it has a very sensible and very speedy flux and reflux. For about two minutes the water rises some inches, after which it falls for about the same period, and then a short rest ensues; so that the total duration is about five minutes. This takes place

twenty times in succession, after which the fountain seems to rest for about two hours, and during that time the water flows in a uniform manner. This, according to the author of the description, is a peculiarity by which this fountain is distinguished from all others that have come within his knowledge. But we have seen that the one of Fontainebleau experiences something of the same kind: a very strong analogy even is remarked between them, and it appears almost evident from the descriptions, that their periodism is not in the spring, but only in the discharge. This is certain, at least in regard to that of Fontainebleau; as the nature of the ground does not permit us to suppose any thing similar to that which requires a periodical flowing in the spring itself.

However, we shall here describe a third fountain, much more considerable than either of the preceding two, and which presents a very striking intermittence; it is situated in Franche-Comté, and a very good description of it was published in the "Journal des Sçavans," for October 1688.

This fountain is, or at least was at that period, near the high road leading from Pontarlier to Touillon, at the extremity of a small meadow, and at the bottom of some mountains which hang over it; it flows from two different places, into two basons, on account of the roundness of which it has acquired the name of *La Fontaine ronde.* The upper bason, which is larger than the other, is about seven paces in length, and six in breadth; and in the middle of it there is a stone cut in a sloping form, which serves to render the motion of its reciprocation sensible.

When the flux is about to commence, a bubbling is heard within the fountain, and the water is immediately seen to issue on all sides, producing a great many air bubbles: it rises a full foot.

During the reflux, the water falls nearly the same time, and by the same gradations. The total duration of the flux and reflux, is about half a quarter of an hour, including about two minutes of rest.

The fountain becomes almost dry at each reflux, and at the end of it is heard a sort of murmuring noise, which announces its cessation.

The small town of Colmars, in Provence, presents also a fountain of the same kind. It is situated in the neighbourhood of the town, and is remarkable for the frequency of its flux. When it is ready to flow, a slight murmur is heard; it afterwards increases for half a minute, and then throws up a jet of water as thick as the arm; it then decreases for five or six minutes, and stops a short time, after which it again begins to flow. In this manner the duration of its flowing and intermittence together is about seven or eight minutes: so that it flows and stops about eight times in an hour. Gassendi and Astruc have given a more detailed account of this fountain; the former in his works, and the latter in his "Histoire Naturelle du Languedoc et de la Provence."

The fountain of Fonzanches, in the diocese of Nismes, deserves also to be mentioned. Fonsanches is situated between Sauve and Quissac, not far from, and on the right of the Vidourle. It issues from the earth, at the extremity of a pretty steep declivity, looking towards the east. Its intermittence is very striking; it flows and stops regularly twice in the course of the day, or of twenty-four hours; the duration of its flux is 7 hours 25 minutes, and that of its intermission 5 hours or nearly; so that its flowing is retarded every day about 50 minutes. But it would be erroneous thence to conclude, that it has any connection either with the motion of the moon, or with the sea, though it has been called *La Fontaine au flux et reflux*. It would be absurd to suppose channels proceeding thence to the sea of Gascony, which is 130 leagues distant. Besides, as the retardation of 50 minutes is not exactly that of the tides, or of the moon's passage over the meridian, the analogy of the one movement with the other can no more be maintained, than if this retardation were much greater or less.

We shall terminate this paragraph with a description of the famous fountain called *Fontestorbe*, situated in the diocese of Mirepoix. The account we shall give of it is extracted from Astruc's description, published in the work before mentioned.

Fontestorbe is situated at the extremity of a chain of rocks, which advance almost to the banks of the river Lers, between Fougas and Bellestat, in the diocese of Mirepoix. At a considerable height above the bed of the river is a cavern, 20 or 30 feet in length, 40 in breadth, and 30 in height. On the right side of this cavern is the fountain in question, in a triangular aperture of the rock, the base of which is about 8 feet in breadth. It is through this aperture that the water issues, when the flux takes place. What characterizes its intermission, in a very singular manner, is, that it is intermittent only during the time of drought; that is to say, in the months of June, July, August, and September: it then flows for 36 or 37 minutes, rising 4 or 5 inches above the base of the triangular aperture, after which it ceases to flow for 32 or 33 minutes. If it happens to rain, the time of intermission is shortened, and when it has rained three or four days in succession, it becomes annihilated; so that the fountain then continues, though with a periodical increase: but at length, when the rain has lasted a considerable time, the flux is continued and uniform, and remains in this state throughout the winter, until the return of dry weather, when the fountain again becomes periodical and intermittent, by the same gradations inverted.

The reason of the greater part of the phenomena here described, may be deduced from the principles explained in the preceding problems. For this purpose, nothing is necessary but to conceive a cavity of greater or less extent, formed by the sinking down of a bank of clay, and which serves as a reservoir to a collection of water, furnished by a spring. Let this cavity have a communication outwards by a kind of crooked channel, the interior aperture of which is near the bottom of the cavity, and the exterior one much lower; this channel will evidently perform the part of the siphon of Prob. LXII. Fig. 37, and will produce the same phenomena, supposing however that the exterior air has access to the cavity.

If the spring then which fills the cavity, here described, always furnishes less water than the supposed siphon can evacuate, the water will flow only periodically; for before it can issue, it must rise to the summit or angle of the two branches of the siphon; it will then flow and evacuate the water contained in the cavity, and it will again stop till more water rises.

But, if the concealed spring, which feeds the reservoir, be variable: that is to say, if it be much more abundant in winter, and during rainy weather, than in summer, or during dry weather, the apparent spring will be intermittent only during the latter; the duration of its intermissions or rest will decrease, according as the concealed spring becomes more abundant; and when the concealed spring gives as much water as the siphon can evacuate, the apparent spring will become continued: it will at length gradually resume its intermittence, according as the interior spring decreases in volume.

Here then the phenomena of the spring of Fontestorbe are explained, by the same mechanism as that of the other springs purely intermittent. It appears, that in the latter the concealed spring derives its origin from subterraneous water, which receives little or no augmentation from exterior water; and that, on the contrary, the spring of Fontestorbe is fed by water arising from rain and melted snow.

We shall add only a few words more, respecting some fountains of this kind, mentioned in various authors. Such is that in the environs of Paderborn, called Bullerborn, which flows, it is said, for twelve hours, and rests during the same period: that of Haute-Combe, in Savoy, near the lake of Bourget, which flows and stops twice in an hour; that of Buxton, in the county of Derby, mentioned by Childrey in his "Curiosités d'Angleterre," which flows only every quarter of an hour; one near the

lake Como, celebrated in the time of Pliny the younger, which rises and falls periodically, three times a day, &c.

III. We shall now describe phenomena of another kind, namely, those exhibited by certain wells or springs, which rise and fall at certain periods, while no place is known by which the water is discharged. There is a well near Brest subject to this periodical falling and rising, the explanation of which has afforded considerable occupation to philosophers. The description we shall give of it is extracted from the Journal de Trevoux, October 1728; it was written by Father Aubert, a jesuit, who appears to have been a very correct and well informed philosopher.

This well is situated at the distance of two leagues from Brest, on the border of an arm of the sea, which advances as far as Landernau. It is 75 feet from the edge of the sea at high water, and nearly double that distance at low water. It is 20 feet in depth, and its bottom is lower than the surface of the sea at high water, but higher than the same surface at low water.

It would not be astonishing, or rather would be altogether in the natural order of things, if the well should sink down at low water, and rise at high water; but the case is quite contrary, as will be seen by the following detailed account of the phenomena observed.

The water of the well is lowest, that is to say is only 11 or 12 inches above its bottom, when the sea is at its highest. It remains in that state about an hour, reckoning from the time of high water: it then increases for about two hours and a half, during the time the sea is ebbing; after which it remains stationary for about two hours. It then begins to decrease for about half an hour before the time of low water, and this continues for the first four hours of the sea's flowing. In the last place, it remains in the same state of falling for about three hours, that is during the last two hours of the sea's rising, and the first hour of its ebbing; after which it again begins to rise, as before mentioned. It was observed during the great drought, in the year 1724, that this well was for some hours dry, while the sea flowed, and that it became full as the sea ebbed. We do not know whether this well be still in existence. What adds to the singularity of the phenomenon is that the neighbouring wells, which might be supposed to experience the same vicissitudes, are subject to nothing of the kind.

According to Desaguliers, a small lake at Greenhithe, between London and Gravesend, exhibits the same phenomena; and this author adds, that he heard at Lambourn, in Berkshire, of a spring which is full in dry weather, and dry during rainy weather. It is much to be wished that he had ascertained the truth of these circumstances.

IV. But every thing hitherto said, though very remarkable, is nothing when compared with the singularity of the famous lake of Tschirnitz. This lake, which is of considerable extent, is situated near a small town of the same name, in Carniola. It is about three French leagues in length, and one and a half in breadth, having a very irregular form.

The singularity of this lake consists in its being full of water during the greater part of the year; but towards the end of June, or the first of July, the water runs off by eighteen holes or subterranean conduits, so that what was the abode of fish and abundance of aquatic fowls, becomes the haunt of cattle, which repair thither to pasture on the grass which is found there in great plenty. Things remain in this state for three or four months, according to the constitution of the year; but after that period, the water returns through the holes by which it had been absorbed, and with so considerable a force that it spouts up to the height of several feet, so that in less than twenty-four hours, the lake has resumed its former state.

It is however to be observed, that there are some irregularities in the time and duration of this evacuation. It sometimes happens that the lake is filled and emptied

two or three times in the year. One year it experienced no evacuation, but it never remains empty above four months. Notwithstanding these irregularities, the phenomenon deserves a place among the most extraordinary singularities of nature. See on this subject a work by M. Weichard Valvasor, a learned man of that country, entitled "Gloria ducatus Corniolæ," &c., 1688, 4to. This author enters into details which entitle him to credit; and besides this, it is a fact well known, and mentioned by various intelligent travellers.

M. Valvasor deduces, with great probability, the phenomena of this lake from subterranean cavities, which communicate with it, by the apertures already mentioned, and which are full of water supplied by the rain. When the rain ceases for a considerable time, so that the water is evacuated to a certain point, a play of siphons takes place, by which means the whole lake is emptied. But for the details of this explanation we must refer to the work before mentioned, or to the Acts of Leipsic for the year 1688.

PROBLEM LXVI.

Of the Speaking Trumpet, and ear trumpet. Explanation of them. Construction of the enchanted Head.

As the sight is assisted by telescopes and microscopes, so similar instruments have been contrived for assisting the faculty of hearing. One of these, called the speaking trumpet, is employed for conveying sound to a great distance: the other, called the ear-trumpet, serves to magnify to the ear the least whisper.

Among the moderns, Sir Thomas Moreland bestowed the most labour in endeavouring to improve this method of enlarging and conveying souud, and on this subject he published a treatise, entitled "De Tuba Stentorophonica," a name which alludes to the voice of Stentor, celebrated among the Greeks for its extraordinary strength. The following observations on this subject are in part borrowed from that curious work.

The ancients, it would seem, were acquainted with the speaking trumpet: for we are told that Alexander had a horn, by means of which he could give orders to his whole army, however numerous. Kircher, on the authority of some passages in a manuscript, preserved in the Vatican, makes the diameter of its greatest aperture to have been seven feet and a half. Of its length he says nothing; and only adds that it could be heard at the distance of 500 stadia, or about 25 miles.

This account is no doubt exaggerated; but however this may be, the speaking trumpet is nothing else than a long tube, which at one end is only large enough to receive the mouth, and which goes on increasing in width to the other extremity, bending somewhat outwards. The aperture at the small end must be a little flattened to fit the mouth; and it ought to have two lateral projections, to cover part of the cheeks.

Sir Thomas Moreland says, that he caused several instruments of this kind to be constructed of different sizes, viz. one of four feet and a half in length, by which the voice could be heard at the distance of 500 geometrical paces; another 16 feet 8 inches, which conveyed sound 1800 paces; and a third of 24 feet, which rendered the voice audible at the distance of 2500 paces.

To explain this effect, we shall not say, with Ozanam, that tubes serve, in general, to strengthen the activity of natural causes; that the longer they are the more this energy is increased, &c.; for this is not speaking like a philosopher; it is taking the effect for the cause: we must reason with more precision.

The cause of this phenomenon is as follows. As the air is an elastic fluid, so that every sound produced in it is transmitted spherically around the sonorous body; when a person speaks at the mouth of the trumpet, all the motion which would be communicated to a spherical mass of air, of four feet radius for example, is com-

municated only to a cylinder, or rather cone of air, the base of which is the wider end of the trumpet. Consequently, if this cone is only the hundredth part of the whole sphere of the same radius, the effect will be as great as if the person should speak a hundred times as loud in the open air; the voice must therefore be heard at a distance a hundred times as great.

The ear trumpet, an instrument exceedingly useful to those almost deaf, is nearly the reverse of the speaking trumpet; it collects, in the auditory passage, all the sound contained within it; or it increases the sound produced at its extremity, in a ratio which may be said to be as that of the wide end to the narrow one. Thus, for example, if the wide end be 6 inches in diameter, and the aperture applied to the ear 6 lines, which in surfaces gives the ratio of 1 to 144, the sound will be increased 144 times, or nearly so; for, we do not believe that this increase is exactly in the inverse ratio of the surfaces; and it must be allowed that, in this respect, acoustics are not so far advanced as optics.

The tube of the ear trumpet is now often made of india-rubber covered with an ornamental net work. It is made of considerable length, and being flexible, the wearer can converse with a person across a table by passing over the end to which the mouth-piece is attached, and applying the other end to his ear.

Remark.—It is a certain fact, proved by experience, whatever may be the cause, that sound confined in a tube, is conveyed to a much greater distance than in the open air. Father Kircher relates, in some of his works, that the labourers employed in the subterranean aqueducts of Rome, heard each other at the distance of several miles.

If a person speaks, even with a very low voice, at the extremity of a tube, some inches in diameter, another who has his ear at the extremity will hear distinctly what is said, whatever be the number of the circumvolutions of the tube.

This observation is the principle of a machine, which excites great surprise in those unacquainted with the phenomena of sound. A bust is placed upon a table; from one or each of its ears a tube is conveyed through the table and one of its feet, so as to pass through the floor, and to end in a lower or lateral apartment. Another tube, proceeding from the mouth, is conveyed in a similar manner, into the same apartment. A person in company is desired to ask the figure any question, by whispering into its ear. A confederate of the one who exhibits the machine, by applying his ear to the extremity of the first tube, hears very plainly what has been said: and placing his mouth at the aperture of the other tube, returns an answer, which is heard by the person who proposed it. If motion be communicated at the same time, to the lips of the machine, by any mechanical means, the ignorant will be much surprised, and inclined to believe that this phenomenon is the effect of magic. It may be easily seen, however, that the cause is very simple.

PROBLEM LXVII.

When boys play at Ricochet, or duck and drake, what is the cause which makes the stone rise above the surface of the water, after it has been immersed in it?

This play is well known, as most boys amuse themselves with it, when near a piece of water of any extent. But the cause why the stone rebounds, after it has touched the surface of the water, seems to be involved in a certain degree of obscurity; and we will even say that some philosophers have mistaken it, by ascribing it to the elasticity of the water. As water has no elasticity, it is evident that this explanation is not well founded.

This rebounding however depends on a cause which approaches very near to elasticity. It is the effort made by every column of water, depressed by a shock, to rise up and resume its former situation, in consequence of a sort of equilibrium which

must prevail between it and its neighbours. But let us enter into a more detailed analysis of what takes place on this occasion.

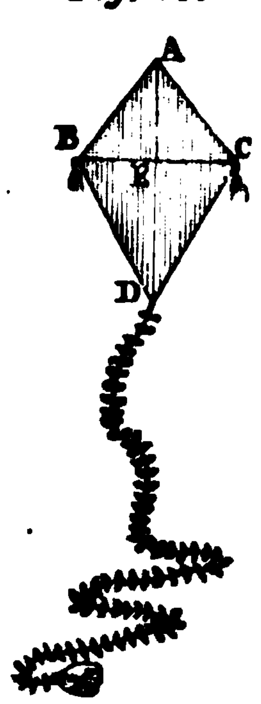
When the stone, which must be flat, is thrown obliquely at the surface of the water, and in the direction of its edge, it is evident that it is carried by two kinds of motion compounded together, one horizontal, which is quicker, and the other vertical, which is much slower. The stone when it reaches the surface of the water, impels it by the effect of the latter only, and depresses a little the column of water which it meets; this produces a resistance which weakens the vertical movement, but without destroying it; so that it continues to dip, depressing other columns; hence there result new resistances, which at length annihilate this motion, so far as it is vertical. The stone has then reached the greatest depth to which it can attain, and must necessarily describe a small curve, the convexity of which is opposite to the bottom of the water; but, at the same time, its motion so far as it is horizontal, has lost little or nothing. On the other hand, the column, depressed by the shock of the stone, reacts against it, being pushed by the neighbouring columns: and hence there arises a vertical motion communicated to the stone, which is combined with the remaining part of its horizontal motion. The result then must be an oblique motion, tending upwards; which causes the stone to rebound above the water, making it describe a very much flattened small parabola; it then again strikes the water obliquely, which produces a second rebounding; then a third, a fourth, and so on, always decreasing in extent and height, till the motion is entirely annihilated.

PROBLEM LXVIII.

Mechanism of paper Kites. Various questions in regard to this amusement.

Every one is acquainted with the amusement of the paper kite, a very curious small machine, which in its mechanism displays great ingenuity. To some however it may appear astonishing that an object of this nature should form the subject of an academic memoir; for there is one on paper kites in the Transactions of the Academy of Berlin for the year 1756. But this surprise will cease when it is known that M. Euler was a profound geometrician, at an age when most young persons see nothing in the paper kite but an object of amusement: to him therefore it could hardly fail of being a subject of meditation. It presents indeed several curious questions, and which for the most part cannot be treated without the higher analysis. This memoir therefore may be ranked among the *juvenilia* of a great mathematician. We shall not follow him in his profound calculations; we shall content ourselves with treating the subject in a less rigorous manner, but much easier to be understood.

Fig. 39.



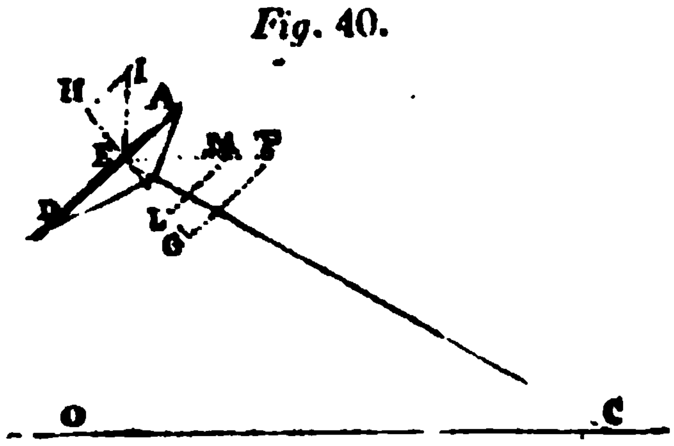
The kite, as is well known, is a plane surface A B C D (Fig. 39.), as light as possible, shaped like an irregular rhombus; that is to say, formed of two triangles, B A C and B D C, in which the angle A of the former is much greater than the angle D of the latter. The head is towards A, and D is the tail, to which is generally affixed a long cord, having pieces of paper attached to it at certain lengths: some much shorter are placed at the corners B and C, which cause the small machine, when elevated, to appear at a distance like a monstrous bird balancing itself in the air by the help of its tail and its wings.

At a point of the axis A D, and towards the point E, is affixed a small cord, some hundreds of feet in length, rolled upon a stick, to be let out or taken in as occasion may require. But it is necessary that this cord should be made fast to the kite in a certain manner; for, in the first place, two other small cords proceeding from a point near the place where it is attached must be extended to the points B and C, to pre-

vent the machine from turning on the axis A D; and secondly, from the same point of the cord, another small cord must proceed to a point near the head A; so that the angle formed by the cord with the axis A D, shall be acute towards A, and invariable: a fourth even is made to proceed from this point of the cord to a point near D.

These arrangements being made; when the kite is to be committed to the wind, an assistant holds the cord at the distance of some yards, and the inferior surface of the kite being exposed to the wind, it is thrown up into the air. The person who holds the cord then begins to run againt the wind in order to increase the action of the air on its surface. If a considerable resistance is experienced, a little of the cord is successively unrolled, and the kite rises: it is necessary to know how to govern it by unrolling or winding up the cord properly; that is to say, letting it go when it is found by the effort experienced that the kite can still rise, and winding it up when it becomes slack. A kite properly constructed, when the time and place are favourable, can rise to the height of three or four hundred feet, and even more.

Fig. 40.



To analyse this amusement, and explain what takes place, let us suppose that A D (Fig. 40.) represents the axis of the kite, to which is attached the cord E C, held at C by the person who directs it. Let F E be the direction of the wind, all the currents of which we suppose united in one, acting on the centre of gravity of the surface of the kite; and which, for the sake of simplifying, we shall suppose not to differ from that of the body itself, or to be very near it.

Let F E represent the force with which the wind, to which the kite is exposed, impels its surface in a perpendicular direction; draw E G perpendicular to that surface, and make E L a third proportional to E F and E G, and draw L M, parallel to G F; E L will represent the force with which the wind impels the lower surface of the kite, in the perpendicular direction, and L M will be the effort exercised by this impulse in the direction M L or A E D.

We shall first observe, that by the latter the kite would tend to be precipitated downwards; but the angle A E C being acute, there thence results an effort in the direction E A, which counterbalances the former; otherwise the kite could not support itself, and this is the reason why this angle must necessarily be acute.

If we now make E H equal to E L, and draw E I perpendicular to the horizon, and H I perpendicular to E H, we shall have two new forces; one of which, I H, will act in the direction E D, and tend to throw down the kite: but this force is annihilated, as well as the former M L, by the power in C, which draws according to the acute angle A E C. The other, E I, is that which tends to make the kite rise in a vertical direction.

Hence, if the force E I be greater than the weight of the kite, it will be raised into the air, and if we suppose that the extremity of the cord is fixed in C, it will turn around the point C as it rises; but by turning in this manner it must necessarily happen that the wind will fall with more obliquity on its surface A D; so that there will at length be an equilibrium. The kite then will rise no farther, unless the cord is let out; in which case it will rise parallel to itself, and as in ascending it will meet with freer air and stronger wind, it will still turn a little around the angle C; or the angle C will become greater, and approach more and more to a right angle.

Such is the mechanism by which the paper kite rises into the air. It may be rea-

dily seen, that if the velocity of the wind, with the surface and weight of the kite, be known, as well as the constant value of the angle A E C, the height to which it will rise may be determined.

A question, which here naturally presents itself, is, what ought to be the value of the angle A E F, in order that the small machine may rise with the greatest facility? We shall not give the analysis of this question, but shall only say, that if the wind be horizontal, this angle must be 54° 44′, or the same which the rudder of a ship ought to make with the keel, that the vessel may be turned with the greatest facility, supposing the currents of water which impel it to have a direction parallel to the keel.

We shall here observe, that it is not absolutely necessary that the angle A E C should be invariable, and determined to be such, by a small cord proceeding from a point of C E to another point near the head; but in this case the point E, where this cord is attached to the kite, must not be the same as the centre of gravity of the surface of the kite, and the centre of gravity must be as far as possible towards the centre of the tail D. It is for this reason, that a cord with bits of paper fixed in it is added to the point D; by which means the centre of gravity is thrown towards that point. Those who amuse themselves with kites, were certainly not conducted to this mode of construction *à priori:* the origin of this appendage must have been a desire to give to the small machine the appearance of a bird with a long tail, balancing itself in the air. But accident on this occasion has been of great utility; for M. Euler found by a calculation, of which no idea can be here given, that this small tail contributes a great deal to the elevation of the kite

In short, this amusement, however frivolous it may appear, presents some other mechanical considerations which require a great deal of address, and a very intricate calculation; but for farther particulars, we must refer to the Memoir of M. Euler before mentioned.

Remark.—By observing the before-mentioned rules, various figures may be given to this small machine; such as that of an eagle, or a vulture, &c. We remember to have once seen a kite which resembled a man. It was made of linen-cloth cut, and painted for the purpose, and stretched on a light frame, so constructed as to represent the outline of the human figure. It stood upright, and was dressed in a sort of jacket. Its arms were disposed like handles on each side of its body, and its head being covered with a cap, terminating in an angle, favoured the ascent of the machine, which was twelve feet in height; but to render it easier to be transported, it could be folded double by means of hinges adapted to the frame. The person who directed this kind of kite was able to raise it, though the weather was very calm, to the height of nearly 500 feet; and when once raised he maintained it in the air by giving only a slight motion to the string. The figure, by these means, acquired a kind of libration like that of a man skaiting on the ice. The illusion occasioned by this spectacle, which might seem fit only for amusing school-boys, did not fail to attract a great number of curious spectators.

PROBLEM LXIX.

Of the Divining Rod; and opinion which we ought to form of it.

In the article which Montucla has given under this head, he expresses strongly his opinion that the performances which are said to have been effected by the aid of the divining rod, are either illusions or philosophical quackeries.

The rod itself is merely a forked hazel twig, and it is said that, if held by the tips between the fingers and thumb there are certain persons in whose hands it spontaneously revolves with considerable force, when near or over either a spring of water or treasure concealed in the earth; and it is said to have been available in discovering criminals.

Montucla mentions a person named d'Aymar, who, by the aid of the divining rod (or *la baguette devinatoire*, as the French call it), pointed out a murderer, of whom he had been sent in pursuit, among many other persons whom he found in prison with him. But the belief among the better informed people of the time was, that d'Aymar had accidentally been a witness of the murder; and gave credit to his rod for the knowledge which he had derived from a more feasible source.

He was a treasure-finder too; but having failed most egregiously in an experiment where he had to experiment *only*, he was thenceforth accounted a cheat, and died in poverty.

Peranque, another of these impostors, exercised his vocation in the southern provinces of France. The wonder in him was his powers of vision. He could *see* springs in the bowels of the earth, even at great depths; and he could trace out their course and give a good estimate of their depth. Another worthy, of the same district, discovered secret springs, by a faculty which manifested itself in a less agreeable manner; he was seized with violent illness when he passed over the place where they were.

A female at Lisbon had the *sharp-sighted* faculty of divination. When but a child she discovered (and it is said truly) that the family cook was *enceinte*, by seeing the child in its mother's belly. The human body was, to her sight, transparent. She was even able to point out to the physicians the viscera affected with disease; but it was essential, to the exercise of her faculty, that the body should be exposed to her view—*naked.*

It is pretended that many wells at Lisbon were dug in consequence of her indications; and that she discovered by her sight an obelisk which had been long buried in the earth, and which was dug out and erected as an ornament to the city.

It is not to be wondered at, that in these times of general credulity, when the little scientific knowledge possessed by the learned was often employed for the purposes of delusion, such follies obtained a certain degree of credence; but it is lamentable to reflect that, even among the most enlightened classes, many were believers in the powers of the divining rod.

In the present edition of Montucla's Recreations, however, the article *Divining Rod* is retained chiefly from a circumstance of which we now proceed to give a short account.

Soon after the publication of the first edition of the English translation of Montucla, Dr. Charles Hutton, the translator and editor, received a clever anonymous letter from a lady, declaring that, incredible as it might seem to the Doctor, and unaccountable as it was to herself, she did actually possess the power of discovering hidden springs by the aid of the *baguette;* that she had known several others who had the property; and detailing very fully the circumstances connected with her becoming aware of possessing such a faculty. She pointed out to the Doctor how he might address a letter to her, and a correspondence ensued, from which it eventually appeared that there was no hoax whatever on the part of the lady, who was a person of exalted rank and superior talents.

On her coming to town with her family, the Doctor waited upon her to pay his respects, when it was arranged that she should visit him at his residence, and give ocular proof of the power of the divining rod in her hands.

We give the result in Dr. Hutton's own words.

" Accordingly, at the time appointed, the lady, with all her family, arrived at my house at Woolwich Common; when, after preparing the rods, &c., they walked out to the grounds, accompanied by the individuals of my own family, and some friends; when Lady —— shewed the experiment several times, in different places, holding the rods, &c. in the manner she had described in her letter. In the places where I had good reason to know that no water was to be found, the rod was always quiescent; but

in the other places, where I knew there was water below the surface, the rods turned round slowly and regularly till the twigs twisted themselves off below her fingers, which were considerably indented by forcibly holding the rods between them.

"All the company present stood close round the lady, with all eyes intently fixed on her hands and the rods, to watch if any particular motion might be made by the fingers; but in vain; nothing of the kind was perceived; and all the company could observe no cause or reason why the rods should move in the manner they were seen to do. After the experiments were ended, every one of the company tried the rods in the same manner as they saw the lady had done, but without the least motion from any of them." The Doctor adds: "In my family, among ourselves, we have since then several times tried if we could possibly cause the rod to turn, by means of any trick in twisting of the fingers, held in the manner the lady did; but in vain; we had no power to accomplish it;" and he expresses his conviction "that there appears to exist such evidences of the reality of the motion, as it seems next to impossible to be questioned.

In conclusion, Dr. Hutton having requested permission to use the *name* of the lady, in connection with an account of the experiment which she had made in his presence, she declined, from a dislike which she had to appearing in print; but added that, "They (the circumstances) are known to *so many*, that I am of opinion they will obtain credit in a great degree, without a *name* being formally attached to them."

Both parties have long been removed beyond the reach of the press; there can therefore be no impropriety in stating now, that the lady in question was the *Hon. Lady Milbanke, wife of Sir Ralph Milbanke, Bart. (afterwards Noel), and mother of the present dowager Lady Byron.*

PART TWELFTH.

OF THE MAGNET, AND ITS VARIOUS PHENOMENA.

Of all the phenomena exhibited to us by nature, magnetism, or the properties of the loadstone, and electricity, may with justice be considered as the most extraordinary, as the causes of the effects produced by them have occasioned the greatest difficulties to philosophers; for it must be confessed that, notwithstanding all their attempts to explain them, we are as yet acquainted only with facts. They have been able indeed to apply certain hypotheses to some of these phenomena; but if we examine these hypotheses with an unprejudiced eye, and without suffering ourselves to be the dupes of illusion, we must perceive that they have little solidity, and that they are subject to difficulties which cannot be removed, as long as we make it a rule to reason only from the known properties of matter, and the laws of motion. Posterity perhaps will be more successful; and, assisted by time and accumulated experiments, will see more clearly into these matters; or perhaps they may for ever remain an impenetrable secret to the human mind.

In this part of our work we shall confine ourselves, in speaking of the magnet, to its properties, and the philosophical amusements which may be performed by their means. Electricity will furnish matter for the succeeding part.

ARTICLE I.

Of the Nature of the Magnet.

The magnet is a metallic stone, commonly of a greyish or blackish colour, compact and very heavy, and is usually found in iron mines. It affects no particular form; exhibits no external marks, to distinguish it from the meanest productions of the bowels of the earth. But its property of attracting or repelling iron, and of directing itself to the north, when at full liberty to move, gives it a title to be classed among the most singular objects of nature.

This stone, properly speaking, is merely an iron ore, but of that kind which is called poor; because it contains only a small quantity of metal. Modern metallurgists indeed have been able to extract iron from it; but, besides that it is difficult to be fused, it is so unproductive, that it would not pay for the expense of working it.

But it may here be asked, why is not every kind of iron ore magnetic? This is a question to which, in our opinion, no answer has ever been given. Its magnetic virtue arises, no doubt, from a peculiar combination of iron with the heterogeneous particles to which it is united; and perhaps it contains some principle which does not enter into the other ores of that metal: but it must be allowed that this does not solve the difficulty. Possibly, chemical analysis may some day discover in what this combination consists; and our profound ignorance respecting the physical causes of the action of the magnet, may arise from chemists having hitherto neglected to make this production of nature a subject of their researches.

Formerly, the loadstone was exceedingly rare. The name *magnes*, by which it was known both among the Greeks and the Romans, seems to have originated in Magnesia, a province of Macedonia, where it was found in great abundance, or which furnished the first magnets known. But the loadstone has been since found in almost every region of the earth, and particularly in iron mines. The island of Elba, so celebrated for its mines of that metal, worked from the earliest ages, is said to furnish the largest and best magnets.

ARTICLE II.

Of the principal Properties of the Magnet.

The ancients were acquainted with no other property of the magnet, than that which it has of attracting iron; but the moderns have discovered several others; such as its communication, its direction, declination and inclination, to which we may add its annual and daily variation.

SECTION I.

Of the attraction which prevails between the Magnet and Iron, or between one Magnet and another.

EXPERIMENT I.

Which proves the attractive power of the Magnet over Iron.

Every person is acquainted with the attractive force which the magnet has upon iron. If filings of that metal be presented to a magnet, and even at some distance, you will see the filings dart themselves towards the stone, and adhere to it. The case will be the same with any small and light bit of iron, such as a needle; this will approach the magnet, as soon as it is within a certain distance of it, according to the force of the stone.

This experiment may be performed also in the following manner. Suspend a long iron needle, in a state of equilibrium, by means of a silk thread, or rather on a point or pivot, so as to leave it at full liberty to move: present to it a magnet, at the distance of several inches, or even of several feet; then, if the magnet be endowed with proper force, you will see one end of the needle turned towards it, until it be as near to it as possible, and then stop in that position; so that if the situation of the magnet be changed, the needle will continually follow it. If the needle float on water, which it may be easily made to do by placing it on a small bit of cork, it will not only turn one of its ends towards the magnet, but it will approach till it come in contact with it.

All these phenomena will take place, even if there be between the magnet and the needle a plate of copper or glass, or a board, or any other body whatever, iron excepted; which proves that the magnetic virtue is not intercepted by any of these bodies but the last.

If the magnetic virtue then is produced by moleculæ agitated, or put in motion in any manner whatever, the tenuity of these moleculæ must be very great, or at least superior to that of any other emanations with which we are acquainted, such as odours; since they freely traverse all metals and even glass. If they produce no effect through iron, it is, in all probability, because they find such a facility of moving in it, or have such affinity with it, that they do not pass beyond it, but are thus intercepted.

We may here remark, that if a light and well-polished sewing needle be dropped gently on water, it will swim on the surface, and thus remain in a state of more delicate suspension than can be attained in any other way, for exhibiting the phenomena of magnetic attraction.

EXPERIMENT II.

To find the Poles of a Magnet.

If a magnet be immersed in iron filings, and then drawn out, it will be found covered all over with them; but it will be observed that there are two places, diametrically opposite to each other, which are the poles, where the filings are closer, and where the small oblong fragments stand as it were upright, while in other parts they lie flat.

By this experiment we are enabled to find the poles of the magnet. Every magnet indeed has two poles, or two opposite points, which, as will be seen hereafter, possess different and peculiar properties. One of these points is called the *north* pole, and the other the *south*; because, if the magnet be freely suspended, the former will turn of itself to the north, and consequently the other will be directed to the south. When it is intended to perform experiments with a magnet, these two points must be first determined.

EXPERIMENT III.

Properties of the Poles of the Magnet, in regard to each other.

Provide a magnet; and having determined its two poles, make it float on the water by placing it on a piece of cork of a proper size; if you then present to the north pole of this stone the same pole of another, the former will be repelled, instead of attracted; but if you present to its north pole the south pole of another, it will be attracted.

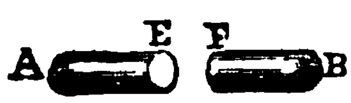
In like manner, if to the south pole of the former you present the south pole of the latter, the first will recede; but if you present to this south pole the north pole of the second, it will approach.

The poles then of the same name, repel; and those of a different name, attract each other.

EXPERIMENT IV.

To produce New Poles in a Magnet.

Fig. 41.



If a magnet be cut in a direction perpendicular to the axis passing through its two poles A and B, Fig. 41, there will be formed by the section two new poles, such as F and E: so that if A be the south pole of the whole stone, E will be a north pole, and F a south pole. By this bisection, therefore, the north side of the stone will acquire a south pole, and the south side a north pole.

This production of magnetic poles may be simply illustrated thus: Suspend from the end of a strong magnetic bar a piece of iron, as a key; and from the lower end of that key, a smaller one may be made to hang in consequence of its induced magnetism. To this may be appended a smaller piece of iron, such as a nail, and we may go on thus adding piece after piece, till the lower one will sustain only such a very small weight as a light needle.

The polarities of the lower end of each piece, if examined before another is added, will be found to be the same as that of the lower end of the magnet, each piece becoming for the time an actual magnet; as are, in fact, the individual particles of iron filings taken up by a magnet placed among them.

This may be further illustrated by the following experiment. Take a piece of iron shaped like the letter Y, and suspend it by one of the branches of the fork to the north pole of a magnet; its lower end will instantly acquire a northern polarity, and will attract and support a small key, or any other small piece of iron. While the key is thus suspended, apply to the other branch of the fork the south

pole of another magnet, and the key will immediately drop off, the latter magnet tending to induce southern polarity at the lower end of the suspended iron, while the other tended to induce northern polarity, and the result is that the polarity at that point is effectually destroyed.

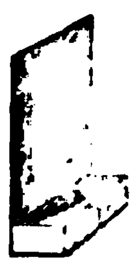
If, on the contrary, the north pole of the latter magnet be applied instead of the southern one, a stronger polarity will be induced at the lower end of the suspended iron; which will therefore, in its turn, support a heavier piece of iron than when the Y shaped piece is suspended by one magnet only.

If the north pole of a magnet be placed in the middle of an iron bar, both extremities of the bar are rendered north poles, while the middle is a south pole. And if the north pole of a magnet be placed perpendicularly in the centre of a round iron plate, the plate will have a south pole at the centre, and every part of its circumference will have the property of a weak north pole. If the plate be cut into the form of a star pointed towards the outer part, each point will be a stronger north pole than the rim of the uncut circular plate.

Remarks.—A magnet, however good it may be, unless it be very large, will scarcely support a few pounds of iron; and, in general, the weight which a magnet can carry, is always very much below its own weight. But means have been found out, by employing what is called arming, to make it produce a much more considerable effect. We shall therefore describe the method of arming a magnet.

First give the magnet a figure nearly regular, and square its sides where the two poles are situated, so that these two sides may form two parallel planes. Then make, of soft iron—for steel is not so good—two pieces, such as that seen Fig. 42, the long and flat side of which may be of the same length and breadth as the faces of the magnet where the poles are situated. The proper thickness however of this side, as well as the projection of the foot, and its thickness, can be found only by repeated trials. These two pieces must embrace the magnet on the two faces where the poles are situated, the feet passing below as if to support it; and they must be fastened in that situation by transverse bands of copper, surrounding the magnet, and compressing the long branches of these pieces against the faces of the poles.

Fig. 42.



Then provide a piece of soft iron, of the form seen Fig. 43, a little longer than the distance between the bands of iron applied to the poles of the magnet, and in thickness somewhat more than the flat faces of the lower part of the feet of the arming. In regard to the height, it must be regulated by what may appear most convenient. Pierce a hole in it towards the middle, to receive a hook for the purpose of suspending from it the weight to be supported by the magnet. Fig. 44 represents an armed magnet, which will be sufficient to give an idea of the whole arrangement, without any farther explanation.

Fig. 43.

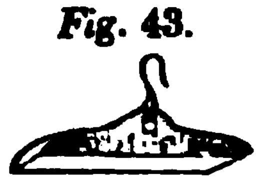
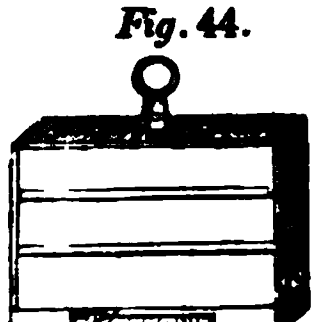


Fig. 44.



A magnet armed in this manner will support a much greater weight, than one not armed. A stone, for example, of 2 or 3 ounces, will by these means support 50 or 60 ounces of iron; that is, 20 or 30 times its own weight.

Lemery says he saw a magnet, of the size of a moderate apple, which supported 22 pounds. Some have been seen, of 11 ounces, which could support 28 pounds. The sum of 5000 livres (above £200 sterling) was asked for it. M. de Condamine, of the Royal Academy of Sciences, possessed one, given to him by Maupertuis, which was capable, we believe, of supporting a weight much greater than any other magnet known. We do not remember its dimensions, or its weight, neither of which was

very considerable; but, if we recollect right, he used to say that it could support sixty pounds.*

II. Researches have been made to discover whether there be any other bodies, besides iron, susceptible of being attracted by the magnet; from which it appears indeed that there are not. Yet Muschenbroeck says, that the magnet seemed to exercise an action on a stone which he calls *Lough-neagh*. We are not acquainted with this stone; but it is probably some kind of iron ore in which that metal is very little mineralised.

In his "Cours de Physique Experimentale," chap. VII., he gives an account of some trials, made on a great many different kinds of matter, to ascertain whether they were susceptible of being attracted by the magnet. He found that this stone, without any preparation, attracted the whole or a great many of the particles in different kinds of sand and earth, which he enumerates. Several others presented no particles susceptible of attraction by the magnet, until they had been exposed to the action of fire, by bringing them to a red heat, and burning them with soap, or charcoal, or grease: after which, says he, they were attracted by the magnet, with almost as much force as iron filings; such, adds he, are the earth of which bricks are made, and which becomes red when burnt; also different kinds of bole and coloured sand. Others, when burnt in this manner, presented only a few particles susceptible of being attracted by the magnet: of these he gives a long enumeration, which we shall not here repeat.

This will not appear surprising, if we compare the two following facts: the first is, that the magnet never attracts iron, but when in its metallic state; and that it has no action on this metal when calcined, or reduced to the state of an oxide: the second is, that iron is universally diffused throughout nature, and in almost all bodies more or less distant from its metallic state, as will be seen hereafter. Bodies which contain it in its metallic state, are all, or in part, susceptible of being attracted by the magnet without any preparation; but in others, it is not susceptible of attraction till it has been burnt with fat matters, which restore it to its metallic state. Such is the only cause of the phenomenon which seems to have embarrassed Muschenbroeck. It would have occasioned no difficulty, had he been as well acquainted with chemistry, as with the other branches of philosophy.

An English navigator says, he observed that grease, which happened to fall on the mariner's compass, disturbed the magnetic needle, and that brass produced the same effect. If this observation be correct, we must conclude that the tallow and brass accidentally contained some ferruginous particles; for in our opinion it may be considered as certain, that iron alone, in its metallic state, has the power of acting on the magnet, and is susceptible of being attracted by it.

EXPERIMENT V.

The direction of the Magnetic Current.

Place an unarmed magnet on a piece of pasteboard, and throw iron filings around it: if you then tap gently on the pasteboard, you will see all the filings arrange themselves around the magnet, in curved lines, which, approaching each other like the meridians in a map of the world, meet at its two poles.

This experiment favours the opinion of those who think that the magnetic phenomena depend on a fluid, which issues from one of the poles of the stone, and enters at the other, after having circulated around it.

* The attractive force of magnetism, temporarily induced by galvanism, is much greater than that of any combination of simple magnets. We believe that at the Royal Gallery of Practical Science in London, 3 or 4 cwt. is often suspended by magnetism induced in this way.

EXPERIMENT VI.

Which proves that the Magnet and Iron have a mutual action on each other.

Place two magnets, or a magnet and a piece of iron, on two bits of cork, made to float in a bason of water. Having then turned the north pole of the one towards the south pole of the other, provided two magnets are employed, if the two pieces of cork be left to themselves, you will see them proceed towards each other; the weaker moving faster than the other. The case will be the same, if a bit of iron be presented to the north pole of the magnet. This attraction then is reciprocal; and it may be said that the iron attracts the magnet, as much as the magnet does the iron. This indeed must necessarily be the case, since there is no action without re-action, and as the latter is always equal to the former.

Remark.—Muschenbroeck endeavoured to determine in what ratio the action of the magnet decreases, according to the distance; and he thought he observed that its attractive force decreased in the quadruple ratio, or as the 4th powers of the distances. Thus, if at a line distance, a particle of iron is attracted with a force equal to 1; at 2 lines distance, that force will be 16 times less; at 3 lines, 81 times less; at 4 lines, 256 times less; and so on of the rest. This action perhaps decreases still more rapidly; for in a ship of war laden with large iron cannon, it is not observed that they have a sensible action on the compass. In our opinion, however, it would be prudent to remove them to as great a distance as possible.

Note.—Recent observations shew that the iron in ships has a very perceptible action on the compass; so much even, in the British Channel, as *a point* of deviation in the direction of the needle having been produced by a change in the direction of the ship's head.

SECTION II.

Of the Communication of the Magnetic Property.

Magnetism, or the property of attracting iron, and of turning towards a certain point of the heavens, is not so peculiar to the magnet, as to be incapable of being communicated; but no bodies have yet been found susceptible of this communication, except iron and steel. About a century ago it was believed that contact alone, or the continued presence of a magnet, could produce this effect; but a method has since been discovered to render a piece of iron magnetic, without the magnet; and these artificial magnets are even susceptible of acquiring a strength rarely found in natural magnets. We shall give an account, in the following experiments, of the different methods of communicating the magnetic virtue.

EXPERIMENT VII.

Method of Magnetising.

Provide a magnet, either armed or unarmed, and make one of the feet of the armour, or one of the poles, to pass over a plate of tempered steel, such as the blade of a knife, but proceeding always in the same direction, from the middle, for example, towards the point. After performing this operation a certain number of times, the plate of iron will be found to be magnetised, and, like the magnet itself, it will attract iron, if placed within the sphere of its attraction.

The case will be the same, if a long slender bit of steel be left a long time attached to a magnet: the steel, by remaining in that situation, will acquire the magnetic property; it will have poles like the magnet, so that the north pole will be at the end which was near the south pole of the stone, and the end which touched the north pole will become the south pole.

EXPERIMENT VIII.

Method of making an Artificial Magnet with bars of steel.

We shall here shew the method of making, with bars of steel, an artificial magnet much stronger than a natural one. For this purpose, provide twelve bars of tempered steel about six inches in length, six lines in breadth, and two in thickness. Care must be taken before they are tempered, to make a mark with a punch, or in any other manner, at one of their extremities. Arrange six of these bars in a straight line, but so as to be in contact, and that the marked ends shall be directed towards the north; take an armed magnet, and place it on one of these bars, with its north pole towards the marked end, and the south pole towards the other end; then move the stone over the whole line, beginning at the unmarked end of the first, and repeat this operation three or four times.

When this is done, remove the two bars in the middle, and substitute them for those at the two extremities, which must be placed in the middle; then move the stone in the same direction over the four bars in the middle only; for it is needless to comprehend those at the extremities; and invert the whole line, that is to say turn up the face which was turned downwards, and magnetise the bars again in the same manner, taking care to transpose the bars at the extremities into the place of the middle ones.

By these means you will have six magnetised bars, which must be formed into two parcels, each containing three. In these parcels the northern extremities must be towards the same side; but when the one parcel is placed upon the other, care must be taken that the northern extremities of the bars of the one may rest upon the south extremities of those of the other. These two parcels must touch at their upper part, and be separated on the other side; this separation may be effected by means of a bit of wood placed between them.

Then place the six bars which were not touched, in the same manner as the preceding six, and magnetise them as above described, by means of the double parcel of the former; that is to say, by drawing the two extremities north and south of this double parcel over the new series of bars: you will thus have six bars much more strongly magnetised than the former. Then make a line of the six former, and magnetise them in the same manner with the double parcel formed of the second, according to the same method, and you will obtain bars of steel capable of supporting 16 times their weight and more.

This is the process of Mr. Mitchell, fellow of the Royal Society of London. Mr. Canton, a celebrated observer of the phenomena of the magnet, has given a method of effecting the same thing; and M. Duhamel, of the Academy of Sciences, another, which may be seen in a small treatise on Artificial Magnets, printed in 1775. We shall say nothing farther on this subject, but only remark, that by these processes the weakest commencement of magnetism is sufficient to produce magnetic bars of the greatest force. It is not even necessary to have a magnet; for, in the following experiment, we shall describe various methods of communicating magnetism without one.

EXPERIMENT IX.

To produce the magnetic virtue in a bar of iron, without the use of a magnet.

To propose communicating the magnetic virtue without the use of a magnet is, no doubt, a sort of paradox. This however has been effected in consequence of some theoretic considerations in regard to the nature of the magnet, and the manner in which the magnetic fluid acts on iron. A magnet therefore is not necessary to produce the commencement of magnetism, which may be afterwards increased to a considerable degree by the process before explained.

Canton, Mitchell, and Antheaume have given different methods for magnetising without a magnet. According to Mr. Canton, take a poker, and having placed it between your knees, in a vertical direction, with the point downwards, affix lengthwise to its upper part, by means of a silk thread, a small plate of soft tempered steel; then holding this apparatus in the left hand by the silk thread, take a pair of tongs, and, holding them almost vertically, rub the small bar from the bottom upwards, about a dozen of times, with the lower end of them: by these means you will communicate to it a magnetic force, capable of making it support a small key.

Mr. Mitchell employed another method. Place, says he, a small bar of steel in a straight line between two iron bars, in the direction of the magnetic meridian, and in such a manner that they shall be somewhat inclined towards the north: then take a third bar, and holding it almost vertically, but with the upper extremity a little inclined towards the south, glide the lower extremity of this bar along the three other bars situated in a straight line, taking care to make it move from north to south: the result will be a commencement of the magnetic virtue in the bar of steel.

M. Antheaume's method is as follows: First fix a board in the direction of the magnetic current; that is, inclined at an angle of about 70 degrees towards the horizon, so that its horizontal projection shall make one of about 20 degrees. Then place in a line, on that board, two square bars of iron, four or five feet in length, or even more, and 15 lines in thickness: they must be filed square at the extremities, which are opposite to each other. Each of these extremities must be furnished with a small square of iron plate, two lines in thickness, so as to project beyond the upper face of the bar the height of a line, and filed square on that side, to form above the bar a kind of knee. The three other sides of this square of iron plate must slightly touch the corresponding faces, and be cut into a bevel. In the last place, a small bit of wood must be placed between the arming of the extremities of these two bars.

When every thing is thus arranged, glide the bar of steel to be magnetised, over the two knees above described, by making it move gently from one of its ends to the other, as an iron bar is magnetised, on the two knees of its arming. M. Antheaume says, he was surprised to find that he could magnetise, by this method, not only small bars of steel, as Messrs. Canton and Mitchell did, but bars of a foot in length, and several lines in thickness.

The same philosopher says, he observed that steel *de caume*, or *à la rose*, and English steel, are the fittest for this purpose; that the operation succeeds best with the first kind, when tempered hard in the usual manner, and that English steel requires to be tempered in bundles: lastly, that if steel tempered and annealed be employed, the temper is a matter of indifference.

Remark.—Even the rubbing of one piece of iron over another is not necessary to produce the magnetic virtue. It has been observed that a bar of iron, kept for a long time in the direction of the meridian, or in a situation nearly approaching to it, acquires the magnetic virtue. The steeple of Notre Dame de Chartres having been considerably damaged by a great storm in 1690, some bars of iron taken from it were found to be magnetic. But what is still more remarkable, pieces of these bars, which were almost destroyed by rust, formed excellent magnets. The Abbé de Vallemont wrote at that time an account of them, which was published in 1692.

Gilbert, an English physician and philosopher, who wrote a treatise on the magnet, in 1640, had then observed that the small bars of an iron window frame, placed north and south, which had remained many years in the same position, were become magnetic. He relates also,* that the wind having bent an iron bar which supported an

* Book iii. chap. 12.

ornament on the church of St. Augustine, at Rimini, when the monks belonging to it were desirous, ten years after, to straighten this bar, they were much surprised to find that it possessed all the properties of a good magnet. Muschenbroek speaks of a similar circumstance, in regard to some pieces of iron taken from the tower of Delft. We read also in the Memoirs of the Academy of Sciences, for the year 1731, that there was at Marseilles a bell which moved on an iron axis, standing in an east and west direction, and resting with its two ends on stone; that the rust of these ends mixing with the dust rubbed from the stone, and with the oil used to facilitate its motion, formed together a hard and heavy mass, which, when detached, was found to possess all the properties of the magnet. It is believed that this bell had existed in that situation 400 years.

It will be found, on trial, that iron palisades, which have been some time up, are strongly magnetical; the upper end being a south pole, and the lower a north pole.

Gilbert observes also, that if a bar of iron, placed north and south, be brought to a red heat in a forge, and be then beat on the anvil in the same position, it will acquire the magnetic virtue; and that if this virtue be not immediately sensible, it will become so by repeating the operation. But it is to be observed, that for this purpose the length of the iron must be 100 or 150 times its diameter. The case is the same with a bar of iron, if it be heated, and then cooled in the direction of the meridian.

The following conjecture of this philosopher, however, has not been verified. He says that if a spherical form be given to a magnet, and if its two poles be at the extremities of a diameter, this spherical magnet, when placed in complete equilibrium, and suspended on its poles, will turn round its axis in twenty-four hours; for as the earth, adds he, is but a large magnet, it must have a similar motion. This would have been a pretty strong proof of the motion of the earth, at least round its axis; but M. Petit, an industrious philosopher of the last century, having taken the trouble to make the experiment proposed by Gilbert, the small magnetic globe remained perfectly motionless. This does not however prevent the motion of the earth from being certain, and it may even be considered as a large magnet, though Father Grandamy concluded, from the failure of Gilbert's experiment, that the earth was motionless.

We add another method of making artificial magnets. Fix the needle to be magnetised horizontally in the magnetic meridian; and apply to its middle a long iron bar, as a poker, held vertically. Immediately opposite, at the lower side of the needle, apply the upper end of another similar poker, or iron bar. Then, keeping the bars vertical, draw them towards the opposite ends of the needle, drawing the upper bar towards the end intended for the south pole. Separate the bars from the needle; and first removing them to a distance, bring them again, as before, to the middle of the needle, and, as before, slide them in opposite directions towards the ends; and repeat the operation several times. By this simple process a small needle may be magnetised to saturation.

To preserve *bar magnets*, keep them in pairs, laid parallel to each other, with their poles turned in opposite directions; and connect each adjoining pole with a piece of soft iron.

When the magnet is made of the horse-shoe form, the poles ought also to be joined by a piece of soft iron.

But magnetism may be imparted by simple percussion. Mr. Scoresby found that a rectangular steel bar acquired a feeble magnetism, by being hammered vertically, the lower end resting on stone or pewter; but that it received a considerable accession of power, when it was similarly hammered while it was placed on a parlour poker, kept also in a vertical position. The poker itself became *strongly* magnetic;

and in this state exerted upon the bar a much more powerful influence than the earth alone could have done.

ARTICLE III.

Of the Direction of the Magnet; and of its Declination and Variation.

EXPERIMENT X.

To find the Direction of the Magnet.

Having found the poles of a magnet, if you place it on a small bit of cork, and make the cork to float on water, it will always place itself in one direction.

The case will be the same with a magnetic needle made to float on water by the same means, or suspended on a fine pivot, so as to be at full liberty to move: it will always assume the same direction.

To make a needle place itself north and south, it is not even necessary that it should be magnetised. When exceedingly light, and perfectly free to move, it spontaneously assumes that direction.

If a very slender common needle be made to float on the surface of water in a state of perfect rest; at the end of some hours it will be found in the same direction as that suddenly assumed by the magnetic needle.

The direction, according to which a needle, whether magnetised or not, arranges itself, is called the *magnetic meridian*, and must be carefully distinguished from the terrestrial or true meridian; for we shall soon shew that, in general, they form an angle with each other. Philosophers agree, almost unanimously, in thinking that this property of the magnet is produced by a current of a particular fluid, surrounding the earth, and which, passing through the magnet lengthwise, or from one pole to the other, makes it assume its proper direction.

What is very singular is, that not only the magnetic meridian, in almost all places of the earth, is different from the terrestrial meridian; declining sometimes to the east and sometimes to the west; but that this declination varies annually, as is proved by the following experiments.

EXPERIMENT XI.

If a magnetic needle be suspended on a pivot, in the direction of a meridian line, traced out with great care, and at a distance from any iron, you will generally find that the direction it takes will form an angle with the meridian. In 1770, for example, it was at Paris 19° 55′ west.

If the experiment be repeated some years after, it will be found that this angle is not the same; but that it is increased or decreased. In 1750, for example, it was at Paris 17° 15′ west; in 1760 it was observed to be 18° 45′; in 1770, 19° 55′, or even 20 degrees and some minutes. And, at London, in 1800, it was about 22° 30′; in 1818, about 24° 30′; and in 1836, about 24°.

It appears also, that the annual change in the variation has been gradually diminishing; from 1622 to 1692, the annual change at London was about 10′; from 1723 to 1753, about 8′; from 1787 to 1795, about 5′; in 1818 it was reduced to zero, and at London at present the variation is decidedly decreasing.

Remark.—In the greater part of our continent, as well as in all North America, except that part of it which is nearest the Gulf of Mexico, the declination is at present west, and goes on continually increasing. In all North America and all the Gulf of Mexico, as well as part of the Pacific Ocean, between the tropics, and on the southern coast, the declination is east, and goes on continually decreasing.

The celebrated Dr. Halley, having taken the trouole to collect a prodigious

number of observations, made by different navigators, published in 1700 a very curious chart, in which he connected, by lines, all those places of the earth where the declination of the magnetic needle was the same. It is there seen, for example, that the line on which the magnetic needle in 1700 had no declination, divided nearly the southern part of the Atlantic Ocean, and cut the equator towards the first degree of longitude, or at its intersection with the first meridian; it thence proceeded in a curved line to New England, and, traversing New Mexico and California, stretched to the north of the Pacific Ocean. In all probability it reached Asia, then passed to the north of Tartary, and proceeding through China, traversed New Holland. On the south and west of this line, the declination was east; on the north and east it was west.

By other observations, made at a later period, it appeared that this line was displaced; and that it had in some manner a motion towards the south-west, changing a little its form. According to those observations collected by Messrs. Mountain and Dodson of the Royal Society, it traversed, in 1744, the middle of the Atlantic Ocean; nearly intersected the equator towards the twelfth degree of longitude, to the east of the first meridian; proceeded thence to the middle of Florida; and passing nearly along the coast of Louisiana, it traversed Old Mexico, from which it extended to the point of California, then to the north of the Pacific Ocean, and intersected the first meridian towards the 44th degree of north latitude; from which it turned southwards, and traversed Japan, the largest of the Philippines, the kingdoms of Pegu and Arracan, and formed a point on the east near the island of Ceylon. It then returned, and traversing the Moluccas, proceeded in a curved line towards the south pole, leaving New Holland on the west. Such was the position of this line in 1744, and thence we may determine nearly its present position.

Dr. Halley's chart exhibited also the line which joined all the points where the declination was 5° to the east or the west; those where it was 10°, 15°, &c. It is observed, at present, that they have all had a motion nearly similar to that of the line without declination.

Dr. Halley's object, in this painful labour, was not mere curiosity: he intended these charts to be employed in determining the longitude at sea. If an accurate chart of these lines of declination were indeed constructed, it is evident that, by observing the latitude and the real declination of the compass, the precise point of the earth, where the observation was made, would be determined. Let us suppose, for example, that the declination has been observed in the Atlantic Ocean, to be 7½° west, the latitude being 32 north. It is evident, in this case, that the ship's place will be the point where the parallel of 32° north, intersects the line of 7½° declination. Nothing then would remain, but to improve the means of determining the declination with great exactness, which is a thing not impossible.

It is to be regretted that we have no old observations of the declination of the magnetic needle. The reason of this no doubt is, that the declination was not properly ascertained by philosophers, till towards the end of the sixteenth century. It is seen however by the observations which have been made, that at Paris, at London, and in great part of Germany, the declination formerly was east; for in 1580 it was found at Paris to be 11° 30′ east. After that time it decreased till 1666, when it vanished entirely; it then became west, increasing continually in that direction; for in 1670 it was observed to be 1° 30′, in 1680 to be 2° 40′, in 1701 to be 8° 25′; in 1770 it was observed to be within a few minutes of 20°. The Royal Society of London recorded their annual observations of the magnetic needle for many years; and it is a great pity that they have of late discontinued such useful observations.

But what is the cause of the magnetic declination? On this subject we shall offer the following conjectures. Messrs. de la Hire, senior and junior, made a curious

experiment, which may serve to throw some light on the cause of this phenomenon. They took a very large magnet, and having given it a globular form as nearly as possible, they sought for its poles, which were found exactly at the extremities of a diameter; and then they traced out on it its equator, and twelve meridians. On this magnetic globe, which was about a foot in diameter, and which weighed nearly a hundred pounds, they applied a magnetic needle, and observed that there were places where it declined towards the west, and others where it had no declination, and which formed one or two continued lines on the surface, as Dr. Halley had determined on the surface of the earth, though of a form absolutely different.

It is more probable, says the historian of the Academy, that the cause of the declination observed on the magnetic globe, was merely the inequality of its contexture, and of the magnetic force of its different parts. There is reason also to conjecture that the earth, being a large magnet, or at least a globe containing in its bosom large magnetic masses, it is the unequal distribution of these masses that occasions on its surface the variety of the direction of the magnetic needle. But there is this difference, that in the bowels of the earth new masses are continually generated; whereas the magnet of Messrs. de la Hire experienced nothing of the kind. Hence it happens that, on the surface of the earth, the direction of the magnet is variable; while on the surface of the magnetic globe, it was necessary that it should be constant.

It must however be allowed, that in this explanation it is difficult to assign a reason why, for two centuries at least, the line without declination has been seen to move constantly from east to west. Effects arising from causes so variable as the destruction and generation of masses in the bosom of the earth, ought to experience greater irregularities, and the progress of the magnetic needle ought to be sometimes east and sometimes west.

Dr. Halley proposed a physical hypothesis, to account for the variety in the magnetic declination. He supposed two fixed magnetic poles, and two moveable, in certain positions. But this hypothesis has been simplified by Albert Euler, in a curious memoir, which may be seen in the Transactions of the Academy of Berlin for the year 1757. Euler supposes only two magnetic poles, one at 14° 53′ from the north pole of the earth, and the other 29° 23′ from the south pole. The meridian in which the former is situated passes through the 258th degree of longitude, and that of the second through the 303rd. He then assumes, as a principle, that the magnetic needle always ranges itself in the plane passing through the two magnetic poles and the place of observation; and he determines, by calculation, the inclination of that plane to the meridian, in the different places of the earth. By means of these data, calculation gives, with great exactness, the quantity of the declination observed of late years, and the position of the lines of declination, as they were found by Messrs. Mountain and Dodson, for 1774, at least in the Atlantic Ocean; for Albert Euler is obliged to consider as false the position given to the line of declination, in the north part of the Pacific Ocean, by those members of the Royal Society, and what he says on this subject is highly probable.

It may be easily conceived, that by making these poles to vary, the lines of declination will vary also, and that according as they approach or recede from each other, they may change their form, as has indeed been observed.

While engaged in investigations connected with the attration of iron on shipboard, Mr. Barlow, of the Royal Military Academy at Woolwich, found that the attractive force of iron on the magnetised needle, depended not upon the *mass*, but upon the *surface;* a shell and a solid ball of the same diameter giving precisely the same results. He found however that a certain thickness of the shell is requisite to a full development of the attractive force; and that when the thickness was less than the thirtieth of an inch, the effect was sensibly weakened.

He found also that a ball, in the neighbourhood of a compass, produces no disturbance in the needle, when the centre of the needle is in any part of a plane passing through the centre of the ball, and perpendicular to the direction of the dipping needle at the place where the experiment is made. This plane forms what may be called, or rather it is parallel to what may be called, the magnetic equator.

Another plane of neutrality is a vertical plane in the line of the dip, or the magnetic meridian.

It is not however the whole of the attractive force of the ball that vanishes in these planes, but the part only which occasions deviations in the natural position of the needle.

Magnetism of non-ferruginous bodies.

Nickel and cobalt have occasionally been found magnetic, and sometimes to exhibit polarity. Brass also, under certain circumstances, has been observed to be magnetic, especially after it has been hammered.

Copper and zinc, the constituents of brass, have never been observed to be magnetic in their separate state.

The magnetism of brass has been found sometimes to interfere with the needle in compasses.

Carbon phosphorous, or sulphur combined with iron, increases its susceptibility for magnetism ; but there is a limit beyond which an excess of these substances renders the compound totally insusceptible of magnetism. On the other hand a small quantity of arsenic has been found to deprive a strongly magnetic mass of nickel of the whole of its magnetism.

Minerals of various kinds, and even some of the precious stones, exert a feeble influence on the magnetic needle; and late inquiries appear to have established the fact that all bodies whatever are, in a greater or less degree, susceptible of magnetism.

Whether this susceptibility arises from iron being a constituent of all bodies, or from the magnetic property being inherent in all, but more decidedly developed in one class of bodies than in others, are questions which have as yet received no satisfactory solution.

In the year 1824, M. Arago shewed that if a plate of copper, or of any other substance, be placed immediately under a magnetic needle, it diminishes sensibly the extent of the needle's oscillations, but does not affect their individual duration; but the needle is brought to rest sooner than when no such substance is placed under it.

When, instead of the needle being made to oscillate, the plate below it is made to revolve with a certain velocity; the needle is found to deviate from its natural position in the magnetic meridian; the deviation increasing as the rotation of the plate becomes more rapid; and if the plate revolve with sufficient rapidity, the needle will at length revolve also, and always in the same direction with the plate.

Conversely, when a circular plate of copper, balanced on a point at its centre, was placed under a strong bar magnet, to which a rapid rotatory motion was given ; the copper soon began to turn in the same direction, and acquired by degrees a very rapid plate velocity.

This tract of investigation has been pursued by many scientific persons, both in this country and on the continent; and the results are in a high degree interesting.

Mr. Canton, a member of the Royal Society of London, discovered some years ago a new motion of the magnetic needle, which is founded on the following experiment.

EXPERIMENT XII.

Diurnal Variation of the Magnet.

Provide a pretty large magnetic needle, 12 or 15 inches in length, and nicely suspended. It must be surrounded by a circle, the centre of which is the point of suspension, divided into degrees, and half degrees, or quarters, at least in that part of its circumference which is opposite to the point of the needle. The whole apparatus must be covered in such a manner, as to prevent it from being subject to any impression from the air.

If this needle be observed at different hours of the day, it will be found that it is scarcely ever at rest. According to Mr. Canton, the declination will be greatest in the morning, and least in the evening: about noon it will be a mean between these two extremes. He assigns also a very probable reason, which is as follows:

It is a fact proved by experience, that a magnet, when heated, loses a little of its force. But as the eastern parts of the earth have noon when the sun rises to us, it is at that time, or nearly so, that they are most heated. The magnetic needle, the direction of which is, in all probability, an effect compounded of the attraction of all the magnetic parts of the earth, will at sun rise be a little less impelled towards the east, than if the sun were not on that side; consequently it will yield to the action of the western parts, and will turn a little more towards that side. Mr. Canton even renders this explanation sensible, by means of two magnets, each of which is heated alternately.

But, whatever truth there may be in this explanation, the phenomenon is now well known: and meteorologists do not fail to observe, at different times, the declination of the magnetic needle, which often varies between morning and night, 20′ and more.*

Mr. Barlow, of Woolwich, has devised a mode of rendering the diurnal oscillations much more perceptible, by diminishing the ordinary directive power of the needle, through the influence of two magnets, so placed with respect to the needle as to counteract or neutralize the terrestrial action. The effort of the ordinary or overmastering action of the earth upon the needle being thus removed, Mr. Barlow anticipated that the cause, whatever it might be, which produced the daily variation, would exhibit effects more perceptibly; and these anticipations have been amply realised by the success of his own experiments, and those of his colleague, Mr. Christie, as detailed in several papers in the Philosophical Transactions.

The general result of the experiments of the latter is, that the deviation of the horizontal needle from its mean position is easterly during the forenoon; greatest about eight o'clock; thence returning quickly to its mean position between nine and ten o'clock; after which it became westerly; at first increasing rapidly, so as to reach its minimum about one o'clock in the afternoon, and then slowly receding during the rest of the day it arrived at its mean place about ten o'clock at night.

Similar changes in diurnal *intensity* have been observed both by Hansteen at Christiana, and Mr. Christie at Woolwich; and the results, on the whole, accord very well, though deduced by totally different methods. Mr. Christie says that he found the terrestrial magnetic intensity the least between ten and eleven o'clock in the morning; between which hours, in this country, the sun passes the magnetic meridian. It increases from this time until nine or ten o'clock in the evening, when it again decreases, till it obtains its minimum, as already stated. It appears, too, that these

* See "Traité de Météorologie du P. Cotte."

variations of magnetic position are in some degree connected with the diurnal changes of temperature.

The mean diurnal changes of variation have been observed to be greatest in June, and least in December.

SECTION III.

Of the inclination or dip of the Magnetic Needle.

EXPERIMENT XIII.

To observe the inclination of the Magnet.

If the needle of a compass, not yet magnetised, be placed in perfect equilibrium on its pivot, so as to remain parallel to the horizon; and then be touched with a magnet, it will lose this equilibrium, and will dip its northern extremity below the horizon.

This experiment is well known to those who construct compasses; for after the needle is magnetised, they are obliged to file the heavier end till it be in equilibrium with the other. The same effect might be produced by loading the other end with a small weight, and it would even be of advantage if this weight were moveable; for as the inclination is variable, different forces are required to form an equilibrium to the effort made by the needle to dip. It is therefore necessary to add a small weight to one of the ends of the needle, according to the different positions of the ship, in order that it may remain perfectly horizontal.

EXPERIMENT XIV.

To observe the inclination of the Magnetic Needle.

Provide a magnetic needle, made of very straight steel wire, terminating in a point at each extremity. The middle of it must be flattened, and formed into a small circle, having its centre in a line with the two points of the needle. A piece of very fine steel wire passing through this circle, in a perpendicular direction, must serve it as a pivot; so that when suspended horizontally in two holes, made in two vertical plates of brass, it may be indifferent to every position, and remain in equilibrium in any situation whatever. These two plates must be affixed to the edge of a brass band, bent into a circular form, and of a diameter somewhat greater than the length of the needle, the pivots of which will be in the centre. This brass circle must be suspended in a ring, and one of its diameters must be placed in a vertical direction. Divide the inside of it into degrees, and quarters of a degree, if possible; but in such a manner that the division beginning with zero, at the extremities of the horizontal diameter, may end with 90 degrees at the extremities of the vertical diameter. The position of this diameter may be ascertained by means of a wire and plummet, suspended from its upper extremity, and which must pass through the lower extremity, that it may be in its true position.

Provide also a wooden stand in the form of an oblong parallelopipedon, in the upper part of which let there be a circular hollow proper for lodging the instrument in the direction of its length. In the last place, there must be a small wedge, for the purpose of being placed under this stand, till the plane of the instrument, or that passed over by the needle in its motion, shall be exactly vertical.

When the needle has been magnetised, apply to the two sides of the instrument, in grooves made for that purpose, two circular pieces of glass, to preserve the needle and its pivots from the contact of the exterior air, and from moisture which is hurtful to magnetism.

By the description of this instrument, it may be readily seen, that it must be disposed in a vertical situation, either by suspending it or placing it on its supporter, which may be easily done by means of the wire and plummet.

The plane of the instrument also, or that passed over by the needle, must be in the plane of the magnetic meridian. For this purpose, lay the instrument flat on a horizontal table; the needle, when it stops, will indicate the magnetic meridian; then draw on the table a line in that direction, and make the long side of the supporter coincide with it. By means of the small wedge and the plumb line, it may then be adjusted in the proper position. The needle, after very long vibrations, will at length stop, and indicate by its point the number of degrees it is distant from the plane of the horizon, which will give the inclination or dip desired.

By these means it is found that the inclination at Paris is at present 72 degrees.

Remarks.—I. Though the construction of such an instrument does not seem difficult, it is shewn by experience that it requires a peculiar kind of skill and dexterity, which few possess. Unless the instrument indeed be perfect, the magnetic needle does not recover its position when displaced, or when the instrument is turned in a contrary direction; that is, in such a manner that the plane which looked towards the east may look towards the west.

II.—The inclination of the magnetic needle is no less variable than its declination. It is observed that it is different in different parts of the earth; but it is erroneous to suppose, as some philosophers in the last century did, that it has any relation to the latitude. It is observed, for example, that it is at Paris at present 72° 25′ North; at Lima, about 18° South; at Quito, about 15° S.; at Buenos-Ayres, about 60½° S.; at the Isle of France, 52¾° North.

This is sufficient to destroy the idea that it has the least relation to the latitude.

The dip, like the variation of the magnetic needle, has undergone changes, since it was first observed. In London, the dip was 73° 20′ in 1680; in 1723, it was 74° 42′; since which time it appears to have been progressively, though by no means regularly, diminishing. In 1773, it was 72° 19′; in 1786, 72° 8′; in 1805, 70° 21′; in 1818, 70° 34′; in 1821, 70° 3′; in 1828, 69° 47′; and in 1830, 69° 38′. At Paris, in 1814, M. Bouvard found the dip at the Observatory, 68° 30′; and in June 1829, M. Arago found it 67° 41′. Captain Sabine, by comparing the present dip with that observed for the last fifty years, concludes that the mean annual diminution may be taken at about 3′; and Mr. Barlow finds that both the dip and variation accord nearly with what would result from a uniform revolution of the magnetic pole round the pole of the earth—the whole period of revolution being about eight hundred and fifty years.

It would appear, then, both from theory and recent observations, that at present the dip is changing more rapidly than the variation; and Mr. Barlow's suggestion would lead to the expectation that they will continue to decrease together till about the year 2085, when the longitude of the magnetic pole will then be 180°, the variation nothing, and the dip only 56°, which will be its minimum. They will then increase together for the next 260 years, when the needle will have attained its greatest easterly variation, and will then return northerly, the dip increasing and the variation decreasing till about 2510, when the magnetic pole will be on the meridian of London, the variation nothing, and the dip 77° 43′. But the whole of these anticipations depend on whether the hypothesis on which they are based has any foundation in nature; and this can be determined only by future experiments.

As observations of the inclination are considered to be of no utility in navigation, it needs excite no astonishment that we have so few. Besides, it is much more difficult at sea to observe the inclination than the declination, on account of the rolling of the ship. Father Feuillée however made a considerable number of them, during his voyages in Europe and America; but, according to all appearance, they are only within a few degrees of the truth. It is nevertheless to be wished that

these observations were more numerous; for though, on the first view, they do not seem of great utility, they may become so hereafter. Let us not cease to accumulate facts, though in appearance useless. Some unexpected light often arises from an observation long considered to be frivolous and unimportant.

III. We may remark also, that the motion of the magnetic needle experiences very singular variations on the approach, or by the effect, of igneous meteors. A needle has been deprived of its magnetic property by thunder, or even magnetised in a contrary direction. The Aurora Borealis seems also to have a very sensible action on the magnetic needle; but for farther information on this subject, we must refer to Father Cotte's "Traité de Météorologie."

ARTICLE IV.

Of certain means proposed for freeing the magnetic needle from its declination, or for making compasses without declination.

It would be so great an advantage to have compasses which should point exactly to the north, that the attempts made to devise combinations to destroy the declination of the needle need excite no surprise; but unfortunately these attempts have hitherto been fruitless, and in our own opinion will always be so. They however deserve to be known, were it only to guard our readers against the illusions of those who imagine that they have solved this problem.

One of these inventions is described by Muschenbroek. It consists in combining, for a determinate place, two needles of equal force, in such a manner, that the one may decline on the one side, and the other on the other side, from the magnetic meridian, by a quantity equal to the declination. Thus, one of them will decline double, and the other will be exactly in the meridian. Let us suppose, for example, that the declination is 20 degrees to the west, as it was at Paris in 1770. If two magnetic needles of the same force be suspended on the same pivot, forming together an angle of 40 degrees, it is evident that neither of them being able to place itself in the magnetic meridian, they will equally decline from it: thus the one will decline 20 degrees to the west of that meridian, or 40 degrees from that of the earth; consequently, the other needle will necessarily be in the meridian, and will have no declination.

It is astonishing that any one should imagine that a combination of magnetic needles, capable of making one of them coincide with the terrestrial meridian, could be obtained in this manner. It may be readily seen, that these two needles, if of equal force, will always arrange themselves in such a manner, that the magnetic meridian will divide into two equal parts the angle comprehended between them. Thus, if we suppose that the magnetic meridian, instead of declining 20 degrees from the terrestrial meridian, declines only 10 degrees to the west, one of the needles will be carried 20 degrees more to the west; consequently will have 30 degrees of declination. At the same time therefore the other needle will be carried 10 degrees from the meridian towards the east.

The last translator of Pliny has given a method, nearly similar, for annihilating the declination, and which differs in nothing from that of Muschenbroek, except that one of the needles must be larger than the other. But Muschenbroek had before proposed and analysed this combination of two unequal needles, and it appeared to him as unlikely to succeed as the other. It is opposed indeed by the same, or by similar reasons, and nothing can rest on slighter grounds, than the physical theory by which the author alluded to appears to have been guided; for he seems to think, that the cause of the declination, is a sort of weakness in the needle, which does not permit it to reach the north. This is an idea not only void of foundation, but even incompatible with the most probable theory of the magnetic motion; for as the

magnetic needle, which during the first half of the 17th century declined to the east, afterwards approached the north, and passed it, to proceed to the west, it would be necessary to say that it was sick; since it was cured about 1660, and that it afterwards got diseased in a contrary direction. We cannot therefore sufficiently wonder at the precipitation of some journalists and some authors, who hastened to announce, with the greatest eulogiums, this pretended discovery, as likely to change the face of navigation. Unfortunately, nothing could be more chimerical; and a better acquaintance with the magnetic phenomena would have preserved both from this error.

We have seen formerly at Paris a Genoese pilot, named Mandillo, who pretended to have found another combination of magnetic needles, proper for correcting the declination. He placed two needles of equal force above each other, but in such a manner that each of them had full liberty to move; he then brought them together, for Paris we shall suppose, so that their deviation was double the inclination observed; for in this position they would each diverge by the effect of the repulsion of their poles or points of the same denomination, and so much the more as they were brought nearer to each other. By these means, one of the needles, as in the preceding process, was carried to the meridian. But the sieur Mandillo pretended that this would every where be the case, which is evidently false; for the deviation of the two needles being the effect of the repulsion of the two poles of the same name, this repulsion, and consequently the deviation, will be the same, whatever be the angle of the magnetic meridian with the terrestrial meridian; otherwise we must suppose that this repulsion decreases at the same time as the declination, which is impossible. This objection we mentioned to Mandillo; but to no purpose. A man who imagines he has found the means of correcting the declination of the magnetic needle, or has discovered the solution of the problem of the longitude,* is as obstinate in his opinions, as he who thinks he has found the quadrature of the circle.

We shall here mention an idea of M. de la Hire on this subject. It was founded on a belief of his having discovered that the poles of a natural magnet had changed their place, as the magnetic poles of the earth had done at the same time. He thence conceived the idea of magnetising steel rings, presuming that their poles would change in the same manner. But it may be readily seen that in this case, the line marked originally north and south on the ring would remain motionless, and would always indicate the real north. This principle however has been found to be false, and even if true, the consequence deduced from it by M. de la Hire did not necessarily follow.

ARTICLE V.

Of certain Tricks which may be performed by means of the Magnet.

For some years past, the properties of the magnet have been employed to perform several tricks, which excited a considerable degree of astonishment in those who first beheld them. No means indeed more secret, and at the same time more proper for action, could be employed than magnetism; since its influence is stopped by no body with which we are acquainted, except iron. This idea was first conceived by the celebrated Comus, who varied, in a singular manner, the different tricks performed by this agent; so that all Paris flocked, with the utmost eagerness, to the places where they were exhibited. He was admired by the ignorant, who considered him as a sorcerer, while the learned endeavoured to discover the artifice, which however was a profound secret, as long as no one suspected magnetism to be the principal cause of it.

* In the time of Montucla, deemed the extreme of absurdity.

We shall here endeavour to give an idea of some of these tricks, as they will form a fund of rational amusement to those who know how to perform them.

SECTION I.

Construction of a Magic Telescope.

Those who exhibit these tricks often employ a pretended magic telescope, by means of which one can see, it is said, through opake bodies. It is nothing else than an instrument in the form of a telescope, at the bottom of which, that is towards the object glass, there is a magnetic needle, which assumes its proper direction, when the telescope is placed upon the side which that object glass forms.

To construct this telescope, provide a turned tube of ivory, wider towards the end where the object glass is placed; but the ivory must be of sufficient thinness to admit the light through to the inside. The narrow end is furnished with an eye-glass, which serves to shew more distinctly the inside of it. The other end also is furnished with a glass, which has the appearance only of an object glass, the posterior surface of it being opake, so as to serve for the base or bottom of a sort of compass or magnetic needle, which turns on a pivot fixed in its centre. When the telescope rests on the end containing the object glass, this needle assumes a horizontal position, and points towards the north, or towards a magnetic needle in the neighbourhood. It is necessary also to have a real telescope, similar in appearance to the other, in order that it may be shewn instead of it, which may be done by dexterously substituting the one for the other.

When you wish to employ the pretended magic telescope, place it with the object glass downwards upon any thing you intend to examine, and if there be a magnet, or piece of magnetised iron below it, the needle will turn to that side.

SECTION II.

Several figures being given, which a person has arranged close to each other in a box, to tell through the lid or cover what number they form.

If you are desirous of employing the ten ciphers, take ten small squares, of an inch and a half on each side, and on the upper face of each make a groove; but let these grooves be in different directions; that is to say, the first intended for the number 1 must proceed directly from the top to the bottom; the second must deviate to the right, so as to form an angle equal to a tenth part of the circumference; the third an angle of two-tenths; and so of the rest; which will give ten different positions. Then introduce into these grooves small bars of steel, well magnetised, taking care to turn their north poles to the proper direction; cover these grooves and the face of the squares with strong paper, in order to conceal the bars. You must also provide a narrow box, capable of containing in its breadth one of these squares, and of such a length that they can all be arranged in it.

Then desire a person in your absence to take several of these squares, and arrange them in the box in any manner, at pleasure, so as to form any number whatever, and to shut the box; after which you are to tell the number which has been formed.

Deposit your pretended telescope on the place of the first square, that is on the left, if the figure below it be unity; the needle will turn in such a manner that the north point or pole will be before you. If the figure be 4, it will turn to the fourth division of the circle, which is equally divided into ten parts; and so of the rest. It will thence be easy to discover the figure in each place, and consequently to tell it.

A word written in secret, with given characters, may be discovered in the same manner; also an anagram, formed of a proposed word, as *Roma*, which gives *amor*,

mora, orma, maro, &c.; or a question which has been selected from several persons, and put into the box. In short, this trick may be varied in a great many ways, exceedingly agreeable, but all depending on the same principle.

The box of metals, for example, is only a similar variation of the same trick. You put six plates of different metals in a box, and bid a person take any one of them, and put it into another box, and shut it. You may then easily tell which one he has taken. These plates are of such a form, that they can occupy in the small box only one position. Each of them, that of iron excepted, contains in its thickness a magnetic bar, arranged in situations which are known, and these situations are discovered by means of the pretended magic telescope; consequently the nature of the metal must be known. No magnetic bar is placed in the plate of iron, because this would be useless; but one side of the plate may be magnetised, or if it be not magnetised, the indeterminate direction of the needle will announce that it is iron.

SECTION III.

The Learned Fly, or the Syren.

This trick is somewhat more complex than the preceding, and depends partly on philosophical principles, and partly on a little deception. You must provide a table, with a box sunk in its thickness, and the box must be furnished with a broad brim, inscribed with numbers, the hours of the day, or answers to certain questions. You then desire a person to point out a number, or to name any hour in the day, or to ask what o'clock it is, or to select any one of certain questions written upon cards which you présent to him. A fly, a syren, or a swan, floating in water, indicates, in their order, the figures of this number, or answers the question proposed.

All this is performed by means of a strongly magnetised bar, supported by a brass circle, concealed in the rim of the bason, which contains the water. It is evident that if the motion, necessary to point out the letters, or numbers, required for the answer, can be given to this bar, the fly or syren, placed on a small boat containing another magnetic bar, will proceed towards it, and appear to answer the question. Such are the philosophical principles of this trick. The deception is as follows.

The table, which is some inches in thickness, is hollow, and the cavity contains a certain mechanism put in motion by a string, which, passing through the feet of the table, traverses the floor, and is conveyed into a neighbouring apartment, separated from that where the trick is exhibited, only by a very slight partition. This string terminates at a sort of table, on which are marked the divisions of the bason; and the whole is combined in such a manner, that when the end of the string is brought opposite to a certain figure, such as 4 for example, the magnetic bar will be under the 4, inscribed on the edge of the vessel.

When the syren then is desired to tell what o'clock it is, the person behind the partition, and who hears the question, has nothing to do but to pull the string, and to bring the end of it opposite to the required hour on the table, which is before him. The magnetic bar will arrange itself below, and the tractable syren, beginning to move, will go and point out the hour.

If a question has been selected, the person who exhibits the trick repeats it under a pretence of interrogating the syren. The confederate, who hears it, causes the magnetic bar to move to the answer.

It would not be difficult to establish between both a secret communication of such a nature, that, without speaking, the syren should appear to guess the question, and to give an answer to it.

The principle works on the magnet are, "A Treatise de Magnete," by Gilbert, an English philosopher, printed in 1633: it contains traces of that spirit of observation which has since caused philosophy to make so great a progress. The "Ars

Magnetica" of Kircher: this is a kind of encyclopædia of everything written before the author's time on that subject, enlarged by a great many of his own ideas, the greater part of which however display more imagination than judgment. The "Magnetologia" of Father Leothaud, 1668, in 4to: a work of very little importance. Father Scarella's treatise, entitled "De Magnete," in four volumes, quarto, printed at Brescia, in 1759, may supply the place of all the preceding, as it contains a comprehensive account of everything useful or solid, said or written on the magnet, till that period; to which the author, a very enlightened philosopher, has added his own ideas. The small treatise on Artificial Magnets, translated from the English, with an historical preface by the translator, will make the reader fully acquainted with that part of the theory of the magnet; or, in want of it, recourse may be had to the "Memoire sur les Aimants Artificiels," by M. Antheaume, which gained the prize proposed by the academy of Petersburgh in 1758. Several papers also by M. Dutour, inserted in the "Mémoires presentés à l'Académie, par des Sçavans étrangers," deserve to be known, and to be studied, by those who may be desirous of cultivating and enlarging this theory.

To the above may be added an excellent treatise on the subject by Dr. Roget, in the Library of Useful Knowledge, and a treatise on the subject by Sir David Brewster, in the Encyclopedia Britannica.

PART THIRTEENTH.

OF ELECTRICITY.

Electricity is an almost inexhaustible source of singular and surprising phenomena, which must excite the curiosity of the most indifferent observer of nature. What indeed can be more extraordinary, and at the same time more difficult to be reconciled with the known laws of natural philosophy, than to see mere friction excite, in certain substances, the power of attracting and repelling such light bodies as are near them; to see this power communicated, by contact, to other bodies, and even to very great distances; to see fire issue from a body in that state; and a thousand other pheuomena, the enumeration of which would be too tedious? We shall mention only the famous experiment of Leyden, where a rank of persons, holding each other by the hands, or having a communication by means of an iron wire, or rod, suddenly receive from an invisible agent an internal commotion, which might even be so violent as to kill those who experience it.

It must however be allowed, that the case has not yet been the same with electricity as with magnetism. The latter, by the invention of the magnetic needle, has served to render navigation more secure, and to discover the new world, a source of new riches, new wants, and of new evils to the old one. But electricity has not yet produced any thing of so much importance to mankind, and to the arts, if we except the analogy now fully proved between the electric fire and lightning: an analogy which has given rise to a pretty sure preservative from the effects of that dreadful meteor; for in regard to the cures effected by electricity, it must be acknowledged that they are either rare, or not well ascertained.

But we must not treat all researches on this subject as useless; for when we consider the phenomena exhibited by electricity, we cannot help allowing that it is one of the most general and most powerful agents in nature. Is it possible to deny that the identity of the electric fire and lightning is a noble and grand discovery? What can we say of a multitude of other analogies observed between electricity and magnetism, the nervous fluid, the principle of vegetation, &c. They seem to promise a copious harvest to those who shall continue to cultivate this fertile field.

SECTION I.

Of the Nature of Electricity; and the distinction between bodies Electric by friction or by communication.

Electricity is a property which certain bodies acquire by friction, that is to say the power of attracting or repelling light bodies which may be near them. If you rub, for example, a stick of Spanish wax, with your hand, or rather with a piece of cloth, and then make the wax pass within a few lines of small bits of paper or straw, you will see them rush towards the wax and adhere to it, as if cemented, until the virtue acquired by the friction is dissipated. The ancients had observed that yellow amber, when rubbed in this manner, attracted light bodies: hence the name of

electricity; for they called that substance *electrum*. But their observations went no farther.

The moderns have found the same property in a great many other bodies; such as grey amber, and in general in all resins, which can bear a certain degree of friction without becoming soft; as sulphur, wax, jet, glass, the diamond, crystal, the greater part of the precious stones, silk, woollen, the hair of animals, and very dry wood.

In regard to bodies which do not acquire electricity by friction, it has been observed that they can acquire it by communication; that is to say by contact, or by being brought very near to those of the first species; and that they can transmit it, by the same means, to other bodies of the same nature. Those bodies, which cannot be rendered electric by friction, are metals, and water, either liquid or congealed;* also earthly and animal bodies. But we must observe that, properly speaking, metals and the aqueous fluid are the only true conductors of electricity; and that the rest are not so, unless they participate in the metallic nature, or contain more or less moisture. Electricity seems even to prefer the metallic bodies, for transmitting itself from one body to another. If you place a body then of the latter kind, such as a metal rod, or a piece of moist wood, in the neighbourhood of a body, or in contact with a body of the first kind, electrified by friction, and with precautions to be mentioned hereafter, it will itself become electric; which may be readily seen by the motion it will communicate to light bodies in its neighbourhood.

All bodies then are susceptible of being electric, but in two different ways: one kind are in some measure electric of themselves, as that virtue can be excited in them by friction; for this reason they are called *electric per se:* the other kind are electric only by communication, and for this reason, they are commonly called electric by communication, or *non-electric;* but it would be better to call them *conductors of electricity:* and this is the appellation which we shall most frequently employ.

It may be here proper to observe, that those of the first class are not susceptible of receiving electricity by communication, or they receive it in that manner with difficulty. Hence it happens that, in the experiments we are about to describe, the bodies to be electrified by communication, must be placed either on cakes of resin, or be suspended by silk strings; otherwise the electricity produced in them would be immediately dissipated, by the contact of bodies susceptible of being electrified by communication, with which they might be in contact.

SECTION II.

Description of the Electric Machine, and of the apparatus necessary for performing electrical experiments.

When philosophers began to cultivate the theory of electricity, they employed nothing for the purpose of exciting it but a glass tube, about 3 inches in diameter, and from 25 to 30 inches in length. It was rubbed lengthwise and in the same direction, with the bare hand provided it was very dry, or wrapped up in a bit of flannel or cloth; and this tube was afterwards presented to the body intended to be electrified. It was in this manner that Gray and Dufay made their first experiments.

A globe suspended between two wooden pillars, which was made to revolve rapidly by means of a handle or wheel, was afterwards substituted instead of this tube: the dry hand was applied to the globe thus arranged, or it was made to rub against a cushion: this operation excited the electricity, which was collected as we

* It has since been observed that glass heated till it becomes red or more, and flame, are conductors of electricity. On the other hand water, which in its state of fluidity is a conductor of electricity, ceases to be so when strongly frozen.

may say by means of a metallic fringe suspended from the globe, or disposed in some other manner.

These machines were succeeded by one much simpler. It consists of a foot or stand A, Fig. 45, upon which are raised two uprights, B and C, secured and united at the top by means of the circular piece D. These uprights must be of a greater or less height, according to the diameter of the circular glass plate E, placed between them; for the edge of the plate must not approach too near the wooden frame, either at the top or the bottom.

Fig. 45.

D

This circular plate of glass E, is the most essential part of the machine. It has a hole in the centre, of sufficient size to admit a steel axis, which turns in the two supporters, and towards the side C, is continued outwards, where it terminates in a square extremity, fitted into a handle, which serves to turn the plate.

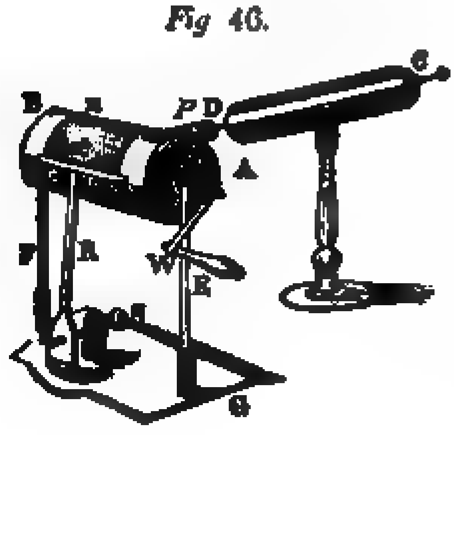
Leather cushions, stuffed with hair, are applied to the supporters on each side, both at the top and bottom; so that the glass plate, as it revolves, is rubbed by the cushions to the distance of some inches from its edge.

On the longitudinal part of the stand is placed a conductor, supported by a glass foot in the form of a column. This conductor is a copper cylinder, terminating at one end in a ball G, of the same metal, and having at the other end two arms bent almost in a semicircular form. At the extremities of these arms are two hemispherical figures, H and I, which present to the glass plate their circular bases, furnished with four sharp steel spikes, all of the same length. The foot of the conductor can be made to advance or recede on the bottom part of the machine, which supports it, in such a manner that the before mentioned spikes can be brought nearer to, or removed from, the surface of the glass plate; for it is these spikes, as will be seen hereafter, that attract the electric fluid, excited or put in motion by the friction of the small cushions on the circular glass plate.

When you are desirous of producing electricity, place the machine on a firm table, and make it fast by means of screws; then fix the conductor in such a manner, that the spikes may approach very near to the glass plate, and put the latter in motion by turning the handle. The conductor will almost immediately exhibit signs of electricity, either by emitting sparks on the finger being applied to it, or by attracting and repelling light bodies placed near it.

The common electrifying machine, consisting of a glass cylinder, was first introduced by the Germans, and for ordinary purposes it may still be considered as the most perfect of any.

Fig 46.



This machine is represented at Fig. 46, where A B is the cylinder of glass supported upon two pillars E F, which may be either of glass, or baked wood, though it is not essential that the pillars should be non-conductors. They are firmly fixed into a stand or horizontal board G H, which may either rest on the floor, or be set upon and clamped to a table. Rotatory motion is given to the cylinder either by a simple winch, as W, or by a multiplying wheel fixed to the lower part of the pillar E.

and acting by means of a band on a smaller wheel affixed to the axis of the cylinder.

A rubber represented at m is supported on a pillar B, fastened to the board G H. By means of a screw s, this pillar can be brought so as to make the cushion apply with sufficient tightness to the cylinder. A silk flap n o, attached to the rubber, passes over the cylinder. The conductor C D is placed upon an insulating stand, and receives the electricity from the cylinder by means of a series of metallic points, as at p.

If the cushion is insulated, a conductor is sometimes applied to it, and while the other conductor is electrified positively by the electricity collected from the revolving cylinder, that applied to the cushion is at the same time exhausted of its electricity, or electrified negatively.

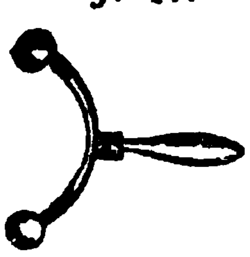
When only the positive conductor is used, and the cushion is insulated, it is necessary that a communication should be established between the cushion and the earth, to afford a supply of electric matter to be put in action by the cylinder.

Remark.—Some other instruments are necessary for electric experiments; but we shall here mention those only which are commonly used, reserving the description of the rest till we come to speak of the different experiments in which they are necessary.

I. You must be provided with a few stools, either square or circular, covered with resin, about 15 or 18 inches in breadth: and, the better to ensure the effect, they may be made to rest on four glass bottles or feet. These stools serve to insulate the persons or bodies intended to be electrified.

II. As it is sometimes dangerous to draw out the electricity by the finger, it will be proper to have an instrument, called the discharger, which is a circular piece of metal, Fig. 47, affixed by the middle to a handle, made of glass or Spanish wax: but the first is preferable and stronger. By touching bodies, electrified in the highest degree, with one of the balls of this instrument, sparks may be extracted from them without danger; because the glass handle intercepts the passage of the electricity, from the discharger to the person who holds it.

Fig. 47.



III. You must have also a chain of metal, or of several pieces of wire connected together. This chain serves for transmitting the electricity to a distance from the first conductor I; which is done by suspending it from silk cords, attached to the ceiling, or extending it between two supporters.

IV. It will be proper to provide likewise a long tube of metal, or of gilt pasteboard, three or four inches in diameter. This tube, having a communication with the conductor by means of a chain, forms a second conductor, which becomes charged with a great deal of electricity, and may be employed in a variety of experiments. The longer and larger this tube is, the greater will be the quantity of electricity with which it will be charged. For reasons which will be mentioned hereafter, it is necessary that it should have no points or sharp eminences.

V. A few glass plates are also necessary, to insulate those bodies which may be required to retain their electricity.

VI. You must provide likewise a few pieces of metal, some pointed and others terminating in spherical or round ends; some affixed to glass handles, and others furnished with handles of some substance that transmits electricity, as before mentioned.

VII. The cushions must be occasionally rubbed over with a kind of amalgam which serves to increase the friction. That which appears to answer best, is an amalgam of tin and mercury, such as that placed at the back of mirrors, mixed

with one half of chalk or Spanish whitening, the whole reduced to an impalpable powder.

Such are the principal parts of the apparatus necessary for the most common electric experiments; to which we shall now proceed, beginning with those that are simplest.

EXPERIMENT I.

The Electric Spark.

Every thing being arranged, as above described, and the air in the apartment being dry, put the machine in motion. If a person then present his finger to the conductor, at the distance of two or three lines, or more, according to the strength of the electricity, two sparks will proceed at the same time, from the conductor and the finger, accompanied with a snapping noise, which will even occasion some pain. When the person, whom we suppose to be on the floor, touches the conductor, it will give no more signs of electricity, because he will then be in communication with the whole mass of non-electric bodies with which he is in contact.

If a piece of chalk be placed on the conductor it will be found that considerably longer sparks may be drawn from the chalk than the conductor itself; and that the sparks are of a zig-zag form; and are indeed beautiful miniature representations of forked lightning.

EXPERIMENT II.

Communication of Electricity to Several Persons.

Cause the person, above mentioned, to stand on a cake of resin, by which means he will be insulated from the floor, and put the machine in motion. This person, as well as the conductor, will thus be electrified; so that all those, not in the same state, who happen to touch each other, will elicit electric sparks.

Twenty persons, and more, may be electrified in this manner, provided they are insulated.

EXPERIMENT III.

Attraction and Repulsion.

Present to the person electrified, or to the conductor, some gold leaf or tin-foil, or bits of straw, or paper, or other light non-electric bodies. When at a proper distance, you will see them dart towards the electrified body; but as soon as they have touched it they will be repelled. If you then receive them on a non-electric body, as soon as they touch it, they will return towards the electrified body, and will be again repelled; and so on alternately.

EXPERIMENT IV.

Some Electric Amusements, founded on the preceding property. The Gold Fish; the Electric Dance; the Luminous Rain.

The property which electric bodies have of repelling each other, when in that state, and of attracting each other when one of them only is electrified, has given rise to several very agreeable amusements, which we shall here explain.

I. Cut a piece of strong gold leaf into the form of a rhombus, two opposite angles of which may be very obtuse, while the other two are very acute. Present this metallic leaf to the conductor, in such a manner that one of the acute angles shall be first raised, and immediately put below it a metallic plate. You will then see the leaf place itself between the conductor and the plate, and remain in that state almost motionless.

Cut leaves of metal of this kind into the shape of the human figure, having an acute angle at the top, like a pointed cap, and lay them flat on the plate: if you then

place them below the conductor, on another plate, you will see them start up, leap towards the conductor, then fall down, turning round with more or less rapidity, so as to represent a kind of dance; and if the experiment be performed in the dark, you will observe luminous aigrettes dart alternately from the head and the feet, which will form a very agreeable spectacle.

II. Cut a piece of the same gold leaf into a figure very much lengthened on one side, but on the other much less acute; to this part if you choose you may give the form of the head of a fish. If you lay hold of it by the acute angle, and present the obtuse one to the conductor, at the distance of a foot, if the electricity be strong, it will escape from your fingers, and will fly with an undulating motion towards the the conductor, above which it will place itself at the distance of the eighth part of an inch, turning towards it the obtuse angle. Sometimes it will approach so near as to come into contact with it, and will be immediately repelled, forming the representation of a small fish going to attack or bite the conductors. This amusement therefore has been called the *Gold Fish.*

III. The luminous rain may be produced in the following manner. Suspend from the conductor a circular plate, 5 or 6 inches in diameter; then provide a metallic plate in the form of a saucer, and surround the edge of it with a glass cylinder 5 or 6 inches in height. Cover this plate with very fine light shavings of metal, and place it under the plate suspended from the conductor. When the latter is strongly electrified, you will see all these small leaves of metal ascend from the lower to the upper plate, and sparkle; being then repelled to the lower one, they will again sparkle, or will sparkle between the plates, when one which is electrified meets with one not electrified: by these means the glass cylinder will be filled with a great deal of light, which will exhibit the appearance of a shower of fire.

EXPERIMENT V.

Repulsion between bodies equally electrified.

Suspend from the extremity of the conductor two threads of any non-electric matter; such as flax, hemp, or cotton, which will hang down perpendicularly, and touch each other, if their upper extremities are in contact. If you then work the machine, and produce electricity in the conductor and these threads, you will see them repel each other, and form an angle of greater extent, as the electricity is stronger. When the electricity decreases, they will approach each other.

This experiment proves a very important fact in the theory of electricity; which is, that two bodies, similarly electrified, repel each other: and hence we are enabled to explain several electric phenomena and amusements.

EXPERIMENT VI.

Construction of an Electrometer.

By the preceding experiment we are furnished with the means of determining the strength of electricity; and the two threads, above mentioned, may be considered as a sort of electrometer. However, as two threads of this kind may be subject to various movements, independent of electricity, electricians have almost universally adopted the following instrument, which is equally simple.

The whole of this machine consists of two small balls, two lines in diameter, made of cork or the pith of the elder tree, and fixed to the two extremities of a thread capable of conducting electricity. This thread is made to pass over the conductor in such a manner, that the two balls hang at the same height. As soon as electricity is produced in the conductor, and consequently in the small balls, they diverge from each other, and the magnitude of the angle formed by the threads will convey some idea of the intensity of the electricity. We say conveys an idea of this intensity; for it is not possible, either by this or any other method with which we are acquainted

to determine when the electricity is double, triple, or quadruple, &c.; but we are at least enabled to conclude, that one degree of electricity is greater or less than another; or that two degrees of electricity are equal, according as the divergency of the balls is greater or less, or the same, which in general is all that is necessary to be known.

EXPERIMENT VII.

To kindle Spirit of Wine by means of the Electric Spark.

When a person is electrified, if another standing on the floor approaches him, having in his hand a spoon filled with spirit of wine, well dephlegmated, and somewhat heated; and if the electrified person presents his finger to the spirit of wine, or, what is still better, the point of some blunt instrument, or the point of a sword, an electric spark will proceed from the liquor, and set it on fire.

If the painful sensation produced by the electric spark, could leave any doubt of its being real fire, this experiment must be a convincing proof of it.

EXPERIMENT VIII.

Properties of sharp Points or Spikes.

Instead of a conductor, such as that above described, if you employ an angular bar of metal, or a bar terminating in one or more points; on approaching your finger to one of the angles or points, when the machine is put in motion, but not in such a manner as to produce an electric spark, you will feel something exhaled like a gentle breath of wind, and even with a sort of crackling noise.

But, if the experiment be performed in the dark, you will enjoy a very beautiful spectacle; for when the electricity is strong, you will see luminous gerbes issue from the angles of the conductor, and these gerbes will be considerably increased on presenting your finger to them.

You will discover, at the same time, that the cause of this gentle breath, accompanied with noise, is nothing else than the eruption of the electric fluid, whatever it may be, from the electrified body, which rushes towards your finger. Hence it follows that it is a body, since it re-acts against another body. It will be found also that this dispersing electricity has a peculiar smell.

It is to be observed, that when the electrified body is angular, it loses much sooner the electricity which has been communicated to it. These angles and points seem to be so many spontaneous discharges of the electric matter; they ought therefore to be avoided in bodies intended to be electrified, and in which you are desirous to maintain the electricity as long as possible.

EXPERIMENT IX.

Difference between Pointed and Blunt Bodies.

Electrify strongly in the dark a common conductor, or any other body whatever, not angular, and when it is strongly charged, present to it a blunt body, such as the finger or a spike rounded at the end, holding it so near it as to elicit the electric spark. But if you present to it a very sharp instrument or spike, you will see a luminous star formed at the extremity of it, even before it is brought so near; and if the electrified body does not every moment receive a fresh supply of electricity, it will thus soon be deprived of it.

If this spike be supported by a cake of resin, it will itself become electrified; but the electricity of the conductor will not be entirely destroyed.

It appears from this experiment, that if the luminous gerbes, in the preceding one, are formed by a matter which flows off from the electrified body, the case in the present instance is contrary: they are formed by a matter which throws itself towards the point presented to the electrified body. What indeed can be said when we observe a non-electrified body become so by this method; but that the electric matter, fire,

or fluid, whatever it be, proceeds from the electrified body to another, especially as it is certain that the former thereby loses either the whole or a part of its electricity, according to circumstances; that is to say, according as the other stands on the floor, or is insulated?

But however this may be, the following is a singular and remarkable property of pointed bodies. The extraordinary use which Dr. Franklin made of it will be shewn hereafter.

EXPERIMENT X.

Method of knowing whether a body be in a state of Electricity.

When two bodies are equally electrified, if they be brought into contact with each other, no sign of electricity will be manifested between them, by sparks or any electric emanation.

This may be easily proved; for if a person electrified by touching the conductor, gives his hand to another electrified in the same manner, there will be no spark.

These two persons, however, may know that they are electrified by the following sign. Let each of them take in his hand a thread, made of any non-electric substance, or a cork ball suspended from such a thread; if these two balls or two threads repel each other, it may be concluded that the persons are in an electric state.

EXPERIMENT XI.

Distinction between the two kinds of Electricity.

Provide two electric machines, one of them constructed as they were formerly, that is, with a glass globe, and the other with a globe of sulphur instead of glass: if a conductor be then electrified by each of them at one of its ends, you will see with astonishment, if the machines are moved with equal velocity, that scarcely any sparks can be extracted from the conductor. The case certainly would not be the same, if the conductor were electrified by means of two glass globes at the same time, or with two globes of sulphur; the sparks would be much stronger than if one globe had been put in motion.

Remark.—This experiment, which Dr. Franklin says he made at the request of his friend Mr. Kinnersley, seems to me to leave no doubt in regard to the difference between electricity communicated by glass and that communicated by sulphur; and consequently establishes the distinction of vitreous and sulphureous or resinous electricity; a distinction before asserted by Dufay.

Dufay indeed had observed, that though two bodies electrified by glass or sulphur mutually repelled each other, yet when one of them was electrified by the one of these substances, and the other by the other, instead of repelling they attracted each other. We do not think that any stronger proof of the two states can be desired.

If to this be added the above experiment of Dr. Franklin, how can we elude the consequence which he and Dufay deduce from it? For, it is well known that two bodies equally electrified by a glass globe, may touch each other without producing a spark, and without the electrical virtue being diminished in either of them. Since these bodies then electrified, one by the glass and the other by the sulphur, mutually destroy each other's electricity, the one must be of a nature contrary to the other, and entirely different.

Some able philosophers, however, notwithstanding these reasons, persist in rejecting this distinction; but in our opinion they labour under the influence of prejudice, or, being seduced by peculiar ideas, keep their eyes shut against the light. We are inclined to think, that if the Abbé Nollet had not previously formed his system on electricity, he would have adopted the distinction of the two kinds.

However, as this is the proper place, we shall here give an idea of Dr. Franklin's system in regard to electricity. According to this celebrated philosopher, all bodies in their natural state contain in their substance, or on their surface, a certain quantity of a fluid, which is the electric fluid. The air, which when dry is not a conductor of electricity, prevents its dispersion. But the friction of certain bodies, glass for example, collects on the surface of them a greater quantity of the fluid; so that if the glass be in contact, or very near to a body electric by communication, such as a mass of iron, the fluid accumulated on the surface of the glass tends to pass into the mass of iron, in order to preserve an equilibrium. By these means this mass acquires a greater quantity of the electric fluid, and is then electrified *positively*. But if the electric body, instead of acquiring by friction a greater quantity of the electric fluid, loses some of what it had, as is the case with sulphur, the body in contact with it will lose a part of its own natural electricity, and will then be electrified *negatively*. The one will have more electric fluid than it has in its natural state, and that of all the bodies which have a communication with the earth; the other will have less. Such is the nature of *positive* and *negative* electricity.

It must, however, be allowed that it does not clearly appear how friction should accumulate, on the surface of the rubbed body, a greater quantity of the electric fluid. It is not even known whether the effect of friction is to accumulate the fluid on the glass, or to diminish the quantity of it; whether it lessens it on sulphur and resins, or increases it. Hence it is uncertain which is the positive and which the negative electricity; but we know beyond a doubt that their effects are contrary, and this is sufficient. Several reasons, however, make it probable that the electricity produced by the friction of glass, is the positive or accumulated electricity.

Notwithstanding this uncertainty, Franklin's theory has a great advantage over that of the Abbé Nollet. The latter supposes a matter diffused throughout all bodies, and even in the atmosphere, which with all other electricians he calls the *electric fluid*. In this he agrees with Dr. Franklin; but he thinks the effect of friction is to make this fluid sometimes issue from the pores of the body rubbed, and sometimes to attract it. Electricity therefore, or the electric fluid, is sometimes *effluent* and sometimes *affluent*; and it is by means of this effluence or affluence that this philosopher explains all the phenomena of electricity. But the great defect of this system is, that every thing in it is, as we may say, arbitrary. What cannot be explained by the affluent fluid, may be explained by the effluent. These are the different matters of Descartes, or his subtle matter, which may be applied to every thing. On the other hand, in the system of Franklin, the effects are much better connected with the causes, even supposing them hypothetical. Why does a spark issue when a body, positively or negatively electrified, is brought near to another which is in its natural state? The answer is easy. The electric fluid accumulated on the one side, and extended in the form of an atmosphere, as it were, on the surface of a body, puts itself in equilibrium, when it comes in contact with another electric atmosphere less condensed: the fluid divides itself equally between the two bodies; which cannot be done without an exceedingly rapid movement that produces light. But what is most remarkable in the hypothesis of Franklin, and is almost the touchstone of truth, is, that even the bare description of the simplest experiment, to those who have properly comprehended this hypothesis, is sufficient to enable them immediately to guess the result. The case is not the same with the system of the Abbé Nollet: none of the effects about to be produced are foreseen, and if every thing be explained, it is because no effect is connected with its cause. Had the phenomenon been quite contrary, it might have been explained with the same ease: effluence could be employed instead of affluence; one is the remedy or supplement of the other.

We must however acknowledge, that there are some facts difficult to be reconciled with the motion of the electric fluid, which is a necessary consequence of Franklin's system.

For example, when the finger is brought near to a body, electrified either positively or negatively, why do we see a double spark proceed from each of these bodies? It would appear that it ought to proceed only from that which is endowed with positive electricity.

In a certain experiment, in which a quire of paper is pierced by the electric spark, why is the rough edge of the hole turned in a direction contrary to that in which it ought to be, if the fluid accumulated on the surface of the electrified body were the only one that proceeded to the body negatively electrified? We shall omit several others, which have been remarked by the partisans of the Abbé Nollet, and only observe that there is still reason to suspend our opinion on the mechanism of this phenomenon.

EXPERIMENT XII.

The Leyden Flask, and Shock.

There is not perhaps in natural philosophy a phenomenon more astonishing than that which we are about to describe. Provide a flask of very thin white glass, with a long neck, and fill about two thirds of it with water, or metallic filings, or raspings of lead. Close it with a cork stopper, and introduce into it, through the cork, an iron wire, so as to be immersed with one end in the water or filings, while the other projects some inches beyond the cork, and terminates in a blunt or crooked extremity.

When the flask is thus prepared, lay hold of it by the belly, and present the iron wire to the conductor of the electrifying machine while in action. By these means the flask will be charged. While the wire is in contact with the conductor, if you then endeavour to touch the latter, or the iron wire, with the other hand, you will experience throughout your whole body a violent shock, which will seem to affect more particularly, sometimes your breast, sometimes your shoulders, and sometimes your arm or wrist.

The same effect will be experienced if you retire, holding the bottle by the belly with one hand, and touch the iron wire with the other.

Nay, a chain may be formed of as many persons as you choose, holding each other by the hand, and without being insulated. The first person holds the bottle in his hand, or only touches it, while the iron wire is in contact with the conductor; and the last touches the conductor; all those who form the chain will experience the before-mentioned shock at the same instant. When the flask is of considerable size, and has been well charged, the shock is sometimes so violent, that those subjected to it suffer a momentary loss of respiration. The celebrated Muschenbroek, to whom M. Cuneus exhibited this phenomenon, which he had discovered by accident, received, according to every appearance, a violent shock; since in announcing it to the French philosophers, he protested that he would not expose himself to it a second time for the whole kingdom of France. It is however probable that he afterwards became bolder. As this singular experiment was first performed at Leyden, it is generally called the *Leyden experiment*, and the bottle, so prepared, is distinguished by the name of the *Leyden flask* or *phial.*

The French philosophers once formed a chain 900 toises in length, by means of 200 persons, all connected by iron wires; and all these persons experienced the shock at the same instant. Another time they tried to transmit the shock along an iron wire 2000 toises in length, and the experiment succeeded, though the wire passed over the wet grass, and newly ploughed land. In short, they comprehended

in the chain the water in the grand bason of the Tuilleries, which is nearly an acre in extent, and the shock was transmitted very well across it.

Remarks.—I. As some inconvenience resulted from the weight of the water or filings, put at first into the flask, the idea was afterwards conceived of covering the inside of it with a metallic coating. This may be done in several ways. The most simple is to pour into the bottle strong gum water, and to moisten with it the part intended to be coated. The superfluous gum water is then poured out, and very fine copper filings are put into the bottle; these filings adhere to the gum water, and form an internal coating, which must be in contact with the iron wire, that the bottle may be charged.

The effect of the Leyden flask may be increased also by covering a great part of the outside of it with tin-foil.

II. The flask may be charged in another manner, that is to say, externally. For this purpose, hold it suspended in the one hand by the hook or iron wire on the outside, and bring the outside into contact with the electrified conductor. If it be then touched on the outside with the other hand, you will experience a shock. You may form, in like manner, a chain of several persons, the last of whom, or the one farthest distant from the person who holds the iron wire, by touching the outside of it, will produce the same phenomenon throughout the whole chain.

III. Dr. Franklin observed the following very singular circumstance, which takes place in performing the Leyden experiment: if you are desirous of charging the inside of the jar or flask, the outside must communicate with some body which is a conductor of electricity; for if the flask be placed on a cake of resin, it will be in vain to electrify, by the conductor of the machine, the wire which touches the water or metallic coating in the inside: the flask will not become charged. Is it necessary, before it can be charged, that in proportion as the electricity is accumulated on one side, it should be diminished on the other? This is the conclusion which Dr. Franklin forms, and which appears indeed to be agreeable to reason. But how is it that the electric fluid is expelled from one side, while the other becomes more highly charged with it? This appears to us to be a considerable difficulty.

IV. The jar seems to be impermeable to electricity, at least when cold, or when it has only the temperature of the atmosphere. Dr. Franklin once tried to grind away the belly of a charged flask, which was of the usual thickness. He ground down ⅔ of its thickness, without its being discharged, which would have been the case if the fluid in the inside had communicated with that on the outside. It is to be wished that this philosopher had continued to diminish the thickness, until a discharge had taken place.

But when the glass is dilated and softened by a heat which brings it nearly to a state of fusion, it then not only becomes a conductor of electricity, but the charged jar discharges itself spontaneously.

V. If a chain, suspended from the conductor, be introduced into the flask while held in the hand, it becomes charged in the like manner; but if the flask be lowered in such a manner, as no longer to support any portion of the chain, it then gives no more signs of electricity.

There is reason thence to conclude that the electricity with which the inside of the bottle is charged, ought to have for its support some non-electric matter, or a conductor of electricity. It would be in vain to attempt to charge an empty bottle, or a bottle not covered in the inside with a metallic coating.

VI. If the inside of the bottle be charged, making it communicate by the hooked wire with a conductor positively electrified, the outside will then be negatively electrified; for the outside will attract the small ball of cork suspended from the

conductor, while the hook of the flask repels it. But it is well known that an electrified body repels another electrified like itself; it attracts only bodies not electrified, or electrified in a contrary sense. Since the outside of the bottle then, electrified by the hook, attracts the small ball of cork, the electricity of which is of the same nature as that of the conductor, or of the inside of the jar, the exterior electricity must be of a nature entirely different.

VII. If you have two equal flasks, equally charged, and in the same manner, and if you then bring them near to each other, so as that the hooks or the sides of them be in contact, they will not be discharged; but if you apply the hook of the one to the side of the other, a discharge will immediately take place.

If one of the flasks be charged by a globe of sulphur, and the other by a globe of glass, if the hook of the one be then made to approach the hook of the other, or the side of the one the side of the other, they will be discharged.

VIII. If several persons, instead of holding each other by the hands, present to each other the tips of their fingers, at the distance of one or two lines, at the moment when the last touches the conductor, you will observe between all the fingers an electric spark, and each will experience a shock.

IX. If the persons, instead of holding each other by the hands, form a communication with each other by holding glass tubes filled with water, and stopped with a cork, through which passes an iron wire immersed in the fluid, and which is in contact with each person, at the moment when the last person touches the conductor, or the wire immersed in the flask, you will perceive a train of light in the water in each tube, which will be wholly illuminated by it.

X. The chain being formed, if one or two or more persons form a new one, connected on one side with those who form the first chain, and on the other side with another person of the same chain, those of the latter will experience nothing; the electric fluid seems to proceed from one end to the other of the first chain, by the shortest route.

XI. Some beautiful effects of colour may be obtained by passing the shock through different sorts of wood. And if the shock be passed through a lump of sugar, or over a piece of chalk, they will, in the dark, exhibit a sort of phosphoric illumination for some time.

EXPERIMENT XIII.

Another method of giving the shock, namely, by an electric pane of glass.—To pierce a quire of paper by the electric spark.

Does the singular effect observed in the preceding experiment depend on the figure of the Leyden jar, or merely on the nature of the glass? This question, which naturally occurs, is answered by the following experiment: it proves that the effect alluded to depends entirely on the nature of the glass.

Take a pane of glass of any dimensions, and cover both sides of it with tin-foil, leaving on each side a border of the glass uncovered; place the glass horizontally on a non-electric supporter, and make the chain of the conductor to fall on its surface. If you then put the machine in motion, the glass will become charged like the Leyden flask; that is, if resting one side of the discharger on the upper surface, the other be applied to the lower one, you will extract a strong and powerful spark. If the glass be large, it will be dangerous to touch one of the surfaces with the one hand, and the other with the other.

If you are desirous to pierce a quire of paper with the electric spark, you must proceed as follows. Extend a piece of iron wire on a table, and place over it the pane of glass, so that the end of the wire shall touch the coating of the lower side: on the upper coating place a quire of paper; then electrify the upper surface by means of the chain of the conductor, which must be made to fall upon it. When

you think the electricity is very strong, bring one of the ends of the discharger into contact with the wire, and apply the other end to the paper. A very strong spark will issue from it, with a noise like the report of a pistol; and the quire of paper will be pierced through and through.

If the experiment be made with a piece of glass of about 35 inches square, 150 sheets of paper, and even more, may thus be pierced.

A very remarkable circumstance will be noticed in making this experiment. There will not be an indentation on one side of the paper, and a protrusion on the other, but a protrusion on both sides; as if the electricity had passed out of the paper on both sides.

This method of performing the Leyden experiment is attended with the advantage of greatly increasing its effect; for the surface of the largest flask can contain no more than two or three square feet. But a plate of glass, 36 inches in every direction, contains 9 square feet, and the effect is thereby increased in the same proportion.

It may be easily seen, that, in performing this experiment, you must take care not to stand in the circle between the upper and lower surface, otherwise you might run the danger of being killed.

EXPERIMENT XIV.

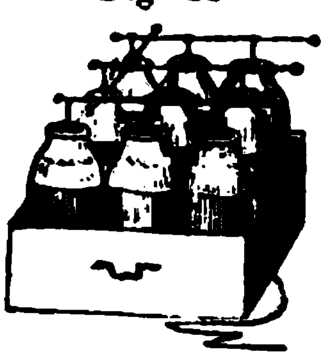
Means of increasing, as it were indefinitely, the force of electricity.—The Electric Battery.

A single flask, charged with electricity, does not produce a very great effect: but this effect is increased according as the volume of the jar is augmented. It would however be inconvenient, and perhaps impossible, to obtain jars beyond a certain size; for this reason several jars have been substituted in the room of one; but the united effect of them is exceedingly dangerous, unless great precautions are employed.

For this purpose, instead of long-necked flasks, several large cylindric jars, of a much greater length than breadth, must be employed. It is not necessary however that their diameter should be very great, because cylinders of a small diameter have, in proportion to their solidity, a greater surface, and surface is what is required to be increased. They are lined on the inside with a coating of tin-foil, which covers the bottom and sides to within two inches of the brim. And they are coated, in the same manner, on the outside: after which, they are arranged close to each other in a box, lined also in the inside with tin foil and copper filings. The tin-foil communicates with a wire ring projecting from each jar, and to these rings is attached the chain, by means of which a communication is established between any body and the exterior part of the battery.

To establish a communication with the inside of the jars, a piece of iron wire, twisted at the lower end, and terminating at the upper end in a ring, must be made to pass through the cork stopper of each jar, so as to descend to the bottom of it. A metal rod, having a ball at each end, passes through all the rings of the same row: and, to establish a communication between the rods, the chain of the conductor is made to rest on them; in consequence of this arrangement, you may charge, if necessary, only one or two rows of the jars, by making the chain to rest on one rod only, or on two, &c.

Fig. 48.



Such is the construction of an electric battery, the representation of which, supposing it to consist of only nine jars, each 15 inches in height, 3 inches in diameter, and containing 12 inches of coated surface, which gives altogether 6¾ square feet, is seen Fig. 48. A similar battery of 64 jars would give 48 square feet, and yet form only a box of two feet some inches

square, and from 15 to 18 inches in height. The effect of such a battery would be prodigious.

The method of using this apparatus is as follows. To charge the jars, make the chain, which proceeds from the conductor of the machine, to rest on the rods, and turn the glass globe or plate for some time. Experience will shew how many turns of the machine will be necessary for this purpose; if the jars are over-charged, they will discharge themselves with a loud report. When they are in the proper state, if you wish to discharge them, nothing is necessary but to lay hold of the chain, which communicates with the outside by means of the discharging rod, and to bring the end of it into contact with the conductor; a strong spark will be elicited, and the jars will be discharged.

If a person, holding the end of the chain, should touch with his finger, either the conductor of the machine, or one of the rods which touch the inside of the jars, he might be killed in consequence of the terrible shock he would experience. If a flask indeed, 5 or 6 inches in diameter, strongly charged, give by its discharge a violent shock in the arm and breast, we may form some notion of the effect which would be produced by the discharge of 12, 15, 20, 30 or 50 square feet. Electricians therefore ought to be very attentive to themselves, and to the spectators, for fear of some fatal accident.

All philosophers, who perform electrical experiments on a large scale, have at present a similar apparatus, of greater or less size. It is by these means that they fuse metals, which can be reduced even to a calx; that they communicate the magnetic virtue to a needle, or change its poles, or imitate the effects of thunder, and so on, as will be seen hereafter.

EXPERIMENT XV.

To kill an animal by means of electricity.

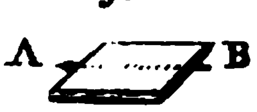
Affix the chain, which communicates with the outside of the battery, to one of the animal's feet; and then with the discharging rod form a communication between the animal's skull and one of the rods that communicate with the inside: the animal, if it be even a sheep or perhaps an ox, will be struck dead.

Remark.—It has been observed that the flesh of animals, killed in this manner, is immediately fit for being eaten: for the shock which kills them is similar to that of lightning, and it is a well known fact that animals killed by lightning pass very speedily to a state of putrefaction. This artificial lightning might therefore be employed to kill those animals which are intended to be immediately used as food: they will be what is called mortified in a minute. But as the operation is dangerous, Dr. Franklin humorously advises philosophers to be on their guard, lest in attempting to mortify a pullet they should mortify their own flesh.

EXPERIMENT XVI.

Production of Magnetism by Electricity.

Fig. 49.



Provide a steel needle, like that of a compass, some inches in length, and place it between two plates of glass, so that its two ends, A and B, shall project a little beyond them. Then make one of its ends A communicate with the conductor of the electric machine, or any one of the transverse rods of the electric battery; charge the battery strongly, and discharge it through the needle by means of the discharging rod, bringing the end of the chain, which communicates with the outside of the jars, to the end B of the needle. All the electric fluid will pass through the needle, entering by the end A, and issuing at B; and the needle will then be magnetized in such a manner that the end A will turn to the north.

When a needle has been magnetised, if the end A be turned towards the north,

performing a contrary operation, that is to say, making the electric fluid pass from B to A, the needle will be un-magnetised; and by repeating the same operation it will be magnetised in a contrary sense; that is, in such a manner that the end B will turn to the north.

It may be readily conceived, that this will depend on the quantity of the electric fluid. If it be less in the second operation than in the first, some small portion of magnetism may remain; if it be much more considerable, the poles may be changed by the first shock.

EXPERIMENT XVII.

To Fuse Metals by means of Electricity.

This experiment is one of the most curious that are performed by electricity. Take an iron wire half a line in diameter, and suspend from it a weight of about 6 pounds; then by means of a battery, consisting of from 16 to 25 jars, make the electric fluid pass along it; the wire will immediately stretch itself, and sometimes will break. But this could not be the case, were it not softened or mollified in some part of its extent.

Another Method.—Take a piece of very thin gold leaf, and having cut it into a slip of two inches in length, and a line in breadth, put it between two plates of glass, very close to each other; then place these plates in such a manner as to form part of the electric circle of a strong battery, consisting of 50 or 60 jars. The gold leaf will pass through the state of fusion; and what proves it is, that several of its parts will be incorporated with the glass itself.*

But if you place between the glasses and the gold leaf two bits of card, and squeeze the plates of glass closely together, the electric spark made to pass, as above, through the gold leaf in the direction of its length, will reduce it, in a great measure, into that kind of purple powder, known in chemistry under the name of the *precipitate of Cassius;* because this preparation was first invented or simplified by that chemist. The two cards will be dyed of the same colour, which may be heightened by repeating the operation with new bits of gold leaf.

Silver leaf treated in the same manner gives a powder of a beautiful yellow colour.

Copper leaf gives a green powder.

Tin-foil gives a white powder.

Platina, treated in this manner, is reduced, after repeated shocks, to a blackish powder, which when applied to porcelain produces a dark olive colour.

In short, we are assured, by different chemical proofs, that these calces are exactly the same as those produced by longer processes.

For these experiments we are indebted to M. Comus, so celebrated for his industry and address, and who united to the most extraordinary talents in this way the most profound knowledge in different parts of philosophy. A circumstantial and truly interesting detail of them may be seen in the "Journal de Physique" for the year 1773.

EXPERIMENT XVIII.

Which proves the identity of Lightning and the Electrical Spark.

On a high insulated place, such as the summit of a tower, fix in a vertical direction an iron rod, terminating in a very sharp point. The higher this point is in the atmosphere, the better will the experiment succeed. This bar must be supported by some base to insulate it from every body capable of conducting electricity.

* We have seen this effected by a battery formed of four phial glasses, each about two inches in diameter, and eight inches high.

Then wait till a storm takes place, and when a thunder cloud passes over the rod, or near it, touch it with an iron rod attached to a glass handle. You will not fail to extract sparks from it, sometimes very large, and accompanied with a loud noise. It will be dangerous however to approach too near it; for the rod is sometimes so highly charged with electricity, that the sparks proceed to the distance of some feet, and make a noise like the report of a pistol. Mr. Richman, professor of mathematics at Petersburgh, and member of the Imperial Academy of that city, fell a victim, as is well known, to an experiment of this kind; for in a moment of forgetfulness, having approached too near the machine, he was struck dead, and all those effects observed in persons killed by lightning were seen on his body.

This accident has induced some philosophers, who study electricity, to arrange their machine in such a manner, that it can never become too much charged with electricity. For this purpose, they place at some distance from the rod a piece of sharp pointed metal, which communicates with the floor or the mass of non-electric bodies. This point, when the electricity is moderate, will attract none of it; but when very strong, it will draw it off, as we may say, and discharge it insensibly; so that it will accumulate only in such a moderate quantity as to be incapable of doing michief. The nearer the point is to the rod, the more it will absorb of the electricity.

By its means it may be known in obscurity whether the cloud be electrified positively or negatively; for in the first case you will observe at the point a simple luminous star, or very short gerbe; in the second, you will observe a large and beautiful gerbe.

It is customary also to place near the bar a metal ball suspended by a silk tnread; and, at a little farther distance, a bell commuuicating with the body of the building. The use of this apparatus, is to inform the observer that the bar is electric; for at the moment when it is charged with electricity, it attracts the ball which possesses none, electrifies it, and propels it against the bell; the sound of which announces that the electric cloud has produced its effect. The degree of the electricity also is indicated by the same means; for if it be very strong, the vivacity of the ringing is proportioned to it, and the observer is warned to be on his guard.

This experiment, without either a tower or a terrace, may be performed in a chamber. Nothing will be necessary but to place in the chimney a bar of iron, insulated by means of silk strings, which keep it firm on every side. The point of this bar must rise some feet above the top of the chimney: 12 or 15 feet, and even less will be sufficient. Every time then that an electric cloud passes over the chimney, the bar will emit signs of electricity, if touched with caution, or by means of a few electric bells arranged near it.

Instead of this apparatus, Father Cotte, an assiduous observer of all meteorological phenomena, places in a transverse direction, between two elevated places, an iron chain, the links of which are furnished with sharp points. The two ends of the chain are supported by silk strings, covered with pitch. From the middle of the chain proceeds another, of the same form and size as those used for electric experiments, which is conveyed into the apartment, either through the chimney or the window, by means of silk strings which support it. At the end it ought to be furnished with a metal ball, which will produce sparks much more considerable than the chain itself would do. The multitude of points, with which the chain is covered, furnish such a quantity of electric matter, that the ball must not be touched without great circumspection.

Remark.—This curious experiment, highly interesting to philosophy, was proposed and announced by the celebrated Dr. Franklin, in letters addressed to Mr. Collinson, fellow of the Royal Society; but it was performed, for the first time, at Marly, by M. Dalibart and M. Raulet the Curé of that place, on the 10th of May 1752. It was

afterwards exhibited before the king and the whole court. Since that time it has been repeated by various philosophers, and at present nothing is more common than this electrical apparatus, which shews the identity of the electric fluid and lightning. But it is to America, and to Dr. Franklin in particular, that we are originally indebted for it.

From this discovery we can deduce the explanation of several phenomena; for which no proper cause had been before assigned. Of this kind are those fires often observed during storms, on the top of steeples, at the extremity of the masts and yards of ships, which the ancients distinguished by the names of Castor and Pollux, and which are known to the moderns under that of the fire of St. Elmo. It is nothing else than the electric fluid of the clouds attracted by the points of these steeples, or the iron at the summits of the masts. Cæsar relates, that a great storm having come on while his army was arranged in the order of battle, flames were seen to issue from the points of the soldiers' pikes. This phenomenon has nothing wonderful in it to those acquainted with electricity. The flames observed were the electric fluid, which escaped from these points; the clouds, in all probability, being electrified negatively, which, according to Dr. Franklin, is often the case.

EXPERIMENT XIX.

Which proves the same fact in another manner.—The Electric Kite.

It is sometimes difficult, if not impracticable, to raise an iron rod to a great height; and therefore another artifice has been devised to deprive the clouds, in some measure, of their electric fluid or lightning. It is by means of the paper kite, a small machine more employed before that time by young persons and school-boys than by philosophers; but the use made of it by some of the latter has in some measure ennobled it.

Provide a kite, covered with silk, of a pretty large size, such as 5 or 6 feet in length at least; for the larger it is the higher it will rise, on account of the weight of the cord being less, in comparison of the force with which the wind tends to elevate it. Adapt to the head of it, a very delicate rod of iron, extending on the one side, along the lower axis of the kite, to the point where the cord is affixed to it, and, on the other, terminating in a sharp point projecting beyond the kite; so that when the machine is at its greatest height, it may be nearly vertical, and rise above it about a foot. The cord may be of common pack-thread, with a very flexible copper-wire twisted round it, nearly in the same manner as on the lower strings of some musical instruments, but much closer. This is done, because hemp, unless moistened, is a bad conductor of electricity. To the extremity of this rod is attached another of silk, some feet in length, to insulate the kite, when it has reached its greatest height; and near this silk string is connected with the cord of the kite a small tube of tin-plate, about a foot in length and an inch in diameter, for the purpose of drawing sparks from it.

When these arrangements are made, expose the kite to the wind, when you observe a storm approaching, and suffer it to rise to its greatest height. If the silk string be then made fast to some fixed object, and in such a manner that the string shall not be moistened by the rain, you will not fail to observe very often exceedingly strong signs of electricity, and sometimes so powerful that it would be dangerous to touch the string or tube without the utmost caution.

For this purpose, affix to the end of a glass tube, or a large stick of Spanish wax of about a foot long, a piece of iron some inches in length, having a small metal chain hanging down from it to the earth. Without this precaution, weak sparks only would be elicited, because the piece of iron, being itself insulated, would on the first touch be electrified like the cord of the kite.

M. de Romas, the first person in Europe who employed this method of drawing electricity from the clouds, caused a kite 7½ feet in length, and 3 in breadth, at its

widest diameter, to rise to the perpendicular height of 550 feet, and produced by it the most extraordinary effects. Having imprudently touched the tube of tin-plate with his finger, he received a violent shock; but happily the electricity had not nearly acquired its utmost strength; for the storm increasing, some time after he felt, at the distance of more than three feet from the cord, an impression similar to that made by a spider's web; he then touched the tube of tin-plate with the discharging-rod, and extracted a spark of an inch in length and three lines in diameter. The electricity then increasing in a very great degree, at the distance of more than a foot, he extracted a spark three inches in length and three lines in diameter, the snapping of which was heard at the distance of 200 paces.

But what was most remarkable in this experiment is, that while the electricity was nearly at its highest degree, three straws, one of them a foot in length, stood upright in consequence of the attraction of the tin-plate tube, and balanced themselves for some time between it and the earth, always turning round, till one of them at length rose to the tube, and produced an explosion in three claps which were heard in the middle of the town of Nerac, the experiment having been performed in the suburbs. The spark which accompanied this explosion, was seen by the spectators like a spindle of fire, 8 inches in length, and 4 or 5 lines in diameter. The straw which had occasioned this spark at last moved along the string of the kite, sometimes receding from it, and sometimes approaching it, and producing a very loud snapping when it came near it. Some of the spectators followed it with their eyes, to the distance of more than 50 toises.

Farther details respecting this experiment, no less interesting than curious, may be seen in the "Mémoires des Sçavans Etrangers," published by the Royal Academy of Sciences, vol. ii. It was followed by a great many others of the same philosopher, which prove that, even during calm weather, a kite of this kind is sometimes so highly electrified, as to make the cord to sparkle, and to give violent shocks to those who inadvertently touch it.

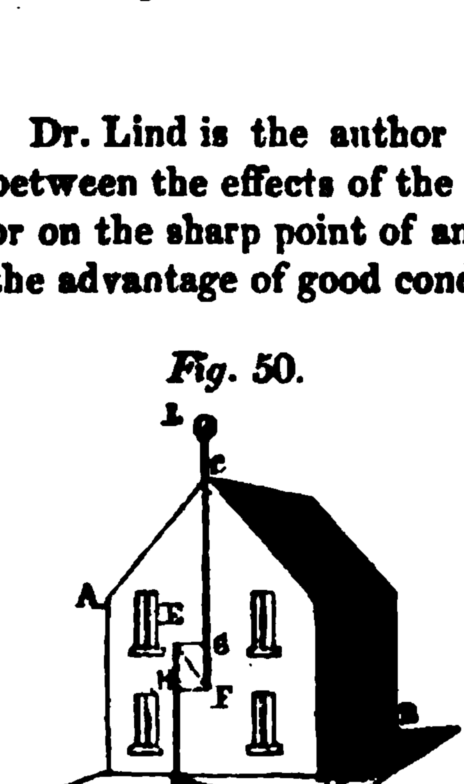
We have already said that M. de Romas was the first person in Europe who made this curious experiment; but it had been made some months before in Pennsylvania by Dr. Franklin; for he sent an account of it to Mr. Collinson, his correspondent at London, in the month of October, 1752. This discovery however was not known in France till a long time after; and M. de Romas even announced it enigmatically to the Academy of Sciences, about the middle of the same year. Thus, while we adjudge the first merit of the invention to Dr. Franklin, we cannot help acknowledging that M. de Romas concurred in this respect with the celebrated philosopher of Philadelphia.

EXPERIMENT XX.

The House struck by Lightning.

Dr. Lind is the author of this experiment, which serves to prove the difference between the effects of the explosion of thunder, when received on a blunt end or ball, or on the sharp point of an uninterrupted conductor. It displays in the fullest light the advantage of good conductors for preserving houses from lightning.

Fig. 50.



A B (Fig. 50.) is the model of a small house, of which C is the summit of the roof; A D is a wall in which is formed the square hole, G F H E, destined to receive a square board; in this board is placed diagonally an iron rod, which according to the position of the board can be disposed in the direction F E or G H, as seen in the figure. L G is an iron rod terminating in a ball L, which ends at the point G. From H to I is another rod of the same kind, the end of which I terminates in a chain of a length proper for the purpose intended.

When this arrangement has been made, place the board

in such a manner that the rod, sunk in it, may be in the direction F E; leaving an interruption from G to H. Then make the chain pass round the body of a jar, like those used for an electric battery, and charge the jar as highly as possible. Then affix to one of the legs of the discharging rod, furnished with a glass handle, the chain of the conductor, and with the other leg of the rod touch the ball L, which rises above the roof of the house, and the rod G C. An electric circle being thus formed, a strong explosion will ensue, and the board F G H E will be thrown from its place on account of the jump which the electric matter must make from G to H, to reach the conductor, which is interrupted in that place.

But, instead of a rod terminating in a ball L, substitute a rod ending in a sharp point, and place the board F G H E in such a manner, that the small rod E F may be in the direction G H; if you then repeat the same operation as before, the electricity will silently pass along the rod L G H I, without displacing any thing.

This is an exact representation of what takes place when a building is struck by lightning. The top of the building receives the shock, and the lightning follows the first metallic conductor it finds, without doing it any damage, provided it be of a sufficient size; but if this conductor be any where interrupted, it makes an explosion, and blows to pieces the walls, the wainscoting, &c., till it finds a new conductor. At every interruption a new explosion takes place, to the great danger of those who are in the neighbourhood; for as the body of a man is a pretty good conductor of electricity, on account of the fluids with which it abounds, it attacks him in preference, and infallibly destroys him.

But if the rod, elevated above the house, terminates in a sharp point, and if the conductor is not interrupted, nothing of this kind will take place. There may be some slight explosion at the point of the bar; but the electric fluid or lightning thence follows the conductor to its extremity, which is sunk in the earth to a depth sufficient to reach moisture.

M. Sigaud de la Fond, professor of natural philosophy, rendered this experiment still more sensible, by the disposition he gave to the small house. It was such that the electric explosion blew up the roof, and separated the walls.

EXPERIMENT XXI.

The Ship struck by or preserved from Lightning.

This experiment, in some respects, is merely a variation of the preceding. We have introduced it, however, because it is no less amusing, and is equally calculated to prove the utility of uninterrupted metallic conductors, for preventing damage by lightning.

In the middle of a small boat, representing the hull of a vessel, raise a tube about eight inches in height, and half an inch in diameter, to represent the main-mast. Fill this tube with water; and, having closed both its extremities with two pieces of cork, introduce through them two pieces of iron wire, so that the ends of them shall be at the distance of half an inch from each other, in the inside. The lower piece of wire must be immersed in the water in which the vessel floats, and the upper one must terminate, without the tube, in a small knob.

Now, if a communication be established between the exterior surface of an electric battery and the lower wire, and if the end of the iron chain, which is connected with the inside of the battery, be applied to the end of the upper wire, the explosion of the electric matter, passing from the end of one wire to that of the other in the tube, will be such, even if a small part only of the chain be employed, that it will shatter the tube to pieces; and the bottom of the vessel being pierced, it will sink. Such is the manner nearly in which the main-mast of a ship is shivered by lightning, so that the vessel is in danger of being lost.

But if instead of two wires, one only be made to pass through the two pieces of cork and the water with which the tube is filled, and if the same communication be established with the electric battery, the charge of sixty-four jars may be transmitted through the tube, without doing it any injury. Sometimes however the force of the electric matter, or of this small flash of artificial lightning, will be so great, as to destroy the metallic wire.

This experiment was invented by Mr. Edward Nairne, and might easily be adapted to represent, in a manner more agreeable to reality, the phenomena of a vessel struck by lightning; but we have chosen rather to describe it as given in the Philosophical Transactions for the year 1773. It clearly shews how dangerous the interruption of metallic conductors is, and how the smallest conductor, if properly continued, will carry off the electric fluid.

GENERAL REMARK.

On the analogy between lightning and the electric fluid.—The means of securing houses from the effects of lightning.

Though the identity of lightning and electricity is sufficiently proved by the preceding experiments; to establish it more completely, we shall mention some of the phenomena most commonly observed in the progress of lightning, when it falls on a house or any other object.

The first of these phenomena, or what takes place most frequently, is, that the lightning runs along metallic bodies, wherever it meets with them in its way. For want of metallic bodies, it explodes, or attaches itself to moist bodies, or to animals, which are composed almost entirely of fluids. Hence it is often observed, when the lightning falls on steeples, that from the weather-cock or cross on the summit, which receives the first shock, it runs along the iron work, proceeding thence to the roof or to the inside of the building, and there explodes; for as it no longer meets with any thing besides wood or stone, which are bad conductors, it cannot conveniently pursue its course; it therefore often strikes men who are in the steeple, in consequence of a bad custom which prevails of ringing the bells on such occasions. Sometimes it falls on the bell, and follows the rope to its extremity; and if the rope at that moment is held by a man, he seldom escapes destruction; for being a better conductor than hemp, the lightning seems to give him a fatal preference.

It very often happens that the lightning melts the lead of the cross, which it strikes rather than other bodies, which are worse conductors.

We may thence explain also, why it has happened that a man with a sword by his side has been struck by lightning, without sustaining any hurt, and that the point of the sword had been found melted in the scabbard: it is because the electric fluid preferred passing through the sword, entering at the hilt, and issuing at the extremity; and as this extremity terminates in a sharp point, it found it more compact, and reduced it to a state of fusion. This effect may be imitated, by causing a large quantity of the electric matter to pass through a sharp-pointed wire.

When lightning falls on a tree, if there be any animals beneath, they rarely escape, especially if the tree be of a resinous or oily nature. The reason of this is, that wood is a bad conductor; the lightning therefore abandons it if there be a better conductor, such as an animal, in the neighbourhood. Hence it happens that the walnut tree is reckoned to be particularly dangerous: its oily sap renders it a worse conductor of electricity than any other.

But it is when lightning falls on a house that its predilection for metallic bodies principally appears. Almost all the accounts of the effects of lightning agree, in representing the electric matter as preferring to run along the wires fixed to the bells, or the metallic edges of cornices, or looking-classes, or pictures, &c., exploding

every time that this route, which it finds most commodious, is interrupted. It has been seen to pass in this manner, through several apartments, and even through several stories. This route indeed is so well established by the observations which have been made, that there is every reason to believe, that if these metallic conductors had been wanting, or had been insufficient, it would have occasioned great damage.

One of the best related, and most remarkable accounts of such events, is that of the lightning which struck the hotel occupied by Lord Tilney at Naples, on the 20th of March 1773. We are indebted for it to Sir William Hamilton, who was present in the apartment, through which the lightning passed, together with M. de Saussure, professor of Natural history of Geneva, and they both soon after examined the whole hotel with the utmost care, in order to trace out the progress of the meteor. The circumstances of this event were as follow.

The apartments of Lord Tilney, which consisted of nine rooms on a floor, were decorated with great elegance, like most of those belonging to persons of rank in that country. A very large cornice went round all the rooms, and this cornice was gilt in the Italian manner; that is to say coated with tin-foil, covered by a yellow varnish, in imitation of gold. From this cornice proceeded a great number of platbands, which served as frames to the tapestry, and which were gilt in the same manner, as well as the borders of the panels of the wainscoting, the frames of the pictures, and the mirrors, the door-posts, &c. The apartments above were ornamented in the same style. This hotel had a profusion of such ornaments, and it is to be observed that all the rooms communicated with each other by means of the bell-wires, which, for the sake of convenience, were very numerous.

Lord Tilney had a party to dinner, and Sir William Hamilton says, that on this occasion there were in the hotel upwards of 500 persons, including the domestics. A loud clap of thunder was heard, and in an instant the whole apartment, where the company were assembled, seemed to be on fire. Every one thought himself struck by the lightning, and the terror and confusion which this circumstance produced may be easily conceived. No person however was either killed or wounded; and this no doubt was owing to the prodigious quantity of metal conductors, which enabled the lightning to pass through them.

Sir William Hamilton and M. de Saussure, having examined, soon after, and next day, the different apartments, observed that the greater part of the extensive cornices were damaged, and black in a number of places, particularly at the corners, and where the bell-wires passed through them; the gilt varnish was detached in many parts, and thrown down in the form of powder; in some other places the cords of the bells were burnt. In one room where two paintings were suspended, one above the other, between the cornice and the door, the lightning had passed from the cornice to the gilding of the frame of the picture immediately below; then to that of the second picture, and thence to the frame of the door, and its course was marked on the wall, which was whitened according to the custom of the country, by the impressions of the smoke. In another room, the lightning had also passed from the cornice to the frame of a picture, which was in contact with it, and thence to the interior border of the frame of the door, making an explosion each time; it had then descended along the frame of the door, and had split part of a small socle, at which the mouldings terminated. The same phenomena nearly had taken place in the upper story.

It may be seen, by this description, that the lightning had preferred passing through all these metallic materials; and there can be no doubt that it was owing to the great profusion of gilding, and to the wires of the bells, that some of the persons present escaped being killed.

The predilection which the electric matter, or lightning, shews for metallic con-

ductors, induced Dr. Franklin, about the year 1752, to propose, at Philadelphia, a new method of preserving edifices from this destructive meteor. It consists in placing on the tops of the houses an iron rod, terminating in a point, and continued downwards by several more rods joined together. The lowest rod ought to be sunk in the earth to a sufficient depth to meet with moisture, which, being a good conductor, will convey off the electric matter, by transmitting it to the whole mass in the earth. In regard to the size of the rod, Dr. Franklin observes that three or four lines in diameter will be sufficient.

In the year 1755, a great many houses in North America, and particularly in Pennsylvania, Maryland, and Virginia, where thunder is very common, and often falls on buildings, were furnished with conductors of this kind. It is allowed that several of these houses were struck by lightning; but the circumstances always observed in America, were as follow: 1st, that the damage done to these houses was less. 2d, that when they were struck, the lightning, instead of occasioning the same havoc as in others, passed off by the conductors, leaving only a slight impression in the neighbouring parts. In these cases the point of the conductor was, for the most part, found to have been fused.

The object of these rods is not indeed, as was at first supposed in Europe, to deprive an immense cloud of its electricity, but to furnish a conductor to that electric matter, when by an accident, which cannot always be avoided, a cloud highly charged with electricity has fallen on an edifice.

This expedient, however, found powerful opponents, especially in France. One of the most conspicuous was the celebrated abbé Nollet, the rival of Dr. Franklin in the theory of electricity; but it must be allowed that nothing could be weaker than the arms with which the French philosopher combated the American. They were mere assertions, unsupported by any proofs, or rather contrary to the result of experiments. According to Nollet, these pointed rods of iron are more calculated to attract the lightning than to preserve from it; and it is not a rational project for a philosopher, says he, to exhaust a stormy cloud of the electric matter it contains. To answer these assertions, it is sufficient to be acquainted with the effects of lightning. They prove, in the most evident manner, that if the places where it fell had been furnished with good conductors, no explosion would have taken place. Besides, it is not true that a sharp-pointed rod attracts the electricity of a storm-cloud; on the contrary, if a sharp-point be presented to a flake of cotton, suspended from the conductor of the electric machine, it immediately repels it. Is it therefore better to wait till a storm-cloud, charged with electricity, and driven by the wind against a building, shall explode and pour into it a torrent of the electric fluid, than to draw it off by degrees, so that when it approaches the edifice, it shall be entirely deprived of it? In regard to the impossibility of freeing a cloud entirely from its electric matter, it is needless to make many observations; as all that is meant is merely to supply the electric matter, poured forth from a storm cloud, with the easy means of escaping.

But, when it is considered, that every time almost, that lightning has fallen in any place, without doing damage, it has followed conductors as small as a bell-wire, or gilding., &c, and that it has never exploded but when its course was interrupted, there can be no doubt that a rod, half an inch or an inch in diameter, would afford a passage to all the electric fluid that could be produced by the largest cloud.

Sharp-pointed conductors, considered as preservatives against the effects of lightning, were opposed in England by the noted electrician Wilson, on the following occasion. The method proposed by Dr. Franklin, for preventing the effects of lightning, having excited the attention of the government in 1772, the Royal Society of London were consulted on the means of securing from this destructive agent the new powder magazines at Purfleet. The Society having appointed Mr. Cavendish, Dr. Watson, Dr. Franklin, Mr. Wilson, and Mr. Robertson to examine this subject;

four of these gentlemen were of opinion, that the magazines should be furnished with sharp-pointed conductors. Mr. Wilson alone maintained that the points of the conductors ought to be blunt, and he refused to sign the report. It may be easily seen that Mr. Wilson's motive was an apprehension that sharp-pointed conductors might attract the electric fluid at too great a distance. Dr. Franklin, in a paper written on purpose, which contains an account of new and ingenious experiments, endeavoured to make him change his opinion; but did not succeed. The magazines of Purfleet however were furnished with conductors according to the idea of Dr. Franklin and the other three commissioners.

The sequel of this business gave rise to the most extraordinary transactions that ever occurred in the Royal Society.

EXPERIMENT XXII.

Of some amusements founded on electric Repulsion and Attraction.—The Electric Spider, &c.

Cut a small bit of cork, or of the pith of the elder tree, into the form of a spider, and fix to it six or eight cotton or linen threads, a few lines in length. Suspend this small figure from a hook by a silk thread, and place on one side of it, and at the same height, the knob of a small jar positively charged, and on the other that of a jar negatively charged, or merely a similar knob not electrified, and communicating with the general mass of non-electric bodies. You will then see this figure first attracted towards the electrified knob, and then repelled by it: and as the filaments of the threads will mutually repel each other, the spider will appear as if at work, and extending its legs to lay hold of the second knob. But it will have no sooner touched it, than it will seem to fly from it; for when deprived of its electricity by the touch, it will be attracted by the first knob, from which it will be afterwards repelled; and this play will continue as long as there is any electricity in the jar.

A common conductor charged with electricity will supply the place of the electrified jar; and, instead of the electrified knob, the finger may be employed. The spider, after having touched the conductor, will appear to throw itself on the finger, as if to seize it, and will embrace it with its legs.

EXPERIMENT XXIII.

The Electric Wheel and Turnspit.

Construct a wheel consisting of eight or ten glass radii, implanted in a common centre, about six or eight inches in length, and each furnished at the extremity with a ball of lead.

This wheel must be placed, in perfect equilibrium, on a small vertical axis, which turns in a piece of glass, so that the slightest impulse can put it in motion. The stand, by which it is supported, must be susceptible of being insulated.

Then provide two jars, one charged positively and the other negatively; and, having insulated the above wheel, place the two jars one on each side of it, so that the balls shall pass at the distance of a quarter of an inch from the knob of each jar.

It may be easily conceived, that if this small machine be in perfect equilibrium when one of the balls approaches one of the knobs, that for example which belongs to the flask charged positively, it will be attracted by it, and the machine will tend to turn round; but the ball, by passing near to that knob, will be electrified positively, and consequently will be immediately repelled.

The same thing will take place in regard to the flask which is charged negatively: the non-electrified ball will be attracted by it, and in passing near it will be electrified negatively; it will therefore be repelled, as soon as it has passed it.

As the same thing takes place in regard to all the other balls, the result will be

a circular motion, which will be accelerated more and more, and will continue as long as the two jars are in a state of electricity. But they may be easily kept in motion, by making the knob of a jar strongly charged touch that of one of them, and the knob of the other the side of the same jar: by these means the one will be charged positively, and the other negatively.

When the electricity is very strong, and the machine is well constructed and in equilibrio, it acquires a motion capable of turning a weight of several pounds placed on its vertical axis.

The electricians of Philadelphia employed this apparatus to turn a spit, when a party of them met to amuse themselves with philosophical experiments. Being persuaded, no doubt, that Reason must sometimes throw itself into the arms of Folly, they assembled on the banks of the Skuylkill, a river which runs past Philadelphia, and having killed a turkey, by the electric shock, they placed it on a spit adapted to an electrical jack, and roasted it at a fire kindled by the electric spark; they then drank to the health of the European and American philosophers who cultivated electricity, not amidst the noise of musketry, but of electric batteries discharged at each toast. Dr. Franklin, the first of the philosophical electricians, calls this an *electrical feast*.

EXPERIMENT XXIV.

The Electric Alarum and Electric Harpsichord.

Suspend from the conductor of an electric machine, three bells, at the distance of about an inch from each other; but the outer ones must be suspended by threads which transmit the electric fluid, and that in the middle by a silk thread, or other electric subtance. The bell in the middle must communicate, at the same time, with the floor, by means of a small chain or metallic wire.

At equal distances, between each of these three bells, suspend by silk threads two small balls of metal, in such a manner, that when pulled a little to the right or left, they shall strike against the bells.

If the conductor be now electrified, you will immediately see these small clappers put themselves in motion, and strike the bells alternately, which will form a small alarum; and if the electric apparatus be concealed it will be difficult for those present to discover the cause of it.

The cause however of this continued play may be easily discovered; for, by the construction of the apparatus, the two lateral or outer bells are electrified, as soon as the electric machine is put in motion. The small balls suspended between them and that in the middle will therefore be attracted by these bells; but as soon as they touch them they will be repelled, being electrified in the same manner as they are; they will then be carried towards the middle bell, which having a communication with the floor, will immediately deprive them of their electricity. They must therefore fall back towards the electrified bells, which will attract them again; and this play will continue as long as the electric machine is kept in action.

Remark.—On this principal, an instrument called the *electrical harpsichord* has been invented. The following is a short account of this ingenious machine, for which we are indebted to Father de la Borde, a Jesuit, who gave a description of it in a small work published in 1759.

Suppose an iron rod, supported by silk strings, and furnished with two rows of bells, each two of which are capable of emitting the same sound, because two will be required for each tone. One of these bells must be suspended from the rod by a wire, so that when it is electrified the bell may be electrified also. The other must be suspended by a silk string, and between each pair of bells a small ball of steel must be suspended by the same means.

The bell suspended from the bar at the top, by the silk string, is furnished with a

wire which proceeds downwards, and is fixed by another silk string. To its lower extremity is fastened a small lever, which in its usual position rests on another insulated bar, communicating as well as the other with the conductor of an electrical machine.

In the last place, below this second bar, is a harpsichord so disposed, that when one of its keys is touched, the other extremity of it raises up the corresponding lever; this intercepts the communication of the bell with the electrified conductor, and establishes a communication with the general mass of the earth.

After this description, it may be easily conceived, that if one of the keys be touched, while the electric machine is in motion, one of the bells being electrified, the steel ball will immediately advance towards the other, and being electrified by it, will be repelled towards the first, which will deprive it of its electricity, and therefore it will return to the other. This motion will take place indeed with great rapidity, and the result will be an undulating sound resembling the vibrations of an organ. When the lever falls down, the two bells are equally electrified, and in a moment the steel ball stops.

Father de la Borde, having constructed this machine, could by practice perform on it, with a considerable degree of correctness, simple airs. But was it of sufficient importance to be made the subject of a particular treatise, since neither the science of music nor the theory of electricity could be much benefited by it?

EXPERIMENT XXV.

The Electrical Horses which pursue each other; or the Electrical Horse-race.

With two small iron plates, or two small wires, construct a sort of cross, having a piece of copper in the centre, so as to represent two magnetic needles crossing each other at right angles. The ends of these four branches must terminate in a point, and about an inch of the extremities of them, more or less according to the size of the machine, must be bent back so as to form somewhat less than a right angle. On these bent ends fix a small bit of light card, and place on each the figures of horses having their tails turned toward the points. Then arrange the whole on a steel pivot, raised in a perpendicular direction, that the cross with its load may preserve itself in a horizontal direction, and have a very easy rotary motion.

Having then insulated the machine and its plate, if the latter, or the point of steel, be made to communicate with the electrified conductor, you will soon see these four branches of iron assume, as if spontaneously, a rotary motion in a direction contrary to that in which their extremities are bent; so that the four horses will seem to pursue each other in a circular course. And this play will continue as long as the electricity lasts, and even longer, on account of the acquired velocity.

If the experiment be made in the dark, and without the horses, that is to say with the four points alone, you will see pencils of light or electric fire issue from them, which will form a very agreeable spectacle, as the result will be a ring of fire; and this ring may be rendered larger by giving unequal lengths to the branches of the cross.

Several stages of wires, placed in the form of a cross, each stage always decreasing in size, might be constructed, and by these means a luminous pyramid would be formed.

The cause of this apparently spontaneous motion may be easily conceived. It is the impulse of the electric matter, issuing from the points, which meeting with the air, experiences a reaction, and is impelled backwards.

Remark.—Some have pretended to deduce from this experiment a pretty strong objection against the hypothesis of Dr. Franklin; for whether this small machine be electrified either positively or negatively, the motion takes place in the same direction, which has astonished those even who are decided Franklinians. To us this objection appears to have little weight; for in our opinion it might be said, that in

the case of negative electricity, the electric fluid which is thrown into the points, cannot be absorbed, without communicating to them an impulse, which acts exactly in the same direction as the repulsion experienced by the electric fluid, on issuing from the points when they are electrified positively.

EXPERIMENT XXVI.

To cause writing in luminous characters to appear suddenly, by the means of electricity.

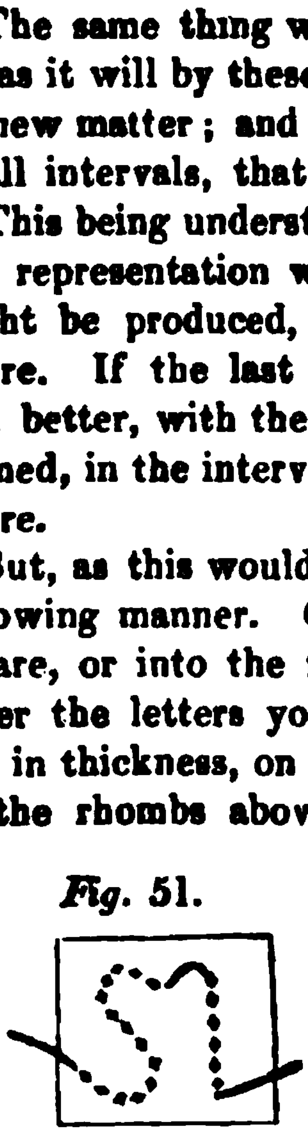
This electric amusement is founded on a well known observation, that if several metallic wires be disposed together in such a manner that their ends, without being in contact, shall approach very near to each other, so as to be at the distance of a line or half a line; if the first be electrified, while the last has a communication with the mass of non-electric bodies. sparks will continually be emitted between the ends of these wires.

The same thing will take place if the last of these wires terminate in a point; for as it will by these means lose its electricity, there must be a continual afflux of new matter; and this cannot be the case without causing a spark in each of the small intervals, that separate the ends of the wires.

This being understood, it may be easily conceived that a series of sparks, forming any representation whatever, with some limitations, which will be seen hereafter, might be produced, by arranging the ends of the wires along the outlines of any figure. If the last of the wires be then touched by the finger, or, what will be still better, with the exterior furniture of the Leyden flask, there will be instantly formed, in the intervals between these wires, sparks representing the contour of the figure.

But, as this would be attended with difficulties, it may be easily executed in the following manner. Cut a leaf of tin-foil into small pieces, of a line or half a line square, or into the form of a rhombus somewhat elongated. Then delineate on paper the letters you intend to represent, and having put a plate of glass, about a line in thickness, on the drawing, cement to the glass the small squares of tin-foil, or the rhombs above described, according to the outlines; taking care that the angles correspond to angles, and that they be at the distance from each other of about half a line, as seen in the delineation of the letter S, (Fig. 51.) Then connect the extremity of one letter with the commencement of the following one, by a small bent metallic plate, terminating on both sides in a sharp point, as seen in the same figure; a small plate of the like kind at the commencement of the first letter, and another from the end of the last, must proceed to the edge of the glass, and beyond it.

Fig. 51.



Let us now suppose that the first of these small plates has a communication with the electric conductor, and that you touch the second, or *vice versâ;* each angle of the small squares will convey the electric fluid by a spark to its neighbour; and if the experiment be performed in the dark, these two letters will be perceived as if delineated by a series of luminous sparks.

If the last plate communicate with a mass of non-electric bodies, and if the electricity be strong, an explosion will take place between each square, which will render this writing luminous. A very brilliant effect is produced by passing an electric *shock* through the circuit proposed as above.

Remark.—But it is to be observed, that all the letters of the alphabet cannot be represented in a manner so simple as the two here given by way of example. Thus the O cannot be represented by this method, as the electric fluid, instead of going

round in a circle, would proceed from the first to the last square. The A also would remain truncated at its upper part, as the electric matter would pass through it. A particular artifice therefore is necessary to obviate this inconvenience, which occurs in regard to a great many other letters, such as E, F, H, &c.

This artifice consists in delineating one half of the letter on one side of the glass, and the other half on the other, and in forming a communication between them by a small metallic band, which, proceeding from the upper to the lower side of the glass, may convey the electric matter from the last square of the first half of the o (Fig. 52.), for example, to the first square of the second half of the same letter, and then uniting, by a similar band, the last square of that second half with the first square of the following letter. If Fig. 52 be carefully examined, the mechanism of this amusement will be easily comprehended. The letters, or parts of letters, represented on the upper side of the glass, must be strongly shaded, and those on the lower lightly. As the propagation of the electric fluid is instantaneous, no inconvenience can arise from this method of transmission.

Fig. 52.

It may be readily seen that such an artifice, in the times of superstition, migh have been employed to terrify the ignorant. If a number of people, assembled in a dark place, should see, after a clap of thunder, a luminous writing on the wall, containing a pretended decision of the Deity, what would they not be capable of doing? and with what terror would a man be struck, who on waking should see written on the glass, *This day thou shalt die?*

EXPERIMENT XXVII.

Electric Fire Works.

We shall here describe a new kind of spectacle. We will not absolutely warrant its success, but we are inclined to think that our idea is susceptible of being carried into execution.

An exhibition of fire-works is generally composed of a fixed decoration, representing an edifice, suited to the subject, and of various moveable pieces of fire; such as rockets, gerbes, cascades, fixed or revolving suns, &c.; and all these pieces, in our opinion, may be represented by merely electric fire.

Let us first assume, by way of example, a decoration of architecture, which is illuminated by a series of lamps, disposed in such a manner as to trace out its principal parts. Now, might not a series of points, rendered luminous by electricity, be substituted instead of these lamps? The preceding experiment will furnish us with the means of accomplishing this end; for since letters, the figures of which are much more complex, may be rendered apparent by a series of similar points, lines, the greater part of which are straight and parallel or perpendicular to each other, may be represented with much more ease, by attending to the directions given in that experiment. The following however is another method.

On a piece of very dry and well planed resinous wood, trace out the design of your decoration, and mark by points the places where lamps would be suspended, were it to be illuminated; then place at each of these points a piece of iron wire, one or two lines in length, and terminating on the outside in a very delicate sharp point; and form a communication between all these wires, by a long piece of wire connected with them. If the electric matter be then excited in a powerful manner, there is no doubt that each of these points will emit in the dark a small luminous gerbe, which will trace out the design of your architectural decoration; for it is well known that a bar of iron, when strongly electrified, throws out in the dark

from all its angles large luminous gerbes, which are sometimes several inches in length.

Nothing is easier than to represent a gerbe of fire: a group of iron wires, terminating in a point, will produce an assemblage of small gerbes, which together will form one of a considerable size.

If a fixed sun is to be represented, it may be done by means of ten or twelve points disposed in the form of radii, at the extremity of a wire which terminates in a button, and if twelve points be arranged in a proper manner, they may form a star by the emanation of the electric fluid: nothing will be necessary but to dispose them in the same manner as the rockets are in common fire works, to represent the same thing.

If several pieces of iron wire, terminating in a point, and having a communication with a common handle, be arranged in a semicircular form, in a direction inclined to the horizon, they will represent a cascade, by the electric gerbes which issue from their points.

If the figure of a revolving sun be required, you must construct a cross similar to that described in the 25th experiment; but instead of making it turn round on a vertical axis, it must be brought into perfect equilibrium on a horizontal one. The luminous gerbes which issue from the bent points will form a circle of fire, if the motion be rapid, or something that has a near resemblance to a sun.

What may give to this exhibition an air of reality, is that it is possible to accompany it with the noise of an electric battery, which will excite the idea of marroons and saucissons, a discharge of which accompanies in general other fire-works, if not continually, at least at certain intervals. This might be done by means of small electric batteries, discharged partially and successively.

This, as already said, is merely an idea, which has need of being subjected to experiment; but in our opinion an ingenious artist might turn it to advantage. It may easily be conceived, that the electricity in this case ought to be strong; but what could not be done by one electric machine, might be performed, in all probability, by two, or three, or four.

EXPERIMENT XXVIII.

On the Electricity of Silk.

We shall here present the reader with a few more singular experiments made by Mr. Symmer, who published them in the Philosophical Transactions for the year 1759.

1st. During exceedingly dry, cold weather, when a north or north-east wind prevails, take two new silk stockings, the one black and the other white, and after having heated them well, put them on the same leg: the action of putting them on will itself electrify them. If you then pull them off, one within the other, making them both glide at the same time on the leg, they will be found so much electrified, that they will adhere to each other with a greater or less force. Mr. Symmer once saw them support, in this manner, a weight which was equal at least to sixty times that of one of them.

2d. If you draw the one from within the other, pulling one by the heel and the other by the upper end, they will still remain electrified, and you will be astonished to see each of them swell in such a manner as to represent the volume of the leg.

3d. If one of these stockings be presented to the other at some distance, you will see them rush towards each other, become flat, and adhere with a force of several ounces.

4th. But, if the experiment be performed with two pairs of stockings, combined in the same manner, the one white and the other black, on presenting the white stocking to the white, and the black to the black, they will mutually repel each

other. If the black be then presented to the white, they will attract each other, and become united, or will tend to unite, as in the third experiment.

5th. The Leyden flask may be charged by these stockings.

It appears thence to result, that silk rubbed against silk is capable of electrifying; but for this purpose a preparation must be given to one of the pieces: for two white or two black stockings, placed one within the other, cannot electrify. But it is not the black as black, opposed to the white as white, which produces this effect. The Abbé Nollet has shewn, that the preparation here alluded to, is the operation of galling, which precedes that of dying black; for two white ribbons, one of which only is galled, if rubbed against each other properly, will produce the same phenomena of adhesion, attraction, and repulsion. There can be no doubt that the case is the same in regard to stockings.

The partisans of the Franklinian doctrine respecting electricity, will not find it difficult to explain the other phenomena which have been mentioned. Each stocking is electrified in a different manner, one positively and the other negatively; it appears that it is the white which is electrified positively, or in the manner of glass. The swelling up, observed in each of the insulated stockings, is only the effect of repulsion between the bodies similarly electrified; for all the parts of the same stocking have received the same electricity. For the like reason, two stockings of the same colour necessarily repel each other.

But, if a black stocking be presented to a white one, as their electricities are different, these two bodies will attract each other; this phenomenon is well known, and if not general does not fail to take place between two bodies electrified one positively the other negatively, or the one in the manner of glass, and the other in that of sulphur.

A very remarkable phenomenon here is, that two bodies, the one electrified positively, and the other negatively, according to the language of the Franklinians, may be applied to each other, without their electricity being destroyed. Mr. Symmer remarks this with some astonishment; and hence he was induced to deviate from the Franklinian doctrine in assigning reasons for it, which, as the Abbé Nollet observes, approach near to the explanation of the latter.

It has since been remarked, that the surfaces of two bodies, one of which is electrified positively, and the other negatively, may be applied to each other, without their electricity being destroyed. This is the principle of the electrophorus, an electric instrument, invented a few years ago. Nay more, these two surfaces applied in this manner, retain their electricity much longer; but it does not manifest itself when they are separated. Electricity is a mine, which, the more it is searched, presents new phenomena difficult to be explained. How this is to be explained, according to the Franklinian theory, we do not know; and, though attached to the science we shall not attempt it.

EXPERIMENT XXIX.

Which proves that electricity accelerates the course of fluids.

Provide a capillary tube; or a tube terminating in an aperture so narrow, that the water which runs through it can issue only drop by drop. If this water be electrified, you will immediately see it run out in a continued stream.

REMARKS.—*On the consequences of this experiment, and the cures performed, or said to be performed, by electricity.*

It is probable that it was this experiment which gave rise to the application of electricity to medicine; for it was natural to reason in this manner; as electricity accelerates the course of fluids, it is probable that it will accelerate that of the blood, and of the nervous fluid in animals. But there are certain diseases which appear

to be merely the consequence of an accumulation of the nervous fluid, such as the palsy, and different maladies depending on the same cause; as deafness, blindness, &c; consequently, electricity, by accelerating either the course of the blood, or that of the nervous fluid, may remove that accumulation, and so will produce a cure of the disease.

Some physicians therefore began to electrify patients attacked by the palsy; and it must be allowed, as we have the testimony of persons free from every kind of suspicion, such as M. Jallabert of Geneva, and others, that the attempt was attended with some success. It is certain that this celebrated professor, and citizen of Geneva, if he did not cure radically, at least greatly relieved a person of the name of Noguez, afflicted with a paralytic disorder. This man, who was incapable of raising up his arm, after being electrified three months, was able to raise up a large hammer.

This cure, which was published in some of the journals, made a considerable noise, as may be well supposed; and a multitude of electricians, in various parts of Europe, undertook the cure of paralytics, the dumb, the blind, &c. We have a collection, in three volumes, by M. Sauvages, not of these cures, for there are few to which that appellation belongs; but of the progress of this application of electricity. There were some however very well established, such in particular is that of a young man of Colchester, to whom Mr. Wilson restored the use of his eyesight, which he had lost after a violent fever. In regard to the greater part of the rest, the application was of no effect.

It cannot however be denied, that on the first application of the electricity, the patients in general experienced some relief. Paralytics felt, in the palsied part, a kind of heat and pricking, which seemed to announce the return of sensation; the blind sometimes saw sparks of light; but in general nothing farther took place, and these beginnings, which seemed to promise the greatest success, were not followed by happy consequences.

Some Italian philosophers have asserted something more marvellous. About the year 1750 they announced, at Padua, that electricity exalted and attenuated odours to such a degree, that they passed through glass; that purgative drugs put into a vessel carefully, and hermetically sealed, produced their effect on the person who held the vessel in his hand, while it was electrified. This no doubt would have been a noble discovery in medicine; but unfortunately this pretended discovery, announced with great solemnity to all Europe, vanished entirely before the enlightened eyes of the Abbé Nollet, who undertook a journey to Italy to examine it. He found, at least, that there was precipitation and misconception in all these fine assertions, which could not be realized in his presence. Having repeated the experiment himself several times, in his closet, he never found that the most penetrating odour passed through the pores of a glass vessel, when properly closed, whether electrified or not electrified; and the case was the same with the purgative emanations of cassia and rhubarb.

M. le Roy, one of the French philosophers, who cultivated this branch of philosophy with the greatest care, was induced to try the effects of electricity on some of his patients; the first of whom had been afflicted with a hemiplegia for three years; another with a gutta serena; and a third with deafness. The electric matter conveyed several times through the palsied parts of the first, seemed in the commencement to revive sensation; the patient perspired a great deal, an effect which could not be produced by all the medicines before administered to him. After being electrified four or five months, sensation and the power of motion returned to the palsied fingers, and the patient could lay hold of a glass, and convey it to his mouth; he could even raise a weight of 40 or 50 pounds; but this commencement of a cure was all that could be effected; and as the patient received no

additional benefit from a continuation of the same treatment, for four months more, it was laid aside as entirely useless.

M. le Roy received as little satisfaction in regard to his blind patient, though in order to free the optic nerve from its obstruction, he had invented an apparatus, by means of which he gave him gentle shocks through the head. The patient, at the moment of the electric explosion through his head, perceived a flash, but after the electricity had been applied some months, M. le Roy became tired, as before, of administering a useless remedy.

The patients labouring under deafness were not more fortunate. M. le Roy directed the electric fluid from one ear to the other. At each shock a kind of noise was heard in the head, which one of them compared to all the petards of La Grève. But the auditory nerves were not cleared, nor was the deafness removed. An account of the treatment of these patients may be seen in the Memoirs of the Academy of Sciences, for the year 1755.

We have read, somewhere in the Philosophical Transactions, of an intermittent fever being cured by electricity. This is not impossible; as the electricity, by accelerating the motion of the fluids, might act as a tonic.

Some years ago, the Abbé Sans, Canon of Perpignan, announced at Paris several cures which had been performed in his country by electricity. He published them in a particular work, with various certificates annexed; but his operations on M. de la Condamine, attacked with total insensibility in one half of his body, and total deafness, were attended with no effect. These infirmities indeed had taken root for several years, and therefore it would have been unjust to require success in any attempt made to remove them. But we never heard that this electrician had ever much success at Paris.

To conclude, it appears to us that too great hopes were at first conceived in regard to the application of electricity to such diseases; but that it was not entirely without effect, and that in recent cases it might be tried with some hope of success. The rheumatism, according to M. le Roy, seemed to oppose the least resistance to this remedy, which perhaps acted by re-establishing perspiration. He obtained profuse sweats to the greater part of his patients. In short, there can be no doubt that it occasions in the human body an universal orgasm, which under certain circumstances might be critical and advantageous.

EXPERIMENT XXX.

Natural and Animal Electricity.

During cold weather, lay hold of a cat, and draw your hand over her back several times, in a direction contrary to that of the hair; by these means you will often excite very strong sparks, which will emit a snapping noise.

Remark.—This however is not the only animal which exhibits the electric phenomena by friction: even men, under certain circumstances, emit sparks which are absolutely of the same nature. There are few people to whom this circumstance is not known by experience. It is during cold winters, and after being well heated, that they exhibit this phenomenon. On these occasions, if they throw off their shirt in the dark, sparks more or less vivid, and accompanied with a sensible snapping, will issue from it. Some, in consequence of a particular temperament, are more subject to this phenomenon than others. In all probability, these persons are very hairy; for hair, as it approaches the nature of silk, is electric by friction; and according to every appearance, it is the friction of the dry, warm linen, against the hair, which is also dry and warm, that produces this electricity, and the sparks which accompany it. In combing a person's hair of a morning, in dry frosty weather, it becomes very elastic, and manifests strong signs of electricity.

These luminous appearances were formerly classed among the phosphoric phenomena; but since the new discoveries in electricity, there can be no doubt that they belong to the latter.

It would have been easy for us to enlarge this part of the Philosophical Recreations much more, by introducing into it a great number of other curious and surprising experiments relating to the theory of electricity; but as we must confine ourselves within narrow limits, we shall conclude this part with a list of the principal works to which those who are desirous of obtaining a thorough knowledge of electricity may recur. Of this kind are "Essai sur Electricité de M. l'Abbé Nollet," and in particular a work entitled "Recherches sur les Causes particulières des Phénomènes électriques, et sur les Effets nuisibles ou avantageux qu'on peut en attendre," Paris 1754, 12mo; and to these may be added "Lettres sur l'Electricité," 3 vols. 12mo. Though the Franklinian theory seems, in general, to have many more partisans than that of the Abbé Nollet, it must be allowed that the latter applied to the study of this branch of science with the greatest success. Besides the above, we may refer also to various memoirs of the same author, in which he discusses the theory of Dr. Franklin, published in the "Mémoires de l'Academie, for 1755 and 1760," &c.; and "Recherches sur les Mouvements de la Matière électrique, par M. Doutour," 1760, 12mo. These are the best of the works in which the theory of the French philosopher is illustrated and defended.

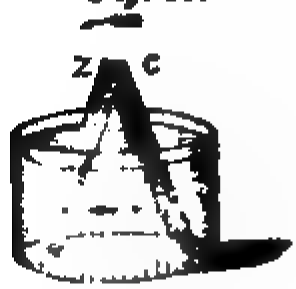
The Franklinian theory was made known for the first time, in France, by a work entitled "Expériences et Observations sur l'Ectricité, faites à Philadelphie en Amerique par M. Benjamin Franklin," Paris 1756, 12mo; translated from the English. We have since seen an edition of Dr. Franklin's works, in two vols. 4to.; the first of which contains all his experiments, and a great many interesting facts in regard to electricity. From these works the reader may soon acquire a sufficient knowledge of the theory of electricity. We ought to add also different memoirs of M. le Roy, one of the principal partisans of Dr. Franklin, published in the "Memoires de l'Académie, for 1754," &c. A very interesting work also on this subject, is a Treatise by Father Beccaria, entitled "Dell' Electricismo naturale è artificiale,' which appeared at Turin in 1759, 4to. It contains experiments which are strongly in favour of the Franklinian theory, and a great many new observations concerning the electricity of the clouds. We must not here omit to mention, that Father Beccaria is one of those philosophers who were most successful in cultivating the science of electricity, and that he discovered a number of new and very extraordinary phenomena. In some of the volumes of the Philosophical Transactions, likewise, are a great many curious papers on different electric phenomena, by Mr. Nairne and Mr. Wilson, Drs. Lind, Watson, &c.; but it would be too tedious to particularise them.

In the year 1752, a work entitled "Historie de l'Electricité," was published in three small vols. duodecimo. The author has collected pretty well every thing that had been done or said in regard to electricity before that period; but intermixed with insipid witticisms and illiberal sarcasms. Since that period, however, Dr. Priestly, one of the ablest of the English philosophers, has given a new History of Electricity, which is much better and more instructive. A French translation of it appeared at Paris in 1771, 3 vols. 12mo. To these we may add the following works:—Adams's Essay on Electricity, 8vo. 1787; a complete Treatise on Electricity by Cavallo, 3 vols. 8vo; and a learned Work on the subject, in 4to. 1779, by Lord Mahon, now Earl Stanhope.

A most excellent Treatise on Electricity, by Dr. Roget, Secretary to the Royal Society, has recently been published in the Library of Useful Knowledge.

To the preceding articles on Magnetism and Electricity, we shall here add a short account of the modern discoveries in kindred branches of science.

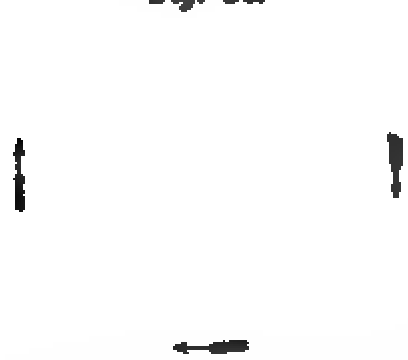
Fig. 53.



If we take a plate of zinc z, (Fig. 53.) and one c of copper, and partially immerse them in diluted sulphuric acid, keeping their upper edges in contact, but their lower edges apart, we shall find that a continued current of electricity passes from the zinc to the acid, from the acid to the copper, and from the copper to the zinc, and so on through the acid, copper, and zinc in continued succession, in the same direction.

If, instead of joining the metal plates, they be connected by a wire, as in Fig. 54, the same effect will be produced.

Fig. 54.



The course of the electric current will be, in the fluid from the zinc towards the copper, from the copper through the wire to the zinc, &c. By separating the wire as at Y, the current may be passed through any body which it may be wished to subject to its action. The direction of the current is indicated by the arrows. When united, the wire W, proceeding from the copper, is imparting electricity to X, which is in connection with the zinc plate. The wire W is therefore considered as being in a positive and X in a negative state of electricity. Either of these arrangements is called a galvanic circle.

Though the electrical effects of this simple apparatus are in general very feeble, it has been found possible, by a small apparatus of this kind to exhibit some of the more energetic effects of galvanism. In Dr. Thompson's Annals of Philosophy, Dr. Wollaston has described what he calls an elementary battery, by which he raised the temperature of a slender piece of wire to a red heat.

The following is Dr. W.'s description of the miniature instrument :

"The smallest battery I have formed of this construction consisted of a thimble without its top, flattened till its opposite sides were about [illegible] of an inch asunder. The bottom part was then nearly one inch wide, and the top about [illegible], and as its length did not exceed [illegible] of an inch, the plate of zinc to be inserted was less than [illegible] of a square inch in dimensions.

"Previously to insertion, a little apparatus of wire, through which the communication was to be made, was soldered to the zinc plate, and its edges were then coated with sealing wax, which not only prevented metallic contact at those parts, but also served to fix the zinc in its place, by heating the thimble so as to melt the wax.

"A piece of strong wire, bended so that its two extremities could be soldered to the upper corners of the flattened thimble, served both as a handle to the battery, and as a medium to which the wires of communication from the zinc could be soldered.

"The conducting apparatus consisted, in the first place, of two wires of platina about [illegible] of an inch in diameter, and one inch long, cemented together by glass in two parts; so that one end of each wire was united to the middle of the other. These wires were then turned, not only at their extremities for the purpose of being soldered to the zinc and to the handle, but also in the middle of the two adjacent parts for receiving the fine wire of communication.

"One inch of silver wire $\frac{1}{100}$ of an inch in diameter, containing platinum at its centre [illegible] part of the silver in diameter, was then bended so that the middle of the platina would be freed from its coating of silver by immersion in dilute nitrous acid. The portion of silver remaining on each extremity served to stretch the fine filament of platina across the conductors during the operation of soldering; a little sal-ammo-

niac being then placed on the points of contact, the soldering was effected without difficulty, and the two loose ends were readily removed by the silver attached to them.

"It should here be observed, that the two parallel conductors cannot be too near each other, provided they do not touch, and that, on this account, it is expedient to pass a thin file between them, (previously to soldering on the wire) in order to remove the tin from the adjacent surface. The fine wire may thus be made as short as from $\frac{1}{30}$ to $\frac{1}{20}$ of an inch in length; but it is impossible to measure this with precision, since it cannot be known at what points the soldering is in perfect contact.

"The acid which I have employed with this battery consists of one measure of sulphuric acid, diluted with about 50 equal measures of water; for though the ignition effected by this acid is not permanent, its duration for several seconds is sufficient for exhibiting the phenomena, and for shewing that it does not depend upon mere contact, by which only an instantaneous spark should be expected.

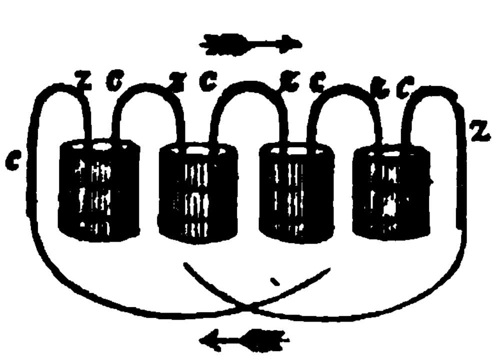
"Although in this description I have mentioned a wire $\frac{1}{3000}$ of an inch in diameter, I am doubtful whether this thickness is the best. I am, however, persuaded that nothing is gained by using a finer wire; for though the quantity of matter to be heated is thus lessened, the surface by which it is cooled does not diminish in the same ratio; so that where the cooling power of the surrounding atmosphere is the principal obstacle to ignition, a thicker wire, which conveys more electricity in proportion to its cooling surface, will be more heated than a thin one; a fact which I not only ascertained by trials on these minute wires, but afterwards took occasion to confirm on the largest scale, by means of the magnificent battery of Mr. Children, in the summer of 1813."

By increasing the size of the plates, batteries of this kind have been constructed, capable of producing very powerful effects.

Many effects of galvanism require for their production the combined influence of several pairs of plates, arranged so as to form what has been called a *compound galvanic circle*.

Let there be taken an equal number of silver coins, round pieces of zinc, and circular discs of paper, somewhat smaller than the metallic ones, but soaked in salt water. Of these form a pile, beginning with silver s, then zinc z, then card c, and so on; s, z, c, successively, as often as you think proper, finishing however with z. If then the uppermost disc be touched with the finger of one hand, previously wetted, while a finger of the other hand, also wetted, touches the lowest disc, a distinct shock will be felt similar to that from a Leyden phial, or still more resembling that from a weakly charged electric battery. By retouching, as often and as quickly as you please, a succession of shocks is received; the strength being greater, the greater the number of plates; and the direction of the fluid is from the zinc at the top of the pile, to the silver at the bottom.

Fig. 55.



Another arrangement was adopted by *Volta*. Taking a number of glasses filled with an acid or saline solution, he placed in each a plate of zinc and a plate of copper (or silver) connecting them as in Fig. 55. The course of the electric current is from the last plate of copper on the right through the zinc plate outside, and the wires, to the copper c on the left; the end of the battery terminated by the zinc plate—being that from which the electricity is given out to the wire—is, consequently, the *positive* end or *pole* of the battery; and the opposite end, terminated by a copper plate, is the *negative pole*.

In the pile described above the zinc also is the positive, and the silver or copper the negative pole.

The apparent discordance between the direction of the electric current in the *simple* and the *compound* galvanic circles is explained by the consideration that in the former the conducting wire communicated directly with the plate which is immersed in the fluid; but in the latter it proceeds, not from the immersed plate, but from one outside associated with it, and therefore of a different kind.

The form of the galvanic battery has been variously modified, according to the views of experimenters; but in the space to which we must confine ourselves we have not room for further details.

A striking difference observable between electricity, as excited by galvanism, or as produced by friction of electric or non-electric bodies, is, that though the action of the latter, while it lasts, may have any degree of energy, that energy is exerted only for an instant; it is accumulated by degrees, and is discharged at once; and the effect of the discharge is beyond the control of the operator; while in *galvanic* (or *Voltaic*) electricity, the battery continues to supply while in action, uninterruptedly and indefinitely, vast quantities of electricity, which is not lost by returning to its source (the earth), but circulates with undiminished force in a perpetual stream. The effect of this continuous current in bodies subject to its action is therefore more definite; and as it increases with the time, it may at length acquire a force incomparably greater than even that produced by electric explosion.

But the *intensity* of electricity developed by galvanic combinations is increased according to the number of alternations in the elements which form those combinations; and is totally independent of the extent of surface exposed to the action of the fluid in which the plates are immersed; though the *quantity* developed in a given time is greater, the more extensive that surface is.

If the Voltaic battery is of sufficient size, its electricity may be transferred to a common electric battery, by connecting the inner and outer coatings of the electric battery respectively with the poles of the Voltaic one, when the charge will be instantly communicated to the former. On removing the Voltaic battery, this communicated electricity may be discharged; and on renewing the communication, a similar charge will be received; and the same process may be repeated indefinitely. Instead of removing the Voltaic battery, we may allow it to remain connected with the electric battery, when a rapid succession of sparks may be obtained from it by connecting a wire with the outer coating, and presenting the other end of the wire to the knob of the phial. If the Voltaic battery is very powerful, these rapid explosions are so powerful as to throw the iron wire into a state of intense combustion. With a series of a thousand plates, each spark, or discharge, is attended with a sharp sound, and will burn thin metallic leaves. It is remarkable that this often happens when the same Voltaic battery has not power to produce such effects by itself, unconnected with the electrical battery. The Voltaical battery imparts in an *instant* to the electrical one the whole of the charge it is capable of communicating.

When a Voltaic battery is composed of a considerable number of alternations of plates, on bringing together the wires from the opposite poles, the transfer of electricity begins while they are at a sensible distance from each other, and is accompanied, as in ordinary electricity, by vivid light; the sparks occurring every time the contact is broken, as well as when it is renewed.

We extract the following interesting paragraph from the article Galvanism, in the Library of Useful Knowledge; recommending, at the same time, the article itself to the attention of our readers.

"The most splendid exhibition of electric light is obtained by placing pieces of charcoal, shaped like a pencil, at the end of two wires, in the interrupted Voltaic

circuit. When the experiment was tried with the powerful battery of the Royal Institution, a bright spark passed between the two points of charcoal, when they came within the distance of about the thirtieth of an inch; and immediately afterwards more than half of each pencil of charcoal (the length of which was an inch, and the diameter one sixth of an inch), became ignited to whiteness. By withdrawing the points from each other, a constant discharge took place through the heated air, in a space of at least four inches, forming an arch of light in the form of a double cone, of considerable breadth, and the most dazzling brilliancy. Any substance introduced into this arch instantly became ignited; platina melted in it, as wax in the flame of a candle; quartz, the sapphire, magnesia, lime were fused; fragments of diamond, charcoal, and plumbago, seemed converted into vapour, apparently without having undergone previous fusion."

A battery of a hundred pairs of plates, of six inches square, will exhibit these phenomena on a smaller scale. Charcoal prepared from the harder woods is to be preferred.

Light thus obtained from voltaic electricity is the most intense that art has yet produced. It often assumes in succession different prismatic colours, and exhibits some of the rays which are deficient in the solar beams. Its brightness distresses the eye, even by a momentary impression, and effaces the light of lamps in an apartment otherwise brilliantly illuminated, so as to leave an impression of darkness on the sudden cessation of the galvanic light.

The light is given out with equal splendour whether the experiment is made in air or in gases which contain no oxygen; such as azote or chlorine; it is therefore independent of combustion, and it is found too, that during the ignition neither the gas nor the charcoal undergoes any chemical change.

In common electricity, heat is not evolved when the fluid passes freely through a conducting substance, but only when some resistance is opposed to its passage, and then its equilibrium is suddenly restored by an effort accompanied with light, heat, and sound. But in using the voltaic battery, heat is evolved when the connection is perfect, and the stream of electricity is conducted from one pole to the other in the most silent manner. If the connecting wire pass through water contained in a vessel, the water becomes heated even to boiling; and the ebullition continues as long as the connecting wire passes through the vessel. Even the battery itself is heated when the apparatus is in an active state.

As was remarked of the *light* produced by voltaic agency, so it may be said of *heat* produced by it, that it is more intense than any other which art has yet produced.

We shall now advert to a few of what may be called the *magnetic* effects of voltaic electricity.

Let the poles of a powerful voltaic battery be connected by a metallic wire, a part of which is made straight. Take a magnetic needle nicely balanced on its pivot, and allowing it to settle in the magnetic meridian, bring over and parallel to it the aforesaid straight part of the rod in the galvanic circuit, and it will be found that the needle will instantly change its position, its ends deviating from their natural position, towards the east or west, according to the direction of the electric current in the wire; so that by reversing the direction of the current, the deviation of the needle is also reversed, the end of the needle next the negative pole of the battery being always deflected towards the west.

The deviation of the needle is the same while the uniting wire is parallel to the needle and above it; even when it is not directly over it, but the direction of the deviation is reversed when the uniting wire is parallel to and below the needle.

If the wire is not parallel to the needle, various phenomena both with respect to deviation and dip are presented; but for an account of these phenomena we

must refer to works on this comparatively new branch of science, and conclude with a short account of *Electro Magnetic Induction*.

During the passage of an electric current through a conducting wire, it tends to induce magnetism in such bodies in its vicinity, as are capable of being excited. The connecting wire in action has a sensible attraction for iron filings, which it holds in suspension like an artificial magnet, while the electric current circulates through the wire; but the moment the galvanic circuit is interrupted, the filings fall off.

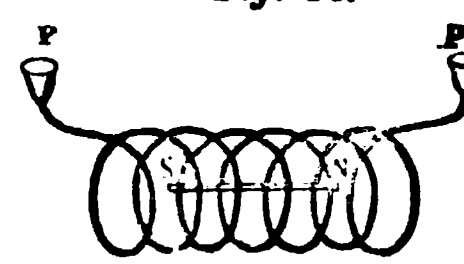
Mr. Watkins of Charing Cross, a most successful cultivator of this branch of science, first noticed that if a thick copper wire be extended between the poles of a voltaic battery, and some fine iron filings be gently sifted upon it, they adhere to the wire all round in distinct transverse bands, the particles of which cohere as long as the current is maintained. On a broad and thin copper ribbon, substituted for the wire, the filings arrange themselves in parallel bands across the ribbon.

To try whether steel might not receive permanent magnetism from voltaic action, Sir Humphrey Davy fastened several steel needles in different directions by fine silver wire, to a wire of the same metal about the thirtieth of an inch in thickness, and eleven inches long, some parallel, others transverse, above and below, in different directions, and placed them in the electrical circuit of a battery of thirty pairs of plates, each of 45 square inches, and tried their magnetism by iron filings. They were all magnetic. Those that were parallel to the wires attracted filings in the same way as the wire itself; but those in transverse directions had each two poles. All the needles that were placed under the wire, when the positive end of the battery was east, had their north poles on the south side of the wire, while those that were placed over the wire had their south poles to the south, whatever was the inclination of the needle to the horizon. On breaking the connection all the steel needles that were on the wire in a transverse direction retained their magnetism, which was as powerful as ever; while those that were parallel to the silver wire appeared to lose it at the same time as the wire itself.

Contact with the wires was not necessary for magnetising the needle, the effect being produced instantaneously by mere juxta-position of the needle in a transverse direction, even through a thick pane of glass.

The efficacy of electric magnetic induction is greatly increased by placing the needle or bar to be magnetised within a helix of wire, as represented in Fig 56. It is not necessary that the bar should be placed in the axis of the helix, as it may lie in any situation within it, or be enclosed in a tube of glass, which will also be convenient as a support for the coils of wire, and for the introduction of different needles in succession. The needle should not be allowed to remain more than a moment in the tube; for the magnetising effects of the helix are produced almost instantaneously; and it has been observed, that a needle left in the helix for a few minutes sometimes has its first acquired polarity impaired, or even destroyed.

Fig. 56.



If the needle to be magnetised is not very hard, its whole length need not be inserted in the glass tube; for if held in the hand, so that only one half of it is within the helix, it will become equally magnetic with one that has been wholly acted upon. It will be understood that the ends of the helix P and p are connected with the two poles of the galvanic battery. The two cups at P and p have a little mercury put in them to render the contact with the wires from the battery and the helix more complete.

A very powerful temporary magnet may be obtained by bending a thick cylinder of soft iron into the form of a horse shoe, and surrounding it with a coil or helix of thick copper wire; the coils being prevented from touching each other by a covering

of silk or some other non-conducting material. When this wire is made part of the galvanic current of a battery, even of moderate power, the iron is rendered so highly magnetic that it will lift up a very heavy weight by means of a piece of iron applied to its poles, which acts precisely for the time like those of a horse-shoe magnet.

Fig. 57.

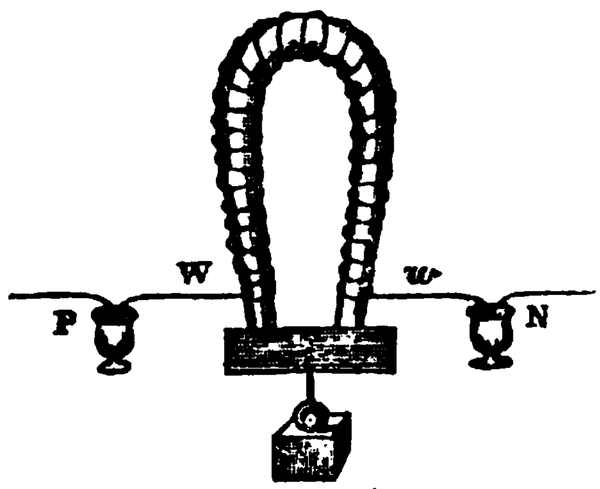


Fig. 57. exhibits an arrangement of this kind, W *w* being the two ends of the wire coiled round the iron to be magnetised, and bent so as to dip into the cups, P and N, for forming connexions with the battery.

More recent discoveries in this branch of science have gone far to show that light, electricity, and magnetism are essentially the same, but developed under different circumstances. We have seen that galvanism produces electricity, and that both are capable of imparting to iron the magnetic properties of attraction and polarity; and perhaps the most singular of all the discoveries that have been made with reference to these subjects is, that magnetism, under certain circumstances, is capable of exciting electric action, to produce electric sparks, and even a continuous stream of electric fluid.

But we must here close our brief notice of this interesting subject; and refer our readers to works which are expressly devoted to its discussion.

ELECTROTYPE.

A most unlooked-for and wonderful application of voltaic electricity has recently been made. We copy the following account of it from the "Athenæum Journal" of October 26th, 1839; merely remarking that the applications of the principle which have since been made fully justify the anticipations of the journalist.

"We lately published M. Jacobi's letter to Mr. Faraday, in which he described his attempts to copy in relief engraved copperplates, by means of voltaic electricity. We have since received a communication from Mr. Thomas Spencer of Liverpool, from which it appears that that gentleman has for some time been independently engaged on the same subject; and that he has not only succeeded in doing all that M. Jacobi has done, but he has successfully overcome those difficulties which arrested the progress of the latter. It is unnecessary here to enter on the question of priority between these gentlemen. To Mr. Spencer much credit is certainly due for having investigated, and successfully carried out, an application of voltaic electricity, the value of which can hardly be questioned. The objects which Mr. Spencer says he proposed to effect were the following: 'To engrave in relief upon a plate of copper—to deposit a voltaic copper-plate, having the lines in relief—to obtain a *fac-simile* of a medal, reverse or obverse, or of a bronze cast—to obtain a voltaic impression from plaster or clay—and to multiply the number of already engraved copper-plates.' The results which he has obtained are very beautiful; and some copies of medals which he has forwarded to us are remarkably sharp and distinct, particularly the letters, which have all the appearance of being struck by a die.

"Without entering into a detail of the steps by which Mr. Spencer brought his process to perfection, many of which are interesting, as shewing how slight a cause may modify the result, we shall at once give a description of his process.

"Take a plate of copper, such as is used by an engraver; solder a piece of copper wire to the back part of it, and then give it a coat of wax; this is best done by heating the plate as well as the wax; then write or draw the design on the wax with a black lead pencil or point. The wax must now be cut through with a graver or steel

point, taking special care that the copper is thoroughly exposed in every line. The shape of the tool or graver employed must be such that the lines made are not V shaped, but as nearly as possible with parallel sides. The plate should next be immersed in diluted nitric acid; say three parts water to one of acid; it will be at once seen whether it is strong enough, by the green colour of the solution and the bubbles of nitrous gas evolved from the copper. Let the plate remain in it long enough for the exposed lines to get slightly corroded, so that any minute portions of wax which might remain may be removed. The plate thus proposed is then placed in a trough separated into two divisions by a porous partition of plaster of Paris or earthenware; the one division being filled with a saturated solution of sulphate of copper, and the other with a saline or acid solution. The plate to be engraved is placed in the division containing the solution of the sulphate of copper, and a plate of zinc of equal size is placed on the other division. A metallic connexion is then made between the copper and zinc plates, by means of the copper wire soldered to the former; and the voltaic circle is thus completed. The apparatus is then left for some days. As the zinc dissolves, metallic copper is precipitated from the solution of the sulphate on the copper-plate, wherever the varnish has been removed by the engraving tool. After the voltaic copper has been deposited in the line engraved in the wax, the surface of it will be found to be more or less rough, according to the quickness of the action. To remedy this, rub the surface with a piece of smooth flag or pumice stone with water. Then heat the plates, and wash off the wax ground with spirits of turpentine and a brush. The plate is now ready to be printed from at an ordinary press.

"In this process, care must be taken that the surface of the copper in the lines be perfectly clean, as otherwise the deposited copper will not adhere with any force, but is easily detached when the wax is removed. It is in order to ensure the perfect cleanness of the copper, that it is immersed in diluted nitric acid. Another cause of imperfect adhesion of the deposited copper Mr. Spencer has pointed out, is the presence of a minute portion of some other metal, such as lead, which, by being precipitated before the copper, forms a thin film, which prevents the adhesion of the subsequently deposited copper. This circumstance may, however, be turned to advantage in some of the other applications of Mr. Spencer's process, where it is desirable to prevent the adhesion of the deposited copper.

"In copying a coin or a medal, Mr. Spencer describes two methods: the one is by depositing voltaic copper on the surface of the metal, and thus forming a mould, from which fac-similes of the original medal may readily be obtained by precipitating copper into it. The other is even more expeditious. Two pieces of clean milled sheet-lead are taken, and the medal being placed between them, the whole is subject to pressure in a screw-press, and a complete mould of both sides is formed in the lead, shewing the most delicate lines perfect (in reverse.) Twenty, or even a hundred of these, may be so formed on a sheet of lead, and the copper deposited by the voltaic process with the greatest facility. Those portions of the surface of the lead which are between the moulds may be varnished, to prevent the deposition of the lead; or a whole sheet of voltaic copper having been deposited, the medals may be afterwards cut out. When copper is to be deposited on a copper mould or medal, care must be taken to prevent the metal deposited adhering. This Mr. Spencer effects by heating the medal, and rubbing a small portion of wax over it. This wax is then wiped off, a sufficient portion always remaining to prevent adhesion.

"Enough has been said to enable any one to repeat and follow up Mr. Spencer's interesting experiments. The variations, modifications, and adaptations of them are endless, and many new ones will naturally suggest themselves to every scientific reader; and for their gratification, the medals produced by this process, and forwarded to us by Mr. Spencer, will be left with our publisher for some days, and open to inspection."

PART FOURTEENTH.

CHEMISTRY.

THOSE who are only initiated in Chemistry must conceive an idea of this science very different from that which is entertained by the vulgar. According to the common class of mankind, Chemistry is the chimerical art of transmuting metals, or at most of producing some extraordinary phenomena, rather curious than useful. But in the eyes of the philosopher, to whom it is known, it is the most extensive and most interesting of all the branches of natural philosophy. We may even venture to assert, that it is doubtful whether the appellation of a great philosopher can be given to any person unacquainted with chemistry. It is at least certain, that though without its assistance we can account for some of the phenomena of nature, such as the motion of the celestial bodies, the effects of the gravity of the air, &c.; yet there are a far greater number which can be explained by chemistry alone. Indeed chemistry is not less extensive than nature itself. Animals, vegetables, and minerals, all fall within its province. It is it that analyses them, combines their principles, examines the phenomena resulting from them; and thus renders us more intimately acquainted with their nature. Hence are deduced a multitude of useful processes; so that it may with truth be said, that many of the arts are nothing else than a continual application of chemistry. Of this kind are the arts of glass-making, dyeing, metallurgy, &c. We may even add, that the most common of the arts, those that are most necessary, are often chemical processes; such, for example, is that of washing; respecting the truth of which the person who practises it entertains no doubt; though still it is an operation which chemistry alone can explain. This explanation consists in the property which fixed alkalies have of rendering fat substances soluble in water, and of forming a soap with them. Those who know that one of the first operations of this art is to soak the linen in a sort of ley, made of wood ashes or vegetable fixed alkali (pot-ash), will readily be convinced of the truth of what we have advanced. In this part of our work we need not adduce any other examples.

Chemistry also, of all the branches of philosophy, exhibits the most singular and most curious phenomena. Who will not be astonished to see iron filings, immersed in a liquid as cold as itself, immediately produce a violent ebullition, and vapours susceptible of inflammation? Can any one, without admiring the operations of nature, see this metal, so solid, afterwards destroyed in some measure by the above fluid, and united with it in such a manner as to pass with it through the closest filtre? Who will not be filled with wonder, on seeing another limpid liquor suddenly dissolve this union, and cause the iron to fall to the bottom of the vessel, in the form of an impalpable powder? It is needless, and would require too much time, to enumerate here any more of these phenomena, as we shall hereafter give a particular account of those that are the most remarkable and most interesting. But it will first be necessary to bring the reader acquainted with the principal substances employed as agents in these operations.

ARTICLE I.

Of Salts.

The name of Salts, or saline matters, is given to all those bodies which, when immersed in water or exposed to damp air, resolve themselves into liquids: of this we have an example in the well known marine salt, in nitre or saltpetre, in alum, vitriol, tartar, sal ammoniac. &c. When immersed in water, these disappear, and become intimately mixed with the whole liquor. This is what is called *solution* or *dissolution*.

Salts, to be dissolved entirely, require a greater or less quantity of water, according to the nature of them. Marine salt, to be dissolved entirely, requires twice its weight of water; alum, twelve times its weight; selenite, six or seven hundred times, &c.

Though acids, alkalies, and their mixtures, generally speaking, are called saline substances, it is necessary in chemistry to consider them separately, in order to obtain a knowledge of their principal properties.

SECTION I.

Of Acids.

We shall not here say, with the author of the "Dictionnaire de Physique portatif," * and some of the old chemists, that an acid is formed of long, sharp, and cutting particles; for nothing is more destitute of foundation; and by the help of such a definition it would be impossible, in an apothecary's shop, to distinguish an acid from an alkali, or from a neutral salt. The following is a more correct description.

Acids in general have a liquid form, and leave on the tongue an impression of sourness and coldness. They possess also the property of changing the colour of blue vegetable juices to red; and they combine readily with alkalies, earths, and metals, with which they form neutral salts.

To try an acid by vegetable colours, take sirup of violets, or blue paper, and pour the liquid acid over it; the violet or blue colour will soon assume a red hue. When any liquid therefore, poured over sirup of violets or blue paper, changes the colour to red, we may be assured that it is an acid, or that an acid predominates in it.

Acids are by modern chemists divided into mineral, vegetable, and animal, according as they are derived from one or the other of these sources.

The old chemists were acquainted with no more than three mineral acids, namely, the vitriolic, the nitric, and the marine, so called because obtained from vitriol, nitre, and marine salt. The mineral acids now known are much more numerous. The principal of these acids are the sulphuric or vitriolic, the nitric, the muriatic or marine, the carbonic, the boracic, the fluoric, the succinic, the arsenic, the molybdic, and the tungstic.

The vegetable acids are also now pretty numerous. The following are known: the acetic acid, or pure vinegar; the tartareous, extracted from tartar; the gallic, from gall-nuts; the citric, from lemons; the malic from apples; the benzoic, from flowers of benjamin; the camphoric, from camphor oxygenated to acidity; the pyrotartareous, the pyroligneous, and the pyromucous.

The three last mentioned, however, have been proved to be merely acetic acid

* According to this author, iron is composed of vitriol, sulphur, and earth; fermentation is a movement occasioned by the introduction of acids into alkalies; when alkalies coagulate they form crystals; sulphur is an inflammable mixture composed of fire, oil, water, and earth; copper is a compound of sulphur, vitriol, &c. !!

rendered impure by empyreumatic oil; and ought therefore to be no longer considered as distinct acids.

The following animal acids are known: the phosphoric, or phosphorus and oxygen in union to saturation; the lactic, obtained from milk; the saccholactic, extracted from sugar of milk; the formic, from ants; the sebacic, from fat; the bombic, from silk-worms; the uric, from the human calculus; the prussic, which is the colouring matter of Prussian blue.

Acids formerly were considered either as simple bodies, or various hypotheses were admitted in regard to their origin and composition. It is now however proved by ex-experiment, that every acid consists of a peculiar body combined with oxygen, which is the basis of oxygen gas, and thence has been called oxygen, that is the generator of acidity. The bodies which enter into union with oxygen, and form acids, are called the radicals, or bases of the respective acids, which they constitute.

When the bases are perfectly saturated with oxygen, the acids are called perfect acids, and are denoted by the termination *ic*, to distinguish them from the imperfect acids, in which the bases predominate, and to which the termination *ous* is given. Hence we have nitric and nitrous, sulphuric and sulphurous acids, &c.

Some bases can combine with an excess of oxygen; the acids thence resulting are called oxygenated or oxygenised.

Of the Vitriolic Acid.

The vitriolic or sulphuric acid, the most powerful of all, is furnished by vitriol either green or blue, or by alum; for vitriol is a salt formed merely by the combination or union of an acid with iron and copper; alum, in like manner, is the combination of an acid with argillaceous earth; and the acid in these three substances is absolutely the same.

It will be sufficient here to observe, that the acid is extracted by means of fire. These matters are inclosed, with certain precautions, in a retort, and being exposed to a strong fire, the violence of the heat obliges the acid, which is susceptible of being reduced to vapour, to abandon the metal or earth, to which it was united, and to pass over into the receiver in the form of a liquid.

Another method, still more simple, of obtaining the vitriolic acid, is by the combustion of sulphur; for it has this substance for its base, and on that account it is now known by the name of the sulphuric acid.

If sulphur be burnt slowly under a bell glass, the fumes arising from it will be condensed on the interior surface of the vessel, and run down in the form of a liquid, which will be an acid with an excess of base, that is sulphurous acid. In this process, the sulphur, by its combustion, receives oxygen from the atmospheric air, which is a mixture of oxygen gas and azotic gas, and at the same time moisture, which enables it to assume the liquid form.

In the large way of manufacture, the sulphuric acid is obtained by burning sulphur, with the addition of a little nitre, in an apartment lined all round with lead, a metal on which that acid exercises little or no action. The floor is covered to some depth with water, and the aperture or door is shut as soon as the materials are inflamed. The nitre is employed to facilitate the combustion. The nitric acid, one of the principles in the nitre, is in a great measure decomposed, and gives up its oxygen to the sulphur, which by this supply, and the oxygen it derives from the air in the room, becomes acidified. When the combustion is over, and the fumes have had sufficient time to be absorbed by the water in the apartment, the door is opened to admit a fresh supply of air, and to introduce a new charge of sulphur and nitre, which is deflagrated as before. This operation is successively repeated for several weeks, till the water, now mixed with acid, is found by examination to be of a certain specific gravity. The fluid is then distilled or simply boiled, which drives off a por-

tion of the redundant water, and leaves the acid in a concentrated state. When this is properly done, its weight is equal to about double the weight of an equal volume of water.

The sulphuric acid of commerce is never quite pure; always containing a portion of sulphate of lead and sulphate of potash. The former comes from a partial dissolution of the lead in the chamber in which it is made, and the latter from the nitre used in the process.

The vitriolic acid is the most powerful of all, and is able to separate all the others from their combinations with alkalies, earths, and metals. In a subsequent article we shall give some experiments to illustrate this chemical play, which is exceedingly curious, and the cause of a thousand singular effects in nature.

Sulphurous acid, like sulphuric, is a combination of sulphur and oxygen; having less of oxygen, or more sulphur, than sulphuric acid. In the gaseous state it is invisible, but of a strong suffocating smell. It is readily absorbed by water, and then forms liquid sulphurous acid. It unites with various bases, forming the salts called *Sulphites*.

Of Carbonic Acid.

Carbonic acid is a combination of carbon and oxygen, there being in it 27·5 parts of carbon to 72·5 of oxygen. In a state of vapour, it is invisible, and unfit for combustion or respiration. By pressure, water may be made to absorb three times its bulk of this gas; and it imparts to the water an acidulous and not unpleasant taste. In the proportion in which it exists in atmospheric air, it is favourable to the growth of vegetables; but in a large proportion it is highly injurious. In combination with earth, alkalies, and metallic oxides, it forms with them salts called *Carbonates*.

Carbonic acid gas is found in abundance in many natural waters, as those of Pyrmont, Spa, and Selzer. Various imitations of these pleasant waters, sold under the names of single and double soda water, are manufactured by houses in London, equal in every respect to the natural waters imported from the continent.

Of the Nitrous Acid.

Nitre or saltpetre, a substance well known, furnishes the nitric acid. Saltpetre indeed is the result of the union of this acid with a matter to which chemists give the name of *fixed vegetable alkali*, because the ashes of vegetables yield the same substance. They are separated from each other by processes which it is not our object here to describe; and in this manner is obtained the liquor called *nitrous acid;* from which the *nitric acid* is procured merely by separating, by means of heat applied to it in a proper apparatus, that portion which has not been fully saturated with oxygen. It is lighter, and in general less active than the vitriolic acid. It is commonly of a dark yellow colour; and when well concentrated, continually emits reddish vapours, which seem to circulate in the vessel that contains it. The ratio of its weight to that of water is then as 3 to 2.

The nitrous acid, when in a moderate state of concentration, is called also *aqua-fortis*. It is the proper solvent of silver and copper. When fully saturated with oxygen, or in the state of *nitric acid*, it is colourless.

Nitric oxide gas is formed by depriving nitric acid of some portion of its oxygen. It is obtained by putting some diluted nitric acid into a retort, and then adding to it some bits of copper, iron, silver, or some other metal. The metals are dissolved by the acid, and a gaseous product is obtained, which is *nitric oxide gas*. It does not possess acid properties, and if inhaled it is destructive to animal life. If suffered to escape into the air, it attracts oxygen, and a red vapour is formed, which is *nitrous acid gas*.

When nitrous oxide gas is received into the lungs, it does not at once prove fatal, because it meets there, and is diluted with the atmospheric air which is present in

that organ. The sensations produced by it are highly pleasurable. Great exhilaration, an irresistible propensity to laughter, a rapid flow of vivid ideas, and a propensity for jumping and other muscular exertions, are its ordinary effects. It has therefore been called *laughing gas*. We would however advise all who desire to eschew *apoplexy*, to resort to other means of exciting pleasurable sensations.

Nitric acid may be used to determine whether an instrument is made of steel or wrought iron. For a drop of the acid let fall on steel will, when it has dried, leave a *black mark;* but not if dropped upon iron.

Of the Marine or Muriatic Acid.

Marine salt, so generally employed, and so well known, is the substance from which the marine acid is extracted; for common salt is a combination of this acid with a substance called by chemists the *fixed mineral alkali*, or *soda*. They are separated from each other by certain processes, and the liquor thence resulting is marine or muriatic acid.

The marine acid has characters and properties, which render it very distinct from the preceding ones. In its highest state of concentration, it is only a little heavier than water, being in the ratio of about 19 to 17. Its colour is a lemon yellow, and its smell approaches to that of saffron.

Of Fluoric Acid.

Fluoric acid is of a very peculiar nature; it is found in fluor spar. It is obtained by pouring sulphuric acid upon the powdered spar in a leaden retort, and applying a gentle heat; the sulphuric repels the fluoric, and unites with the lime in its stead. The gas evolved is condensed in water to form liquid fluoric acid. To preserve the liquid acid, it must be kept in bottles lined on the inside with wax dissolved in oil; or in vessels of lead or platina. It dissolves silica, so that it cannot be kept in a glass vessel.

If the surface of a pane of glass be covered with wax, and a drawing of any kind be made upon it by cutting out the wax with proper instruments,—and the piece being put into a leaden receiver, and the gas disengaged from fluor spar by an Argand lamp thrown upon it, the drawing made on the glass will be etched in, and the drawing will be as permanent as the glass itself.

This acid is also used for destroying the polish on glass for *lamp shades*, &c.

Remark.—To enter into a particular account of all the mineral acids, would require more room than the nature of this work will admit; and it is the less necessary, as information respecting them may be found in almost every work on chemistry.

Of the Vegetable Acids.

The preceding remark must be applied also to this class of acids, as we mean to confine ourselves to the acetous, which is well known. Vinegar may be deprived of its superfluous water in a great many different ways; and in this state its strength is little inferior to that of many of the mineral acids.

The following is a simple method of concentrating it. Expose vinegar to the severe cold during winter: it will then in part freeze, and if the ice be removed the remainder will be vinegar of a much superior quality. By repeating this operation several times, you will obtain vinegar more highly concentrated, as the cold has been more intense. Artificial cold may be afterwards employed, and in a much greater degree than the greatest cold ever experienced in our climates.

If vinegar be mixed with charcoal dust, and exposed to a strong heat in a distilling apparatus, the water will be first thrown off when a boiling heat is attained;

and then by a stronger heat the pure concentrated acetous acid will be drawn over. This and the Prussic acid are the only vegetable acids that rise in distillation combined with water.

SECTION II.

Of Alkalies.

Alkalies are divided into *fixed* and *volatile*. The former have no smell: the latter have a pungent and penetrating odour. The general properties of pure alkalies are:

1st. That they have a peculiar caustic or burning taste, termed *alkaline*.

2d. That they change the blue juices of vegetables green; the yellow infusion of turmeric, brown; and the red infusion of Brazil wood, violet.

3d. That combining with acids they form neutral salts.

When a liquor therefore poured over sirups or infusions of the above kind, or upon paper that has been tinged with them, changes the colour in the manner described, it is alkaline, or an alkali predominates in it.

Alkalies however are seldom pure in the strict sense of the word, being generally combined with carbonic acid, or fixed air; and in this state they are termed mild, to distinguish them from caustic or pure alkalies, that is alkalies which have been freed from the fixed air.

The common method of depriving them of their fixed air is by throwing into an alkaline solution, or ley, a quantity of quick lime. The lime, by its greater affinity for the fixed air, seizes on it, and leaves the alkali in a caustic state; for limestone or chalk is pure lime saturated with fixed air, which it gives up when exposed to a strong heat; and it is then called quicklime. When saturated with the carbonic acid or fixed air, from the alkali it again becomes mild lime or chalk, and falls to the bottom of the vessel.

Of fixed Alkalies.

There are two kinds of fixed alkali, namely, the mineral or soda, and the vegetable or potash. They are called fixed, because though exposed to a strong heat they are not dissipated, but fuse like metals, and like them can endure a red heat. They facilitate the fusion of stones, earth, and sand; and for this reason are much used, especially the first, in glass houses. They are both of extensive use in the arts.

Fixed mineral alkali is obtained from marine salt; for when this salt is deprived of the acid which enters into its composition, the remainder is fixed mineral alkali; but to extract it from this substance requires a tedious and expensive process. The most common method of obtaining it, is to burn certain plants which grow on the sea coast, or are washed on shore by the tide. Of this kind is the plant kali or glass wort, from which the denomination of alkali is derived, and various other marine plants, such as *varec* or sea-wrack, *fuci*, &c. The ashes of these plants contain abundance of fixed alkali, which may be extracted and purified by lixiviating them, and then evaporating the ley. This is what is known in commerce under the name of *soda*.

Fixed vegetable alkali is commonly obtained by the combustion of the greater part of other plants, and of wood, such as common fire wood. A great deal is made in this manner, in different forests, where immense quantities of wood are burnt for that purpose in pits; the ashes which remain contain abundance of this fixed alkali, known in the shops under the name of *potashes*. By lixiviating them, and then evaporating the ley, an alkali much more active may be obtained.

Another method of obtaining fixed vegetable alkali, much purer, is to employ wine lees, and the tartar which adheres to the sides of wine casks. These substances

are formed into packets or masses of the size of the fist, and then burnt until they have assumed a white colour. By these means very pure fixed alkali may be produced. It is known in the shops under the name of *salt of tartar*, or *alkali of tartar*. It is absolutely the same as common potash. In this state however they are not entirely free from a combination of carbonic acid.

Common potash is composed of the metal called *potassium*, and oxygen, in the proportion of 100 of potassium to 20 of oxygen. Potassium is procured in large quantities by keeping potash in fusion with intensely hot iron. Potassium is solid at the usual temperature of the atmosphere, but quickly becomes soft and ductile when held in the hand. Its specific gravity is to that of water as 865 to 1000. It is perfectly white, and has the lustre of polished silver.

The two fixed alkalies, the vegetable and mineral, differ from each other principally by one peculiar property. The fixed vegetable alkali attracts the moisture of the air so strongly, that to preserve it in a solid state it must be put into well closed vessels. If left exposed to the air, it deliquesces of itself, and in this state is called *oil of tartar per deliquium;* an appellation very improper, for it is not an oil.

On the other hand, fixed mineral alkali, instead of attracting moisture, loses its own, and effloresces; that is, falls into dust: and for this reason it can be preserved with much more convenience than the other.

Volatile Alkali.

This alkali, or ammonia, is produced by the combustion of most animal matters, or by the putrefaction of animal or vegetable substances. The smell of putrefied bodies is owing to the alkali disengaged from them during that operation, by which nature reduces them in some measure to their first principles, in order that they may serve for new compositions. The strong odour which proceeds from privies, is a highly volatile alkali. It is called *volatile*, because a heat, inferior even to that of boiling water, is sufficient to disperse it in vapours, which always discover themselves by their penetrating smell.

Ammonia is a compound of hydrogen, and nitrogen in the proportion of one part, by weight, of the former, to four parts of the latter. It may be in a short time formed by the following process, so as to become evident to the sense of smelling.

On some filings of tin or zinc pour some moderately diluted nitrous acid, and after a short time stir into the mixture some quick lime, and a very strong pungent smell of ammonia will be produced.

All animal and vegetable substances, in a state of putrefraction, furnish ammonia; but in England it is generally procured by a dry distillation of bones, horns, and other animal substances. It is found in mineral waters, and whenever iron rusts in water, having a free communication with the air. Carbonate of ammonia is procured in large quantities from the waste liquor which is collected in the manufactories of coal gas.

Ammoniacal gas for chemical experiments may be procured by mixing one part of powdered sal ammoniac with two parts of quick lime in a retort, and applying the heat of a lamp which will disengage the gas in abundance.

When liquid ammonia is combined with carbonic acid, it takes a concrete form, and a beautiful white colour being then what is called *volatile salts*.

Muriate of ammonia is formed by combining ammonia with muriatic acid. It is known in commerce by the name of *sal ammoniac*.

SECTION III.

Of Neutral Salts.

When a salt is neither acid nor alkaline, if it neither turns sirup of violets nor blue paper red or green,* it is called *neutral.* The reason of this appellation is evident. Of this kind are marine salt, nitre, the different species of vitriol found in a natural state, and various other salts, both natural and artificial.

A neutral salt consists of an acid combined with an alkali, an earth, or a metal. We shall here give a few examples, enumerating the most common combinations of the three principal mineral acids, with different substances.

Thus, vitriolic acid, combined with zinc, forms white vitriol (sulphate of zinc);

With copper, blue vitriol;

With iron, green vitriol;

With argillaceous earth, alum;

With calcareous earth, selenite;

With volatile alkali, ammoniacal vitriol;

With fixed mineral alkali, Glauber's, Epsom, or Seidlitz salt;

With fixed vegetable alkali, vitriolated tartar.

Salts so formed are called *sulphates;* as, sulphate of iron, of copper, of zinc, of ammonia, &c. according as one or other of these substances is united with the acid.

The nitric acid, combined with fixed vegetable alkali, forms nitre;

With fixed mineral alkali, quadrangular or cubic nitre;

With volatile alkali, a nitrous ammoniacal salt;

With silver, a peculiar salt fusible by a moderate heat, known under the name of *lapis infernalis,* on account of its causticity.

These combinations are called *nitrates* of the substances joined to the nitric acid.

The marine acid, combined with fixed mineral alkali, forms common marine salt, or muriate of soda;

With fixed vegetable alkali, febrifuge salt of Sylvius, or muriate of potash;

With volatile alkali, sal ammoniac;

With mercury, if digested on it long enough to oxidate the metal completely, it forms corrosive sublimate.

When this muriate is not perfectly saturated with oxygen, it forms a salt called *mercurius dulcis.*

All neutral salts, composed of marine or muriatic acid, combined with alkalies, earths, or metals, are called *muriates.*

The vegetable acid, called the tartareous, combined with fixed vegetable alkali, forms tartar:

With fixed mineral alkali, the salt called *vegetable salt, Seignettes salt, or Sal polychrest:*

Acetic acid, when combined with vegetable alkali, forms a salt called *foliated earth of tartar* (acetate of potash):

With copper it forms verdigris (acetate of copper); a salt well known in commerce, and a violent poison:

With lead it forms a salt called *saccharum saturni* (acetate of lead), which is also a poison, and employed in the arts:

With mercury, acetate of mercury, a salt of great use in cases of syphilis.

We shall confine ourselves to this brief account of the best known compositions of the different acids with different substances. The number however might have been considerably increased; for every acid may be combined with almost all the alkalies, earths, and metals.

* This rule is liable to some exceptions. It may however be followed without any danger of much mistake.

ARTICLE II.

Of Oxygen.

We have already had occasion to mention this substance, for a knowledge of which we are indebted to modern chemistry. It has never been obtained alone; but its presence and effects are now so well known, that its existence may be considered as demonstrated. We shall here observe that almost all the phenomena explained formerly, by admitting the agency of an ideal principle, to which the old chemists gave the name of *phlogiston*, are now known to be produced by a play of chemical affinities, in which oxygen performs a conspicuous part. Thus, for example, in what used to be called the *calcination*, but now the *oxydation*, of metals, it was believed that the metals parted with phlogiston. It is however shewn by experiment, that instead of parting with anything, they receive an increase of weight; and it is known also, by taking proper care to perform the experiments in close vessels, that the air not only loses in its volume, but suffers also a diminution of its weight equal to what the metal has gained. But besides, the oxygen may be again separated from the metal, which yields a quantity of gas equal in weight and volume to what had been taken from the air by the metal.

The oxygen gas, thus separated from an oxide, is found to be that part of the atmosphere which serves for respiration; it is therefore called also *vital air*. When a metal, such as mercury for example, has been converted into an oxide, by exposure with heat, to atmospheric air, in a close vessel, the air which remains after all the oxygenous part has been taken up by the metal, is found to be a suffocating gas, distinguished by the name of *azotic gas*. It is not only unfit for respiration, but incapable of maintaining combustion; whereas oxygen gas, which may be separated from the oxide of mercury, or of manganese, &c., is better adapted for promoting combustion than atmospheric air itself; so that many substances, which will not inflame in common air, burn with great violence in oxygen gas. We shall mention one for the sake of illustration. If a piece of small iron wire, with a bit of lighted tinder to begin the combustion, be introduced into a vessel containing oxygen gas, it will burn with much greater intensity than flax or cotton, even though dipt in oil, will do in atmospheric air.

From this, and other considerations, which it would be too tedious here to enumerate, it is inferred that the oxygen in calces or oxides, in acids, and in all other combinations, the gaseous ones excepted, is in a concrete state; but that in the gaseous state it is united to something else, namely the matter of heat, or of light, or of both: and from this theory the process of combustion is thus explained: during the process, the oxygen of the oxygenous part of the atmosphere, unites itself to the burning body; and the heat and light which were united with the oxygen, and which held it in chemical solution, are liberated, producing that phenomenon commonly called burning. The heat and light therefore are not furnished by the burning body, but by the air.

When carbonaceous matters, such as coals, wood, &c., are employed in the process, a portion of the liberated heat is again taken up by a new gas which has been formed, namely carbonic acid gas, or fixed air, which we have already had occasion to mention.* It is the same, in every respect, as the gas that may be separated from limestone or chalk, by exposing it to heat. The matter of heat or *caloric* gives it the gaseous form; and oxygen communicates to it its acid properties. All acids indeed, as already mentioned, are indebted for their acidity to oxygen. Acidifiable bases, fully saturated with oxygen, produce acids; others, such as the metals, become oxides; but if means could be devised to saturate them completely it is probable they would all produce acids. Some of them indeed have been brought to this state.

* There are a variety of gases which all differ in many of their properties from common air; but to enter upon them would extend the present article beyond the limits which must be allotted to it.

Having had occasion, in speaking of the vitriolic acid, to mention the part which oxygen has in the formation of acids, it is needless to say any thing farther on that subject.

ARTICLE III.

Of Affinities.

It is also necessary that we should here say a few words respecting Affinities; as they are a key which serves to explain a great number of chemical compositions and decompositions.

Affinity is the force with which two substances tend to combine and maintain themselves in a state of union. Thus, for example, if vitriolic acid be poured over chalk, it drives out the carbonic acid, or fixed air, before united to it; lays hold of the calcareous earth; combines with it, and forms a mixture which is neither earth nor acid; but if fixed alkali, whether vegetable or mineral, be added to the solution, the calcareous earth will be expelled from its place; the vitriolic acid will seize on the fixed alkali, abandoning the former, and will thus form a new salt—an alkaline sulphate.

The moleculæ of the calcareous earth, and those of the vitriolic acid, have therefore a stronger tendency to unite, than those of the same earth and carbonic acid. The union is so perfect, that the fluid, though one of the ingredients in the compound be an earth, passes through a filtre. And hence it appears, that this result is not a mere division and interposition of the particles of the stone, between those of the solvent as was supposed, and is still believed by some who are not acquainted with the principles of chemistry. But, we might ask such persons, why do the particles of iron, dissolved in an acid, maintain themselves in the liquor, notwithstanding their excess of specific gravity? for, according to their philosophy, this is inexplicable. But if each particle of the iron be united to each particle of the solvent, the difficulty will vanish; and if we admit this principle, and also an inequality of force in the above tendency, all the phenomena of chemistry may be so easily explained, that the existence of such a force in the particles of bodies cannot be denied.

Besides, we have positive proofs of the force with which polished surfaces adhere, independently of any surrounding fluid. Nothing then is more natural, than to conceive a similar force between the minute particles of bodies: it would be sufficient to suppose them to have small facets of different forms and sizes, by which they adhere with a force that may be subject to very complex laws; since it may depend on the extent of the facet, and the density and form of the particle; for these may produce a great many variations.

These affinities or tendencies are indeed very unequal, and by way of example we shall observe, that the force with which calcareous earth combines with vitriolic acid, is less than that with which it combines with any alkali. On this account an alkali may be substituted instead of calcareous earth. All acids, in general, have more affinity for alkalies than for calcareous earths; for the latter than for metals; and for some metals than for others; which furnishes an easy method of decomposing certain mixtures. In the course of this Part we shall give a few curious and instructive examples.

Chemical affinity is sometimes rendered evident by the heat which is produced on mixing two cold bodies. A mixture of equal parts of water and oil of vitriol instantly acquires the temperature of boiling water. If potassium be dropped upon ice, the ice will be partially melted, the water decomposed, and a brilliant flame engendered by the action of the two substances upon each other.

Sometimes the effect of the admixture of different bodies is to produce another in which the properties of the constituents are totally lost. For example,— Equal parts of tin and iron, malleable and ductile metals, form an alloy which is neither

malleable nor ductile, being a very brittle metal. Liquid ammonia and muriatic acid, both strong smelling fluids, if mixed in equal proportions, produce muriate of ammonia, a fluid devoid of smell.

It often happens, if one body be added to a mixture of two others, that it will combine with one in preference to the other. This property which bodies possess of singling out those substances with which they form the strongest affinity, is called *elective* attraction.

ARTICLE IV.

Solution and Precipitation.

Solution is an operation, by which a fluid combines with the moleculæ of a solid, or of another fluid; so that each particle of the one contracts an adhesion with each particle of the other. This union or adhesion is produced by the affinity of these particles for each other; because, if a greater or less affinity does not exist, there can be no solution.

Solution does not consist in a mere attenuation of the body dissolved, and an interposition of its moleculæ between that of the fluid. When there is only an interposition of this kind, a separation soon takes place.

Precipitation is effected when the moleculæ of the dissolved body, being abandoned by the solvent, fall to the bottom of the liquor. This happens sometimes in consequence of a mere diminution in the force of the solvent, produced by diluting it with a great deal of water; but it is produced, for the most part, by presenting to the solvent some body for which it has a greater affinity than for the body already dissolved. For example, if a fixed alkali be poured into nitrous acid, holding in solution calcareous earth, the acid will seize on the alkali, on account of its greater affinity, and will abandon the earth, which will fall to the bottom of the vessel.

At other times, precipitation takes place, in consequence of presenting to the solution a body which, by combining with the dissolved body, forms a new mixture, insoluble in the solvent. Of this we have an example in the following operation: If calcareous earth be dissolved in nitrous or marine acid, on pouring into the solution vitriolic acid, the latter will seize on the earth, and form with it sulphate of lime, well known under the name of selenite. But as selenite is not soluble in these acids, nor even in water, unless it be in very large quantity, it falls to the bottom. The same thing takes place when vitriolic acid is poured into a solution of mercury in nitrous acid: the particles precipitated form what is called *white precipitate.*

ARTICLE V.

Of Effervescence and Fermentation.—Difference between them.

Nothing is more common than for those who have but little knowledge of chemistry, to confound these two phenomena; which however are essentially different: and it must be allowed that, till within a few years, the French chemists confounded these terms, though they did not confound the operations which they denote.

Effervescence is the motion, accompanied with heat, which often takes place during a solution. Thus, for example, when a little nitrous acid is poured over copper filings; or when vitriolic acid is poured over those of iron; or when a small quantity of the same acid is poured over calcareous earth, a violent ebullition is excited, till the combination is formed, when the commotion subsides, and the liquor becomes transparent. Such, in a few words, is effervescence.* Hence it is said that acids, in general, effervesce with alkalies, with metals, and with calcareous earths.

* In this case, one of the gases, alluded to in a former note, is produced, namely, *hydrogen gas*, or inflammable air. It has the property of burning when once inflamed: if mixed with atmospheric air, and then set fire to, it explodes like gunpowder.

But fermentation is entirely different: it is the intestine and spontaneous motion, produced in certain liquors, extracted from vegetable matters; and which, from being sweet and insipid, renders them spirituous and vinous. *Must*, for example, or the expressed juice of grapes, is not wine; there is not a single drop of spirit in it; but when exposed to a moderate heat, a play of affinities takes place, by which the liquor becomes turbid of itself, is internally agitated, throws up bubbles, which are found to be fixed air, and when the disengagement of these bubbles ceases, it is entirely a new liquor, spirituous, intoxicating, &c. The case is the same with beer, produced by the fermentation of malt, or the strong decoction of barley, prepared in a certain manner. This, as may be seen, is an operation very different from effervescence, as above described. When a person, therefore, speaking on chemical subjects, confounds these two words, the ears of the enlightened chemist are as much shocked as those of a philosopher would be, were he to hear abhorrence of a vacuum employed to explain any of the phenomena of nature.

ARTICLE VI.

Of Crystallization.

This appellation is given to that peculiar arrangement, which the greater part of salts, and even other bodies, affect to assume, after their solution in a liquid, when their parts, brought sufficiently near to each other by evaporation, dispose themselves in groups. As rock crystal was the first body in which this regular arrangement was observed, its name has been given to that discovered in many other bodies, and particularly salts, by the subsequent researches of chemists and naturalists.

Dissolve common salt in water, and evaporate the solution to a certain degree; if it be then left at rest in a cool place, the saline particles, brought near to each other, and falling together to the bottom of the vessel, or attaching themselves to the sides of the vessel, will form masses, in which the cubic figure will be easily distinguished; as prisms of six sides terminating in pyramids, implanted in each other, are distinguished in rock crystal. If crystallization be promoted at the surface, by evaporating the liquid, it takes place in the form of truncated square pyramids, composed of small cubes, heaped together in a certain order, one upon the other; as has been shewn by M. Rouelle, who has explained the phenomenon with great ingenuity.

If the salt held in solution be saltpetre, the crystals formed will be hexagonal prisms, terminated by hexagonal pyramids.

In short, each salt affects a peculiar form.

Alum crystallizes in exact octaedra, that is, figures formed of two quadrangular pyramids, having a common square base.

Vitriol of iron forms crystals which are oblique-angled cubes, or cubes the six faces of which are rhombuses with unequal sides.

The crystals of blue vitriol are compressed dodecaedra, the form of which cannot be described in a few words. Verdigrise, or the salt produced by a combination of vinegar and copper, forms crystals which are oblique-angled parallelopipedons.

Crystallized or candied sugar forms quadrangular prisms, cut obliquely by an inclined plane.

But, as before observed, there are a multitude of other bodies, besides salts, which possess the same property of forming themselves into masses, and affecting the same regular figures. Most ores and pyrites are distinguished by their particular form: mineralized lead, for example, has a great tendency to the right-angled or oblique-angled cubical form. Even stones, in such cases, observe a certain regularity. The crystals of gypsum, or plaster of Paris, are shaped like the point of a lance; and gypsum therefore is properly a salt. Calcareous spar, known under the name of Icelandic crystal, is always an oblique-angled parallelopipedon, inclined in the direction of its diagonal, and at determinate angles. In short, when metals cool slowly, their

particles are at liberty to arrange themselves in a regular form ; and it was long ago remarked that this was the case with antimony, and has since been observed in regard to iron, copper, zinc, &c.

As this phenomenon is one of the most curious in chemistry, it would afford matter for a very long article ; but as we have given a short view of the subject, we must refer the reader, for further information, to the "Essai de Cristallographie" of Romé Delisle, which appeared in 1772, in 8vo, and to the Abbé Haüy's late work on the same subject.

We shall now give a series of chemical experiments, which will be partly an application of the before-mentioned principles, or which will exhibit curious phenomena.

ARTICLE VII.

Various Chemical Experiments

EXPERIMENT I.

How a body of a combustible nature may be continually penetrated by fire without being consumed.

Put into an iron box a piece of charcoal, sufficient to fill it entirely, and solder on the lid. If the box be then thrown into the fire it will become red, and it may even be left in it for several hours or days. When opened after it has cooled the charcoal will be found entire, though there can be no doubt of its having been penetrated by the matter of the fire, as well as the whole metal of the box which contains it.

The cause of this effect is as follows. Before charcoal, or any other combustible body can be consumed, oxygen must have access to it ; but the contact of the atmosphere is prevented by the iron case being made quite close.

Hence too it happens, that coals covered with ashes require much longer time to be consumed, than if they were exposed to the open air. This phenomenon, though well known, could not be explained by any philosopher unacquainted with the nature and properties of oxygen.

EXPERIMENT II.

Apparent Transmutation of iron into copper or silver.

Dissolve blue vitriol in water, till the latter is nearly saturated ; and immerse into the solution small plates of iron, or coarse filings of that metal. These small plates of iron, or filings, will be attacked and dissolved by the acid of the vitriol ; while its copper will be precipitated, and deposited in the place of the iron dissolved.

If the bit of iron be too large to be entirely dissolved, it will be so completely covered with the cupreous particles, that it will seem to be converted into copper. This is an experiment commonly exhibited to those who visit copper mines. At least we have seen it performed at Saint-Bel in the Lyonnese. A key immersed some minutes in water, collected at the bottom of a copper-mine, was entirely of a copper colour when drawn out.

If you dissolve mercury in marine acid, and immerse in it a bit of iron ; or if this solution be rubbed over iron, it will assume a silver colour. Jugglers sometimes exhibit this chemical deception, at the expence of the credulous and ignorant.

Remark.—In this case, there is no real transmutation, but merely the appearance of one. The iron is not converted into copper ; the latter, held in solution by the liquor impregnated with the vitriolic acid, is only deposited in the place of the iron with which the acid becomes charged. Every time indeed that a menstruum, holding any substance in solution, is presented to another substance, which it can dissolve with more facility, it abandons the former, and becomes charged with the latter. This

is so certain, that when the liquor which has deposited the copper is evaporated, it produces crystals of green vitriol, which, as is well known, are formed by the combination of the vitriolic acid with iron. And this also is practised in the mine before mentioned. The liquor in question, which is nothing but a pretty strong solution of blue vitriol, is put into casks, or large square reservoirs; and pieces of old iron being then immersed in it, are at the end of some time converted into a sort of sediment, from which copper is extracted. The liquor thus charged with iron is evaporated to a certain degree, and wooden rods are immersed in it, which become covered with crystals of green vitriol.

This experiment may be made also by dissolving copper in the vitriolic acid, and then diluting the solution with a little water. This is a new proof that the liquor only deposits the copper with which it is charged.

EXPERIMENT III.

Different Substances successively precipitated, by adding another to the Solution.

In the former experiment we have seen copper precipitated by iron; we shall now shew iron itself precipitated. For this purpose, throw into a solution of iron a small bit of zinc; and in proportion as the latter dissolves, the iron will fall to the bottom of the vessel: it may easily be known to be iron, because it will be susceptible of being attracted by the magnet.

If you are desirous to precipitate the zinc, nothing will be necessary but to throw into the solution a bit of calcareous stone, such as white marble, or any other stone capable of making lime; the vitriolic acid will attack this new substance, and suffer to be deposited at the bottom of the vessel a white powder, which is zinc.

To precipitate the lime or calcareous earth, pour into the solution liquid volatile alkali (spirit of hartshorn); the earth, being abandoned by the acid, will deposit itself at the bottom of the vessel.

The calcareous earth may be precipitated also, and much better, by pouring into the liquor a solution of fixed alkali, such as fixed vegetable alkali; or by throwing into it fixed mineral alkali.

Remark.—It is by a similar effect that hard water decomposes soap, instead of dissolving it, and suffers to be deposited a greater or less quantity of calcareous earth. The manner in which this takes place is as follows.

Water in general is hard only because it holds in solution selenite or gypsum (a combination of vitriolic acid with calcareous earth), which it has dissolved in its passage through the bowels of the earth, or which has been formed by the water first becoming impregnated with vitriolic salts, and afterwards in its course meeting with and dissolving a portion of calcareous earth.

On the other hand, soap is an artificial combination of fixed alkali with oil, or with some other greasy substance, and which have no great affinity.

When soap therefore is dissolved in water impregnated with selenite, the vitriolic acid of the latter, having a greater tendency to unite with the fixed alkali than with the calcareous earth, which enters into the composition of the selenite, abandons that earth, and combines with the fixed alkali, in such a manner that the soap is decomposed; and as the oil is immiscible with water it is diffused through it in the form of white flakes, while the calcareous earth of the selenite falls to the bottom.

Here we have a new example of the use of chemistry, to account for certain common effects which are inexplicable to the philosopher unacquainted with that science.

EXPERIMENT IV.

By the mixture of two transparent Liquors to produce a blackish Liquor.—Method of making good Ink.

Provide a solution of green or ferruginous vitriol, and an infusion of gall-nuts, or of any other astringent vegetable substance, such as oak-leaves, well clarified and filtred; if you then pour the one liquor into the other, the compound will immediately become obscure, and at last black.

If the liquor be suffered to remain at rest, the black matter suspended in it will fall to the bottom, and leave it transparent.

Remark.—This experiment may serve to explain the formation of common ink; for the ink we use is nothing else than a solution of green vitriol mixed with an infusion of gall-nuts, and a little gum. The blackness arises from the property which the gall-nuts have of precipitating, of a black or blue colour, the iron held in solution by the water impregnated with vitriolic acid; but as the iron would soon fall to the bottom, it is retained by the addition of gum, which gives to the water sufficient viscidity to prevent the iron from being precipitated.

The reader perhaps will not be displeased to find here the following recipe for making good ink.

Take one pound of gall-nuts, six ounces of gum arabic, six ounces of green copperas, and one gallon of common water or beer: pound the gall-nuts, and infuse them in a gentle heat for twenty-four hours, without bringing the mixture to ebullition; then add the gum in powder. When the gum is dissolved, put in the green vitriol: if you then strain the mixture, you will obtain very fine ink.

EXPERIMENT V.

To produce inflammable and fulminating Vapours.

Put into a moderately sized bottle, with a short and wide neck, three ounces of oil or spirit of vitriol, and twelve ounces of common water. If you then throw into this mixture at different times, an ounce or two of iron filings, a violent effervescence will take place, and white vapours will arise from it. On a taper being presented to the mouth of the bottle, these vapours will inflame, and produce a violent detonation, which may be repeated several times, as long as the liquor continues to furnish similar vapours.

EXPERIMENT VI.

The Philosophical Candle.

Provide a bladder, into the orifice of which is inserted a metal tube, some inches in length, and so constructed, that it can be fitted into the neck of a bottle containing the same mixture as that used in the preceding experiment.

Having then suffered the atmospheric air to be expelled from the bottle by the elastic vapour of the solution, apply to the mouth of it the orifice of the bladder, after carefully expressing from it the common air.* The bladder, by these means will become filled with inflammable air; which if you force out against the flame of a taper, by pressing the sides of the bladder, will form a jet of a beautiful green flame. This is what chemists call a *philosophical candle.*

EXPERIMENT VII.

To make an Artificial Volcano.

For this curious experiment, which enables us to assign a very probable cause for volcanoes, we are indebted to Lemery.

* Great care must be taken not to omit this precaution; for a mixture o. inflammable and atmospheric air will explode with violence, instead of burning.

Mix equal parts of pounded sulphur and iron filings, and having formed the whole into a paste with water, bury a certain quantity of it, forty or fifty pounds for example, at about the depth of a foot below the surface of the earth. In ten or twelve hours after, if the weather be warm, the earth will swell up and burst, and flames will issue out, which will enlarge the aperture, scattering around a yellow and blackish dust.

It is probable that what is here seen in miniature, takes place on a grand scale in volcanoes; as it is well known that they always furnish abundance of sulphur, and that the matters they throw up abound in metallic and perhaps ferruginous particles; for iron is the only metal which has the property of producing an effervescence with sulphur, when they are mixed together.

But it may be easily conceived, from the effect of a small quantity of the above mixture, what thousands or millions of pounds of it would produce. There is no doubt that the result would be phenomena as terrible as those of earthquakes, and of those volcanic eruptions with which they are generally accompanied.

EXPERIMENT VIII.

To make Fulminating Gold.

First to make Aqua Regia, by mixing four parts of spirit of nitre with one of sal ammoniac. Put some fragments of pure gold into this liquor, and when they are dissolved, add to the liquor a solution of fixed alkali, called also *oil of tartar per deliquium*. The gold will be precipitated to the bottom in the form of a yellow powder, which must be collected by pouring off the supernatant liquid. If this powder be then washed in warm water and dried, you will have fulminating gold.

To make it detonate, put a very small quantity of it on the point of a knife, and hold it over the flame of a taper. As soon has it has acquired a certain degree of heat, the powder will inflame with a terrible explosion, far greater than that which would be produced by a like quantity of gunpowder.

Gold prepared in this manner does not detonate by the application of heat alone; mere touching is sufficient to produce the same effect. We have seen some particles of fulminating gold, which had got between the neck of a bottle and the stopper, during the act of shutting it, make a sudden explosion, so as to break the bottle to pieces, and wound the person who held it. The same thing would infallibly take place if this gold were triturated in a mortar; or if one should attempt to fuse it, in order to reduce it to a metallic mass, without the necessary preparations.

Remarks.—The gold would not be fulminating, if the aqua regia were made by a mixture of spirit of nitre and spirit of marine salt, or spirit of nitre to which marine salt had been added, (for it is prepared in all these ways); and if the gold were precipitated by fixed alkali, in order to render the gold fulminating, volatile alkali must be united either with the aqua regia or the precipitant.

If aqua regia, therefore, made with spirit of nitre and spirit of marine salt, be employed to dissolve gold, it must be precipitated by volatile alkali. By these means you will still obtain fulminating gold.

To deprive it of its fulminating property, pour over it vitriolic acid, or a solution of fixed alkali. By this process a combination is formed which destroys in the gold its detonating quality; if it be then washed, it will be found in a powder, which may be reduced without any danger by the usual means.

EXPERIMENT IX.

To make Fulminating Powder.

Mix together three parts of nitre, two of well dried alkali, and one of sulphur; if a little of this mixture be put into an iron spoon over a gentle fire capable how-

ever of melting the sulphur, when it acquires a certain degree of heat, it will detonate with a loud noise, like the report of a small cannon.

This would not be the case, if the mixture were exposed to a heat too violent; the parts only most exposed to the fire would detonate, and by these means the effect would be greatly lessened.

If thrown on the fire it would not detonate, and would produce no other effect than pure nitre, which indeed detonates, but without any explosion.

Mix eight grains of nitrate of potash with four grains of phosphorus, place the mixture on a warm anvil, and striking it smartly with a warm hammer, a violent detonation will be produced.

EXPERIMENT X.

A liquor which becomes coloured and transparent alternately, when exposed to or removed from the contact of the external air.

Digest copper, that is to say dissolve it slowly by means of a gentle heat, in a strong solution of volatile alkali. As the solvent attacks the copper, it will acquire a beautiful blue colour. If you pour some of this liquor into a small bottle till it is nearly full, and then close it well with a stopper, the colour will gradually become fainter, and at last disappear. On opening the bottle, the colour will return; and this alternation may be produced as often as you choose.

EXPERIMENT XI.

Pretended Production of Iron.

Take clay, or the ashes of burnt vegetables or animals, and draw over them an artificial magnet. By these means you will often attract some particles of iron, which will adhere to the magnet. You may then rest assured that no iron, in a metallic state, remains in the earth or ashes.

Then mix the remaining earth or ashes with pounded charcoal, and having formed the mixture into a paste with linseed oil, put the whole into a crucible, and expose it to a red heat for some time, but not intense enough to produce vitrification. When the mass is cold, and reduced to dust, if the artificial magnet be again drawn over it, a great many iron particles will be attracted by it, and will adhere to it.

Remark.—Some have pretended to give this experiment as a proof that iron may be produced by clay and linseed oil. A celebrated chemist, member of the Academy of Sciences, even entertained this idea, notwithstanding the opposition he experienced from one of his brother members. But, in our opinion, no chemist at present will see in it a production of iron.

It would indeed be wrong to suppose, after the iron found at first by the magnet has been extracted, that no more remains in it. The magnet attracts iron only in its metallic state, or when it approaches near to it; but some still remains in the state of an oxide, and in this state it is not susceptible of being attracted by the magnet, as may be proved by an experiment made on oxide formed artificially by the torrefaction of iron, or on the rust of that metal.

Besides, it is well known that of all the metals iron is that most universally diffused throughout the earth; it is the colouring principle of common or red clay, and when clay is so coloured it contains iron.

What then is the effect of the torrefaction of clay with charcoal dust and linseed oil, or any other fat oily body which has a strong affinity for oxygen? Such bodies having a greater affinity for that principle than iron has, by the aid of a proper temperature, they decompose the oxide, and seizing on its oxygen leave the iron in a metallic state, and consequently susceptible of being attracted by the magnet. This is the whole secret of the operation.

But it may be said, what reason is there to think that these wood-ashes contain iron? In answer to this question, we shall observe, that iron being diffused in great abundance throughout all nature, it is a constituent part in a great many vegetable productions: in some of them it is found even in a metallic state; that it is susceptible of great attenuation; and that when dissolved in any liquid it passes with it through the filtre, at least in part. Hence it may easily ascend with the sap of plants; it circulates in the human body with the blood: in short, some even assert that plants are coloured by this metal with the concurrence of light; so that without iron and light all plants would be entirely white.

EXPERIMENT XII.

With two liquids mixed together, to form a solid body, or at least a body which has consistence.

Make a highly concentrated solution of fixed alkali, and another of nitrate of lime: if the two solutions be mixed together, there will be an abundant precipitation of a matter which will assume a sort of solidity.

This phenomenon appeared so wonderful to the old chemists, that the operation by which it is produced was called the *chemical miracle*. There is however nothing wonderful in it; for what takes place is as follows:

The two solutions being mixed, the nitrous acid abandons the earth, to seize on the fixed alkali, and with it remains in solution and transparent in the supernatant liquor. The earth is then precipitated, and forms the solid body which results from the mixture.

We shall here give another operation, which might with more justice be called a *chemical miracle*. We are indebted for it to a remark of M. de Lassonne, first physician to the queen.

EXPERIMENT XIII.

To form a combination, which when cold is liquid and transparent; but which when warm becomes thick and opake.

Put equal quantities of fixed alkali, either mineral or vegetable, and of well pulverised quick-lime, into a sufficient quantity of water, and expose it to a strong and speedy ebullition. Then filtre the product, which at first will pass through with difficulty, but afterwards with more ease, and preserve it in a bottle well stopped. This liquor, when made to boil either in the bottle or in any other vessel, will become turbid, and assume the consistence of very thick glue; but when cold it will recover its fluidity and transparency; and this alternation may be repeated as often as you choose.

M. de Lassonne made many experiments to discover the cause of this singular phenomenon; and he assigns one which may be seen in the "Mémoires de l'Académie des Sciences" for 1773.

EXPERIMENT XIV.

To make a flash, like that of lightning, appear in a room, when any one enters with a lighted candle.

Dissolve camphor in spirit of wine, and deposit the vessel containing the solution in a very close room, where the spirit of wine must be made to evaporate by speedy and strong ebullition. If any one then enters the room with a lighted candle, the air will inflame; but the combustion will be so sudden, and of so short duration, as not to occasion any danger.

It is not improbable that the same effect might be produced, by filling the air of an apartment with the dust of the seed of a certain kind of lycoperdon, which is inflammable; for this seed, which is exceedingly minute, and like fine dust, inflames in

the same manner, as the pulverised resin used for the torches of the Furies, and for representing lightning at the opera. And perhaps it would be better to substitute it for resin, as it does not produce that strong smell which results from the latter when burnt, and which is so disagreeable to the spectators.

EXPERIMENT XV.

Of Sympathetic Inks; and some tricks which may be performed by means of them.

Sympathetic inks are certain liquors which alone, and in their natural state, are colourless; but which, by being mixed with each other, or by some particular circumstance, assume a certain colour.

Chemistry presents us with a great many liquors of this kind, the most curious of which we shall here describe.

1st. If you write with a solution of green vitriol, to which a little acid has been added, the writing will be perfectly colourless and invisible. To render it visible, nothing will be necessary but to immerse the paper in an infusion of gall nuts in water, or to draw a sponge moistened with the water over it. Those who have observed the fourth experiment may readily see, that in this case the ink has been formed on the paper. In the making of ink the two ingredients are combined before they are used for writing; here they are not combined till the writing is finished: this is the whole difference.

2d. If you are desirous of having an ink that shall become blue, you must write with a solution of green vitriol, and moisten the writing with a liquor prepared in the following manner:

Make four ounces of tartar, mixed with the same quantity of nitre, to detonate on charcoal; then put this alkali into a crucible with four ounces of dried ox blood, and cover the crucible with a lid, having in it only one small aperture; calcine the mixture over a moderate fire, till no more smoke issues from it; and then bring the whole to a moderate red heat. Take the matter from the crucible, and immerse it, while still red, in two quarts of water, where it will dissolve by ebullition; and when the liquor has been reduced to one half, it will be ready for use. If you then moisten with it the writing above mentioned, it will immediately assume a beautiful blue colour. In this operation, instead of black ink, there is formed Prussian blue.

3d. If you dissolve bismuth in nitric acid, and write with the solution, the letters will be invisible. To make them appear, you must employ the following liquor.

Boil a strong solution of fixed alkali with sulphur, reduced to a very fine powder, until it dissolves as much of it as it can: the result will be a liquor which exhales vapours of a very disagreeable odour, and to which if the above writing be exposed, it will become black.

4th. But of all the kinds of sympathetic ink, the most curious is that made with cobalt. It is a very singular phenomenon, that the characters or figures traced out with this ink, may be made to disappear and to re-appear at pleasure: this property is peculiar to ink obtained from cobalt; for all the other kinds are at first invisible, until some substance has been applied to make them appear; but when once they have appeared they remain. That made with cobalt may be made to appear and to disappear any number of times at pleasure.

To prepare this ink, take zaffre, and dissolve in it *aqua regia* (nitro-muriatic acid) till the acid extracts from it every thing it can; that is the metallic part or the cobalt, which communicates to the zaffre its blue colour; then dilute the solution, which is very acrid, with common water. If you write with this liquor on paper, the characters will be invisible; but when exposed to a sufficient degree of heat, they will become green. When the paper has cooled they will disappear.

It must however be observed, that if the paper be heated too much, they will not disappear at all.

Remark.—With this kind of ink, some very ingenious and amusing tricks, such as the following, may be performed.

1st. *To make a drawing which shall alternately represent summer and winter.*

Draw a landscape, and delineate the ground and the trunks of the trees with the usual colours employed for that purpose; but the grass and leaves of the trees with the liquor above mentioned. By these means you will have a drawing, which at the common temperature of the atmosphere will represent a winter piece; but if it be exposed to a proper degree of heat, not too strong, you will see the ground become covered with verdure and the trees with leaves, so as to present a view in summer.

Screens painted in this manner were formerly made at Paris. Those to whom they were presented, if unacquainted with the artifice, were astonished to find, when they made use of them, that the views they exhibited were totally changed.*

2d. *The Magic Oracle.*

Write on several leaves of paper, with common ink, a certain number of questions, and below each question write the answer with the above kind of sympathetic ink. The same question must be written on several pieces of paper, but with different answers, that the artifice may be better concealed.

Then provide a box, to which you may give the name of the Sybil's cave, or any other at pleasure, and containing in the lid a plate of iron made very hot, in order that the inside of it may be heated to a certain degree.

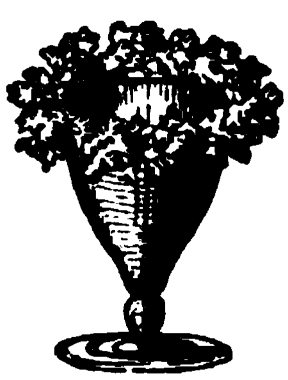
Having selected some of the questions, take the bits of paper containing them, and tell the company that you are going to send them to the sybil or oracle, to obtain an answer; introduce them into the heated box, and when they have remained in it some minutes take them out, and shew the answers which have been written.

You must however soon lay aside the bits of paper; for if they remain long in the hands of those to whom the trick is exhibited, they would see the answers gradually disappear, as the paper becomes cold.

EXPERIMENT XVI.

Of Metallic Vegetations.

To see a shrub rise up in a bottle, and even throw out branches, and sometimes a kind of fruit, is one of the most curious spectacles exhibited by chemistry. The operation by which this delusive image is produced, has been called chemical or metallic vegetation, because performed by means of metallic substances: and it is not improbable that some respectable persons, who thought they saw a real palingenesy, have been deceived by a similar artifice. However this may be, the following are the most curious of these vegetations, which in fact are only a sort of crystallizations.



Arbor Martis, Tree of Mars.

Dissolve iron filings in spirit of nitre (aquafortis) moderately concentrated, till the acid is saturated; then pour gradually into the solution a solution of fixed alkali, commonly called *oil of tartar per deliquium.* A strong effervescence will take place; and the iron, instead of falling to the bottom of the

* These drawings are now very commonly exhibited for sale in the shop windows in London.

Fig. 59.

vessel, will afterwards rise, so as to cover its sides, forming a multitude of ramifications heaped one upon the other, which will sometimes pass over the edge of the vessel, and extend themselves on the outside, with all the appearance of a plant. If any of the liquor be spilt, it must be carefully collected, and be again put into the vessel, where it will form new ramifications, which will contribute to increase the mass of the vegetation.

Two of these vegetations, copied from a memoir of M. Lemery, junior, and inserted among those of the Academy of Sciences for 1706, are represented Fig. 58 and 59. A very probable explanation of the phenomenon may be found among those of 1707.

Arbor Dianæ, Tree of Diana.

This kind of vegetation is called the Tree of Diana, because it is formed by means of silver; as the former is called the Tree of Mars, because produced by iron. We shall here give two processes for this purpose, one of them by Lemery, and the other by Homberg.

Dissolve an ounce of pure silver in a sufficient quantity of aquafortis, exceedingly pure, and of a moderate strength, and having put the solution into a jar, dilute it with about twenty ounces of distilled water. Then add two ounces of mercury, and leave the whole at rest. In the course of forty days, there will rise from the mercury a kind of tree, which throwing out branches will represent a natural vegetation.

Should this process, which is in other respects very simple, be thought too tedious as to time, the following one of Homberg may be employed.

Form an amalgam of a quarter of an ounce of very pure mercury, and half an ounce of fine silver reduced into filings or leaves; that is to say, mix them together by trituration in a porphyry mortar, by means of an iron pestle. Dissolve this amalgam in four ounces of very pure nitric acid, moderately strong, and dilute the solution in about a pound and a half of distilled water, which must be stirred, and then preserved in a bottle well stopped. Pour an ounce of this liquor into a glass, and throw into it a small bit of an amalgam of mercury and silver, similar to the former, and of the consistence of butter. Soon after you will see rising from the ball of amalgam a multitude of small filaments, which will visibly increase in size, and throwing out branches will form a kind of shrubs.

Homberg, in the Memoirs of the Academy for 1710, gives a method of making a a similar vegetation, either with gold or silver, in the dry way; that is to say by distillation without any solution.

There is still another kind of vegetation, mentioned by M. de Morveau, which he calls *Jupiter's beard*, because tin forms a part of its composition: the process for making it may be seen in his "Essais Chimiques."

Non-metallic Vegetation.

Cause to decrepitate, on burning charcoal, eight ounces of saltpetre, and place it in a cellar, in order that it may produce oil of tartar per deliquium: then gradually pour over it, to complete saturation, good spirit of vitriol, and evaporate all the moisture. The result will be a white, compact, and very acrid saline matter. Put this matter into an earthen dish, and having poured over it a gallon of cold water, leave it exposed to the open air. At the end of some days the water will evaporate, and there will be found all around the vessel ramifications in the form of needles, variously interwoven with each other, and about 15 lines in length. When the water is entirely evaporated, if more be added the vegetation will continue.

It may be readily seen that this is nothing but the mere crystallisation of a neutral salt, formed by the vitriolic acid and the alkali of the nitre employed; that is to say vitriolated tartar.

EXPERIMENT XVII.

To produce heat, and even flame, by means of two cold liquors.

Put oil of guiacum into a bason, and provide some spirit of nitre, so much concentrated, that a small bottle, capable of holding an ounce of water, may contain nearly an ounce and a half of this acid. Make fast the bottle containing the acid, to the end of a long stick; and after taking this precaution, pour about two thirds of the acid into the oil in the bason; the result will be a strong effervescence, followed by a very large flame. If an inflammation does not take place in the course of a few seconds, you have nothing to do but to pour the remainder of the nitrous acid over the blackest part of the oil; a flame will then certainly be produced, and there will remain, after the combustion, a very large spongy kind of charcoal.

Oil of turpentine, oil of sassafras, and every other kind of essential oil, may be made to inflame in the like manner. The same phenomenon may be produced with fat oils, such as olive oil, nut oil, and others extracted by expression, if an acid, formed by equal parts of the vitriolic and nitrous acids well concentrated, be poured into them.

EXPERIMENT XVIII.

To fuse iron in a moment, and make it run into drops.

Bring a rod of iron to a white heat, and then apply to it a roll of sulphur; the iron will be immediately fused, and will run down in drops. It will be most convenient to perform this experiment over a bason of water, in which the drops that fall down will be quenched. On examination they will be found reduced to a kind of cast iron.

This process is employed for making shot used in hunting, as the drops, by falling into the water, naturally assume a round form.

We shall here add two little experiments, merely because they are usually given in books of Philosophical Recreations.

EXPERIMENT XIX.

To fuse a piece of money in a walnut-shell, without injuring the shell.

Bend any very thin coin, and having put it into the half of a walnut shell, place the shell on a little sand, in order that it may remain steady. Then fill the shell with a mixture made of three parts of very dry pounded nitre, one part of the flowers of sulphur, and a little saw-dust well sifted.

If you then inflame the mixture, as soon as it has melted you will see the metal completely fused in the bottom of the shell in the form of a button, which will become hard when the burning matter around it is consumed. The shell employed for the operation will have sustained very little injury. The cause of this, no doubt, is, that the activity of the fire, assisted by the vitriolic acid contained in the sulphur, acts with such rapidity, that it has not time to burn the shell.

A small ball of lead, closely wrapped up in a bit of paper,* may be fused in the same manner, by exposing it to the flame of a candle. The paper will not be hurt, except in the bottom, where it will have a small hole through which the metal has run.

* If the paper be not in perfect contact with the lead, the experiment will not succeed.

EXPERIMENT XX.

Action of quicksilver upon nitrous acid.

Put about half an ounce of quicksilver into an oil flask, and on it pour about an ounce of diluted nitrous acid. The nitrous acid will be decomposed by the quicksilver with great rapidity; the bulk of the acid will be changed into a beautiful green, while its surface will appear of a dark crimson colour; and a vivid and pleasing effervescence will go on while the acid acts on the quicksilver. When a part of the metal is dissolved, the acid will by degrees become paler, till it is as pellucid as water.

EXPERIMENT XXI.

Throw a little pulverized muriate of lime into a spoonful of burning charcoal, and a beautiful red flame will be produced, the colour of which may be heightened by agitating the mixture during inflammation.

EXPERIMENT XXII.

To split a piece of money into two parts.

Fix three pins in the table, and lay the piece of money upon them; then place a heap of the flowers of sulphur below the piece of money, and another above it, and set fire to them. When the flame is extinct, you will find, on the upper part of the piece, a thin plate of metal, which has been detached from it.

It is to be observed that the value of about three-pence might be detached in this manner from a piece of gold such as a guinea, by employing sulphur to the value of fifteen or twenty pence; so that this experiment can never become dangerous to the public. Besides, the piece of money loses, in a great measure, the boldness of its impression: those therefore who might attempt to debase the current coin in this manner, would become victims to their dishonesty.

EXPERIMENT XXIII.

Illustration of continued identity under apparent change of form.

Dissolve separately equal weights of sulphate of copper and crystal of carbonate of soda, in sufficient quantities of boiling water; pour them together, while hot, into a flat pan, and when the water has evaporated a little, and the whole is suffered to cool, the salts will shoot; the sulphate of copper in blue, the soda in white crystals, similar to what they were before they were dissolved.

EXPERIMENT XXIV.

Examples of different Fluids forming Solids, when mixed.

Dissolve muriate of lime and carbonate of potash, separately, in water, so as to form a saturated solution of each. On pouring the two transparent fluids together, muriate of potash and carbonate of lime will be formed; and if the mixture be well stirred, a solid mass will be produced.

Again: Take a saturated solution of sulphate of magnesia (Epsom salts), and pour into it a like solution of caustic potash, or soda, and the mixture will immediately become solid.

These instances of sudden conversion of two fluids into a solid, have been called *chemical miracles.*

What we have said is sufficient to inspire our readers with a desire to become better acquainted with this useful science. We shall therefore point out to them a few works which will assist them in prosecuting that design. Though chemistry has experienced a complete revolution since the discoveries of Lavoisier, Priestley, Black, Higgins, Cavendish, and others, some useful information may still be ga-

thered from the works of the old chemists: we therefore hope we may be allowed to introduce here "Les Elemens de Chimie Theoretique et Practique" of Macquer, 3 vols. 12mo; the first of which contains the theoretical, and the other two the practical part. To this work we may add the "Manuel de Chimie" of Baumé, which contains a great many curious processes useful in the arts. Boerhaave's Elements of Chemistry were formerly held in great estimation; but at present this work is of less value: it may however serve as a good introduction to the modern chemistry. Wiegleb's Chemistry, transl ate dby Hopson, though founded on the old theory, may also be perused with advantage, by those who are desirous of acquiring a thorough knowledge of this science. Also the chemical works of Bishop Watson. The best systems of chemistry, and elementary works according to the new theory, are those of Lavoisier, Fourcroy, Chaptal, Gren, Jacquin, Buillon-Lagrange, Brisson, and Thomson. As books of reference we can recommend Henry's Manual and Parkinson's Chemical Pocket Book. A great many curious and interesting papers on chemistry in general, and its application to the arts, may be found also in the Philosophical Magazine.

DISSERTATION ON THE PHILOSOPHER'S STONE; ON AURUM POTABILE; AND ON PALINGENESY.

We have here mentioned the two most celebrated chimeras of the human mind: for though the quadrature of the circle in geometry, and the perpetual motion in mechanics, have been much celebrated also on account of the fruitless attempts made to solve these two problems, their celebrity is inferior to that of the first two above mentioned, nearly in the same ratio as the importance of squaring the circle is inferior to that of acquiring immense riches, or of rendering ourselves almost immortal. A great many persons therefore, seduced by these chimeras, have at all times bestowed incredible labour on researches respecting them.

Such is the character of the human mind,

> ——— Quid non mortalia pectora cogunt
> Auri sacra fames, vitæque immensa cupido!

We shall therefore here offer a few observations on these chemical problems, either because they afford matter which comes within our province, or because what we shall say may serve as a preventive of that illusion to which so many persons have been dupes.

SECTION I.

Of the Philosopher's Stone.

The philosopher's stone (formerly called *the work*, by way of excellence, or *chrysopea*,* the transmutation of the base and imperfect metals into gold or silver) has since time immemorial been an object to which the attention of multitudes of people, either versed in chemistry or scarcely initiated in the science, has been directed. The vulgar even think that it is the sole object of chemistry; and it must indeed be allowed that it was in some measure the fault of those who first cultivated that noble branch of philosophy: there were few of them who did not suffer themselves to be blinded by the illusion of attempting to make gold.

But at present fewer people are infatuated with the philosopher's stone; at any rate

* Χρυσοποιια, auri fabricatio, or gold-making.

few of the enlightened chemists employ themselves with the means of making gold; but there are still many other persons, who though they have scarcely an idea of the simplest operations of chemistry, waste their time in vain attempts to regenerate that precious metal. They are often seen proceeding at hazard, and still imagining themselves on the point of succeeding; in the midst of poverty consoling themselves with the agreeable idea, that this indigence will be succeeded by the possession of immense treasures. They call themselves *adepts*, because they pretend to have reached to the summit of philosophy, *quasi summam sapientiam adepti;* they speak enigmatically and in an unintelligible manner, because mankind in general do not deserve to possess such a secret. Filled with empty pride, they cast a sardonic smile of contempt on the rational chemists, and on those who endeavour to deduce phenomena from clear and established principles.

We might say to the searchers after the philosopher's stone, Before attempting to make gold, first decompose and re-compose it; for if there be any method of ascertaining and demonstrating the constituent principles of any substance, it is that of decomposition and re-composition. We might say also to those alchemists, Before you make for us the precious metals, such as gold and silver, make for us only lead;* for before you proceed to the most difficult, method requires that you should execute the easiest. But we are acquainted with no chemical operation which resolves either of these problems. Gold, as stubborn in regard to decomposition as to composition, always remains the same, in whatever manner it be treated: it is only more or less attenuated, but is never in the state of calx. It has been kept for several years in fusion, without losing the least part of its weight.

But let us hear the alchemists, and learn what are their pretensions in regard to the formation of metals.

According to them, metals are all formed of an earth, which they call *mercurial*, but more or less mature, more or less mixed with heterogeneous matters; so that, to convert the imperfect into perfect metals, nothing is necessary but to free them from these heterogeneous matters, and to mature them.

All this is very fine: but who has proved the existence of this mercurial earth? who has proved that the difference among metals consists in this greater or less maturity? by what means is it to be produced? To these questions no solid answer can be given. The partisans of this idea, seduced by words, have no just and precise conception of what they say.

According to other alchemists, mercury contains in principle all the perfect metals; it has the splendour of them, and nearly the weight; it is even heavier than silver. If it is fluid and exceedingly volatile, it is because it is alloyed with impurities which degrade it. The question then is to fix the mercury, by freeing it from these impurities. We should then have the mercury of the philosophers, which would require only a certain degree of baking to be brought to a red heat, and the result would be gold; brought to a white heat, it would furnish silver; nay, this matter would have such an activity on the impure parts of other metals, that by throwing a pinch of it into a crucible filled with melted lead, it would transmute it into silver or gold, according as it had been carried to a white or a red heat. But the great matter is, how to destroy the impurities by which quicksilver is debased. Aristeus, a celebrated adept, teaches us the process in the clearest manner, in his "Code de Verité." "Take," says he, "king Gabertin, and the princess Beya his sister, a young lady, beautiful, fair, and exceedingly delicate: marry them together, and Gabertin will die almost immediately. Be not however alarmed; after eighty days, Gabertin will revive from his ashes, and become more beautiful and more perfect than he was before his death; will beget with

* Some have pretended to make iron; but it is now proved that, in the operation employed for this purpose, the iron is only restored to its metallic form.

Beya a red child, more beautiful and perfect than themselves." After this, will any one pretend to say that the alchemists explain themselves obscurely? What true adept, for there are true and false, and every one thinks himself among the former, will not evidently see in this allegory the whole process of the fixation of mercury and of the powder of projection?

This language, and this affectation of obscure allegories, are no doubt very proper for making these pretended adepts be considered as finished and contemptible quacks, or perhaps as people whose brains have been deranged by the heat of their furnaces. But the partisans of their researches and follies allege pretended facts; and it is our business to make them known.

It is related that Helvetius, a physician and celebrated professor in Holland, having declaimed one day with great violence, in one of his lectures, against the vanity and absurdity of pretending to make gold, was visited by an adept, who gave him a certain powder, a pinch of which, thrown into a crucible filled with melted lead, would transform it into gold: that the learned Dutchman did so, and obtained from his lead a considerable quantity of that metal. Helvetius then hastened to find the adept; but the latter had given him a false address, and was not to be found; for the chemists of this order never fail to disappear at the moment when they have given a proof of their profound knowledge.

The same thing occurred, it is said, to the emperor Ferdinand. An adept came to him, and offered to transform mercury into gold. Mercury was put into a crucible in the presence of the prince, and the adept having performed certain operations, a button of gold was found in the bottom of the vessel. But while those present were employed in examining and assaying the gold, the adept disappeared, to the great regret of the emperor, who already beheld in idea the immense treasures which he hoped to obtain by the acquisition of this grand secret.

At the sale of the effects left by M. Geoffroy, in 1777, there were three nails, which, as it was said, were a proof of the possibility of at least transmuting silver into a common metal, such as iron. They were the work, as asserted, of a celebrated adept, who wished to prove to Geoffroy the possibility of the transmutation of metals. One of these nails was converted into silver, by being dipped in an appropriate liquor; the head of the other only having been dipped, the remainder of it was iron; and the point of the third having been dipped, that part was silver, and the head iron.

Notwithstanding these authorities we have no belief in the philosopher's stone. It is very probable, that in all these pretended transmutations, there was some deception, even if the above accounts were true. In short, we shall believe in the philosopher's stone when we have seen any adept perform before us the same operations; but he must permit us to furnish him with the crucibles, rods, and ingredients; for it is more than probable, that if gold has been made in this manner, it either existed in the matters employed, or some of it was introduced into them by sleight of hand.

However, the alchemists pretend that all the fables of antiquity are nothing else than the process of the grand work explained symbolically. The conquest of the golden fleece, the Trojan war, the events which followed it, and the whole mythology, are only emblems of the chrysopea, prudently veiled by the ancient philosophers, who did not wish that their secret, become common, should be employed to produce an immense increase of the precious metals, which must then have lost their value, and have ceased to be the medium of commerce among mankind. The reader may see, in a curious work by Dom Pernetty, entitled "Les Fables Egyptiennes et Grecques," 3 vols. 8vo., including the "Dictionnaire Mytho-hermetique," how far human sagacity may be extended, to find an explanation of such fables. But every thing may be explained in the same manner. We have heard of an adept, in the Faubourg

Saint Marceau, who, being persuaded that the whole Roman history was a fiction, intended to give a chemical explanation of it, which would serve as a supplement to the Fables Egyptiennes et Grecques. We have even heard that the history of the combat of the Horatii and the Curiatii was explained in it, with an appearance of truth, capable of making us doubt whether that famous circumstance in the Roman history ever really took place.

SECTION II.

Of Potable Gold.

If there be no reason to think that gold will ever be made, is it not possible to employ this precious metal for prolonging life? Gold is a metal unalterable, and as difficult to be destroyed as to be made; it is the sovereign of the metallic world, as the sun, to which it is assimilated, is in the system of the universe. Nature therefore must have concealed in this valuable body the most useful remedies; but to make it useful, in this respect, it is necessary that it should be introduced into the body in a liquid form; it must in short be rendered potable: let us endeavour then to make potable gold. A life indefinitely prolonged is certainly worth all the treasures in the world. Such is in substance the reasoning of the alchemists, and therefore they have subjected gold to a multitude of operations, by means of which they have pretended to render it soluble, like a salt in water. The substance they produce has indeed the appearance of it; but to speak the truth, it is only gold very much attenuated, and by these means suspended in the liquid: in short, it is in no manner combined with the fluid, and it even gradually deposits itself at the bottom in the metallic form.

However, the following is a process for making a kind of potable gold. We shall examine afterwards, supposing it to be a real solution of gold, whether it would possess properties so marvellous, and so salutary to the human body, as is pretended.

First dissolve gold in aqua regia: mix this solution with fifteeen or sixteen times the quantity of any essential oil, such as that of rosemary, stirring it round, and separate the aqua regia, which occupies the bottom, from the essential oil. If you then dissolve this essential oil in four or five times its weight of well rectified spirit of wine, you will have a yellowish liquor, known under the name of the potable gold of Mademoiselle Grimaldi.

Vitriolic ether, and ethereal liquids of different kinds, possess the same property as essential oils; namely, that of seizing on the gold dissolved in the aqua regia. A kind of potable gold therefore may be made with ether. This gold may then be taken in drops on sugar, in the same manner as when ether is taken; for this liquor is not miscible with water.

The celebrated drops of General Lamotte are not different from the potable gold of Mademoiselle Gramaldi. It has been remarked that one gross of gold was diluted in 216 gross of spirituous liquor, and as the bottles must have weighed two gross, and as General Lamotte sold his for 24 livres, it results that with one gross of gold he made at least 108 bottles, from the sale of which he received at least 2592 livres. In reality he made 136, which were worth to him 3264 livres.

It hence appears, that if General Lamotte's drops were not useful to the health, they were exceedingly useful to his purse. But what will not quackery effect among mankind, when supported by ignorance and the love of life?

But let us examine whether there be any foundation for the wonderful properties ascribed to potable gold. A very little reasoning will shew that nothing can rest on a slighter foundation. What proofs indeed can the alchemists produce, that potable gold is salutary to the human body? Because gold is the most fixed of all metals, because it has the beautiful colour of the sun's rays, because it is represented in

chemical characters by the characteristic sign of that luminary, are we thence to conclude that, when reduced to a liquid form, and conveyed into the blood, it regenerates that fluid, renovates youth, and restores health? What person, accustomed to deduce just consequences from any principle, will ever form such a conclusion? All the virtues of potable gold are founded merely on analogies, invented without any physical foundation, by fervid imaginations, and by heads deranged by the heat of their furnaces. This is the most favourable opinion that can be entertained; for it is probable that such ideas are as much connected with imposture, as with credulity and want of reasoning.

SECTION III.

Of Palingenesy.

Palingenesy is a chemical operation, by means of which, a plant, or an animal, as some pretend, can be revived from its ashes. This, if true, would no doubt be one of the noblest secrets of chemistry and philosophy. If some authors are to be credited, several learned men of the 17th century were in possession of it; but at present, as this pretended secret, in consequence of the great progress made in chemistry, is considered as a mere chimera, we shall here confine ourselves to examining the foundation of those principles which have induced some respectable authors, such as the Abbé Vallemont* and others, to believe in the possibility of this process.

According to the honest Abbé, nothing is simpler and easier to be explained. We are indeed told, says he, by Father Kircher, that the seminal virtue of each mixture is contained in its salts, and these salts unalterable by their nature, when put in motion by heat rise in the vessel through the liquor in which they are diffused. Being then at liberty to arrange themselves at pleasure, they place themselves in that order in which they would be placed by the effect of vegetation, or the same as they occupied before the body, to which they belonged, had been decomposed by the fire: in short, they form a plant, or the phantom of a plant, which has a perfect resemblance to the one destroyed.

This reasoning is worthy of an author who could believe that he who robs another of his money can exhale corpuscles different from those exhaled by a man who carries his own, and thereby make the divining rod turn towards the places where he has passed, or remained for some time. Does it not shew great weakness to believe that the mere immorality of an action can produce physical effects? It would indeed be offering an insult to our readers, to attempt to shew the folly and absurdity of the above reasoning of the good Abbé, and of Father Kircher. Let us therefore only examine the facts which he relates.

An English chemist, named Coxe, asserts that having extracted and dissolved the essential salts of fern, and then filtered the liquor, he observed, after leaving it at rest for five or six weeks, a vegetation of small ferns adhering to the bottom of the vessel. The same chemist, having mixed northern potash with an equal quantity of sal ammoniac, saw some time after a small forest of pines, and other trees, with which he was not acquainted, rising from the bottom of the vessel.

The following fact is considered by the author as more conclusive. The celebrated Boyle, though not very favourable to palingenesy, relates, that having dissolved in water some verdigris, which, as is well known, is produced by combining copper with the acid of vinegar, and having caused this water to congeal by means of artificial cold, he observed at the surface of the ice small figures, which had an exact resemblance to vines.

Notwithstanding these facts, and others quoted by the Abbé from Daniel Major, Hanneman, and various authors, if the partisans of palingenesy can produce none

* See "Les Curiosités de la Végétation," &c.

more conclusive, it must be confessed that they support their assertions by very weak proofs. Every true chemist sees in these phenomena nothing but a simple ramified crystallization, which may be produced by different well known compositions: the most beautiful of these crystallizations, called improperly vegetations, are produced by the combination of bodies from the animal kingdom.

The last experiment, related by Boyle, might occasion more embarrassment ; but as it is the only one, of a great many, made with the essential salts of a variety of plants, that succeeded, there can be no doubt that the figures he saw were the mere effect of chance ; for how many other philosophers, who made the same attempt, saw nothing but what is exhibited by the surface of frozen water, which sometimes forms ramifications exceedingly complex ?

The partisans of palingenesy however quote other authorities, to which they attach great importance. We are told by Sir Kenelm Digby, on the authority of Quercetan, physician to Henry IV. of France, that a Pole shewed twelve glass vessels hermetically sealed, each containing the salts of different plants: that at first these salts had the appearance of ashes, but that when exposed to a gentle and moderate heat, the figure of the plant, as a rose for example, if the vessel contained the ashes of a rose, was observed gradually to rise up, and that as the vessel cooled the whole disappeared. Sir Kenelm adds, that Father Kircher had assured him, that he performed the same experiment, and that he communicated to him the secret, but it never had succeeded. The story of this Pole is related by various other authors, such as Bary, in his Physique, and Guy de la Brosse, in his book on the Nature of Plants.

Lastly, we are told by Kircher himself, in his Ars Magnetica, that he had a long-necked phial, hermetically sealed, containing the ashes of a plant which he could revive at pleasure, by means of heat; and that he shewed this wonderful phenomenon to Christina, queen of Sweden, who was highly delighted with it; but that having left this valuable curiosity one cold day in his window, it was entirely destroyed by the frost. Father Schott also asserts, that he saw this chemical wonder, which, according to his account, was a rose revived from its ashes; and he adds, that a certain prince having requested Kircher to make him one of the same kind, he chose rather to give up his own than to repeat the operation.

The process, indeed, as taught by Kircher, is so complex and tedious, that it would require no small patience to follow it. Father Schott relates it at full length, in his work entitled " Jocoseria Naturæ et Artis," and he calls it the Imperial secret, because the emperor Ferdinand purchased it from a chemist, who gave it to Kircher.

This emperor was exceedingly fortunate; for it was to him that the philosopher, who had the secret of the philosopher's stone, addressed himself, and gave a proof of his art by transmuting, as is said, in his presence, three pounds of mercury into two pounds and a half of gold.

We must however content ourselves with pointing out the places where the curious may find this singular process; for besides the length of the description, nothing seems less calculated to succeed. Digby, therefore, and many others who followed this method, did not obtain a favourable result; and there is reason to believe that their zeal for palingenesy would induce them to omit nothing that was likely to insure them success.

Dobrezensky, of Negropont, has also given a process for the resurrection of plants, which seems to have been attended with no better success. We are at least told by Father Schott, that the attempts of Father Conrad proved ineffectual, and he therefore supposes that Dobrezensky did not reveal all the circumstances of the process, but kept the most important to himself.

What then can be said in opposition to all these authorities? In our opinion, the the Polish physician was a quack, and we shall describe hereafter a method of producing a false palingenesy, which, if performed with art and in a proper place, may

impose on credulous persons. To be convinced that Dobreczensky of Negropont was a mere impostor, we need only read the "Technica Curiosa," or the "Jocoseria Naturæ et Artis," of Father Schott; for he had the impudence to pretend that he could pull out the eye of an animal, and in the course of a few hours restore it, by means of a liquor, which he no doubt sold as a remedy for sore eyes. He even tried it on a cock. A person who could assert such an impudent falsehood, in regard to one fact, would do the same in regard to another.

The authority of Father Schott will certainly be of little weight with those who have read his works.

In regard to the testimony of Kircher, we confess that we find some embarrassment: a Jesuit certainly would not wilfully have told a falsehood. But Kircher was a man of a warm imagination, passionately fond of every thing singular and extraordinary, and who had a strong propensity to believe in the marvellous. What can be expected from a man of that character? He often thinks he sees what he does not see, and if he deceives others he is first deceived himself.

Some persons go still farther, and assert that an animal may be revived from its ashes. Father Schott, in his "Physica Curiosa," even gives the figure of a sparrow thus revived in a bottle. Gaffarel, in his "Unheard of Curiosities," believes in this fact, and considers it as a proof of the possibility of the general resurrection of bodies. This pretended revival, however, is a chimera, still more absurd than the former; and which, at present, it would be ridiculous to attempt seriously to refute.

In short, what reasonable man can with Kircher believe, that if the ashes of a plant be scattered on the ground, plants of the like kind will spring up from them, as he says he frequently experienced? Who can admit as a truth, that if crabs be burnt, and then distilled, according to a process given by Digby, there will be produced in the liquor small crabs of the size of a grain of millet, which must be nourished with ox's blood, and then left to themselves in some stream? yet we are told by Sir Kenelm that this he himself experienced. It is therefore impossible to clear him from the charge of imposture, unless we suppose that by some means or other he was led into error. However, it is certain that Digby, with great zeal and a considerable share of knowledge, had a strong propensity to all the visions of the occult and cabalistic sciences. In our opinion, he was one of those visionaries known under the name of *Rosicrusians.*

An illusory kind of Palingenesy.

We have already mentioned a kind of sleight of hand, by means of which, credulous people might easily be imposed on, and induced to believe in the reality of palingenesy. We shall now discharge our promise by describing it.

Provide a double glass jar of a moderate size, that is, a vessel formed of two jars placed one within the other, in such a manner, that an interval of only a line in diameter may be left between them. The vessel may be covered by an opake top or lid, so disposed, that by turning it in different directions, the inner jar may be raised from or brought nearer to the bottom of the exterior one. In the interior jar, on a base representing a heap of ashes, place the stem of an artificial rose. Into the lower part of the interval between the two jars introduce a certain quantity of ashes, or some solid substance of a similar appearance; and let the remainder be filled with a composition made of one part of white wax, twelve parts of hog's lard, and one or two of clarified linseed oil. This oily compound, when cold, will entirely conceal the inside of the jar; but when brought near the fire, if done with dexterity, it will dissolve, and by shaking the lid, under a pretence of hastening the operation, the compound may be made to fall down into the bottom of the exterior jar. The rose in the interior one will then be seen; and the credulous spectators, who must not be

suffered to approach too near, will be surprised and astonished. When you wish to make the rose disappear, remove the jar from the fire, and by a new sleight of hand make the dissolved semi-transparent wax flow back into the interval between the jars. By accompanying this manœuvre with proper words, the gaping spectators will be more easily deceived, and will retire firmly persuaded that they have seen one of the most curious phenomena that can be exhibited by the united efforts of chemistry and philosophy.

SUPPLEMENT I.

OF THE DIFFERENT KINDS OF PHOSPHORUS, BOTH NATURAL AND ARTIFICIAL.

ONE of the most interesting objects in chemistry is Phosphorus; for it is a very singular and curious spectacle, to see a body, absolutely cold, emit a light of greater or less vivacity, and others kindle of themselves, without the application of fire. What person, who has any taste for the study of nature, will not be struck with astonishment, on viewing such phenomena?

These phenomena are the more remarkable, as most philosophers have hitherto failed in their attempts to explain them. We, however, except the artificial kinds of phosphorus, respecting which they have advanced things consistent with probability, and founded on chemical causes fully established. But in regard to the natural kinds nothing satisfactory has yet been offered. The explanation of them depends, no doubt, on a more profound knowledge of the nature of fire and light.

Some kinds of phosphorus are natural; others are the production of art, and particularly of chemistry. Hence we are furnished with a natural division of this Supplement. We shall begin with the Natural.

ARTICLE I.

Of the Natural Kinds of Phosphorus.

SECTION I.

Of the Luminous appearance of the Sea.

Though navigators must have observed this phenomenon for many centuries past, as it is common to every sea, and as there is scarcely any climate where it does not, under certain circumstances, present itself, it appears that very little attention has been paid to it till within a late period. Most sea-faring people believed that this light was merely the reflection of that of the stars, or of that of the vessel itself; others, considering it as a real light, imputed it to the collision of sulphur and salts; and, satisfied with this vague explanation, they scarcely condescended to pay attention to the phenomenon.

As it is highly worthy of profound research, and is attended with very remarkable circumstances, we shall here give a description of it, as it appeared to us on our passage from Europe to Guyana, in the year 1764.

I do not recollect that we beheld the sea luminous till our arrival between the tropics; but at that period, and some weeks before we reached land, I almost constantly observed that the ship's wake was interspersed with a multitude of luminous sparks, and so much the brighter as the darkness was more perfect. The water around the rudder was, at length, entirely brilliant; and this light extended, gradually diminishing,

along the whole wake. I remarked also, that if any of the ropes were immersed in the water they produced the same effect.

But it was near land that this spectacle appeared in all its beauty. It blew a fresh gale, and the whole sea was covered with small waves, which broke after having rolled for some time. When a wave broke, a flash of light was produced; so that the whole sea, as far as the eye could reach, seemed to be covered with fire, alternately kindled and extinguished. This fire, in the open sea, that is at the distance of 50 or 60 degrees from the coasts of America, had a reddish cast. I have made this remark, because I do not know that any person ever examined the phenomenon which I am about to describe.

When we were in green water,* the spectacle changed. The same fresh gale continued; but in the night time, when steering an easy course, between the 3d and 4th degree of latitude, the fire above described assumed a tone entirely white, and similar to the light of the moon, which at that time was not above the horizon. The upper part of the small waves, with which the whole surface of the sea was curled, seemed like a sheet of silver; while on the preceding evening it had resembled a sheet of reddish gold. I cannot express how much I was amused and interested by this spectacle.

The following night it was still more beautiful; but at the same time more alarming, in consequence of the circumstances under which I then found myself. The ship had cast anchor at a considerable distance from the land, waiting for the new moon, in order to enter the harbour of Cayenne. Being anxious to get on shore, I stepped into the boat with several other passengers; but scarcely had we got a league from the ship, when we entered a part of the sea where there was a prodigious swell, as a pretty smart gale then prevailed at south-east. We soon beheld tremendous waves rolling in our wake, and breaking over us. But what a noble spectacle, had we not been exposed to danger! Let the reader imagine to himself a sheet of silver, a quarter of a league in breadth, expanded in an instant, and shining with a vivid light. Such was the effect of these billows, two or three of which only reached us, before they broke. This was a fortunate circumstance, for they left the boat half filled with water, and one more, by rendering me a prey to the sharks, would certainly have saved me from the trouble of new modelling the work of the good M. Ozanam.

There is scarcely a sea in which the phenomenon of this light is not sometimes observed; but there are certain parts where it is much more luminous than in others. In general, it is more so in warm countries, and between the tropics, than any where else; it is remarkably luminous on the coasts of Guyana, in the environs of the Cape Verd islands, near the Maldives and the coast of Malabar, where, according to M. Godeheu de Riville, it exhibits a spectacle very much like that above described

A phenomenon so surprising could not fail to excite the attention of philosophers; but till lately they confined themselves to vague explanations; they ascribed it to sulphur, to nitre, and other things, of which there is not a single atom in the sea, and they then imagined that they had reasoned well.

M. Vianelli, an Italian philosopher, is the first person, it seems, who endeavoured, by the help of observation, to explain the cause of this light; and he was thence conducted to a very singular discovery. Observing that the sea water, near Chi-

* The water of the sea, at least of the Atlantic Ocean, at a distance from the coasts, is of a dark blue colour; but near land, that is to say, 20 or 25 leagues from the coast of Guyana, the water suddenly changes its colour, and becomes a beautiful green. This is a sign of being near land. This change, in all probability, is produced by the muddy, yellowish water of the river of the Amazons; for it is well known that blue and yellow form green. But a remarkable circumstance is, that this change is absolutely abrupt; it does not take place by degrees, but suddenly and in an interval, which appeared to me, from the deck of the vessel, to be scarcely a foot in extent.

oggia, had a very luminous appearance, and that the light was concentrated into small brilliant points, he conceived the idea of examining it with a microscope. By these means he found that those luminous points were small insects, resembling worms, or rather caterpillars, composed of twelve articulations; that they differed from our luminous worms in this respect, that the light proceeded from every part of their bodies, and that when at perfect rest the light ceased, but that it re-appeared when they were put into a state of agitation. This explains why certain parts are made to sparkle by the strokes of an oar, the dashing of water against the rudder, and the breaking of the waves, while the rest of the water remains dark. These observations were confirmed by the Abbé Nollet, who soon after undertook a tour to Italy.

It appears however that the insect which gives a luminous appearance to the water of the sea, is not every where the same. M. Godeheu de Riville, having observed some of these luminous points in the Indian seas, between the Maldives and the coast of Malabar, saw an insect quite different from Vianelli's worm with twelve rings. This insect has a near resemblance to that called the water flea, and is enclosed between two transparent shells, somewhat like a kidney half open. The luminous matter seems to be contained in a vessel, which may be compared to a bunch of grapes; it consists of small round grains, and when the insect is pressed it emits a luminous liquor. It then mixes with the water, and, as it is of an oily nature, it collects itself on its surface in the form of small round drops. According to every appearance the insect suffers this phosphoric liquor to escape only in consequence of some shock or agitation, or of other circumstances; and hence the reason why the sea is not luminous except when agitated, and at certain times more than others.*

M. Rigault observed, in the seas between Europe and America, another insect, different from the worm of Vianelli, or the water flea of M. Godeheu, being rather a kind of polype, almost spherical, and with only one arm.

In the last place, M. Leroy, a physician of Montpellier, observed, in sea water, globules of a phosphoric matter, on which he made different experiments to ascertain what circumstance rendered them luminous, and by what means they were deprived of that quality. From these experiments he was induced to conclude, that though Vianelli and others have, on good grounds, ascribed the luminous appearance of the sea to insects, or to a liquor which they contain, and which they emit on certain occasions, this is not the only cause; but that it may arise also from a phosphoric matter in the water of the sea, and which is produced there by a peculiar combination of the principles dispersed throughout it; that this matter is not always luminous, but becomes so from different causes; such as the shock of the particles of the water against each other, the contact of the air, and its mixture with certain liquors.†

SECTION II.

Of some Luminous Insects.

If those beings which we tread under foot hold a very low, and, we might even say, contemptible rank in the animal kingdom, Nature, which seems to observe a general system of compensation, has given to several of them properties very extraordinary, and for which the largest animals might envy them: such is that of emitting light, with which many of them are endowed. We are acquainted indeed with no large animal which enjoys it while living; but there are several insects which are luminous, and it appears that they can become so at pleasure. Of what utility is this light to them, and how is it produced? These are problems which we shall not attempt to solve; we shall only confine ourselves to facts.

* See "Mémoires des Sçavans étrangers," vol. iii. † Idem.

1. *Of the Glow-worm.*

Every person is acquainted with this small insect, the light of which is often observed under the hedges in the fine evenings of summer.

This insect, called by the Greeks *lampyris*, and by the Latins *cicindela*, exhibits nothing remarkable in its external appearance. It has a pretty near resemblance to the sea louse, only that it is much smaller, and proportionally thinner. It is its last ring, where the anus is situated, that emits the light, by which it is distinguished from other animals of the same class. This light is of a pale greenish colour, and the animal can shew or conceal it at pleasure. It is supposed that it is by this light, which is peculiar to the female, that it attracts the male: the latter has wings, and is destitute of this luminous quality. This however is rather a matter of conjecture, and is contested by Baron de Geer, a celebrated Swedish naturalist, in consequence of some observations made by him.

An insect so singular was, no doubt, worthy of being celebrated by the poets; and indeed M. Huet, bishop of Avranches, has made it the subject of a small poem, entitled *Lampyris*, which is much esteemed by those who are fond of Latin poetry. It begins in the following manner.

> Quæ nova per cæcas splendescit stellula noctes
> Sepibus in nostris? An ab æthere lapsa sereno
> Astra cadunt, tacitis an captant frigora silvis,
> Si quando ardentis cœperunt tædia cœli?
> Non ità, sed duris frustrà exercita matris
> Imperiis, sentes lustrat Lampyris opacos,
> Si fortè amissum possit reperire monile.

The poet then feigns that the nymph Lampyris, having lost her necklace, is expelled by her mother, and that she wanders about through the woods, searching for it by the help of a lantern. These ideas were much applauded a century ago; but we do not know whether the case be the same in the present one.

2. *Of the Fire Fly of warm climates.*

Such is the luminous insect of our climates; but the warmer countries have been more favoured by nature. Their luminous insects have wings. They are found in Italy after the Alps are crossed; and they become more frequent, according as the traveller approaches the southern parts of Italy. They exhibit a very curious spectacle, during the fine nights of summer, when they are seen flying about in every direction, and one cannot move a step in a meadow without observing some of these small animals, whose route is marked by a train of light. I never enjoyed this spectacle in Italy; but I have seen it in South America.

It appears, however, that the fire flies of Italy, and those of America, are entirely different from the luminous insect of our countries. I confess that during my residence in America I did not pay much attention to them. I was employed with occupations of greater importance; but I know with certainty that this insect emits light as it flies about. The part of its body which is luminous seems to be concealed by its wings, or by the covering of them when closely applied to the body. I have never seen a good description of this insect, which has a great resemblance to a common fly.

It may be readily conceived, that these luminous insects must have inspired some persons with the hope of obtaining from them a perpetual phosphorus. Many attempts have been made for this purpose; but though the posterior part of the animal, when it is cut in two, retains its light for some time, it gradually becomes extinct: and every effort hitherto made to retain it has proved fruitless. Some authors

indeed have proposed means for accomplishing this object; but these people were either impostors, or labouring under a deception: it is certain that their pretended means will not succeed.

3. *Of the Cucuyo of America.*

A valuable acquisition of this kind, possessed by America, is the Cucuyo. The Caribs have given this name to a large beetle, found in the islands of the Gulf of Mexico, and even in Mexico itself. Its luminous quality is seated in the eyes, and in two parts of its body covered by the sheaths of its wings. It is asserted that five or six of these beetles will afford a sufficient light to enable a person to walk in the darkest night; that the natives of the country tie them together alive, and by these means form them into a sort of necklaces, to guide them through the woods, and that they employ them in their huts to give them light to perform their nocturnal labours. But this we can hardly believe.

4. *Of the Beetle of Guyana.*

A luminous insect, which had a great resemblance to the Cucuyo, and which perhaps was the same, was some years ago brought to France by a very singular accident. A great deal of wood for cabinet-makers having been imported from Cayenne, in 1764, and the following years, a cabinet maker purchased a piece of it, and kept it by him till he should find use for it. His wife hearing some noise one night, like the buzzing of an insect when flying, observed soon after a strong light adhering to the window. Recovering from the terror which this spectacle at first inspired, she ran up to it, and found an insect of the coleoptera kind, (that is, insects whose wings are covered by a sheath,) which emitted from the posterior parts of its body a bright light that illuminated the whole of the apartment. The insect was put into the hands of M. Fougeroux, who wrote a description of it, which was inserted in the Memoirs of the Academy of Sciences for 1766.

There is great reason to think, or rather it is certain, that the animal had been brought over in the piece of wood in the state of nymph, concealed in some hole; the time of its development being arrived, it issued from its retreat, and appeared under the form of a beetle.

If this insect was not the Cucuyo of the American isles, or of New Spain, it must be considered as a fourth kind endowed with the property of emitting light.

SECTION III.

Of some other Phosphoric Bodies.

We hall here take a cursory view of a great number of other phosphoric bodies.

1. *The Eyes of different animals.*

As several animals, such as the tiger, and the cat, which is only a tiger in miniature, the wolf, the fox, &c., among quadrupeds; and the owl, and others, among birds, have been destined by nature to search for their food in the night time, it was necessary they should have a lamp to guide them. This lamp is contained in their own eyes; for they are luminous, and it is no doubt by this light that they are guided in the dark. As their retina is exceedingly sensible, the light of their eyes renders objects to them sufficiently luminous. Besides, nature has favoured them with a very large aperture in the pupil, so that the quantity of light which reaches the retina is increased. Such, in all probability, is the mechanism by which these animals see in the night time: the extreme sensibility of their retina renders the light of the day incommodious to them, and even blinds some of them.

It appears that these animals have it in their power to render their eyes luminous

at pleasure. I have often seen those of a cat, which I kept, entirely destitute of light, while at other times they were like a burning coal.

The dog also is endowed in some measure with the same property. I have several times seen the eyes of that animal sparkle.

In short, it is asserted that some men also are endowed with this property. Tiberius, it is said, could see in the night time; and the same thing is related of many others. The most singular instance of this faculty is that of a hermit, who, according to Moschus, in his "Pré Spirituel," had never occasion for a lamp while reading at night, or employed with any other occupation. Those who could believe such ridiculous tales, would almost deserve to be sent to feed *cum asinis et jumentis*.

2. *Clayton's Diamond.*

This diamond was much celebrated; and if it was not one of the finest of its kind, this defect was more than compensated by the singular property it possessed. When rubbed in the dark againt any dry stuff, or against the fingers, it shone with a faint whitish light. The celebrated Boyle made a great many observations on this diamond, an account of which he communicated to the Royal Society in 1668; and he does not hesitate to call it a precious stone, unique of its kind,—*gemma sui generis unica;* for at that time no other was known which possessed the same property. We have however heard that, since that time, other diamonds have been found, which could be rendered brilliant in the dark by friction. This singular diamond was purchased by Charles II.

We shall here say a few words respecting the carbuncle, which, as some pretend, shines also in the dark; but we must observe, that this property ascribed to it is absolutely fabulous. The carbuncle is a ruby; but no ruby, nor any other kind of precious stone, shines in the dark; and this supposed phenomenon is merely a popular tale.

We may make remark also that this light is not properly phosphoric, but is of the same nature as electric light. The diamond indeed is susceptible of becoming electric by friction; its light is of the same nature as that emitted by sugar when grated, and by various other bodies when rubbed.

3. *Rotten Wood.*

It is not uncommon to find, in the forests, pieces of rotten wood, which emit a very vivid light of a white colour, inclining to blue; it has even sometimes happened that this light has been the occasion of great terror.

Unfortunately, every kind of rotten wood is not phosphoric; and the cause which renders it so is not known.

We must class also, among the number of puerile tales, what is related by Josephus of a plant called *Baaras*, said to be luminous in the dark. This plant, it is said, cannot be plucked up without the most imminent danger; but when the root of the plant has been loosened, a dog is tied to it, and the animal, by making efforts to join its master, at length tears it up. Is it possible that authors can thus sport with the credulity of mankind!

We must place in the same rank what is related by Pliny of another plant, called *Nyctegretum*, which grows, it is said, in Gedrosia, and which, when torn up by the root and dried in the sun's rays for a month, becomes luminous in the night time. This it not absolutely impossible; but if so, the plant would be known to our naturalists, as well as the Aglao-phytis, and the Lunaris, to which the same property was ascribed, according to the testimony of Ælian. When a circumstance is related by Ælian, one may bet a hundred to one that it is a fable.

4. *The Worms in Oysters.*

This natural phosphorus was first remarked by M. de la Voye, who communicated his discovery to M. Auzout, in 1666.

Small oblong worms, which shine in the dark, are often engendered in oysters. According to the description given of them, some are as large as a small hair pin, and about five or six lines in length: others are much smaller. He found also three kinds; the first with legs, to the number of about twenty-five on each side; the second kind were red, and similar, except in size, to our common glow worms; the third were of a singular form, having a head like that of the sole. They readily resolve, on the least touch, into a viscid matter, which retains its luminous property for about twenty seconds.

Such are the observations of M. de la Voye, which do not entirely agree with those of M. Auzout, who observed only a viscid matter, extended in length. But it is to be remarked, that the latter made his experiments only on old oysters; whereas the former made his on oysters quite fresh.

5. *Putrid Flesh.*

Putrid flesh is also susceptible of becoming sometimes luminous in the dark. Lemery says that a great quantity of such luminous flesh was seen at Orleans, in 1696; some of it was entirely so; other pieces were luminous only in some points, which had the appearance of small stars. People were at first afraid to eat of it; but they soon learnt by experience that there was no danger, and that it was as good as any other. It was remarked that in some butchers' shops the meat was almost all luminous; in others only part of it was so.

Fabricius ab Aquapendente relates the same thing of a lamb, purchased by some young men at Rome. One half of it, which was left, being put by, they observed in the evening that several parts of it were luminous. They immediately sent for the above physician, who having examined the phenomenon with attention, observed that the flesh and the fat shone with a silver coloured light; and that a piece of goat's flesh, which had touched the lamb, shone in the same manner. The fingers of those who touched it became luminous also. He observed likewise that the luminous places were softer. This phenomenon would, no doubt, be observed more frequently if the butchers' shops, and places where meat are kept, were oftener visited in the dark.

6. *Different kinds of Fish, or the Parts of Fish.*

But this phenomenon is most frequently exhibited by fish, and their different parts.

It is generally when fish, or any of their parts, approach the state of putrefaction, that they acquire this phosphoric property. Leo Allatius, in a letter to Fortunio Liceti, says that he was once much frightened by fresh water crabs thrown into a corner by a careless servant. He describes the whole of this adventure at great length; but want of room will not permit us to enter into any farther details respecting it.

According to Pliny, and other authors, the sea-worm is susceptible of shining in this manner. Those who reside near the sea coast may have an opportunity of ascertaining the truth of this fact.

The celebrated Thomas Bartholin observed the same thing, in regard to some polypes, which he was dissecting: he gives this name to the fish called at present the cuttle fish, since he says that it contains a black liquor, which may be employed as ink. This light, adds he, flowed from beneath the skin, and was the more abundant the nearer the animal approached to a state of putrefaction.

We shall conclude this subject by mentioning some experiments of Dr. Beale, in-

serted in the Philosophical Transactions, for the year 1666. Fresh mackarels having been boiled in water with salt and herbs, the cook stirring the water a few days after, to take out some of the fish, observed that on the first movement of the water it became luminous, as well as the fish, which emitted a strong light through it: the water also appeared to be transparent: whereas in the day time it was opake.

Drops of this water were exceedingly luminous; and wherever they fell, they left a luminous spot, as large as a six-pence. On rubbing the hands with it, they became entirely luminous.

We have here confined ourselves to facts; for nothing farther can be said on the subject, as it is difficult to assign any probable or well-founded cause for this light. The globulous matter of Descartes was exceedingly convenient for explaining all these phenomena; for it was sufficient to say, that putrid fermentation, being a kind of intestine motion, this motion, according to every appearance, put in action the globulous matter in which light consists. But unfortunately this matter, at present, is considered as a chimera.

ADDITION BY THE FRENCH CENSOR.

There are some inaccuracies in what has been before said in regard to the luminous insects, in the 1st, 2d, 3d, and 4th paragraphs of the second section. The author seems to have trusted too much to his memory, which has led him into error, and not to have been acquainted with every thing written on the subject. We shall therefore supply this deficiency.

1st. The male of the glow worm is a winged insect of the class of the coleoptera, or insects which have sheaths to their wings. It is not entirely destitute of the property of being luminous. M. Fougeroux says, that he often caught in the dark some of these males, which were attracted by the light of the female, and he observed that they emitted light themselves after copulation.

2d. The fire-fly of Italy, commonly known under the name of the *lucciola*, is not a fly; it is also an insect with sheaths to its wings, and in form approaches near to the male of the glow worm. At first one might be induced to believe that it is the same insect, to which the climate has given the property of shining in the dark, as is the case with that of our country under certain circumstances. But there are some differences, which will not allow of their being confounded; and what seems absolutely to exclude this identity is, that in places were the *lucciola* is found, the common glow worm is never seen, though it exists also in Italy.

In regard to the luminous insect of the warm countries of America, I must remark, with the author, that I am unacquainted with any correct description of it.

3d. What has been said, in regard to the luminous insect of Cayenne, is not entirely correct; and the account given of its being discovered requires to be rectified. In the month of September, 1766, two women, in the Faubourg Saint-Antoine, saw this insect in the evening flying through the air, like a stream of light, and at length settle on a window. They at first thought it was one of those falling stars so common during the summer nights; but as the light continued, they went to inform the occupiers of the house against which the animal rested. It was caught, and given to M. Fougeroux, in order that he might examine it. That it came from Cayenne was only conjecture; but by comparing it with the insects of that country, it appeared to be an inhabitant of the same or a neighbouring climate. It was a coleoptera, known under the name of *mareschal*, and of the class of those which, when placed on their back, dart into the air like a spring which unbends itself: on this account it has been distinguished by the name of *elater*. This insect is an inch and a half in length; its light is contained in two elongated protuberances, placed on the posterior and lateral part of its corselet. It emits light also in certain positions, by the separation of its body from the corselet; and in all probability by that of the rings of its body from

each other. This light is of a beautiful green colour, and of such strength, that if the insect be put into a paper cornet, one can see to read the smallest characters by it, at the distance of some inches. The insect is found also in Jamaica, and has been described by Brown, under the denomination of *elater major fuscus phosphoricus.* Another smaller kind of phosphoric fly is found in Jamaica, and also in Saint Domingo.

4th. What the author says of the cucuyo of America, that it emits light from its eyes, and two parts above the sheaths of its wings, is not correct. It is possible that travellers, unacquainted with natural history, who have spoken of it, may not have examined it with attention.

5th. There are some other luminous insects, which the author has not mentioned. The lantern bearer, or *acudia*, which Reaumer places in the class of the *pro-cigales*, (cicada spumaria), or a class which approaches near to that of the *cigales*, (grasshopper), the *vielleur* beetle of Surinam; like the author we are unacquainted with any description, sufficiently correct, to enable us to determine in what they differ from the *cucuyo*, and from each other. Such also is the lantern-bearer of China, described by Linnæus, in the Transactions of the Academy of Stockholm; but as the animal was dead, that learned naturalist had no opportunity of ascertaining what part of it is luminous; he suspects it is the proboscis, which does not appear improbable. There is also an insect of the same kind in Madagascar, known under the name of *hese-cherche*, which shines in the night-time. But we have never seen a description of it.

6th. Clayton's diamond was long considered to be the only one which had the property of shining in the dark. But M. Dufay found, by a great number of experiments made on different diamonds, that several of them possessed the same property; though he was not able to discover the cause, why some of them possessed it, while others were destitute of it. Beccari, a celebrated philosopher of Bologna, made at the same time similar experiments, which confirm the discovery of Dufay. This philosopher found that the class of phosphoric bodies is much more considerable than is commonly supposed, and it follows from his experiments, that the phosphoric bodies which have chiefly attracted the attention of philosophers, did so, not on account of that property maintaining itself for a longer time, but because a very great number of bodies appear luminous to an eye immersed in profound obscurity, when they are speedily removed from the light to a place of darkness.

7th. The sea worms or borers possess this property in an eminent degree; not when they approach to a state of putrefaction, as has been before said, but when living and fresh, so as to be fit for eating. The observations of Beccari, Monti, and Calcati of Bologna, on these marine fish, are very old; and they confirm and illustrate what has been said by Pliny on the same subject.*

ARTICLE II.

Artificial kinds of Phosphorus.

What nature produces under certain circumstances, art, assisted by observation, has found means to imitate, in the artificial kinds of phosphorus. But before we explain these curious operations, we must make a distinction which the modern chemists have introduced, and which is necessary.

The appellation Phosphorus is still given to those bodies which emit light without any sensible heat; but when a body emits light, and at the same time inflames of itself, when exposed to the air it is called pyrophorus. Hence we say the pyrophorus of Homberg, to denote that composition of alum and animal or vegetable matter which takes fire when exposed to the air. The English phosphorus is both phosphorus and pyrophorus; for when exposed to the air in a mass, it burns, and consumes

* See a Memoir of M. Reaumur on the same subject, "Mem. de l'Acad." 1723.

like sulphur, of which it is a singular species, but very much attenuated; and when mixed with a liquor it becomes luminous, without heat.

SECTION I.

Phosphoric experiment: or how to burn gunpowder without an explosion.

Expose a towel or cloth to a strong heat, till it becomes very hot, and carry it into a dark place. While it is cooling, throw upon it, from time to time, some grains of gunpowder, which at first will inflame. Leave it to cool a little, till the powder no longer detonates. If you then cover it with powder, the latter, when it acquires the same heat as the cloth, will emit in the dark a faint light or weak flame, which will consume all the sulphur, without causing the nitre to detonate.

It is hence seen, that common sulphur is susceptible of two combustions, one gentle and calm, which is not capable even of kindling the charcoal, otherwise the nitre would detonate; and the other violent, which burns and kindles such combustible bodies as are in contact with it.

SECTION II.

Of the Bologna Stone.

This kind of phosphorus is called the Bologna stone, because first made from a stone found only at the bottom of mount Paterno, near that city. A shoemaker, named Vincenzo Casciarolo, was the first who observed the property which these stones have, of shining in the dark, after they have been calcined. He was employed on the grand work, as it was called; and from the brilliant appearance of these stones, he conceived an idea that they contained either metals, or some principle by which he should obtain what he was in quest of. He therefore brought them to a red heat in a crucible, and having afterwards carried them into a dark place, he was struck by their luminous appearance, and published an account of his discovery. This phosphorus is made by the modern chemists in the following manner.

They take one of these stones, and having freed it from all its heterogeneous parts, file it all round with a large file, in order to obtain a certain quantity of dust. They then dip the stone in the white of an egg, and roll it in the dust, until it be entirely covered with it to a certain depth. When the stone is dry, it is placed in a furnace filled with charcoal, in such a manner as to be completely surrounded by it. The charcoal is then kindled, and when the whole is consumed, the stone is found calcined according to the required degree. If it be carried into a dark place, it will be seen to shine with a singular brilliancy, which however becomes gradually weaker, and after some minutes entirely ceases. But this brightness may be renewed by exposing the stone for some time to the day-light. These stones are preserved in a dry place wrapped up in dry cotton. They however gradually lose their property of imbibing the light; but it may be restored to them by a second calcination.

The Bologna stone, according to the observations made by naturalists, is one of those known under the name of *fusible spar*. Vitriolic acid enters into their composition, and this inspired Margraf, a celebrated chemist, with the idea of trying whether all the other spars were not endowed with the same property. He found that when treated in a proper manner they all become luminous. The process for calcining and preparing them, according to his method, is as follows.

When properly freed from their heterogeneous parts, they are brought to a red heat in a crucible, and then reduced to very fine powder in a glass or porphyry mortar. This powder is formed into small cakes, a line or more in thickness, and of any size at pleasure, by mixing it with gum tragacanth and the white of an egg; and these cakes are then calcined in the following manner, after they have been dried in a strong heat.

A common reverberating furnace is filled to three fourths of its height with charcoal; the cakes are laid flat above it, and are covered with more charcoal. The furnace is then kindled, and when the whole charcoal is consumed, and the furnace has cooled, the cakes are found calcined. After being cleaned from the ashes, by means of a pair of bellows, they are preserved for use, as before described. When an experiment is to be made, they are exposed for some time to the light, after which they are carried into a dark place, where they exhibit the appearance of burning coals to those who have kept their eyes shut for a few minutes.

The most probable cause of this phenomenon, according to the ablest chemists, is as follows:

When it is considered that phosphorus of the same kind is made only by burning with charcoal stones which contain vitriolic acid,* there is reason to think that in this operation there is formed a kind of sulphur, exceedingly combustible, and in which the action of the light alone is capable of producing that slow combustion, almost without heat, of which common sulphur, as already seen, is itself susceptible. This combustion manifests itself only by the faint light it emits. It ceases with the absence of the cause which produced it; and the stone no longer is luminous.

Among several reasons which confirm this explanation, there is one which seems to be of great weight: after the stone has ceased to shine, if placed in a dark place on a plate of iron, which has been heated, but not to such a degree as to emit light, it immediately becomes luminous, without having been exposed to the action of the sun's light. To this reason we may also add, the odour exhaled by the Bologna stone; for it is exactly the same as that of sulphur. But in regard to this subject we must refer the reader to Macquer's "Dictionnaire de Chimie," under the head Stony Phosphorus, where explanations will be found, which, on account of their length, cannot be admitted into this work.

SECTION III.

Baldwin's Phosphorus.

This phosphorus, as well as the following, has a great affinity to the Bologna stone. The method of making it is as follows:

Dissolve very pure white chalk in good spirit of nitre; filter the solution, and evaporate the liquor till the residuum is very dry. Then put the residuum into a good crucible of a proper size, and place it for an hour in a reverberating furnace. If the matter calcined in this manner be put into a bottle, with a glass stopper, you will have Baldwin's phosphorus.

This phosphorus has the property of shining in the dark, like the Bologna stone, if the bottle containing it be exposed open to the light. But as it has the fault of attracting moisture, it soon loses this property.

SECTION IV.

Homberg's Phosphorus.

Take one part of sal ammoniac in powder, and two parts of quick-lime slaked in the open air: mix them well together; and having filled a crucible with the mixture, place it over a slow fire. As soon as the crucible is red, the mixture will begin to fuse; but as it rises up and swells in the crucible, it must be stirred with an iron rod, to prevent it from running over. When the matter is fused, pour it into a copper bason, and when cool it will appear of a grey colour, and as if vitrified. If it be struck with any thing hard, such as a piece of iron, copper, or other substance of the like kind, the whole part which has been struck will for a moment seem on

* Margraf at least asserts so, though Dufay says he made Bologna phosphorus with stones purely calcareous.

fire. But as this matter is very brittle, the experiment cannot be often repeated. To remedy this defect, M. Homberg immersed in the crucible, containing the fused matter, small rods of iron or copper, which by these means became covered with it as with enamel. The experiment may be performed on rods incrusted in this manner, as they will bear to be struck several times, without the matter being deranged.

It is to be observed that the phosphoric enamel, which adheres to these rods, readily attracts the moisture of the air: for this reason they must be deposited in a dry warm place, where they will retain their property for a long time.

SECTION V.

Canton's Phosphorus, or Phosphorus in Powder.

Another kind of phosphorus, analogous to that of Baldwin, and to the Bologna stone, may be made in the following manner:

Calcine oyster shells, by keeping them in a common fire for half an hour, and then pulverize them; mix the finest part of this powder with one third of its weight of fine flour of sulphur, and put the mixture in a crucible, filling it to the brim, and keep it for half an hour at least in the midst of burning coals, till it is perfectly red. Then suffer it to cool, and having once more pulverized the matter it contains, if necessary, you will obtain a phosphorus, which, if exposed for a few minutes to the daylight, will appear luminous in the dark.

Those who have comprehended the nature of the Bologna phosphorus, will readily see that Canton's phosporus is properly the same thing; for the Bologna stone and all the fusible spars, which have been found to possess a phosphoric property, are nothing but combinations of vitriolic acid with calcareous earths.

SECTION VI.

Homberg's Pyrophorus.

The following chemical discovery was entirely owing to chance. The celebrated Homberg had been assured that a white oil, in no manner fetid, which had the property of fixing mercury, could be extracted from human excrement. He therefore subjected this matter to experiment; and extracted from it a white oil without any odour. It did not fix mercury; but having exposed the residuum of his distillation to the air, he was surprised to see it take fire. Such is the origin of his pyrophorus.

It has however been since found, that it is not necessary to employ matters so filthy as those from which Homberg first extracted this pyrophorus. The common process for this purpose is simple, and is as follows:

Mix in an iron pan, placed over the fire, by means of an iron spatula, three parts of alum and one of sugar, until the matter becomes perfectly dry, and reduced to a blackish carbonaceous substance: if there be any lumps in it of considerable size they must be broken. Put this matter into a matrass with a narrow neck, about eight inches in length, and place the matrass in a crucible, capable of containing the belly of it when surrounded by half an inch of sand. Then immerse the crucible in burning coals, and bring both it and the matrass to a state of ignition, heating it at first gradually, and then strongly urging the fire, till a sulphurous flame is seen to issue from the neck of the matrass. The fire must be maintained in this state for about a quarter of an hour; then suffer the fire to become gradually extinct, and when the neck of the matrass is no longer red, close it with a cork stopper, otherwise the pyrophorus would inflame.

When the whole is perfectly cold, pour the pyrophorus speedily into several phials, which can be well shut, and instantly close them. Sometimes it inflames in passing from the matrass to the bottle; but this is of no consequence, as it is extinguished as soon as the bottle is closed.

To make an experiment with pyrophorus, put a small quantity of it upon a piece of paper. Soon after, it inflames, becomes red like a burning coal, and sets fire to any combustible bodies it may be in contact with. The inflammation may be accelerated by putting it on paper somewhat moist, and breathing upon it.

SECTION VII.

Of the Phosphorus, or Pyrophorus of Kunckel, called also the English Phosphorus.

This is the most curious composition of modern chemistry. Who would have believed that a luminous body could be extracted from putrid urine? Nay more, a body susceptible of inflammation, and capable of inflaming, by its contact, other combustible bodies? Such, however, is the origin of this phosphorus, which in some measure may be considered as abject; but to the philosopher nothing is abject in nature; and the most disgusting object sometimes contains principles capable of producing the most singular and uncommon effects.

The discovery of the phosphorus of urine, like many others, was the effect of chance. A citizen of Hamburgh, an enthusiast in regard to the philosopher's stone, was making some experiments with urine. He was not the first nor the only person who imagined that the substance proper for fixing mercury ought to be found in human excrement; and by repeated trials on this matter he found phosphorus. This discovery made a great noise in the chemical world. But Brandt, the author of it, was not disposed to part with his secret for nothing. Kunckel, an able chemist, united with one Krafft to endeavour to draw from him the process; but Krafft deceived Kunckel, purchased from Brandt the secret of making this phosphorus, and being desirous to carry on a lucrative traffic, refused to impart it to his associate. The latter, incensed on account of this treachery of Krafft, and knowing that he had made great use of human urine, endeavoured by researches to discover the secret, and at length found it. The honour of it therefore has remained with him; for this phosphorus is commonly called *Kunckel's Phosphorus.**

On the other hand, Krafft went to England, and having shewn this phosphorus to the king and queen, the celebrated Boyle, whose curiosity was highly excited by so singular a phenomenon, endeavoured also to discover the secret. He knew only, like Kunckel, that Krafft laboured on urine. He began therefore to make experiments on that matter, and found out likewise the method of extracting phosphorus from it. He communicated the process to the public, in the Philosophical Transactions for 1680, and according to every appearance taught the different operations more particularly to a German chemist, settled at London, named Godfreyd Hanckwitz; for he was a long time the only person who made phosphorus.

Though Boyle published the process for making this phosphorus in 1680; though Homberg taught it in 1692; and though described in various books, phosphorus was made only in England, and by Hanckwitz alone. A foreigner, who came to France in 1737, offered however to disclose the whole process, and the ministry promised him a reward for it. Several chemists and members of the Royal Academy of Sciences were requested to be present at the operation, which was performed at the Jardin Royal des Plantes, and attended with perfect success. M. Hellot wrote an account of the process, and published it in 1738, in the Memoirs of the Royal Academy of Sciences. Since that time only the method of making phosphorus has been known; but it is one of the nicest operations of chemistry, and does not succeed but in very expert hands.

But none of the modern chemists has paid so much attention to the composition of phosphorus as Margraf, who has rendered the process more certain, more exact

* Leibnitz asserts that this account, generally given in regard to Brandt, is entirely void of foundation. He gives a history of phosphorus, which may be seen in his works, vol. ii.

and less tedious; for which reason we shall take him as our guide, in what we are going to say on this subject.

1st. Provide good urine, and let it purify itself; then put it into a glass vessel placed over the fire, and evaporate the phlegm, till it be reduced to the consistence of honey or of cream.

It must here be observed, that this matter contains a particular salt, called *fusible salt of urine;* that this salt is composed of an acid different from all the rest, called the *phosphoric*, because it is a necessary ingredient in phosphorus, by its combination with another principle, and because this acid is extracted by the deflagration of phosphorus, as the vitriolic acid is by that of common sulphur.

2d. Then mix four pounds of minium with two pounds of sal ammoniac in powder, and distil the mixture, which will furnish a volatile alkali highly concentrated. This alkali however is useless. But marine acid will attack the minium or calx of lead, and will form with it a compound, known to chemists under the name of *corneous lead.* Corneous lead, ready made, may be employed; but we have thought proper to describe the method of making it, because all our readers may not be chemists.

3d. Mix this corneous lead, by little and little in an iron pan, with eight or nine pounds of the extract of urine, mentioned in the first article, taking care to stir it continually; add to it half a pound of charcoal dust, and continue to dry it till it be reduced to a black powder. Then throw the matter into a retort to distil it in a moderate heat, and extract from it all the products; which are volatile alkali, a fetid oil, and a kind of sal ammoniac, which adheres to the neck of the vessel. Then bring the retort to a moderate red heat, and when nothing more passes over, unlute the apparatus, and reserve the residuum, which is a kind of *caput mortuum.* This residuum contains the phosphorus, and must be distilled in a much more violent heat. It is a sign that it is well prepared; as a small bit of it, when thrown on the coals, exhales an odour of garlic, and burns with a small lambent flame.

4th. Put the residuum into a good Hessian retort. M. Margraf recommends those of Waldenbourg as the best; but none of them are brought to France. Those of Hesse will answer the purpose, though they suffer the phosphoric matter to transpire a little; but this defect may be in part remedied, by a luting of earth mixed with cow's hair.

5th. Having filled three fourths of the retort with the above matter, place it in a furnace, having over it a chimney or tube, five or six inches in diameter, and eight or nine feet in height. The chimney serves to increase the activity of the fire, by the rapidity of the current of air, and for introducing through the door of it, at different times, the quantity of charcoal necessary to carry on the operation about six hours.

6th. Lute the neck of the retort with that of a middle sized balloon, half filled with water, and having in it a small hole, by means of fat luting; and secure the joint by a bandage of linen, covered with a luting of lime and the white of an egg. The hole in the balloon affords a passage to the vapours, which otherwise would occasion it to burst. It may be slightly closed with a pellet of paper, which must be occasionally taken out during the distillation. Care must be taken also to close with a luting of clay the notch of the furnace, through which the neck of the retort passes; and to raise between the furnace and the balloon a brick wall, to prevent the heat from being communicated to the former.

7th. These arrangements having been made twenty-four hours before, set fire to the furnace, and expose the retort to a gradual heat for an hour and a half; after which the fire must be excited, to bring it to a white heat. This operation will cause to pass into the balloon, first luminous vapours, and then drops of pure phosphorus, which falling into the water in the balloon will become fixed. This operation

will be completed in four or five hours; whereas by Hellot's process it would require twenty-four.

8th. As the phosphorus obtained by this violent distillation is black, on account of the fuliginous vapour it carries along with it, you must distil it a second time in a smaller retort, and in a moderate heat. This heat will be sufficient to render it pure, for when once formed it is exceedingly volatile.

9th. The phosphorus must then be formed into small sticks, by putting it into glass tubes somewhat conical, and immersed in tepid water; for in such a heat it runs like tallow. These operations must be performed in water, to prevent inflammation of the phosphorus, and when the water is cold the phosphorus will be fixed in sticks, which must be taken out and put into phials filled with water, and carefully shut.

It has not yet been discovered to what useful purpose the English phosphorus can be applied, except that its nature and composition have thrown light on certain points of chemistry. But it may be readily conceived, that it may be employed for performing various philosophical tricks, exceedingly curious, such as the following.

To write in characters which will appear luminous in the dark.

For this purpose, it will be first necessary to make liquid phosphorus. Place a grain of phosphorus at the bottom of a small bottle, and having bruised it, pour immediately over it about half an ounce of very clear oil of cloves. If the whole be then put to digest in a gentle heat, such as that of a dunghill, the phosphorus will be almost entirely dissolved. When the bottle is taken out, the matter it contains will shine in the dark, on the bottle being unstopped or agitated a little.

If you dip a small brush in this oil, and describe it with any characters, on a wall, they will appear luminous in the dark.

You may also, if you choose, render your face and hands luminous. Nothing will be necessary but to rub over them a little of this oil, which has no sensible heat, because the phosphoric fire is very much rarified.

This phosphorus amalgamates very well with mercury, and forms a luminous compound. Put ten grains of phosphorus into a pretty large long phial, with two ounces of oil of lavender. The phosphorus will dissolve in it, provided it be exposed to a gentle heat. If you then add half a dram of quicksilver, you will obtain an amalgam, which will be entirely luminous in the dark.

For the same purpose, phosphorus may be mixed with pomatum; it will become luminous, and may be rubbed over the face and hands without any danger.

SECTION VIII.

Composition of a kind of a pyrophorus, which emits flames when brought into contact with a drop of water.

For this composition we are indebted to the celebrated chemist Glauber. Mix together iron filings, cadmia, tartar, and nitre; then form them into a paste, and dry it well in a strong heat, such as that of a potter's furnace. If a few drops of water be thrown over this mass, it will emit flames and sparks. Such is the description given of this process by Beccher. The following is another, extracted from the Natural Magic of Martius. Pulverize quick-lime, tutty, and storax calamite, each an ounce; live sulphur and camphor, each two ounces; and having mixed them well together and sifted them, wrap them up in a piece of very thick linen cloth. Put this cloth into a crucible, cover it with another crucible, which must be tied closely to the former, and lute the joining with potter's earth. When the luting is perfectly dry, put this double crucible into a potter's furnace, and leave it there till the matter is entirely calcined. This may be known by the colour of the crucibles, which ought

to be of a pale red: when the whole is cool, if you throw a drop of water or spirit on this matter, it will emit sparks.

It was no doubt by means of a similar composition, that a German Jew drew sparks from the head of his cane, by spitting on it. This invention indeed is very proper for being employed by jugglers, to excite the wonder of the populace, and extort money from them. The Jew, here alluded to, it seems, turned to great advantage this chemical secret.

Remark.—There are some other pretended kinds of phosphorus; but properly speaking they are not so: they are merely electric phenomena.

Of this kind is the light seen in the inside of certain barometers, called for this reason luminous. In the old editions of the Mathematical Recreations, it was called Dutal's Phosphorus, because that physician, but after Bernoulli, was able to make luminous barometers; it is however now known that this does not arise from phosphorus, but is merely an electric light. M. Ludolff, a German philosopher, has clearly proved that this phenomenon is the effect of electricity, produced in the tube of the barometer, by the friction of the mercury.

The case is nearly the same with mercury, which becomes luminous when inclosed in a very clear glass vessel, exhausted of air. We have already described this phenomenon: it is also electric.

The light emitted by a diamond, rubbed in the dark, or a bit of sugar when grated, is of the same kind.

SUPPLEMENT I.

PERPETUAL LAMPS.

The subject of perpetual lamps has too intimate a connection with that of phosphorus to be here omitted; for if we were urged to explain the accounts, given of fire found in the tombs of the ancients, and from which some pretend to conclude they were acquainted with the art of maintaining a lamp lighted for ages, it would be necessary to have recourse to phosphorus. But these facts rest on so slight a foundation, some of them even bear such evident marks of fiction, and the greater part of those which the honest Fortunio Liceti, a strong partisan of perpetual lamps, has collected as proof of this discovery, are so evident proofs of the contrary, that a moderate degree of acuteness is sufficient to shew that nothing is less entitled to credit. If to this be added the physical reasons which contradict the possibility of an inflammable liquor, burning continually without being consumed, the perpetual lamps must be considered as a chimera, unworthy the attention of a philosopher, and fit only to be banished to the country of potable gold and palingenesy. If we introduce them therefore into this work, it is merely on account of the celebrity of the subject, and because we know that some persons are fond of these singular and extraordinary tales.

ARTICLE I.

Examination of the facts alleged as a proof of the existence of perpetual lamps.

Before the improved state of philosophy had shewn the impossibility of real unextinguishable fire, the learned were much divided in their opinions on this subject; ut of all the champions in favour of perpetual lamps, none has made greater effort obtain credit to their existence than Fortunio Liceti, in his book entitled "De onditis Antiquorum Lucernis."

If credit may be given to this author, nothing was more common among the ancients than perpetual lamps. The lamp of Demosthenes, that which burnt in the temple of Minerva at Athens, the Vestal fire at Rome, all furnish him with so many proofs of the possibility of unextinguishable fire. We cannot help smiling to see so much learning so idly employed: for who does not know that these fires were called perpetual, merely because it was a point of religion to preserve them from being extinguished, and to supply them with continual aliment?

The other partisans of perpetual lamps, while they smile at the simplicity of Liceti, support their reasoning on facts, which seem to carry with them a little more weight; they are as follow.

1. *The Lamp of Tulliola.*

The tomb of Tulliola, the beloved daughter of Cicero, and whose death cost him so many tears, was discovered, it is said, under the pontificate of Pius III. It is pretended that in this tomb there was a lamp actually burning, but which became extinguished on the admission of air.

2. *The Lamp of Olybius.*

But it is the lamp of Olybius, which, above all others, supplies the partisans of perpetual lamps with one of their strongest arguments.

In the year 1500, as we are told, some peasants, digging the earth to a considerable depth at Atesta, in the neighbourhood of Padua, came to a tomb, in which they found two earthen urns, one within the other. The inner, it is said, contained a burning lamp, placed between two phials, one filled with liquid gold, and the other with liquid silver. On the large urn was the following inscription:

Plutoni sacrum munus ne attingite, fures;
Ignotum est vobis hoc quod in orbe latet;
Namque elementa gravi clausit digesta labore,
Vase sub hoc modico, maximus Olybius.
Adsit fecundo custos sibi copia cornu,
Ne tanti pretium depereat laticis.

The second is said to have been inscribed also with these lines:

Abite hinc, pessimi fures;
Vos quid vultis vestris cum oculis emissitiis?
Abite hinc vestro cum Mercurio
Petasato caduceatoque.
Maximus maximum donum Plutoni hoc sacrum fecit.

Such is the manner in which this curious discovery is related by Gesner. But what follows is still stronger. Liceti gives a letter of one Maturantius, who tells his friend Alphenus, that he had got possession of this valuable treasure. "Both the vases," says he, "with the inscriptions, the lamp, and the phials, have fallen into my hands, and are now in my possession. If you saw them you would be astonished. I would not part with them for a thousand crowns of gold." This is no doubt the language of a man who believes he possesses an inestimable rarity. We do not however know that it exists in any collection.

It appears that in this case, as in regard to the tomb of Tulliola, an accident prevented enlightened people from being witnesses to the phenomenon; for we read in the credulous Porta, that as the peasants who found this treasure handled it too roughly the lamp broke in their hands, and was extinguished.

3. *The Lamp of Pallas, the son of Evander.*

We are told also that, about the year 800 of the Christian æra, the tomb of the

famous Pallas the son of Evander, killed as is well known by Turnus, was found at Rome. It was known to be that of Pallas by these verses:

Filius Evandri Pallas, quem lancea Turni
Militis occidit, more suo jacet hic.

It contained a burning lamp, which consequently must have burnt nearly 2000 years, since it was shut up in the year 1170 before the Christian æra.

4. *The Lamp in the temple of Venus.*

This lamp, and the temple of Venus, in which it was suspended, are mentioned by St. Augustine. He says it burnt perpetually, and that the flame adhered so strongly to the combustible matter, that neither wind, rain, nor tempests could extinguish it, though continually exposed to the air, and to the inclemency of the seasons. This author endeavours to explain the mechanism of it, and after offering a conjecture, which in part is pretty correct, namely, that a wick of asbestos was perhaps employed, he concludes by saying that it might have been the work of demons, in order to blind the pagans more and more, and to attach them to the infamous deity worshipped in this temple.

Here then, according to the partizans of perpetual lamps, we have unextinguishable fire, the existence of which is fully confirmed by the testimony of the most enlightened man of his age; and who, notwithstanding his knowledge, is obliged to have recourse to the artifice of demons to explain this phenomenon.

5. *Lamps of Cassiodorus.*

The celebrated Cassiodorus, who, it is well known, was as much respected on account of his employments as of his talents, tells us himself that he made perpetual lamps for his monastery at Viviers. Each monk, it is probable, had one of them for his own use. His words are, "Paravimus etiam nocturnis vigiliis mecanicas lucernas conservatrices illuminantium flammarum, ipsas sibi nutrientes incendium, quæ humano ministerio cessante prolixè custodiant uberrimi luminis abundantissimam claritatem, ubi olei pinguedo non deficit, quamvis jugiter flammis ardentibus torreatur." Some partizans of perpetual lamps may here say: "Is it possible to refuse credit to testimony so authentic, so clear, and so respectable?"

Such are the principal facts adduced in favour of perpetual lamps: but we may venture to say that they will not stand the test of critical examination. In regard to the first three, what dependance can be placed on facts related in so vague a manner, and accompanied with such incoherent and romantic circumstances? None of these facts are supported by any other testimony than that of men who lived a long time after; no person whose testimony is of any weight asserts that he actually saw them. But in disputes which are contrary to the common laws of nature, they must at least be certified by enlightened men, above all suspicion of credulity or ignorance.

The tale respecting the tomb of Tulliola is as old as the year 1345, a period when all Europe was sunk in the grossest ignorance. A body is said to have been found in it; and in that case it could not be the body of Tulliola; for the Romans, in the time of Cicero, always burnt their dead. In consequence of this and similar circumstances, some authors have conjectured that the tomb alluded to was that of the wife of Stilico; but the Christians never placed lamps in their tombs. The account therefore of a lamp found in this tomb has every appearance of a fiction.

But what shall we say of the tomb of Olybius, and the lamp with two phials,

one filled with fluid gold, and the other with fluid silver? This double urn was found by peasants, who according to some authors handled the lamp, contained in the second urn, so clumsily as to break it; and yet Maturantius pretends that he had it in his possession. Who saw the lamp burning? what evidence have we that the peasants beheld it in that state? and whose testimony in this case would be admissible? Some vapour, exhaled from a place shut up for so many ages, might easily impose on rude and ignorant people.

What is the meaning of the inscription? where is thère any allusion in it to perpetual fire? Is it necessary that a gift sacred to Pluto should be a burning lamp? If there be any truth in the discovery of this tomb, it ought only to be concluded that it belonged to some alchemist, of an age not very remote; for it is well known that the Romans had no idea of chemistry, and none of them ever attempted the transmutation of metals. If this folly had then been in existence, some traces of it would certainly be found in their writings; but on this subject they all observe the most profound silence. This chimera was communicated to us by the Arabs, with some real knowledge in regard to chemistry.

But if the Romans were unacquainted with chemistry, how could they construct perpetual lamps, which would be one of the greatest productions of that science?

The story of the tomb of Pallas, the son of Evander, is scarcely worth refutation. Who can be so weak as to believe that the verses, already quoted, were written in the time of Æneas. One needs only to have seen the language of the twelve tables, to be able to judge how little resemblance the ancient language of the Romans, and consequently that of the period of the kings of Alba, bore to these Latin verses.

In regard to the lamp of Venus, which occasioned so much difficulty to St. Augustine, we shall observe that this author does not say that it was never supplied with new aliment. What seems to be most singular is, that it could not be extinguished, either by wind or by rain; but in this there is nothing wonderful, since our oilmen sell flambeaux which have the same property. A method of making similar fire may be found in various books of chemistry. Besides, even admitting that this lamp was perpetual and inextinguishable, who is so ignorant as not to know that the Pagan priests were the greatest impostors, and that they might employ many artifices to supply the lamp with new aliment?

The lamps of Cassiodorus may be explained with equal ease; they were lamps which, like those of Cardan, supplied themselves with oil by means of a reservoir; and Cassiodorus only meant to say, that these lamps lasted a long time, in comparison of the common lamps of that period, which stood frequently in need of having oil poured into them.

These reflections did not escape several ingenious writers, such as Aresi, a bishop, and author of "Symbola seu Emblemata sacra;" Buonamici, a philosopher contemporary with Liceti; and particularly Octavio Ferrari, to whom we are indebted for a curious and learned work "De Veterum Lucernis sepulchralibus." All these authors, and especially the last, overturn the arguments of Liceti, and fully shew that the facts he has adduced, in favour of perpetual lamps, rest on a weak foundation, and that they abound with absurdities and contradictions. They even ridicule the weakness of this learned man, who, by an excess of credulity almost beyond belief, finds, in the lamp of the tomb of the necromancer Merlin, described by the poet Ariosto, a proof of perpetual lamps.

We shall conclude this article with the following very just reflections of Octavio Ferrari, before mentioned, which naturally suggest themselves to the mind. If the secret of constructing perpetual and inextinguishable fire had been known to the ancients, would an art so useful have remained buried in oblivion? but, even admitting that it might be lost, for want of philosophical and chemical knowledge, is it possible that Pliny, who enumerates the common inventions, as well as those most

celebrated, should say nothing of this perpetual fire—a thing so wonderful? When Plutarch makes mention of the lamp of Jupiter Ammon, because it burnt a whole year, is it to be supposed that he would observe silence respecting lamps, in comparison of which the former was a contemptible trifle?

We must therefore say, that both history and sound criticism oppose every idea of such an invention having ever existed. We shall now examine how far it is consistent with the principles of philosophy.

ARTICLE II.

On the Physical Possibility of making a Perpetual Lamp.

Having proved the weakness of all the facts brought as proofs in favour of perpetual lamps, it remains that we should examine how far they are possible, according to the principles of sound philosophy.

To obtain a perpetual lamp, it would be necessary to have as follows:

1st. A wick which could not be consumed.

2d. Some aliment which could not be consumed, or a substance which, after having served as aliment to the fire, should return into the vessel, without losing its inflammable quality.

3d. It would be necessary also that the flame should be able to exist a long time in a place absolutely close, and of small dimensions; for such were the tombs in which these perpetual lamps are said to have been found.

But all these things are impossible, as will be seen by what follows.

SECTION I.

Impossibility of procuring a perpetual Wick.—History of the Amianthus; manner of spinning it, and forming it into a wick; examination of its supposed incombustibility.

The curious properties ascribed to the amianthus, which are in part real, are well known. We shall here give the history of it; but we shall not be so prolix as the inexhaustible Abbé Vallemont. The amianthus, called also *incombustible flax*, and *asbestos*, is a mineral substance found in several parts of the earth. It consists of fibres of a white colour, more or less greyish, which adhere strongly to each other. Means however are found to separate them, and when well washed they have the appearance of the whitest flax. The amianthus is found in the Pyrenées, the Alps, &c. The most beautiful, in our opinion, is that found in or near the mine of Pesey in Savoy. We have seen some, the filaments of which were above a foot in length, and exceedingly white.

But the singular property which characterizes this substance is, that it remains unhurt in the fire, and when taken out is purer and whiter than it was before. This property therefore has been made the basis of a thousand moral and pious comparisons, which we shall not here repeat.

It is proper to observe, that the druggists, a sort of men who throw all natural history into confusion by their corrupted nomenclature, are acquainted with asbestos under no other name than that of *feathered alum*. But this denomination arises from profound ignorance. Alum is a salt, and is soluble in water, whereas the amianthus is insoluble in that liquid. The amianthus therefore is not alum. What has given occasion to this false denomination is, that there is indeed a feathered alum, or alum crystallized in fibres, which has some resemblance to the amianthus; but this alum is exceedingly rare, and the druggists, when it is called for, substitute the amianthus in its stead.

However, it appears that the property of the amianthus has been long known; for we are told by Pliny, in his Natural History, B. xix. chap. 1, that when certain Indian princes died, their bodies were wrapped up in a winding sheet made of live

flax, and that they were thus burnt, in order that their ashes might not be mixed with those of the funeral pile. It is certain that this is not impossible, and we cannot doubt the testimony of Pliny, who says in another place, that he saw cloth and nets, which when dirty required only to be thrown into the fire, and that when taken out they were clean, and not in the least injured. Plutarch asserts the same thing. But Pliny is evidently mistaken, when he says that this live flax was produced from a plant, found only in the hottest districts of India, torrified by the rays of the sun, as if it were fond of growing in the midst of flames. He knew the production, but was deceived in regard to its origin. This custom however seems to have become extinct in India: we know no traveller who speaks of bodies being now burnt in that manner.

It is needless to quote more authorities to prove tne possibility of making incombustible cloth of this kind. We have seen purses, brought from the Pyrenées, which possessed this property. They were indeed exceedingly coarse, but it is certain that they might be made much finer.

It however requires great industry to be able to spin the amianthus, and to form it into cloth. The following is the method, as given by Ciampini, in his treatise "De Incombustibili Lino, deque illius filandi modo;" Romæ, 1691.

To spin this stone, says Ciampini, it must be first soaked in warm water; when it has remained in the water some time, it is taken out, rubbed between the hands, opened and spread out, frequently dipping it in the water, in order to clean it from the earthy particles. This operation is repeated five or six times, until the filaments are well detached from each other, after which they are collected.

They must then be dried on some apparatus, through which the water can easily drain off. The next thing is to provide two small cards, finer than those used to card the wool employed for making hats and stuffs, and the incombustible flax must be placed between these two cards, that a few of the filaments may be drawn out at a time, in order to be spun with a small spindle.

But it is to be observed, that as the filaments of this flax are in general very short, it is necessary to spin along with it some fine cotton or wool, which may embrace and unite them.

Care however must be taken to use always a little more amianthus than of the substance you have chosen to spin along with it. The reason of this is, that when the thread is made into cloth or into purses, the work is thrown into the fire; the cotton or wool then burns, and being consumed, nothing remains but pure amianthus. It is almost in the same manner that gold and silver are spun with silk; and that old gold and silver lace is burnt to obtain the pure metal.

Ciampini says, that those who spin this substance must moisten their fingers, and particularly the thumb and the fore finger, to render the operation easier, and to prevent the fingers from being excoriated, *because the amianthus is corrosive*. He says also that the use of cards may be dispensed with; and that it is sufficient to put the filaments of the amianthus in regular order, in such a manner that they may easily separate to insinuate themselves into the cotton or wool added, in order that they may be spun together. When the cloth or purses are dirty, they are thrown into the fire, and on being taken out are whiter and more brilliant than they were before. He recommends moistening them with a little oil or essence, when they come from the fire; because oil nourishes the amianthus, and causes the thread to remain smoother.

We shall here observe, that to form the amianthus into wicks, it is not necessary that it should be purified or spun. It will be sufficient to take the longest filaments, and to tie them together with a white silk thread, in a quantity proportioned to the size of the wick. It is astonishing to see with what avidity the amianthus attracts

and imbibes the oil. It may be employed as it is found in filaments, in the druggists' shops, and the lamp will not fail to burn and to emit a strong light.

Ciampini however is mistaken when he ascribes to the amianthus a corrosive quality; its stony and no ways saline nature will not admit of such a property

Having given this short history of the amianthus, it remains that we should examine the consequences that may be hence deduced.

If we can believe the partisans of perpetual lamps, since the first step towards the execution of such a work as a perpetual and incombustible wick, we have the object accomplished; for the amianthus supplies us with such a wick, since it is incombustible, and since the trials made of it have been attended with success. Father Kircher assures us, that he had a lamp with a wick of this kind, which answered exceedingly well.

We will not deny that it is possible, by means of the amianthus, to make a wick, which will last for a very long time; but we will assert that it would not be perpetual; for though the amianthus is boasted of as being incombustible, this property is not absolute. We will even venture to say, that the amianthus is at length annihilated by fire, like every other body. It is true that cloth of the amianthus, when thrown into the fire, is taken out sound and entire, but not absolutely so. It is observed that it loses a little of its weight every time it is exposed to the fire. It would therefore be at length destroyed, and perhaps in the course of a very short time, such as a few days, if it were only made red hot and suffered to cool, or if it were left all that time in a very strong fire. Consequently a wick of amianthus would, at the end of a certain period, he entirely destroyed.

Some have tried to make wicks of bundles of gold threads, exceedingly fine. This perhaps would be the means of obtaining a wick almost perpetual in its duration; but it has never been possible to light them; and even if this could be done, another inconvenience would prevent their being of any use: the gold filaments would be fused by the flame, and consequently would be rendered unfit for answering the intended purpose; as it is well known, that if a piece of silver wire be presented to the flame of a taper, it is instantly fused; and the case would be the same with a gold wire, for it is more fusible than silver.

SECTION II.

Impossibility of procuring indestructible aliment for the perpetual lamps.—Pretended recipes for making indestructible oil.

But we shall even suppose a wick absolutely unalterable to have been found, and that it does not become choaked up with fuliginous matter, from the combustible substance by which it is fed. This however would be only a small part of what is necessary for obtaining a perpetual lamp. Some kind of aliment, which shall experience no diminution, or which having served to maintain the flame without experiencing any alteration, may return by a perpetual circulation into the vessel from which it issued will also be requisite. Is all this possible?

But let us now hear the alchemists, or the partisans of perpetual lamps. We shall be entertained with their ideas respecting the manner in which an oil, such as these lamps would require, might be obtained.

Some, considering that the amianthus resists fire, have tried, but without success, to extract an oil from it.

Others, observing that gold and silver, but particularly the former of these metals, are indestructible by fire, conceived an idea of searching in them for that valuable oil which would put us in possession of perpetual lamps. This is the noble secret with which Liceti pretends that Olybius was acquainted; but the metals are as incapable of producing oil as the amianthus.

It may however be said, that if it were possible to reduce gold to a liquid state,

we might perhaps obtain an incombustible oil, as gold is unalterable in the fire. But besides the impossibility of converting gold into a liquid, how do we know that it would be inflammable like oil.

The Abbot Trithemius, or the person who in his name has written a great many falsehoods, pretends to give two recipes for making incombustible oil. We shall here lay one of them before our readers, with the whole process for making a perpetual lamp.

Mix, says that celebrated visionary, or the person who speaks in his name, four ounces of sulphur and four ounces of alum; sublime them, and convert them into flowers. Take two ounces and a half of these flowers with half an ounce of borax and Venetian crystal, and pulverise the whole in a glass mortar. Put the powder into a phial, and having poured into it spirit of wine, four times rectified, cause it to digest. Pour off the spirit of wine, and having added some new, repeat the same thing three or four times, until the sulphur runs without smoke, like wax, on hot plates of brass. You must then prepare a proper wick, which may be done in the following manner: Take filaments of the asbestos stone, of the length of the finger; form them into a packet half as thick as the finger, and tie them with a white silk thread. The wick being thus made, cover it with the sulphur prepared as before described, and immerse it in the sulphur, in a vessel of Venetian glass. Place the whole upon a sand bath for twenty-four hours, so that you may always see the sulphur boil; and the wick being by these means well penetrated and impregnated with that aliment, put it into a small glass vessel with a wide mouth. The match must rise a little above it. Then fill this glass vessel with the prepared sulphur, and place the vessel in warm sand, that the sulphur may melt and surround the wick. If it be then kindled, it will burn with a perpetual flame.

Such is the first kind of fire of the Abbot Trithemius. A very small knowledge of chemistry is sufficient to shew that it would be ridiculous to expect from this process inextinguishable and perpetual fire. None therefore of the partisans of perpetual lamps, not even Liceti himself have any confidence in it, nor yet in the second; from which Liceti concludes that none of the moderns possess, or ever possessed, this valuable secret.

Some alchemists assert that an incombustible oil may be obtained by another process. They pretend that oil of vitriol edulcorated on gold, which they call *oleum vitrioli aurificatum*, will give this valuable liquor. But it is well known that the appellation of oil is here improperly applied; for what is called oil of vitriol has in reality nothing of an oily or inflammable nature; and we shall believe in perpetual lamps when an alchemist shall shew us a common lamp, filled with oil of vitriol, and furnished with any wick whatever, in which the flame shall exist only one second.

SECTION III.

Impossibility of continually maintaining fire in a place absolutely close.

It is a fact, well known to all those who cultivate natural philosophy, that flame cannot be maintained in a close place. If a lighted taper be placed under a glass receiver, so as to prevent its having communication with the external air, the flame will gradually decrease in size, become faint, extend itself in length, and at last be extinguished. Dr. Hales and others have even calculated what quantity of air is consumed in a certain time by the combustion of a taper of certain dimensions; so that we may easily predict in what time the flame will infallibly be extinguished.

A flame however might perhaps be maintained in a large place, though hermetically sealed; but it is well known that the cavities of the ancient tombs were exceedingly small; and to increase the difficulty it is said that the perpetual lamps burnt in the vessels in which they were inclosed. Such at least was the case with that of Oly-

bius; but if the vessel of Olybius had been three feet in diameter, which by no means appears, it is certain that a lamp could not have existed in it two hours without being extinguished

We shall not enlarge farther on this subject, as it would be wasting time to accumulate arguments to combat the chimera of perpetual lamps; and we have reason to think that every enlightened philosopher, at present, will be of the same opinion.

Remark.—Notwithstanding the reasons which are certainly deduced from the principles of sound philosophy, we have seen, in some journal, that a Neapolitan prince was in possession of the secret of perpetual lamps. But as several years have elapsed since this circumstance was announced, and as the secret has not yet been divulged there is reason to think that the information was premature. It is no new thing to see chemists, employed in researches respecting the philosopher's stone, announce their discovery before the operation is finished. Some even in consequence of the good colour of their matter, like that described by Philalethes and the learned Morien,* have gone so far as to purchase estates for a large sum of money.

But, unfortunately, every thing is still deficient, and the good alchemist dies in the hospital, protesting that nothing was wanting to his matter, but an imperceptible degree of coction, to render him the richest man on the earth.

In regard to the perpetual lamp of Naples, we shall change our opinion when we learn with certainty that it has been tried, and that it has burnt only one year.

* Two celebrated adepts.

FINIS.

Printed by Nuttall and Hodgson, Gough Square, London.